Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.2-Algebraic-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1025]. This is test number [52].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.24 (1007)	% 1.76 (18)
Mathematica	% 97.27 (997)	% 2.73 (28)
Maple	% 81.56 (836)	% 18.44 (189)
Maxima	% 28.49 (292)	% 71.51 (733)
Fricas	% 67.8 (695)	% 32.2 (330)
Sympy	% 25.56 (262)	% 74.44 (763)
Giac	% 40.29 (413)	% 59.71 (612)

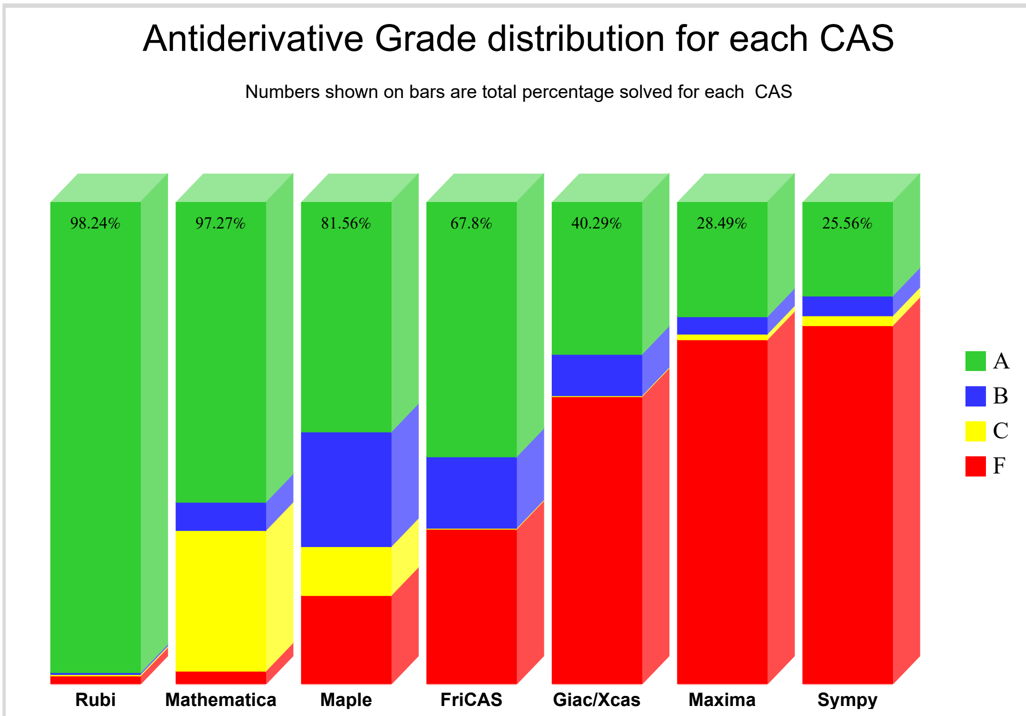
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

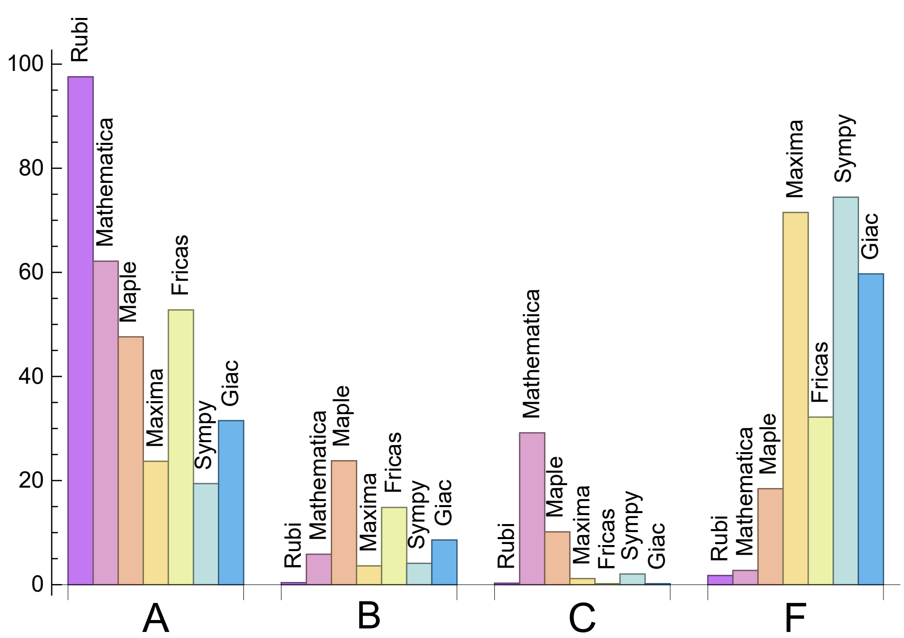
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	97.56	0.39	0.29	1.76
Mathematica	62.15	5.85	29.17	2.73
Maple	47.61	23.8	10.15	18.44
Maxima	23.71	3.61	1.17	71.51
Fricas	52.78	14.83	0.2	32.2
Sympy	19.41	4.1	2.05	74.44
Giac	31.51	8.59	0.2	59.71

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.2	142.3	1.03	84.	1.
Mathematica	0.43	300.1	2.09	86.	1.
Maple	0.02	1478.67	11.97	91.5	1.29
Maxima	1.32	78.92	1.32	41.	1.09
Fricas	3.51	541.4	6.1	185.	3.53
Sympy	10.06	194.52	2.33	57.	0.99
Giac	1.21	198.67	2.27	58.	1.4

1.4 list of integrals that has no closed form antiderivative

{901, 902, 903, 904, 905, 906, 907, 908, 909, 910}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {726}

Mathematica {1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 196, 202, 203, 204, 205, 226, 355, 451, 452, 560, 561, 562, 754, 755, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 778, 780, 785, 786, 790, 791, 795, 798, 799, 801, 802, 804, 885, 1009, 1010, 1018, 1024, 1025}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

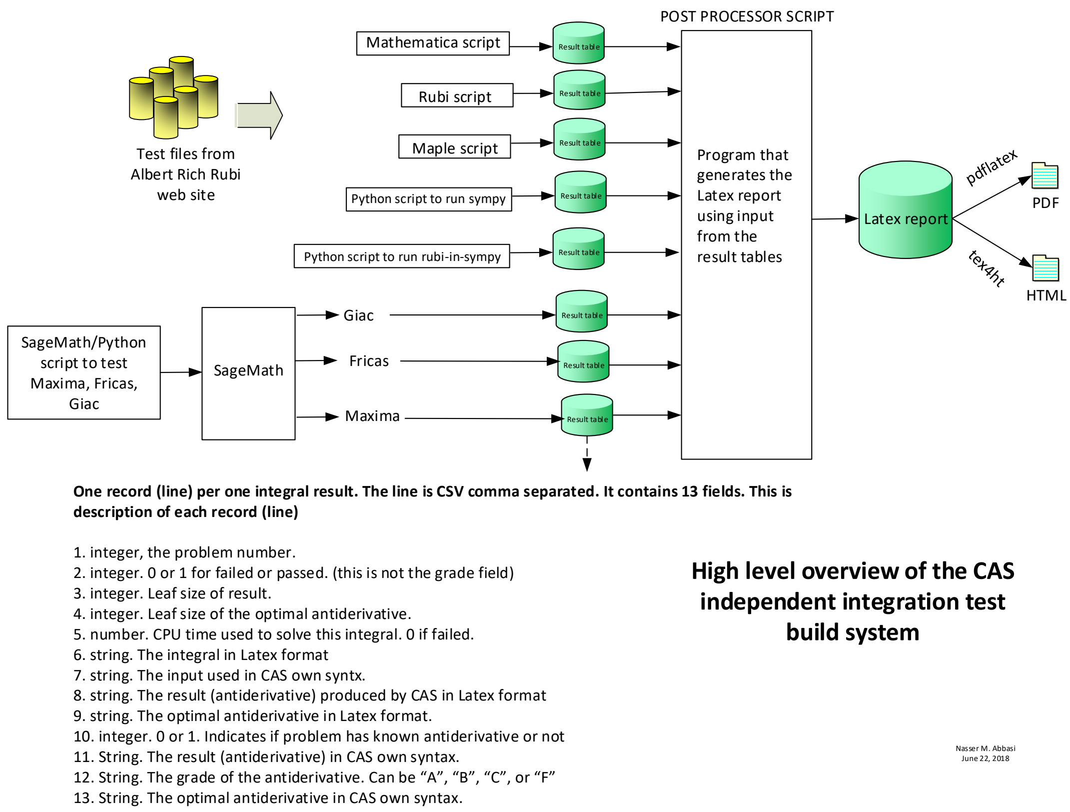
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1019, 1020, 1021, 1022, 1024 }

B grade: { 413, 726, 997, 1025 }

C grade: { 396, 941, 1018 }

F grade: { 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 197, 616, 617, 995, 996, 1017, 1023}

2.1.2 Mathematica

A grade: { 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 247, 249, 250, 251, 252, 253, 259, 263, 264, 265, 266, 267, 268, 269, 272, 273, 276, 277, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 307, 308, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 327, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 347, 348, 349, 352, 353, 356, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 530, 531, 534, 535, 538, 539, 540, 541, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 572, 573, 575, 576, 578, 579, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 664, 665, 666, 667, 670, 671, 672, 673, 674, 678, 679, 680, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 738, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 854, 856, 858, 859, 860, 862, 863, 864, 866, 868, 870, 872, 874, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 915, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 991, 992, 993, 994, 999, 1000, 1001, 1007, 1008, 1016, 1020, 1023 }

B grade: { 261, 408, 413, 524, 562, 623, 624, 681, 682, 737, 739, 740, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 846, 847, 852, 853, 855, 857, 861, 869, 871, 873, 875, 918, 919, 926, 927, 961, 985, 986, 987, 988, 989, 990, 996, 1011, 1012, 1013, 1014, 1015, 1021, 1022 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 241, 242, 246, 248, 254, 255, 256, 257, 258, 260, 262, 270, 271, 274, 275, 278, 283, 284, 285, 286, 287, 288, 302, 303, 306, 309, 313, 314, 315, 316, 317, 325, 326, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 354, 355, 357, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 397, 431, 442, 523, 525, 526, 527, 532, 533, 536, 537, 571, 574, 577, 580, 601, 603, 662, 663, 668, 669, 675, 676, 677, 694, 711, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 796, 797, 798, 799, 800, 801, 802, 803, 804, 813, 865, 867, 916, 917, 942, 984, 998, 1002, 1003, 1004, 1005, 1006, 1009, 1010, 1018, 1019, 1024, 1025 }

F grade: { 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 172, 173, 195, 197, 528, 529, 542, 543, 558, 559, 587, 913, 914, 995, 1017}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 53, 54, 55, 57, 58, 59, 66, 68, 84, 85, 86, 93, 127, 128, 129, 130, 137, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 196, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 270, 271, 272, 273, 274, 275, 285, 286, 287, 289, 290, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 340, 350, 351, 352, 353, 354, 362, 363, 364, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 387, 390, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 426, 427, 428, 429, 430, 431, 438, 439, 440, 441, 444, 445, 446, 448, 450, 451, 454, 455, 456, 474, 475, 530, 531, 538, 539, 565, 566, 567, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 630, 632, 633, 634, 636, 637, 639, 640, 641, 643, 644, 646, 647, 648, 649, 650, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 685, 686, 687, 689, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 733, 735, 736, 738, 741, 742, 743, 744, 748, 749, 750, 751, 752, 756, 757, 758, 762, 763, 805, 806, 807, 808, 809, 810, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 828, 829, 830, 833, 834, 835, 836, 837, 839, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 864, 865, 867, 868, 870, 872, 874, 877, 879, 882, 883, 884, 885, 886, 887, 888, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 967, 969, 970, 971, 972, 975, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1006, 1007, 1020, 1023 }

B grade: { 9, 52, 56, 64, 65, 67, 73, 83, 91, 92, 94, 95, 100, 125, 126, 135, 136, 139, 174, 175, 178, 179, 180, 182, 183, 184, 232, 260, 262, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 355, 356, 357, 358, 359, 360, 361, 365, 369, 408, 413, 422, 424, 432, 433, 442, 443, 447, 449, 452, 457, 458, 459, 473, 476, 477, 478, 487, 523, 524, 525, 528, 529, 534, 535, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 568, 572, 601, 629, 631, 635, 638, 642, 645, 651, 652, 681, 683, 684, 688, 690, 697, 699, 700, 709, 726, 729, 730, 731, 732, 734, 737, 739, 740, 745, 746, 747, 753, 754, 755, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 811, 812, 816, 826, 827, 831, 832, 840, 843, 846, 855, 857, 859, 861, 862, 863, 869, 871, 873, 875, 878, 880, 881, 891, 918, 919, 920, 921, 965, 966, 968, 973, 974, 976, 977, 978, 983, 984, 997, 998, 1015, 1018, 1021, 1022 }

C grade: { 43, 44, 45, 46, 51, 74, 75, 76, 77, 82, 101, 102, 103, 104, 113, 114, 115, 116, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 226, 378, 380, 381, 383, 386, 388, 389, 391, 406, 407, 409, 410, 434, 435, 436, 437, 468, 469, 470, 485, 486, 526, 527, 532, 533, 536, 537, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 796, 797, 798, 799, 800, 801, 802, 803, 804, 838, 866, 930, 954, 1002, 1003, 1004, 1005, 1009, 1010, 1019, 1024, 1025 }

F grade: { 5, 6, 7, 8, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 140, 141, 142, 143, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 253, 254, 255, 256, 257, 258, 293, 393, 453, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 562, 563, 564, 587, 588, 589, 590, 591, 592, 593, 594, 653, 654, 655, 656, 657, 876, 911, 912, 913, 914, 942, 995, 996, 999, 1000, 1001, 1008, 1011, 1012, 1013, 1014, 1016, 1017 }

2.1.4 Maxima

A grade: { 174, 175, 179, 227, 229, 231, 233, 235, 244, 247, 249, 250, 251, 252, 290, 293, 370, 371, 372, 373, 374, 375, 376, 379, 384, 385, 396, 398, 399, 400, 419, 420, 421, 422, 423, 424, 425, 443, 444, 445, 446, 447, 448, 449, 450, 544, 545, 546, 552, 553, 554, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 618, 619, 620, 621, 625, 626, 627, 628, 632, 633, 634, 635, 639, 640, 641, 642, 646, 647, 648, 649, 654, 655, 656, 676, 677, 679, 680, 681, 683, 684, 685, 686, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 724, 725, 733, 734, 735, 737, 738, 739, 741, 742, 743, 744, 747, 748, 749, 762, 763, 805, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 822, 824, 825, 833, 835, 836, 839, 840, 841, 842, 843, 844, 848, 849, 852, 854, 855, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 870, 872, 874, 883, 884, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 935, 936, 937, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 952, 953, 954, 959, 960, 961, 963, 964, 971, 972, 975, 977, 979, 980, 981, 982, 984, 1007, 1020, 1023

B grade: { 178, 182, 183, 238, 369, 564, 566, 574, 580, 612, 613, 614, 615, 616, 617, 653, 675, 678, 729, 730, 731, 732, 736, 746, 750, 829, 845, 846, 847, 880, 948, 957, 958, 974, 976, 978, 1015 }

C grade: { 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 230, 232, 234, 236, 237, 239, 240, 241, 242, 243, 245, 246, 248, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 547, 548, 549, 550, 551, 555, 556, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 622, 623, 624, 629, 630, 631, 636, 637, 638, 643, 644, 645, 650, 651, 652, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 682, 687, 688, 689, 690, 691, 692, 701, 702, 703, 704, 705, 706, 710, 720, 721, 723, 726, 727, 728, 740, 745, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 806, 807, 811, 812, 813, 821, 823, 826, 827, 828, 830, 831, 832, 834, 837, 838, 850, 851, 853, 863, 869, 871, 873, 875, 876, 877, 878, 879, 881, 882, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 930, 931, 932, 933, 934, 938, 939, 940, 955, 956, 962, 965, 966, 967, 968, 969, 970, 973, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

2.1.5 FriCAS

A grade: { 77, 103, 104, 105, 107, 109, 111, 113, 114, 117, 119, 121, 123, 198, 199, 200, 201, 202, 203, 204, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 307, 308, 309, 311, 312, 318, 319, 320, 323, 324, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 349, 355, 356, 359, 366, 369, 370, 371, 372, 373, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 454, 455, 456, 457, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 479, 480, 481, 482, 485, 486, 487, 490, 491, 492, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 514, 515, 516, 518, 519, 520, 522, 524, 525, 528, 529, 530, 531, 534, 535, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 551, 552, 553, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 656, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 685, 686, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 754, 755, 756, 757, 758, 760, 761, 762, 763, 805, 806, 807, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 835, 836, 837, 838, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 862, 872, 874, 877, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 920, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 975, 977, 979, 980, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 1004, 1005, 1007, 1009, 1010, 1011, 1012, 1013, 1014, 1018, 1020, 1021, 1022, 1023, 1024, 1025 }

B grade: { 21, 22, 43, 44, 45, 46, 51, 74, 75, 76, 82, 101, 102, 106, 108, 110, 112, 115, 116, 118, 120, 122, 124, 174, 175, 176, 178, 179, 180, 182, 183, 184, 205, 232, 238, 310, 321, 322, 347, 348, 357, 358, 360, 367, 368, 374, 375, 388, 408, 413, 422, 447, 458, 459, 465, 477, 478, 483, 484, 526, 527, 532, 533, 536, 537, 545, 546, 550, 554, 566, 574, 580, 598, 629, 630, 631, 644, 645, 650, 651, 652, 653, 654, 655, 663, 681, 682, 683, 684, 687, 688, 689, 690, 694, 710, 720, 721, 726, 729, 730, 732, 750, 753, 759, 811, 812, 813, 832, 833, 834, 839, 840, 841, 842, 843, 844, 845, 846, 847, 859, 861, 863, 864, 865, 866, 867, 868, 869, 870, 871, 873, 875, 878, 879, 918, 919, 935, 957, 958, 968, 973, 974, 976, 978, 981, 997, 998, 999, 1006, 1015, 1016, 1017 }

C grade: { 855, 857 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 253, 254, 255, 256, 257, 258, 260, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 393, 394, 395, 396, 397, 453, 472, 488, 489, 493, 494, 499, 500, 505, 506, 510, 511, 513, 517, 521, 523, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 613, 614, 615, 616, 617, 657, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 876, 913, 914, 921, 930, 996, 1000, 1001, 1002, 1003, 1008, 1019 }

2.1.6 Sympy

A grade: { 23, 24, 25, 29, 30, 31, 36, 37, 38, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 174, 175, 176, 180, 206, 212, 244, 245, 250, 251, 252, 289, 290, 379, 388, 403, 408, 412, 413, 421, 422, 443, 446, 447, 452, 454, 455, 456, 466, 504, 524, 525, 527, 528, 529, 531, 537, 539, 545, 546, 547, 553, 554, 555, 564, 565, 567, 569, 570, 571, 573, 575, 576, 577, 602, 606, 607, 608, 609, 610, 611, 618, 619, 620, 621, 622, 634, 635, 636, 641, 642, 643, 658, 659, 662, 663, 664, 665, 668, 669, 670, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 687, 688, 689, 694, 696, 697, 698, 699, 700, 707, 708, 709, 711, 722, 724, 725, 726, 729, 731, 735, 806, 808, 809, 810, 811, 812, 814, 815, 817, 818, 819, 820, 829, 833, 835, 836, 839, 841, 848, 849, 850, 852, 854, 856, 858, 860, 862, 868, 870, 872, 874, 878, 879, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 924, 928, 936, 937, 939, 943, 944, 945, 946, 948, 949, 950, 952, 953, 954, 966, 977, 984, 1015, 1020, 1023 }

B grade: { 177, 181, 197, 416, 417, 418, 468, 469, 470, 487, 498, 526, 530, 532, 533, 536, 538, 566, 580, 603, 604, 605, 623, 624, 685, 695, 714, 762, 763, 837, 840, 843, 846, 877, 933, 957, 958, 959, 963, 964, 967, 1007 }

C grade: { 26, 32, 39, 207, 208, 209, 213, 214, 215, 221, 222, 380, 588, 589, 590, 693, 941, 951, 965, 971, 972 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 210, 211, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 409, 410, 411, 414, 415, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 448, 449, 450, 451, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 534, 535, 540, 541, 542, 543, 544, 548, 549, 550, 551, 552, 556, 557, 558, 559, 560, 561, 562, 563, 568, 572, 574, 578, 579, 581, 582, 583, 584, 585, 586, 587, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 612, 613, 614, 615, 616, 617, 625, 626, 627, 628, 629, 630, 631, 632, 633, 637, 638, 639, 640, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 666, 667, 671, 672, 673, 674, 675, 690, 691, 692, 701, 702, 703, 704, 705, 706, 710, 712, 713, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 730, 732, 733, 734, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 807, 813, 816, 821, 822, 823, 824, 825, 826, 827, 828, 830, 831, 832, 834, 838, 842, 844, 845, 847, 851, 853, 855, 857, 859, 861, 863, 864, 865, 866, 867, 869, 871, 873, 875, 876, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 925, 926, 927, 929, 930, 931, 932, 934, 935, 938, 940, 942, 947, 955, 956, 960, 961, 962, 968, 969, 970, 973, 974, 975, 976, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

2.1.7 Giac

A grade: { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 259, 261, 263, 264, 265, 277, 289, 290, 291, 293, 294, 295, 318, 319, 345, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 416, 417, 418, 419, 420, 421, 422, 443, 444, 445, 446, 447, 452, 454, 455, 456, 473, 474, 475, 530, 531, 538, 539, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 596, 598, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 618, 619, 620, 621, 622, 624, 632, 633, 634, 635, 636, 670, 675, 676, 677, 681, 683, 684, 685, 686, 687, 688, 689, 690, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 707, 708, 709, 710, 711, 712, 720, 721, 722, 723, 724, 725, 731, 733, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 748, 749, 755, 756, 762, 763, 806, 807, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 830, 833, 834, 835, 836, 837, 839, 840, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 868, 872, 874, 877, 882, 883, 892, 893, 894, 895, 896, 897, 898, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 935, 936, 937, 938, 939, 943, 944, 945, 946, 947, 948, 949, 950, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 965, 966, 967, 971, 972, 973, 974, 975, 976, 977, 979, 980, 981, 982, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1007, 1020 }

B grade: { 174, 175, 176, 178, 179, 180, 183, 184, 227, 238, 253, 278, 320, 332, 333, 346, 356, 357, 388, 424, 449, 566, 581, 582, 583, 584, 585, 586, 623, 625, 626, 627, 628, 637, 638, 646, 647, 648, 649, 655, 656, 678, 679, 680, 729, 730, 732, 740, 746, 750, 751, 753, 754, 757, 758, 759, 760, 761, 813, 825, 826, 827, 828, 831, 832, 843, 846, 867, 870, 878, 879, 880, 881, 918, 919, 957, 958, 968, 969, 978, 983, 997, 998, 1006, 1015, 1021, 1022, 1023 }

C grade: { 524, 525 }

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2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	0	0	0
normalized size	1	1.	1.02	0.96	0.	0.	0.	0.
time (sec)	N/A	0.178	0.191	0.117	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	0	0	0
normalized size	1	1.	0.92	0.89	0.	0.	0.	0.
time (sec)	N/A	0.19	0.124	0.085	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0
normalized size	1	1.	0.9	0.88	0.	0.	0.	0.
time (sec)	N/A	0.189	0.143	0.053	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0
normalized size	1	1.	0.96	0.89	0.	0.	0.	0.
time (sec)	N/A	0.178	0.109	0.043	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.334	0.205	0.067	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	0.207	0.052	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.344	0.123	0.049	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	0.145	0.047	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	0	0	0
normalized size	1	1.	0.68	1.99	0.	0.	0.	0.
time (sec)	N/A	0.287	0.207	0.157	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	0	0	0
normalized size	1	1.	0.93	0.9	0.	0.	0.	0.
time (sec)	N/A	0.217	0.209	0.046	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	0	0	0
normalized size	1	1.	0.83	0.87	0.	0.	0.	0.
time (sec)	N/A	0.179	0.127	0.047	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	0	0	0
normalized size	1	1.	0.8	0.79	0.	0.	0.	0.
time (sec)	N/A	0.167	0.164	0.035	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	0	0	0
normalized size	1	1.	0.88	0.89	0.	0.	0.	0.
time (sec)	N/A	0.159	0.106	0.04	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	331	128	123	0	0	0	0
normalized size	1	1.	0.39	0.37	0.	0.	0.	0.
time (sec)	N/A	0.663	0.069	0.019	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	128	133	0	0	0	0
normalized size	1	1.	0.34	0.35	0.	0.	0.	0.
time (sec)	N/A	0.723	0.08	0.023	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	126	124	0	0	0	0
normalized size	1	1.	0.34	0.33	0.	0.	0.	0.
time (sec)	N/A	0.577	0.056	0.018	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	130	133	0	0	0	0
normalized size	1	1.	0.38	0.39	0.	0.	0.	0.
time (sec)	N/A	0.575	0.066	0.023	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.074	0.042	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.063	0.036	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.063	0.044	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	7992	0	0
normalized size	1	0.	0.	0.	0.	54.37	0.	0.
time (sec)	N/A	0.085	0.072	0.055	0.	20.179	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	7996	0	0
normalized size	1	0.	0.	0.	0.	50.29	0.	0.
time (sec)	N/A	0.102	0.085	0.079	0.	14.29	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	498	163	0	0	0	212	0
normalized size	1	1.29	0.42	0.	0.	0.	0.55	0.
time (sec)	N/A	0.397	0.2	0.042	0.	0.	4.418	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	297	142	0	0	0	160	0
normalized size	1	1.23	0.59	0.	0.	0.	0.66	0.
time (sec)	N/A	0.305	0.133	0.03	0.	0.	3.291	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	245	111	0	0	0	114	0
normalized size	1	1.28	0.58	0.	0.	0.	0.59	0.
time (sec)	N/A	0.204	0.078	0.03	0.	0.	2.678	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	207	75	0	0	0	82	0
normalized size	1	1.34	0.48	0.	0.	0.	0.53	0.
time (sec)	N/A	0.15	0.028	0.025	0.	0.	2.237	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	435	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.332	0.048	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	818	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.21	0.035	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	392	0	0	0	206	0
normalized size	1	1.	1.26	0.	0.	0.	0.66	0.
time (sec)	N/A	0.178	0.465	0.034	0.	0.	4.225	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	287	0	0	0	155	0
normalized size	1	1.	1.13	0.	0.	0.	0.61	0.
time (sec)	N/A	0.138	0.377	0.027	0.	0.	3.552	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	201	0	0	0	110	0
normalized size	1	1.	1.37	0.	0.	0.	0.75	0.
time (sec)	N/A	0.102	0.165	0.026	0.	0.	2.6	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	163	0	0	0	78	0
normalized size	1	1.	1.31	0.	0.	0.	0.63	0.
time (sec)	N/A	0.066	0.115	0.02	0.	0.	1.868	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	333	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.062	0.032	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	761	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.337	0.036	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1513	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.475	0.036	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	416	166	0	0	0	204	0
normalized size	1	1.36	0.54	0.	0.	0.	0.67	0.
time (sec)	N/A	0.268	0.145	0.033	0.	0.	4.322	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	239	145	0	0	0	153	0
normalized size	1	1.28	0.78	0.	0.	0.	0.82	0.
time (sec)	N/A	0.237	0.099	0.029	0.	0.	3.261	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	195	95	0	0	0	109	0
normalized size	1	1.38	0.67	0.	0.	0.	0.77	0.
time (sec)	N/A	0.148	0.043	0.03	0.	0.	2.605	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	172	78	0	0	0	78	0
normalized size	1	1.42	0.64	0.	0.	0.	0.64	0.
time (sec)	N/A	0.115	0.032	0.02	0.	0.	1.891	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	332	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.041	0.032	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	760	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.323	0.036	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1357	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.433	0.036	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	326	258	0	198	0	0
normalized size	1	1.	8.81	6.97	0.	5.35	0.	0.
time (sec)	N/A	0.105	0.43	0.034	0.	3.099	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	327	253	0	200	0	0
normalized size	1	1.	8.18	6.32	0.	5.	0.	0.
time (sec)	N/A	0.123	0.365	0.037	0.	2.5	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	325	262	0	717	0	0
normalized size	1	1.	8.55	6.89	0.	18.87	0.	0.
time (sec)	N/A	0.112	0.305	0.029	0.	2.429	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	39	39	328	249	0	718	0	0
normalized size	1	1.	8.41	6.38	0.	18.41	0.	0.
time (sec)	N/A	0.114	0.284	0.032	0.	2.22	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	325	0	0	0	0	0
normalized size	1	1.	5.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	1.101	0.228	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	336	0	0	0	0	0
normalized size	1	1.	5.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	1.119	0.217	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	390	0	0	0	0	0
normalized size	1	1.	5.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.467	0.082	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	375	0	0	0	0	0
normalized size	1	1.	5.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.654	0.082	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	373	889	0	657	0	0
normalized size	1	1.	7.61	18.14	0.	13.41	0.	0.
time (sec)	N/A	0.123	1.066	0.036	0.	3.161	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	336	262	0	0	0	0
normalized size	1	1.	2.13	1.66	0.	0.	0.	0.
time (sec)	N/A	0.213	0.471	0.028	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	335	257	0	0	0	0
normalized size	1	1.	1.94	1.49	0.	0.	0.	0.
time (sec)	N/A	0.242	0.473	0.028	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	333	266	0	0	0	0
normalized size	1	1.	1.89	1.51	0.	0.	0.	0.
time (sec)	N/A	0.225	0.322	0.02	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	338	253	0	0	0	0
normalized size	1	1.	2.	1.5	0.	0.	0.	0.
time (sec)	N/A	0.229	0.327	0.021	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	340	264	0	0	0	0
normalized size	1	1.	2.14	1.66	0.	0.	0.	0.
time (sec)	N/A	0.232	0.47	0.023	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	340	261	0	0	0	0
normalized size	1	1.	1.94	1.49	0.	0.	0.	0.
time (sec)	N/A	0.26	0.489	0.024	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	338	270	0	0	0	0
normalized size	1	1.	1.9	1.52	0.	0.	0.	0.
time (sec)	N/A	0.224	0.351	0.023	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	342	255	0	0	0	0
normalized size	1	1.	2.01	1.5	0.	0.	0.	0.
time (sec)	N/A	0.222	0.444	0.023	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	316	316	336	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.409	1.548	0.071	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	399	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.43	1.271	0.069	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	400	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.413	0.428	0.06	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	387	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.398	0.808	0.059	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	0	0	0
normalized size	1	1.	1.43	3.4	0.	0.	0.	0.
time (sec)	N/A	0.296	1.406	0.007	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	0	0	0
normalized size	1	1.	1.43	1.78	0.	0.	0.	0.
time (sec)	N/A	0.209	0.316	0.022	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	0	0	0
normalized size	1	1.	1.31	1.58	0.	0.	0.	0.
time (sec)	N/A	0.241	0.313	0.021	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	0	0	0
normalized size	1	1.	1.27	1.61	0.	0.	0.	0.
time (sec)	N/A	0.225	0.151	0.02	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	0	0	0
normalized size	1	1.	1.34	1.6	0.	0.	0.	0.
time (sec)	N/A	0.228	0.161	0.019	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	324	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.359	1.056	0.062	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.376	0.897	0.063	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.387	0.309	0.056	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	375	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.37	0.507	0.055	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	0	0	0
normalized size	1	1.	1.51	3.63	0.	0.	0.	0.
time (sec)	N/A	0.271	0.97	0.009	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	46	240	0	117	0	0
normalized size	1	1.	2.	10.43	0.	5.09	0.	0.
time (sec)	N/A	0.059	0.009	0.02	0.	1.995	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	54	240	0	120	0	0
normalized size	1	1.	2.	8.89	0.	4.44	0.	0.
time (sec)	N/A	0.065	0.01	0.024	0.	1.904	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	240	0	105	0	0
normalized size	1	1.	1.	9.6	0.	4.2	0.	0.
time (sec)	N/A	0.059	0.009	0.023	0.	2.259	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	240	0	107	0	0
normalized size	1	1.	1.	9.6	0.	4.28	0.	0.
time (sec)	N/A	0.067	0.009	0.023	0.	2.095	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.025	0.085	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	53	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.023	0.083	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.022	0.074	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.023	0.072	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	650	0	651	0	0
normalized size	1	1.	1.	14.13	0.	14.15	0.	0.
time (sec)	N/A	0.116	0.024	0.143	0.	2.674	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	0	0	0
normalized size	1	1.	1.96	1.77	0.	0.	0.	0.
time (sec)	N/A	0.15	0.297	0.005	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	0	0	0
normalized size	1	1.	1.77	1.61	0.	0.	0.	0.
time (sec)	N/A	0.161	0.271	0.007	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	0	0	0
normalized size	1	1.	1.72	1.58	0.	0.	0.	0.
time (sec)	N/A	0.146	0.178	0.004	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	0	0	0
normalized size	1	1.	1.83	1.64	0.	0.	0.	0.
time (sec)	N/A	0.163	0.192	0.006	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	419	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.315	1.315	0.063	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	304	304	447	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.318	1.219	0.062	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	448	0	0	0	0	0
normalized size	1	1.	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	0.885	0.055	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	422	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.337	0.335	0.055	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	384	661	0	0	0	0
normalized size	1	1.	1.74	2.99	0.	0.	0.	0.
time (sec)	N/A	0.284	1.117	0.008	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	0	0	0
normalized size	1	1.	1.5	1.86	0.	0.	0.	0.
time (sec)	N/A	0.142	0.251	0.005	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	0	0	0
normalized size	1	1.	1.34	1.66	0.	0.	0.	0.
time (sec)	N/A	0.149	0.231	0.006	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	0	0	0
normalized size	1	1.	1.3	1.62	0.	0.	0.	0.
time (sec)	N/A	0.135	0.104	0.004	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	0	0	0
normalized size	1	1.	1.39	1.71	0.	0.	0.	0.
time (sec)	N/A	0.155	0.183	0.004	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	407	0	0	0	0	0
normalized size	1	1.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.28	1.345	0.056	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	0.689	0.055	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.231	0.048	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	410	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.322	0.286	0.051	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	295	653	0	0	0	0
normalized size	1	1.	1.46	3.23	0.	0.	0.	0.
time (sec)	N/A	0.256	0.68	0.007	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	267	245	0	532	0	0
normalized size	1	1.	6.36	5.83	0.	12.67	0.	0.
time (sec)	N/A	0.114	0.434	0.051	0.	1.502	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	269	243	0	533	0	0
normalized size	1	1.	5.85	5.28	0.	11.59	0.	0.
time (sec)	N/A	0.114	0.456	0.049	0.	1.508	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	267	245	0	155	0	0
normalized size	1	1.	6.07	5.57	0.	3.52	0.	0.
time (sec)	N/A	0.104	0.368	0.034	0.	1.605	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	269	247	0	170	0	0
normalized size	1	1.	6.11	5.61	0.	3.86	0.	0.
time (sec)	N/A	0.094	0.373	0.046	0.	1.608	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	322	0	0	3237	0	0
normalized size	1	1.	4.67	0.	0.	46.91	0.	0.
time (sec)	N/A	0.204	0.63	0.143	0.	8.172	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	446	0	0	3351	0	0
normalized size	1	1.	6.28	0.	0.	47.2	0.	0.
time (sec)	N/A	0.192	1.344	0.125	0.	6.663	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	447	0	0	3240	0	0
normalized size	1	1.	6.21	0.	0.	45.	0.	0.
time (sec)	N/A	0.185	0.432	0.07	0.	6.32	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	325	0	0	3348	0	0
normalized size	1	1.	4.51	0.	0.	46.5	0.	0.
time (sec)	N/A	0.169	0.53	0.073	0.	6.398	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	663	0	0	3236	0	0
normalized size	1	1.	9.08	0.	0.	44.33	0.	0.
time (sec)	N/A	0.199	1.407	0.102	0.	4.169	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	648	0	0	3345	0	0
normalized size	1	1.	8.64	0.	0.	44.6	0.	0.
time (sec)	N/A	0.201	1.257	0.092	0.	4.129	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	649	0	0	3239	0	0
normalized size	1	1.	8.54	0.	0.	42.62	0.	0.
time (sec)	N/A	0.192	0.568	0.066	0.	4.138	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	666	0	0	3343	0	0
normalized size	1	1.	8.76	0.	0.	43.99	0.	0.
time (sec)	N/A	0.18	0.873	0.066	0.	4.104	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	269	245	0	155	0	0
normalized size	1	1.	6.4	5.83	0.	3.69	0.	0.
time (sec)	N/A	0.09	0.426	0.019	0.	1.203	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	267	247	0	170	0	0
normalized size	1	1.	5.8	5.37	0.	3.7	0.	0.
time (sec)	N/A	0.102	0.435	0.025	0.	1.675	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	265	245	0	532	0	0
normalized size	1	1.	6.02	5.57	0.	12.09	0.	0.
time (sec)	N/A	0.09	0.29	0.02	0.	1.889	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	271	243	0	533	0	0
normalized size	1	1.	6.16	5.52	0.	12.11	0.	0.
time (sec)	N/A	0.087	0.343	0.015	0.	1.874	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	320	0	0	3240	0	0
normalized size	1	1.	4.64	0.	0.	46.96	0.	0.
time (sec)	N/A	0.176	0.574	0.103	0.	8.397	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	329	0	0	3348	0	0
normalized size	1	1.	4.63	0.	0.	47.15	0.	0.
time (sec)	N/A	0.185	0.806	0.105	0.	8.174	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	330	0	0	3237	0	0
normalized size	1	1.	4.58	0.	0.	44.96	0.	0.
time (sec)	N/A	0.188	0.354	0.074	0.	8.478	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	323	0	0	3351	0	0
normalized size	1	1.	4.49	0.	0.	46.54	0.	0.
time (sec)	N/A	0.168	0.322	0.073	0.	8.271	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	667	0	0	3239	0	0
normalized size	1	1.	9.14	0.	0.	44.37	0.	0.
time (sec)	N/A	0.176	1.244	0.092	0.	5.327	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	649	0	0	3343	0	0
normalized size	1	1.	8.65	0.	0.	44.57	0.	0.
time (sec)	N/A	0.185	1.274	0.09	0.	5.674	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	650	0	0	3236	0	0
normalized size	1	1.	8.55	0.	0.	42.58	0.	0.
time (sec)	N/A	0.185	0.721	0.067	0.	5.526	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	670	0	0	3345	0	0
normalized size	1	1.	8.82	0.	0.	44.01	0.	0.
time (sec)	N/A	0.174	0.782	0.066	0.	6.065	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	0	0	0
normalized size	1	1.	1.86	1.69	0.	0.	0.	0.
time (sec)	N/A	0.24	0.378	0.021	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	0	0	0
normalized size	1	1.	1.84	1.69	0.	0.	0.	0.
time (sec)	N/A	0.228	0.36	0.02	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	0	0	0
normalized size	1	1.	1.68	1.5	0.	0.	0.	0.
time (sec)	N/A	0.247	0.551	0.026	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	291	264	0	0	0	0
normalized size	1	1.	1.56	1.41	0.	0.	0.	0.
time (sec)	N/A	0.277	0.542	0.026	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	289	262	0	0	0	0
normalized size	1	1.	1.52	1.38	0.	0.	0.	0.
time (sec)	N/A	0.246	0.433	0.023	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	0	0	0
normalized size	1	1.	1.6	1.41	0.	0.	0.	0.
time (sec)	N/A	0.236	0.457	0.023	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	332	332	438	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.54	1.767	0.072	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.568	1.649	0.069	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	0	0	0
normalized size	1	1.	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.491	1.101	0.063	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	345	345	441	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.475	0.483	0.061	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	209	255	0	0	0	0
normalized size	1	1.	1.54	1.88	0.	0.	0.	0.
time (sec)	N/A	0.22	0.486	0.022	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	232	257	0	0	0	0
normalized size	1	1.	1.53	1.69	0.	0.	0.	0.
time (sec)	N/A	0.221	0.626	0.02	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	230	255	0	0	0	0
normalized size	1	1.	1.4	1.55	0.	0.	0.	0.
time (sec)	N/A	0.218	0.282	0.017	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	211	253	0	0	0	0
normalized size	1	1.	1.35	1.62	0.	0.	0.	0.
time (sec)	N/A	0.208	0.205	0.018	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	225	255	0	0	0	0
normalized size	1	1.	1.53	1.73	0.	0.	0.	0.
time (sec)	N/A	0.225	0.558	0.022	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	427	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.407	1.198	0.066	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	454	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.413	1.232	0.065	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	455	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.441	0.406	0.058	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	430	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.414	0.492	0.057	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	214	275	0	0	0	0
normalized size	1	1.	0.67	0.86	0.	0.	0.	0.
time (sec)	N/A	1.24	0.633	0.036	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	331	235	264	0	0	0	0
normalized size	1	1.	0.71	0.8	0.	0.	0.	0.
time (sec)	N/A	1.298	0.749	0.044	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	233	273	0	0	0	0
normalized size	1	1.	0.71	0.83	0.	0.	0.	0.
time (sec)	N/A	0.783	0.243	0.036	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	216	266	0	0	0	0
normalized size	1	1.	0.67	0.82	0.	0.	0.	0.
time (sec)	N/A	0.854	0.251	0.039	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	213	275	0	0	0	0
normalized size	1	1.	0.59	0.76	0.	0.	0.	0.
time (sec)	N/A	1.004	0.531	0.02	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	348	348	235	268	0	0	0	0
normalized size	1	1.	0.68	0.77	0.	0.	0.	0.
time (sec)	N/A	0.985	0.67	0.019	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	344	344	233	277	0	0	0	0
normalized size	1	1.	0.68	0.81	0.	0.	0.	0.
time (sec)	N/A	0.696	0.258	0.018	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	364	364	215	266	0	0	0	0
normalized size	1	1.	0.59	0.73	0.	0.	0.	0.
time (sec)	N/A	0.792	0.2	0.019	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	39	132	0	0	56	0
normalized size	1	1.	0.31	1.06	0.	0.	0.45	0.
time (sec)	N/A	0.046	0.026	0.026	0.	0.	3.127	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	40	125	0	0	99	0
normalized size	1	1.	0.29	0.9	0.	0.	0.71	0.
time (sec)	N/A	0.053	0.029	0.032	0.	0.	5.913	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	58	140	0	0	94	0
normalized size	1	1.	0.41	0.99	0.	0.	0.66	0.
time (sec)	N/A	0.05	0.037	0.026	0.	0.	5.932	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	61	135	0	0	61	0
normalized size	1	1.	0.45	0.99	0.	0.	0.45	0.
time (sec)	N/A	0.05	0.038	0.028	0.	0.	3.221	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	41	132	0	0	56	0
normalized size	1	1.	0.32	1.04	0.	0.	0.44	0.
time (sec)	N/A	0.046	0.022	0.017	0.	0.	3.961	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	42	125	0	0	99	0
normalized size	1	1.	0.3	0.89	0.	0.	0.7	0.
time (sec)	N/A	0.051	0.024	0.014	0.	0.	8.378	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	140	0	0	94	0
normalized size	1	1.	0.42	0.97	0.	0.	0.65	0.
time (sec)	N/A	0.047	0.031	0.012	0.	0.	8.451	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	63	135	0	0	61	0
normalized size	1	1.	0.46	0.98	0.	0.	0.44	0.
time (sec)	N/A	0.048	0.032	0.013	0.	0.	4.087	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	194	240	0	0	0	0
normalized size	1	1.	0.58	0.72	0.	0.	0.	0.
time (sec)	N/A	0.64	0.224	0.007	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	195	240	0	0	0	0
normalized size	1	1.	0.51	0.63	0.	0.	0.	0.
time (sec)	N/A	0.7	0.226	0.006	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	375	375	193	240	0	0	0	0
normalized size	1	1.	0.51	0.64	0.	0.	0.	0.
time (sec)	N/A	0.597	0.114	0.007	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	196	240	0	0	0	0
normalized size	1	1.	0.57	0.7	0.	0.	0.	0.
time (sec)	N/A	0.604	0.137	0.005	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	452	452	211	274	0	0	0	0
normalized size	1	1.	0.47	0.61	0.	0.	0.	0.
time (sec)	N/A	1.06	0.544	0.007	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	233	265	0	0	0	0
normalized size	1	1.	0.49	0.56	0.	0.	0.	0.
time (sec)	N/A	1.106	0.695	0.007	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	231	274	0	0	0	0
normalized size	1	1.	0.48	0.57	0.	0.	0.	0.
time (sec)	N/A	0.924	0.298	0.007	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	465	465	213	265	0	0	0	0
normalized size	1	1.	0.46	0.57	0.	0.	0.	0.
time (sec)	N/A	1.034	0.173	0.007	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	34	129	0	0	42	0
normalized size	1	1.	0.28	1.08	0.	0.	0.35	0.
time (sec)	N/A	0.049	0.017	0.004	0.	0.	2.41	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	34	122	0	0	65	0
normalized size	1	1.	0.25	0.91	0.	0.	0.49	0.
time (sec)	N/A	0.059	0.019	0.004	0.	0.	2.617	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	52	129	0	0	60	0
normalized size	1	1.	0.38	0.94	0.	0.	0.44	0.
time (sec)	N/A	0.051	0.027	0.006	0.	0.	2.418	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	56	122	0	0	46	0
normalized size	1	1.	0.43	0.93	0.	0.	0.35	0.
time (sec)	N/A	0.054	0.032	0.004	0.	0.	2.555	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.14	0.039	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.22	0.184	0.039	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	133	451	342	1073	6392	1127
normalized size	1	1.	0.83	2.82	2.14	6.71	39.95	7.04
time (sec)	N/A	0.106	0.125	0.007	1.109	1.218	6.542	1.156

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	104	283	248	744	3703	779
normalized size	1	1.	0.83	2.25	1.97	5.9	29.39	6.18
time (sec)	N/A	0.068	0.078	0.004	1.037	1.157	4.148	1.211

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	167	0	473	1906	487
normalized size	1	1.	1.	1.78	0.	5.03	20.28	5.18
time (sec)	N/A	0.046	0.07	0.004	0.	1.201	3.057	1.188

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	94	0	0	0	741	0
normalized size	1	1.	0.95	0.	0.	0.	7.48	0.
time (sec)	N/A	0.058	0.056	0.027	0.	0.	5.254	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	252	1565	811	3567	0	3591
normalized size	1	1.	0.86	5.32	2.76	12.13	0.	12.21
time (sec)	N/A	0.202	0.278	0.014	1.195	1.057	0.	1.255

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	211	1142	640	2719	0	2746
normalized size	1	1.	0.85	4.6	2.58	10.96	0.	11.07
time (sec)	N/A	0.148	0.203	0.013	1.159	0.971	0.	1.17

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	172	793	0	1963	11664	1994
normalized size	1	1.	0.85	3.91	0.	9.67	57.46	9.82
time (sec)	N/A	0.115	0.167	0.01	0.	0.863	13.977	1.243

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	188	0	0	0	4755	0
normalized size	1	1.	0.9	0.	0.	0.	22.75	0.
time (sec)	N/A	0.127	0.164	0.031	0.	0.	10.485	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	402	3780	1557	8852	0	0
normalized size	1	1.	0.88	8.24	3.39	19.29	0.	0.
time (sec)	N/A	0.317	0.47	0.027	1.152	1.047	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	345	2972	1287	7065	0	6661
normalized size	1	1.	0.87	7.51	3.25	17.84	0.	16.82
time (sec)	N/A	0.264	0.381	0.021	1.095	1.013	0.	1.341

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	290	2280	0	5404	0	5230
normalized size	1	1.	0.86	6.77	0.	16.04	0.	15.52
time (sec)	N/A	0.209	0.359	0.018	0.	0.92	0.	1.371

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	332	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.345	0.027	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	284	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.865	0.599	0.062	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	292	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.862	0.684	0.052	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	239	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.476	0.236	0.046	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.149	0.046	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	237	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.283	0.158	0.046	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	222	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.117	0.046	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	244	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.559	0.247	0.037	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	273	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	0.265	0.056	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.585	0.376	0.047	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.463	0.168	0.053	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1482	1482	820	1126	0	0	0	0
normalized size	1	1.	0.55	0.76	0.	0.	0.	0.
time (sec)	N/A	2.815	2.031	0.151	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	B	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	636	0
normalized size	1	0.	0.	0.	0.	0.	4.71	0.
time (sec)	N/A	0.086	0.049	0.061	0.	0.	56.487	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	296	1640	0	54	0	0
normalized size	1	1.	18.5	102.5	0.	3.38	0.	0.
time (sec)	N/A	0.077	0.829	0.036	0.	1.485	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	280	732	0	69	0	0
normalized size	1	1.	14.	36.6	0.	3.45	0.	0.
time (sec)	N/A	0.085	0.64	0.039	0.	1.375	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	278	1656	0	62	0	0
normalized size	1	1.	15.44	92.	0.	3.44	0.	0.
time (sec)	N/A	0.079	0.228	0.032	0.	1.379	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	298	724	0	65	0	0
normalized size	1	1.	16.56	40.22	0.	3.61	0.	0.
time (sec)	N/A	0.083	0.526	0.035	0.	1.379	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	30	30	424	4397	0	458	0	0
normalized size	1	1.	14.13	146.57	0.	15.27	0.	0.
time (sec)	N/A	0.093	1.204	0.04	0.	1.487	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	38	38	427	1908	0	459	0	0
normalized size	1	1.	11.24	50.21	0.	12.08	0.	0.
time (sec)	N/A	0.108	1.348	0.048	0.	1.506	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	36	36	425	4437	0	458	0	0
normalized size	1	1.	11.81	123.25	0.	12.72	0.	0.
time (sec)	N/A	0.093	0.452	0.028	0.	1.633	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	32	32	426	1888	0	459	0	0
normalized size	1	1.	13.31	59.	0.	14.34	0.	0.
time (sec)	N/A	0.093	0.462	0.035	0.	1.588	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	186	334	0	0	175	0
normalized size	1	1.	0.52	0.94	0.	0.	0.49	0.
time (sec)	N/A	0.233	0.138	0.048	0.	0.	3.844	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	146	310	0	0	138	0
normalized size	1	1.	0.45	0.95	0.	0.	0.42	0.
time (sec)	N/A	0.189	0.135	0.006	0.	0.	3.362	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	109	127	0	0	88	0
normalized size	1	1.	0.69	0.8	0.	0.	0.56	0.
time (sec)	N/A	0.089	0.063	0.004	0.	0.	2.888	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	0	37	0
normalized size	1	1.	0.85	0.81	0.	0.	0.35	0.
time (sec)	N/A	0.019	0.106	0.001	0.	0.	0.658	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	405	565	0	0	0	0
normalized size	1	1.	0.55	0.77	0.	0.	0.	0.
time (sec)	N/A	0.725	0.818	0.015	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1221	1221	382	402	0	0	0	0
normalized size	1	1.	0.31	0.33	0.	0.	0.	0.
time (sec)	N/A	1.8	1.892	0.016	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	157	218	0	0	141	0
normalized size	1	1.	0.53	0.74	0.	0.	0.48	0.
time (sec)	N/A	0.16	0.132	0.014	0.	0.	3.194	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	133	197	0	0	105	0
normalized size	1	1.	0.51	0.75	0.	0.	0.4	0.
time (sec)	N/A	0.124	0.123	0.006	0.	0.	2.664	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	0	61	0
normalized size	1	1.	0.65	0.79	0.	0.	0.5	0.
time (sec)	N/A	0.062	0.042	0.004	0.	0.	1.918	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0
normalized size	1	1.	0.84	0.8	0.	0.	0.41	0.
time (sec)	N/A	0.01	0.033	0.001	0.	0.	0.619	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	200	169	0	0	0	0
normalized size	1	1.	0.49	0.42	0.	0.	0.	0.
time (sec)	N/A	0.272	0.255	0.008	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	425	421	0	0	0	0
normalized size	1	1.	0.7	0.69	0.	0.	0.	0.
time (sec)	N/A	0.76	1.1	0.014	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	513	483	0	0	0	0
normalized size	1	1.	0.78	0.73	0.	0.	0.	0.
time (sec)	N/A	1.157	2.331	0.016	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	126	261	0	0	0	0
normalized size	1	1.	0.42	0.88	0.	0.	0.	0.
time (sec)	N/A	0.136	0.064	0.025	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	108	239	0	0	0	0
normalized size	1	1.	0.4	0.89	0.	0.	0.	0.
time (sec)	N/A	0.117	0.062	0.006	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	59	115	0	0	61	0
normalized size	1	1.	0.52	1.01	0.	0.	0.54	0.
time (sec)	N/A	0.047	0.03	0.004	0.	0.	7.749	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	0	36	0
normalized size	1	1.	0.51	0.87	0.	0.	0.33	0.
time (sec)	N/A	0.019	0.011	0.002	0.	0.	0.68	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	818	818	434	496	0	0	0	0
normalized size	1	1.	0.53	0.61	0.	0.	0.	0.
time (sec)	N/A	0.601	0.883	0.013	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.729	0.393	0.049	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.571	0.224	0.049	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1605	1605	1416	1153	0	0	0	0
normalized size	1	1.	0.88	0.72	0.	0.	0.	0.
time (sec)	N/A	9.68	7.373	0.069	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	205	132	200	161	528	0	479
normalized size	1	1.27	0.82	1.24	1.	3.28	0.	2.98
time (sec)	N/A	0.128	0.093	0.007	1.089	1.54	0.	1.231

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	134	63	60	0	166	0	97
normalized size	1	0.94	0.44	0.42	0.	1.16	0.	0.68
time (sec)	N/A	0.162	0.022	0.004	0.	1.412	0.	1.218

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	187	63	60	63	171	0	97
normalized size	1	1.31	0.44	0.42	0.44	1.2	0.	0.68
time (sec)	N/A	0.105	0.021	0.003	0.997	1.406	0.	1.275

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	78	63	60	0	162	0	65
normalized size	1	1.18	0.95	0.91	0.	2.45	0.	0.98
time (sec)	N/A	0.113	0.021	0.005	0.	1.42	0.	1.217

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	187	63	60	63	167	0	97
normalized size	1	1.31	0.44	0.42	0.44	1.17	0.	0.68
time (sec)	N/A	0.102	0.017	0.006	1.026	1.386	0.	1.227

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	59	0	153	0	34
normalized size	1	1.	0.91	1.84	0.	4.78	0.	1.06
time (sec)	N/A	0.022	0.012	0.003	0.	1.393	0.	1.237

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	175	61	58	58	158	0	62
normalized size	1	1.3	0.45	0.43	0.43	1.17	0.	0.46
time (sec)	N/A	0.053	0.015	0.003	1.035	1.439	0.	1.186

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	183	62	59	0	163	0	99
normalized size	1	1.32	0.45	0.42	0.	1.17	0.	0.71
time (sec)	N/A	0.103	0.023	0.009	0.	1.449	0.	1.229

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	178	62	60	65	151	0	93
normalized size	1	1.33	0.46	0.45	0.49	1.13	0.	0.69
time (sec)	N/A	0.093	0.024	0.005	1.027	1.419	0.	1.25

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	184	65	61	0	163	0	123
normalized size	1	1.31	0.46	0.44	0.	1.16	0.	0.88
time (sec)	N/A	0.105	0.023	0.009	0.	1.451	0.	1.27

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	254	143	236	0	969	0	239
normalized size	1	1.	0.57	0.93	0.	3.83	0.	0.94
time (sec)	N/A	0.246	0.15	0.03	0.	1.982	0.	1.31

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	26	95	181	0	450
normalized size	1	1.	0.91	0.81	2.97	5.66	0.	14.06
time (sec)	N/A	0.027	0.018	0.003	1.075	1.626	0.	1.198

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	208	132	205	0	887	0	207
normalized size	1	1.	0.64	0.99	0.	4.29	0.	1.
time (sec)	N/A	0.073	0.113	0.006	0.	1.717	0.	1.231

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	194	111	221	0	864	0	194
normalized size	1	1.01	0.58	1.15	0.	4.5	0.	1.01
time (sec)	N/A	0.217	0.075	0.013	0.	1.729	0.	1.233

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	209	65	215	0	873	0	250
normalized size	1	1.	0.31	1.03	0.	4.2	0.	1.2
time (sec)	N/A	0.189	0.016	0.008	0.	1.747	0.	1.335

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	204	48	238	0	890	0	204
normalized size	1	1.01	0.24	1.18	0.	4.41	0.	1.01
time (sec)	N/A	0.216	0.019	0.01	0.	1.676	0.	1.256

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	89	59	0	294	0	96
normalized size	1	0.97	1.16	0.77	0.	3.82	0.	1.25
time (sec)	N/A	0.143	0.066	0.01	0.	1.554	0.	1.205

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	26	35	53	38
normalized size	1	1.	1.	1.24	1.24	1.67	2.52	1.81
time (sec)	N/A	0.018	0.005	0.004	0.968	1.495	1.451	1.192

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	0	36	66	38
normalized size	1	1.	1.	1.24	0.	1.71	3.14	1.81
time (sec)	N/A	0.017	0.005	0.003	0.	1.485	1.467	1.253

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	73	38	64	0	289	0	88
normalized size	1	1.03	0.54	0.9	0.	4.07	0.	1.24
time (sec)	N/A	0.136	0.009	0.005	0.	1.531	0.	1.177

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	32	37	62	65	0	109
normalized size	1	1.	0.67	0.77	1.29	1.35	0.	2.27
time (sec)	N/A	0.114	0.008	0.003	1.003	1.48	0.	1.185

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	112	40	79	0	382	0	135
normalized size	1	1.08	0.38	0.76	0.	3.67	0.	1.3
time (sec)	N/A	0.153	0.009	0.007	0.	1.595	0.	1.198

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	152	63	58	115	180	0	81
normalized size	1	1.1	0.46	0.42	0.83	1.3	0.	0.59
time (sec)	N/A	0.188	0.037	0.006	1.013	1.705	0.	1.162

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	113	52	47	86	151	116	62
normalized size	1	1.11	0.51	0.46	0.84	1.48	1.14	0.61
time (sec)	N/A	0.157	0.023	0.006	1.	1.726	117.062	1.198

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	74	41	36	58	119	87	43
normalized size	1	1.12	0.62	0.55	0.88	1.8	1.32	0.65
time (sec)	N/A	0.137	0.02	0.006	0.998	1.689	52.615	1.231

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	26	34	85	58	23
normalized size	1	1.	0.86	0.72	0.94	2.36	1.61	0.64
time (sec)	N/A	0.02	0.007	0.003	0.993	1.699	20.139	1.136

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	141	96	0	0	0	0	257
normalized size	1	1.21	0.82	0.	0.	0.	0.	2.2
time (sec)	N/A	0.158	0.038	0.01	0.	0.	0.	1.176

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	151	50	0	0	0	0	282
normalized size	1	1.14	0.38	0.	0.	0.	0.	2.12
time (sec)	N/A	0.163	0.013	0.008	0.	0.	0.	1.198

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	191	68	0	0	0	0	0
normalized size	1	1.26	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.058	0.008	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	146	52	0	0	0	0	0
normalized size	1	1.23	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.007	0.006	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	142	55	0	0	0	0	0
normalized size	1	1.23	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.01	0.009	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	193	57	0	0	0	0	0
normalized size	1	1.25	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.012	0.007	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	106	122	0	209	0	82
normalized size	1	1.	1.49	1.72	0.	2.94	0.	1.15
time (sec)	N/A	0.025	0.157	0.013	0.	1.272	0.	1.2

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	0	0	0
normalized size	1	1.	1.23	2.38	0.	0.	0.	0.
time (sec)	N/A	0.043	0.057	0.017	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	72	28	0	111	0	30
normalized size	1	1.	2.25	0.88	0.	3.47	0.	0.94
time (sec)	N/A	0.013	0.026	0.003	0.	1.345	0.	1.249

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	0	0	0
normalized size	1	1.	1.5	3.58	0.	0.	0.	0.
time (sec)	N/A	0.013	0.016	0.007	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	198	527	0	1122	0	451
normalized size	1	1.	0.81	2.16	0.	4.6	0.	1.85
time (sec)	N/A	0.333	0.524	0.056	0.	1.537	0.	1.499

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	149	342	0	849	0	257
normalized size	1	1.	0.93	2.12	0.	5.27	0.	1.6
time (sec)	N/A	0.163	0.386	0.012	0.	1.324	0.	1.413

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	143	200	0	656	0	201
normalized size	1	1.	1.39	1.94	0.	6.37	0.	1.95
time (sec)	N/A	0.07	0.284	0.006	0.	1.582	0.	1.349

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	173	179	0	1854	0	0
normalized size	1	1.	1.54	1.6	0.	16.55	0.	0.
time (sec)	N/A	0.128	0.178	0.022	0.	3.018	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	133	326	0	697	0	0
normalized size	1	1.	1.05	2.57	0.	5.49	0.	0.
time (sec)	N/A	0.087	0.101	0.02	0.	3.485	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	174	558	0	890	0	0
normalized size	1	1.	0.84	2.68	0.	4.28	0.	0.
time (sec)	N/A	0.17	0.11	0.023	0.	7.787	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	222	849	0	1162	0	0
normalized size	1	1.	0.7	2.67	0.	3.65	0.	0.
time (sec)	N/A	0.312	0.175	0.027	0.	23.874	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	255	552	0	0	0	0
normalized size	1	1.	0.71	1.55	0.	0.	0.	0.
time (sec)	N/A	0.52	0.479	0.036	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	208	356	0	0	0	0
normalized size	1	1.	0.78	1.34	0.	0.	0.	0.
time (sec)	N/A	0.338	0.288	0.01	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	86	184	0	0	0	0
normalized size	1	1.	0.44	0.95	0.	0.	0.	0.
time (sec)	N/A	0.123	0.055	0.009	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	111	192	0	0	0	0
normalized size	1	1.	0.46	0.8	0.	0.	0.	0.
time (sec)	N/A	0.314	0.251	0.02	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	238	444	0	0	0	0
normalized size	1	1.	0.74	1.38	0.	0.	0.	0.
time (sec)	N/A	0.445	0.64	0.02	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	302	708	0	0	0	0
normalized size	1	1.	0.71	1.67	0.	0.	0.	0.
time (sec)	N/A	0.634	0.568	0.024	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	294	1027	0	1175	0	0
normalized size	1	1.	1.04	3.64	0.	4.17	0.	0.
time (sec)	N/A	0.383	0.57	0.032	0.	17.106	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	191	679	0	886	0	409
normalized size	1	1.	0.96	3.41	0.	4.45	0.	2.06
time (sec)	N/A	0.221	0.621	0.013	0.	5.892	0.	1.392

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	96	432	0	707	0	339
normalized size	1	1.	0.68	3.06	0.	5.01	0.	2.4
time (sec)	N/A	0.09	0.06	0.012	0.	4.255	0.	1.39

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	193	401	0	2226	0	0
normalized size	1	1.	1.28	2.66	0.	14.74	0.	0.
time (sec)	N/A	0.192	1.283	0.015	0.	6.456	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	641	0	748	0	0
normalized size	1	1.	0.88	3.88	0.	4.53	0.	0.
time (sec)	N/A	0.104	0.091	0.015	0.	7.506	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	186	1042	0	925	0	0
normalized size	1	1.	0.73	4.07	0.	3.61	0.	0.
time (sec)	N/A	0.218	0.125	0.016	0.	19.695	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	245	1498	0	1211	0	0
normalized size	1	1.	0.67	4.09	0.	3.31	0.	0.
time (sec)	N/A	0.369	0.192	0.018	0.	93.007	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	290	933	0	0	0	0
normalized size	1	1.	0.74	2.39	0.	0.	0.	0.
time (sec)	N/A	0.678	0.52	0.032	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	235	738	0	0	0	0
normalized size	1	1.	0.76	2.38	0.	0.	0.	0.
time (sec)	N/A	0.438	0.415	0.014	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	206	527	0	0	0	0
normalized size	1	1.	0.79	2.01	0.	0.	0.	0.
time (sec)	N/A	0.202	0.317	0.012	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	228	670	0	0	0	0
normalized size	1	1.	0.74	2.18	0.	0.	0.	0.
time (sec)	N/A	0.45	0.358	0.014	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	275	791	0	0	0	0
normalized size	1	1.	0.72	2.07	0.	0.	0.	0.
time (sec)	N/A	0.632	0.455	0.016	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	357	1197	0	0	0	0
normalized size	1	1.	0.74	2.49	0.	0.	0.	0.
time (sec)	N/A	0.809	0.658	0.017	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	86	52	0	134	39	24
normalized size	1	1.	1.69	1.02	0.	2.63	0.76	0.47
time (sec)	N/A	0.023	0.031	0.02	0.	1.492	36.669	1.196

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	104	78	103	197	66	41
normalized size	1	1.	1.44	1.08	1.43	2.74	0.92	0.57
time (sec)	N/A	0.032	0.047	0.021	1.674	1.543	126.062	1.179

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	86	68	0	139	0	30
normalized size	1	1.	1.62	1.28	0.	2.62	0.	0.57
time (sec)	N/A	0.029	0.032	0.081	0.	1.462	0.	1.179

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	98	80	0	159	0	0
normalized size	1	1.	0.87	0.71	0.	1.41	0.	0.
time (sec)	N/A	0.064	0.037	0.042	0.	1.476	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	109	0	163	225	0	63
normalized size	1	1.	1.03	0.	1.54	2.12	0.	0.59
time (sec)	N/A	0.056	0.055	0.047	1.524	1.516	0.	1.181

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	42	0	88	0	55
normalized size	1	1.	0.94	0.81	0.	1.69	0.	1.06
time (sec)	N/A	0.099	0.016	0.011	0.	1.484	0.	1.167

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	60	0	120	0	82
normalized size	1	1.	0.96	0.88	0.	1.76	0.	1.21
time (sec)	N/A	0.191	0.024	0.028	0.	1.486	0.	1.144

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	224	527	0	1153	0	0
normalized size	1	1.	0.8	1.88	0.	4.1	0.	0.
time (sec)	N/A	0.29	0.389	0.023	0.	1.82	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	172	342	0	886	0	0
normalized size	1	1.	1.02	2.02	0.	5.24	0.	0.
time (sec)	N/A	0.135	0.344	0.01	0.	1.681	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	152	200	0	679	0	0
normalized size	1	1.	1.43	1.89	0.	6.41	0.	0.
time (sec)	N/A	0.068	0.134	0.007	0.	1.597	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	190	179	0	1859	0	0
normalized size	1	1.	1.7	1.6	0.	16.6	0.	0.
time (sec)	N/A	0.106	0.228	0.011	0.	3.026	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	133	326	0	714	0	0
normalized size	1	1.	1.02	2.51	0.	5.49	0.	0.
time (sec)	N/A	0.084	0.122	0.012	0.	3.421	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	173	558	0	926	0	0
normalized size	1	1.	0.79	2.56	0.	4.25	0.	0.
time (sec)	N/A	0.135	0.157	0.013	0.	8.216	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	258	553	0	0	0	0
normalized size	1	1.	0.64	1.37	0.	0.	0.	0.
time (sec)	N/A	0.522	0.48	0.018	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	212	358	0	0	0	0
normalized size	1	1.	0.68	1.15	0.	0.	0.	0.
time (sec)	N/A	0.338	0.273	0.011	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	86	127	0	0	0	0
normalized size	1	1.	0.34	0.5	0.	0.	0.	0.
time (sec)	N/A	0.13	0.057	0.007	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	111	297	0	0	0	0
normalized size	1	1.	0.38	1.03	0.	0.	0.	0.
time (sec)	N/A	0.312	0.258	0.013	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	238	444	0	0	0	0
normalized size	1	1.	0.64	1.19	0.	0.	0.	0.
time (sec)	N/A	0.454	0.404	0.01	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	348	247	1027	0	1652	0	0
normalized size	1	0.98	0.7	2.9	0.	4.67	0.	0.
time (sec)	N/A	0.379	0.502	0.031	0.	18.506	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	190	679	0	1239	0	0
normalized size	1	1.	0.94	3.36	0.	6.13	0.	0.
time (sec)	N/A	0.239	0.332	0.013	0.	7.552	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	86	432	0	914	0	0
normalized size	1	1.	0.59	2.96	0.	6.26	0.	0.
time (sec)	N/A	0.098	0.07	0.01	0.	3.939	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	253	401	0	2674	0	0
normalized size	1	1.	1.66	2.64	0.	17.59	0.	0.
time (sec)	N/A	0.196	0.342	0.016	0.	8.173	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	148	641	0	957	0	0
normalized size	1	1.	0.87	3.77	0.	5.63	0.	0.
time (sec)	N/A	0.111	0.085	0.016	0.	12.369	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	189	1042	0	1284	0	0
normalized size	1	1.	0.74	4.09	0.	5.04	0.	0.
time (sec)	N/A	0.228	0.121	0.018	0.	36.704	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	271	935	0	0	0	0
normalized size	1	1.	0.6	2.06	0.	0.	0.	0.
time (sec)	N/A	0.675	0.504	0.035	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	219	643	0	0	0	0
normalized size	1	1.	0.58	1.7	0.	0.	0.	0.
time (sec)	N/A	0.442	0.389	0.015	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	203	514	0	0	0	0
normalized size	1	1.	0.62	1.57	0.	0.	0.	0.
time (sec)	N/A	0.22	0.465	0.013	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	223	654	0	0	0	0
normalized size	1	1.	0.59	1.72	0.	0.	0.	0.
time (sec)	N/A	0.47	0.365	0.016	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	266	866	0	0	0	0
normalized size	1	1.	0.6	1.95	0.	0.	0.	0.
time (sec)	N/A	0.649	0.466	0.016	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	259	137	533	0	944	0	408
normalized size	1	1.2	0.63	2.47	0.	4.37	0.	1.89
time (sec)	N/A	0.62	0.309	0.05	0.	1.371	0.	1.687

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	181	97	354	0	744	0	219
normalized size	1	1.28	0.69	2.51	0.	5.28	0.	1.55
time (sec)	N/A	0.467	0.174	0.012	0.	1.357	0.	1.368

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	77	180	0	608	0	171
normalized size	1	1.	1.12	2.61	0.	8.81	0.	2.48
time (sec)	N/A	0.054	0.087	0.007	0.	1.412	0.	1.346

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	184	80	235	0	2101	0	0
normalized size	1	1.92	0.83	2.45	0.	21.89	0.	0.
time (sec)	N/A	0.433	0.132	0.02	0.	2.008	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	140	132	454	0	936	0	0
normalized size	1	1.35	1.27	4.37	0.	9.	0.	0.
time (sec)	N/A	0.386	0.356	0.02	0.	2.159	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	218	193	923	0	1218	0	0
normalized size	1	1.25	1.11	5.3	0.	7.	0.	0.
time (sec)	N/A	0.507	0.277	0.025	0.	2.841	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	271	259	1518	0	1601	0	0
normalized size	1	1.02	0.98	5.73	0.	6.04	0.	0.
time (sec)	N/A	0.607	0.459	0.028	0.	5.935	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	478	293	662	0	0	0	0
normalized size	1	1.3	0.8	1.8	0.	0.	0.	0.
time (sec)	N/A	0.718	0.822	0.034	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	370	250	406	0	0	0	0
normalized size	1	1.31	0.89	1.44	0.	0.	0.	0.
time (sec)	N/A	0.517	0.57	0.013	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	279	98	199	0	0	0	0
normalized size	1	1.31	0.46	0.93	0.	0.	0.	0.
time (sec)	N/A	0.203	0.064	0.007	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	353	141	272	0	0	0	0
normalized size	1	1.33	0.53	1.03	0.	0.	0.	0.
time (sec)	N/A	0.51	0.566	0.02	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	472	314	571	0	0	0	0
normalized size	1	1.3	0.87	1.58	0.	0.	0.	0.
time (sec)	N/A	0.63	0.972	0.024	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	598	402	955	0	0	0	0
normalized size	1	1.28	0.86	2.05	0.	0.	0.	0.
time (sec)	N/A	0.812	1.065	0.026	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	311	142	1018	0	954	0	0
normalized size	1	1.25	0.57	4.09	0.	3.83	0.	0.
time (sec)	N/A	0.73	0.342	0.031	0.	3.108	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	222	104	593	0	757	0	598
normalized size	1	1.29	0.6	3.45	0.	4.4	0.	3.48
time (sec)	N/A	0.536	0.207	0.016	0.	2.495	0.	3.185

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	336	0	618	0	510
normalized size	1	1.	0.84	3.57	0.	6.57	0.	5.43
time (sec)	N/A	0.066	0.103	0.011	0.	2.208	0.	3.377

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	206	118	652	0	2414	0	0
normalized size	1	1.63	0.94	5.17	0.	19.16	0.	0.
time (sec)	N/A	0.49	0.234	0.016	0.	2.828	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	170	165	820	0	895	0	0
normalized size	1	1.23	1.2	5.94	0.	6.49	0.	0.
time (sec)	N/A	0.526	0.422	0.014	0.	2.657	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	260	202	1653	0	1191	0	0
normalized size	1	1.27	0.99	8.06	0.	5.81	0.	0.
time (sec)	N/A	0.591	0.288	0.018	0.	5.984	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	287	267	2605	0	1580	0	0
normalized size	1	0.98	0.91	8.92	0.	5.41	0.	0.
time (sec)	N/A	0.725	0.49	0.021	0.	11.616	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	526	308	1098	0	0	0	0
normalized size	1	1.3	0.76	2.71	0.	0.	0.	0.
time (sec)	N/A	0.876	0.848	0.033	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	430	270	823	0	0	0	0
normalized size	1	1.3	0.82	2.49	0.	0.	0.	0.
time (sec)	N/A	0.635	0.782	0.014	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	348	243	515	0	0	0	0
normalized size	1	1.34	0.93	1.98	0.	0.	0.	0.
time (sec)	N/A	0.291	0.543	0.013	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	422	278	873	0	0	0	0
normalized size	1	1.35	0.89	2.8	0.	0.	0.	0.
time (sec)	N/A	0.664	0.703	0.016	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	520	329	1039	0	0	0	0
normalized size	1	1.34	0.85	2.68	0.	0.	0.	0.
time (sec)	N/A	0.869	0.833	0.016	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	648	430	1666	0	0	0	0
normalized size	1	1.31	0.87	3.37	0.	0.	0.	0.
time (sec)	N/A	1.075	1.094	0.019	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	267	140	533	0	961	0	0
normalized size	1	1.19	0.62	2.37	0.	4.27	0.	0.
time (sec)	N/A	0.622	0.27	0.022	0.	1.455	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	189	101	354	0	761	0	262
normalized size	1	1.28	0.68	2.39	0.	5.14	0.	1.77
time (sec)	N/A	0.465	0.164	0.013	0.	1.372	0.	1.337

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	70	184	0	612	0	190
normalized size	1	1.	0.97	2.56	0.	8.5	0.	2.64
time (sec)	N/A	0.052	0.077	0.007	0.	1.325	0.	1.43

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	184	80	312	0	2168	0	0
normalized size	1	1.92	0.83	3.25	0.	22.58	0.	0.
time (sec)	N/A	0.414	0.12	0.012	0.	1.872	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	148	158	452	0	972	0	0
normalized size	1	1.37	1.46	4.19	0.	9.	0.	0.
time (sec)	N/A	0.387	0.214	0.013	0.	1.768	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	218	191	922	0	1251	0	0
normalized size	1	1.23	1.08	5.21	0.	7.07	0.	0.
time (sec)	N/A	0.472	0.324	0.016	0.	2.951	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	498	297	665	0	0	0	0
normalized size	1	1.12	0.67	1.5	0.	0.	0.	0.
time (sec)	N/A	0.709	0.793	0.019	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	398	253	409	0	0	0	0
normalized size	1	1.12	0.71	1.16	0.	0.	0.	0.
time (sec)	N/A	0.521	0.557	0.014	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	319	107	164	0	0	0	0
normalized size	1	1.12	0.37	0.57	0.	0.	0.	0.
time (sec)	N/A	0.209	0.112	0.01	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	387	151	345	0	0	0	0
normalized size	1	1.13	0.44	1.01	0.	0.	0.	0.
time (sec)	N/A	0.483	0.176	0.014	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	486	314	596	0	0	0	0
normalized size	1	1.13	0.73	1.38	0.	0.	0.	0.
time (sec)	N/A	0.637	0.91	0.016	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	323	1215	1240	0	1488	0	0
normalized size	1	1.04	3.92	4.	0.	4.8	0.	0.
time (sec)	N/A	0.783	11.731	0.036	0.	2.859	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	242	133	783	0	1172	0	767
normalized size	1	1.29	0.71	4.19	0.	6.27	0.	4.1
time (sec)	N/A	0.574	0.276	0.014	0.	2.692	0.	8.96

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	50	478	0	871	0	657
normalized size	1	1.04	0.5	4.78	0.	8.71	0.	6.57
time (sec)	N/A	0.073	0.054	0.014	0.	2.133	0.	9.179

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	214	110	1014	0	3229	0	0
normalized size	1	1.6	0.82	7.57	0.	24.1	0.	0.
time (sec)	N/A	0.514	0.472	0.017	0.	2.918	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	174	186	1088	0	1277	0	0
normalized size	1	1.19	1.27	7.45	0.	8.75	0.	0.
time (sec)	N/A	0.449	0.355	0.018	0.	2.648	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	246	202	1947	0	1997	0	0
normalized size	1	1.16	0.95	9.18	0.	9.42	0.	0.
time (sec)	N/A	0.584	0.56	0.018	0.	6.745	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	559	296	1158	0	0	0	0
normalized size	1	1.16	0.61	2.4	0.	0.	0.	0.
time (sec)	N/A	0.906	0.813	0.037	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	475	255	667	0	0	0	0
normalized size	1	1.16	0.62	1.63	0.	0.	0.	0.
time (sec)	N/A	0.661	0.589	0.015	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	411	241	466	0	0	0	0
normalized size	1	1.15	0.68	1.31	0.	0.	0.	0.
time (sec)	N/A	0.309	0.507	0.013	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	476	268	686	0	0	0	0
normalized size	1	1.16	0.65	1.67	0.	0.	0.	0.
time (sec)	N/A	0.68	0.653	0.019	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	567	319	1082	0	0	0	0
normalized size	1	1.16	0.65	2.21	0.	0.	0.	0.
time (sec)	N/A	0.867	0.898	0.017	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	49	64	0	423	0	0
normalized size	1	1.	0.65	0.85	0.	5.64	0.	0.
time (sec)	N/A	0.017	0.02	0.055	0.	1.895	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	57	0	381	0	92
normalized size	1	1.	0.84	1.14	0.	7.62	0.	1.84
time (sec)	N/A	0.012	0.01	0.039	0.	1.862	0.	1.212

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	0	252	0	0
normalized size	1	1.	1.	0.71	0.	10.5	0.	0.
time (sec)	N/A	0.008	0.006	0.03	0.	1.698	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	55	46	0	38
normalized size	1	1.	1.	1.61	2.39	2.	0.	1.65
time (sec)	N/A	0.004	0.004	0.004	1.552	1.224	0.	1.248

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	30	44	68	61	0	63
normalized size	1	1.	0.61	0.9	1.39	1.24	0.	1.29
time (sec)	N/A	0.009	0.006	0.004	1.616	1.302	0.	1.167

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	35	80	0	39
normalized size	1	1.	0.89	0.76	0.95	2.16	0.	1.05
time (sec)	N/A	0.008	0.013	0.01	1.528	1.302	0.	1.132

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	35	80	0	39
normalized size	1	1.	0.89	0.76	0.95	2.16	0.	1.05
time (sec)	N/A	0.013	0.005	0.006	1.688	1.288	0.	1.117

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	44	38	54	116	0	57
normalized size	1	1.	0.62	0.54	0.76	1.63	0.	0.8
time (sec)	N/A	0.015	0.015	0.009	1.673	1.31	0.	1.127

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	37	57	618	0	65
normalized size	1	1.	0.59	0.76	1.16	12.61	0.	1.33
time (sec)	N/A	0.013	0.064	0.003	1.665	1.509	0.	1.134

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	37	57	618	0	65
normalized size	1	1.	0.59	0.76	1.16	12.61	0.	1.33
time (sec)	N/A	0.018	0.015	0.001	1.667	1.411	0.	1.133

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	30	43	43	308	0	51
normalized size	1	1.	0.68	0.98	0.98	7.	0.	1.16
time (sec)	N/A	0.015	0.009	0.011	1.714	1.338	0.	1.129

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	32	26	0	104	0	36
normalized size	1	1.	0.73	0.59	0.	2.36	0.	0.82
time (sec)	N/A	0.006	0.008	0.005	0.	1.295	0.	1.129

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	43	76	0	0	0	0
normalized size	1	1.	0.52	0.92	0.	0.	0.	0.
time (sec)	N/A	0.026	0.008	0.021	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	26	39	20	26
normalized size	1	1.	1.	0.86	1.18	1.77	0.91	1.18
time (sec)	N/A	0.003	0.005	0.001	1.57	1.279	0.402	1.149

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	27	81	0	0	36	0
normalized size	1	1.	0.21	0.62	0.	0.	0.27	0.
time (sec)	N/A	0.082	0.005	0.018	0.	0.	1.035	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	27	62	0	0	0	0
normalized size	1	1.	0.5	1.15	0.	0.	0.	0.
time (sec)	N/A	0.021	0.005	0.024	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	182	0	41
normalized size	1	1.	1.	0.86	0.	8.27	0.	1.86
time (sec)	N/A	0.009	0.004	0.005	0.	1.007	0.	1.127

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	27	116	0	0	0	0
normalized size	1	1.	0.17	0.73	0.	0.	0.	0.
time (sec)	N/A	0.052	0.006	0.026	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	31	65	0	30
normalized size	1	1.	1.	0.86	1.48	3.1	0.	1.43
time (sec)	N/A	0.004	0.004	0.004	1.586	0.983	0.	1.151

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	31	38	47	0	16
normalized size	1	1.	1.	1.24	1.52	1.88	0.	0.64
time (sec)	N/A	0.004	0.005	0.003	1.824	0.992	0.	1.158

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	29	1521	0	0	0	0
normalized size	1	1.	0.1	5.21	0.	0.	0.	0.
time (sec)	N/A	0.232	0.004	0.234	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	29	270	0	0	0	0
normalized size	1	1.	0.11	1.04	0.	0.	0.	0.
time (sec)	N/A	0.061	0.004	0.021	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	321	0	225	14	47
normalized size	1	1.	0.96	13.96	0.	9.78	0.61	2.04
time (sec)	N/A	0.015	0.005	0.063	0.	1.254	1.056	1.172

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	27	232	0	0	0	0
normalized size	1	1.	0.23	2.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.005	0.105	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	177	0	42
normalized size	1	1.	1.	0.79	0.	7.38	0.	1.75
time (sec)	N/A	0.009	0.004	0.006	0.	1.037	0.	1.167

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	27	1784	0	0	0	0
normalized size	1	1.	0.09	5.72	0.	0.	0.	0.
time (sec)	N/A	0.228	0.006	0.069	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	27	353	0	0	0	0
normalized size	1	1.	0.1	1.26	0.	0.	0.	0.
time (sec)	N/A	0.073	0.005	0.02	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.012	0.069	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	35	0	0	0	0
normalized size	1	1.	0.83	0.73	0.	0.	0.	0.
time (sec)	N/A	0.014	0.011	0.04	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	44	37	0	0	0	0
normalized size	1	1.	0.85	0.71	0.	0.	0.	0.
time (sec)	N/A	0.015	0.012	0.064	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	24	0	0	0
normalized size	1	2.35	0.97	0.88	0.71	0.	0.	0.
time (sec)	N/A	0.03	0.029	0.023	1.776	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	0	0	0
normalized size	1	1.	0.93	1.68	0.	0.	0.	0.
time (sec)	N/A	0.056	0.205	0.055	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	23	53	0	27
normalized size	1	1.	1.	1.06	1.44	3.31	0.	1.69
time (sec)	N/A	0.005	0.003	0.002	1.316	1.003	0.	1.142

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	43	93	0	43
normalized size	1	1.	1.	1.04	1.65	3.58	0.	1.65
time (sec)	N/A	0.009	0.007	0.002	1.507	1.02	0.	1.175

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	59	134	0	59
normalized size	1	1.	1.	1.03	1.64	3.72	0.	1.64
time (sec)	N/A	0.016	0.012	0.001	1.442	0.989	0.	1.156

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	90	0	201	0	0
normalized size	1	1.	0.95	0.61	0.	1.37	0.	0.
time (sec)	N/A	0.132	0.171	0.004	0.	0.927	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	66	0	149	0	0
normalized size	1	1.	1.	0.69	0.	1.57	0.	0.
time (sec)	N/A	0.081	0.098	0.004	0.	0.94	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	35	40	0	72	136	0
normalized size	1	1.	0.74	0.85	0.	1.53	2.89	0.
time (sec)	N/A	0.047	0.048	0.003	0.	0.966	0.648	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	73	0	794	0	0
normalized size	1	1.	0.77	0.75	0.	8.19	0.	0.
time (sec)	N/A	0.104	0.072	0.006	0.	1.021	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	99	88	0	980	0	0
normalized size	1	1.	0.96	0.85	0.	9.51	0.	0.
time (sec)	N/A	0.102	0.278	0.013	0.	1.067	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	361	604	0	437	0	0
normalized size	1	1.	1.58	2.65	0.	1.92	0.	0.
time (sec)	N/A	0.373	1.367	0.019	0.	1.016	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	229	431	0	332	0	0
normalized size	1	1.	1.39	2.61	0.	2.01	0.	0.
time (sec)	N/A	0.21	0.871	0.01	0.	0.976	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	114	179	377	0	247	318	0
normalized size	1	1.81	2.84	5.98	0.	3.92	5.05	0.
time (sec)	N/A	0.099	0.569	0.006	0.	0.994	0.992	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	195	258	0	730	0	0
normalized size	1	1.	1.47	1.94	0.	5.49	0.	0.
time (sec)	N/A	0.23	1.064	0.013	0.	1.06	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	205	274	0	873	0	0
normalized size	1	1.	1.45	1.94	0.	6.19	0.	0.
time (sec)	N/A	0.216	0.981	0.013	0.	1.159	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	282	294	0	452	0	0
normalized size	1	1.	0.75	0.78	0.	1.21	0.	0.
time (sec)	N/A	0.372	0.413	0.003	0.	0.946	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	214	222	0	343	942	0
normalized size	1	1.	0.82	0.85	0.	1.31	3.61	0.
time (sec)	N/A	0.236	0.27	0.003	0.	0.999	2.077	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	F	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	151	151	146	0	228	384	0
normalized size	1	2.36	2.36	2.28	0.	3.56	6.	0.
time (sec)	N/A	0.095	0.152	0.003	0.	0.988	1.911	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	142	181	0	1283	0	0
normalized size	1	1.	0.9	1.15	0.	8.17	0.	0.
time (sec)	N/A	0.233	0.197	0.003	0.	1.041	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	223	187	252	0	1558	0	0
normalized size	1	1.38	1.15	1.56	0.	9.62	0.	0.
time (sec)	N/A	0.273	0.596	0.003	0.	1.115	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	45	63	18
normalized size	1	1.	1.	0.67	0.	2.14	3.	0.86
time (sec)	N/A	0.006	0.019	0.002	0.	0.942	0.357	1.157

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	46	63	18
normalized size	1	1.	1.	0.67	0.	2.19	3.	0.86
time (sec)	N/A	0.007	0.02	0.003	0.	1.094	0.374	1.127

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	0	53	51	20
normalized size	1	1.	1.	0.7	0.	2.3	2.22	0.87
time (sec)	N/A	0.023	0.02	0.002	0.	1.222	0.395	1.158

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	42	80	0	76
normalized size	1	1.	1.	0.87	1.11	2.11	0.	2.
time (sec)	N/A	0.114	0.054	0.004	1.691	1.249	0.	1.115

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	59	46	134	0	84
normalized size	1	1.	0.88	1.23	0.96	2.79	0.	1.75
time (sec)	N/A	0.09	0.043	0.007	1.699	1.226	0.	1.117

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	20	62	110	36
normalized size	1	1.	1.	1.26	1.05	3.26	5.79	1.89
time (sec)	N/A	0.054	0.021	0.003	1.607	1.26	79.156	1.11

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	58	23	107	44	43
normalized size	1	1.	0.95	3.05	1.21	5.63	2.32	2.26
time (sec)	N/A	0.025	0.015	0.005	1.904	1.229	24.377	1.161

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	51	55	109	0	0
normalized size	1	1.	1.	1.59	1.72	3.41	0.	0.
time (sec)	N/A	0.089	0.028	0.009	1.769	1.279	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	50	32	112	0	201
normalized size	1	1.	0.85	1.92	1.23	4.31	0.	7.73
time (sec)	N/A	0.08	0.027	0.012	1.518	1.127	0.	1.153

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	45	58	73	109	0	0
normalized size	1	1.	1.32	1.71	2.15	3.21	0.	0.
time (sec)	N/A	0.091	0.041	0.012	1.491	1.278	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	147	90	0	250	0	0
normalized size	1	1.	1.	0.61	0.	1.7	0.	0.
time (sec)	N/A	0.122	0.219	0.004	0.	1.201	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	66	0	186	0	0
normalized size	1	1.	0.74	0.69	0.	1.96	0.	0.
time (sec)	N/A	0.101	0.181	0.003	0.	1.106	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	40	0	109	0	0
normalized size	1	1.	0.83	0.85	0.	2.32	0.	0.
time (sec)	N/A	0.055	0.076	0.002	0.	1.203	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	73	0	393	0	0
normalized size	1	1.	0.77	0.75	0.	4.05	0.	0.
time (sec)	N/A	0.071	0.048	0.003	0.	1.296	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	135	88	0	450	0	0
normalized size	1	1.	1.31	0.85	0.	4.37	0.	0.
time (sec)	N/A	0.095	0.199	0.004	0.	1.205	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	75	120	0	570	0	0
normalized size	1	1.	0.44	0.7	0.	3.33	0.	0.
time (sec)	N/A	0.112	0.089	0.009	0.	1.357	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	238	517	0	1015	0	0
normalized size	1	1.	1.22	2.65	0.	5.21	0.	0.
time (sec)	N/A	0.349	0.676	0.014	0.	1.349	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	177	385	0	813	0	0
normalized size	1	1.	1.25	2.71	0.	5.73	0.	0.
time (sec)	N/A	0.228	0.579	0.008	0.	1.365	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	195	266	0	829	0	0
normalized size	1	1.	1.44	1.97	0.	6.14	0.	0.
time (sec)	N/A	0.181	0.843	0.013	0.	1.457	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	178	272	0	810	0	0
normalized size	1	1.	1.29	1.97	0.	5.87	0.	0.
time (sec)	N/A	0.114	0.721	0.012	0.	1.388	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	109	313	0	308	0	0
normalized size	1	1.	0.89	2.54	0.	2.5	0.	0.
time (sec)	N/A	0.201	0.147	0.015	0.	1.308	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	153	457	0	408	0	0
normalized size	1	1.	0.88	2.63	0.	2.34	0.	0.
time (sec)	N/A	0.223	0.179	0.011	0.	1.304	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	271	246	0	452	0	0
normalized size	1	1.	0.98	0.89	0.	1.63	0.	0.
time (sec)	N/A	0.319	0.39	0.003	0.	1.265	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	120	172	0	331	0	0
normalized size	1	1.	0.74	1.06	0.	2.03	0.	0.
time (sec)	N/A	0.218	0.436	0.003	0.	1.301	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	119	148	0	755	0	0
normalized size	1	1.	0.77	0.95	0.	4.87	0.	0.
time (sec)	N/A	0.202	0.271	0.004	0.	1.319	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	223	192	237	0	641	0	0
normalized size	1	1.42	1.22	1.51	0.	4.08	0.	0.
time (sec)	N/A	0.217	0.812	0.003	0.	1.391	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	275	182	300	0	703	0	0
normalized size	1	1.68	1.11	1.83	0.	4.29	0.	0.
time (sec)	N/A	0.178	0.261	0.003	0.	1.327	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	63	31	122	48	51
normalized size	1	1.	1.	2.03	1.	3.94	1.55	1.65
time (sec)	N/A	0.046	0.015	0.003	1.578	1.296	2.496	1.196

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	42	81	0	76
normalized size	1	1.	1.	0.87	1.11	2.13	0.	2.
time (sec)	N/A	0.325	0.042	0.002	1.688	1.224	0.	1.192

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	59	46	135	0	84
normalized size	1	1.	0.9	1.23	0.96	2.81	0.	1.75
time (sec)	N/A	0.242	0.04	0.001	1.506	1.054	0.	1.216

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	23	63	110	39
normalized size	1	1.	1.	1.24	1.1	3.	5.24	1.86
time (sec)	N/A	0.113	0.018	0.002	1.531	0.945	76.971	1.111

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	59	27	108	46	45
normalized size	1	1.	0.95	2.68	1.23	4.91	2.09	2.05
time (sec)	N/A	0.055	0.015	0.003	1.54	1.011	30.697	1.14

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	51	55	111	0	0
normalized size	1	1.	1.	1.59	1.72	3.47	0.	0.
time (sec)	N/A	0.196	0.03	0.002	1.47	0.92	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	50	32	113	0	201
normalized size	1	1.	0.85	1.92	1.23	4.35	0.	7.73
time (sec)	N/A	0.205	0.025	0.002	1.658	0.969	0.	1.193

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	57	69	108	0	0
normalized size	1	1.	1.39	1.73	2.09	3.27	0.	0.
time (sec)	N/A	0.215	0.04	0.001	1.676	0.98	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	28	28	48	48	0	101	0	0
normalized size	1	1.	1.71	1.71	0.	3.61	0.	0.
time (sec)	N/A	0.321	0.159	0.005	0.	0.923	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	58	62	0	109	226	57
normalized size	1	1.	1.76	1.88	0.	3.3	6.85	1.73
time (sec)	N/A	0.143	0.161	0.004	0.	0.958	25.253	1.194

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	86	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.118	0.01	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	158	175	0	333	279	220
normalized size	1	1.	0.9	1.	0.	1.9	1.59	1.26
time (sec)	N/A	0.133	0.329	0.01	0.	0.998	9.461	1.235

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	126	0	239	116	139
normalized size	1	1.	0.94	0.93	0.	1.76	0.85	1.02
time (sec)	N/A	0.103	0.202	0.003	0.	1.014	3.708	1.178

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	81	75	0	153	54	88
normalized size	1	1.	1.19	1.1	0.	2.25	0.79	1.29
time (sec)	N/A	0.034	0.045	0.003	0.	0.985	2.156	1.156

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	109	1325	0	394	0	0
normalized size	1	1.	0.93	11.32	0.	3.37	0.	0.
time (sec)	N/A	0.094	0.142	0.036	0.	1.063	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	141	4136	0	579	0	0
normalized size	1	1.	0.93	27.39	0.	3.83	0.	0.
time (sec)	N/A	0.113	0.29	0.023	0.	1.125	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	180	9721	0	1081	0	0
normalized size	1	1.	0.93	50.37	0.	5.6	0.	0.
time (sec)	N/A	0.127	0.574	0.036	0.	2.337	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	213	0	0	946	0	0
normalized size	1	1.	0.95	0.	0.	4.2	0.	0.
time (sec)	N/A	0.185	0.344	0.026	0.	1.394	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	175	0	0	752	0	0
normalized size	1	1.	0.96	0.	0.	4.11	0.	0.
time (sec)	N/A	0.15	0.234	0.011	0.	1.353	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	139	0	0	664	0	0
normalized size	1	1.	0.95	0.	0.	4.52	0.	0.
time (sec)	N/A	0.122	0.33	0.007	0.	1.368	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	143	0	0	659	0	0
normalized size	1	1.	0.97	0.	0.	4.48	0.	0.
time (sec)	N/A	0.109	0.25	0.011	0.	1.332	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	167	0	0	1037	0	0
normalized size	1	1.	1.06	0.	0.	6.56	0.	0.
time (sec)	N/A	0.156	0.408	0.01	0.	1.386	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	186	0	0	1683	0	0
normalized size	1	1.	0.93	0.	0.	8.46	0.	0.
time (sec)	N/A	0.177	0.613	0.009	0.	1.56	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	0	0	69	42	0
normalized size	1	1.	0.98	0.	0.	1.68	1.02	0.
time (sec)	N/A	0.016	0.014	0.036	0.	0.978	0.345	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	120	0	0
normalized size	1	1.	0.97	0.	0.	1.74	0.	0.
time (sec)	N/A	0.057	0.057	0.017	0.	0.995	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	80	413	0
normalized size	1	1.	0.78	1.33	0.	1.78	9.18	0.
time (sec)	N/A	0.01	0.066	0.027	0.	1.071	1.137	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	55	0	77	197	0
normalized size	1	1.	1.07	1.34	0.	1.88	4.8	0.
time (sec)	N/A	0.007	0.057	0.017	0.	1.081	1.083	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	64	0	84	197	0
normalized size	1	1.	1.07	1.56	0.	2.05	4.8	0.
time (sec)	N/A	0.007	0.057	0.016	0.	1.184	1.116	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	144	0	0
normalized size	1	1.	0.83	0.	0.	2.18	0.	0.
time (sec)	N/A	0.033	0.218	0.016	0.	1.829	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	0.302	0.014	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	276	685	0	717	0	504
normalized size	1	1.	0.91	2.26	0.	2.37	0.	1.66
time (sec)	N/A	0.384	0.57	0.01	0.	1.876	0.	1.174

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	213	409	0	460	0	302
normalized size	1	1.	0.9	1.73	0.	1.94	0.	1.27
time (sec)	N/A	0.239	0.331	0.005	0.	1.514	0.	1.173

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	120	173	0	258	0	150
normalized size	1	1.	1.02	1.47	0.	2.19	0.	1.27
time (sec)	N/A	0.063	0.197	0.005	0.	1.277	0.	1.185

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	187	4918	0	756	0	0
normalized size	1	1.	0.87	22.87	0.	3.52	0.	0.
time (sec)	N/A	0.193	0.221	0.056	0.	20.717	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	237	58067	0	1673	0	0
normalized size	1	1.	0.89	218.3	0.	6.29	0.	0.
time (sec)	N/A	0.232	0.341	0.036	0.	12.9	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	300	295147	0	3945	0	0
normalized size	1	1.	0.91	894.38	0.	11.95	0.	0.
time (sec)	N/A	0.289	0.747	0.094	0.	96.948	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	357	0	0	2033	0	0
normalized size	1	1.	0.96	0.	0.	5.49	0.	0.
time (sec)	N/A	0.6	1.106	0.029	0.	2.616	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	291	0	0	1439	0	0
normalized size	1	1.	0.96	0.	0.	4.76	0.	0.
time (sec)	N/A	0.415	0.578	0.012	0.	2.592	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	223	0	0	1472	0	0
normalized size	1	1.	0.96	0.	0.	6.32	0.	0.
time (sec)	N/A	0.305	0.402	0.007	0.	2.617	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	238	0	0	1508	0	0
normalized size	1	1.	0.98	0.	0.	6.18	0.	0.
time (sec)	N/A	0.292	0.591	0.015	0.	2.689	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	257	0	0	2992	0	0
normalized size	1	1.	0.96	0.	0.	11.12	0.	0.
time (sec)	N/A	0.368	0.482	0.01	0.	3.459	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	315	0	0	5181	0	0
normalized size	1	1.	0.94	0.	0.	15.47	0.	0.
time (sec)	N/A	0.5	0.846	0.01	0.	6.476	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	138	216	0	363	0	0
normalized size	1	1.	0.84	1.32	0.	2.21	0.	0.
time (sec)	N/A	0.11	0.342	0.044	0.	1.378	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	92	167	0	174	0	0
normalized size	1	1.	0.85	1.55	0.	1.61	0.	0.
time (sec)	N/A	0.063	0.124	0.01	0.	1.353	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	74	2149	0
normalized size	1	1.	0.83	2.31	0.	1.42	41.33	0.
time (sec)	N/A	0.021	0.035	0.007	0.	1.383	3.516	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.025	0.019	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.03	0.013	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	150	0	0	363	0	0
normalized size	1	1.	0.85	0.	0.	2.06	0.	0.
time (sec)	N/A	0.109	0.33	0.019	0.	1.405	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	100	0	0	174	0	0
normalized size	1	1.	0.86	0.	0.	1.5	0.	0.
time (sec)	N/A	0.064	0.118	0.015	0.	1.366	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	76	0	0
normalized size	1	1.	0.89	0.	0.	1.36	0.	0.
time (sec)	N/A	0.023	0.06	0.013	0.	1.338	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.026	0.021	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.025	0.023	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	157	0	0	459	0	0
normalized size	1	1.	0.84	0.	0.	2.45	0.	0.
time (sec)	N/A	0.117	0.328	0.015	0.	1.348	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	111	0	0	244	0	0
normalized size	1	1.	0.85	0.	0.	1.86	0.	0.
time (sec)	N/A	0.094	0.222	0.014	0.	1.36	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	109	0	0
normalized size	1	1.	0.87	0.	0.	1.45	0.	0.
time (sec)	N/A	0.076	0.071	0.014	0.	1.559	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	34	313	0
normalized size	1	1.	1.	0.	0.	2.	18.41	0.
time (sec)	N/A	0.054	0.006	0.018	0.	1.451	3.742	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.025	0.014	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.033	0.016	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	173	0	0	460	0	0
normalized size	1	1.	0.86	0.	0.	2.29	0.	0.
time (sec)	N/A	0.112	0.426	0.023	0.	1.412	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	0	0	246	0	0
normalized size	1	1.	0.87	0.	0.	1.74	0.	0.
time (sec)	N/A	0.095	0.223	0.023	0.	1.147	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	111	0	0
normalized size	1	1.	0.9	0.	0.	1.37	0.	0.
time (sec)	N/A	0.075	0.07	0.022	0.	1.061	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	35	36	0
normalized size	1	1.	1.	0.	0.	1.75	1.8	0.
time (sec)	N/A	0.057	0.007	0.026	0.	0.99	1.818	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.025	0.023	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.03	0.023	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	280	0	0	1384	0	0
normalized size	1	1.	0.77	0.	0.	3.79	0.	0.
time (sec)	N/A	0.474	2.987	0.089	0.	1.23	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	186	0	0	482	0	0
normalized size	1	1.	0.78	0.	0.	2.02	0.	0.
time (sec)	N/A	0.25	0.576	0.007	0.	1.151	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	163	0	0
normalized size	1	1.	0.83	0.	0.	1.52	0.	0.
time (sec)	N/A	0.089	0.367	0.006	0.	1.095	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	0.145	0.105	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.268	0.193	0.068	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	163	0	0
normalized size	1	1.	0.83	0.	0.	1.52	0.	0.
time (sec)	N/A	0.133	0.038	0.006	0.	1.101	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.498	0.037	0.071	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	228	0	0	778	0	0
normalized size	1	1.	0.77	0.	0.	2.62	0.	0.
time (sec)	N/A	0.419	1.245	0.069	0.	1.169	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	135	0	0	261	0	0
normalized size	1	1.	0.79	0.	0.	1.53	0.	0.
time (sec)	N/A	0.325	0.355	0.069	0.	1.102	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	85	0	0
normalized size	1	1.	0.88	0.	0.	2.07	0.	0.
time (sec)	N/A	0.254	0.068	0.069	0.	1.016	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	0.15	0.07	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	85	0	0
normalized size	1	1.	0.88	0.	0.	2.07	0.	0.
time (sec)	N/A	0.444	0.041	0.073	0.	0.994	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	175	0	0	486	0	0
normalized size	1	1.	0.54	0.	0.	1.49	0.	0.
time (sec)	N/A	0.617	0.247	0.075	0.	1.178	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	250	0	0
normalized size	1	1.	0.82	0.	0.	2.69	0.	0.
time (sec)	N/A	0.529	0.098	0.071	0.	1.043	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	152	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.586	0.198	0.07	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	250	0	0
normalized size	1	1.	0.82	0.	0.	2.69	0.	0.
time (sec)	N/A	0.735	0.039	0.076	0.	1.06	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	772	353	0	0	0	0
normalized size	1	1.	4.04	1.85	0.	0.	0.	0.
time (sec)	N/A	0.513	1.582	0.064	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	191	394	0	325	70	4845
normalized size	1	1.	2.36	4.86	0.	4.01	0.86	59.81
time (sec)	N/A	0.058	0.121	0.051	0.	1.417	0.617	2.267

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	233	394	0	327	63	4694
normalized size	1	1.	3.19	5.4	0.	4.48	0.86	64.3
time (sec)	N/A	0.045	0.138	0.048	0.	1.454	0.636	2.228

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	347	70	0
normalized size	1	1.	2.29	1.84	0.	9.13	1.84	0.
time (sec)	N/A	0.063	0.061	0.008	0.	1.476	0.795	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	348	63	0
normalized size	1	1.	2.29	1.84	0.	9.16	1.66	0.
time (sec)	N/A	0.062	0.06	0.009	0.	1.495	0.831	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	A	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	0	78	0	325	139	0
normalized size	1	1.	0.	2.05	0.	8.55	3.66	0.
time (sec)	N/A	0.096	0.275	0.049	0.	1.547	126.247	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	A	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	0	72	0	324	129	0
normalized size	1	1.	0.	1.89	0.	8.53	3.39	0.
time (sec)	N/A	0.095	0.277	0.045	0.	1.565	121.347	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	0	340	78	51
normalized size	1	1.	1.	1.	0.	8.1	1.86	1.21
time (sec)	N/A	0.066	0.021	0.003	0.	1.431	0.527	1.303

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	0	342	75	55
normalized size	1	1.	1.05	0.95	0.	7.77	1.7	1.25
time (sec)	N/A	0.067	0.022	0.003	0.	1.252	0.561	1.365

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	74	0	462	90	0
normalized size	1	1.	2.12	1.85	0.	11.55	2.25	0.
time (sec)	N/A	0.13	0.051	0.112	0.	1.268	1.085	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	77	0	467	80	0
normalized size	1	1.	2.12	1.92	0.	11.68	2.	0.
time (sec)	N/A	0.128	0.052	0.105	0.	1.237	1.091	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	61	42	78	0	321	0	0
normalized size	1	1.45	1.	1.86	0.	7.64	0.	0.
time (sec)	N/A	0.219	0.317	0.053	0.	1.38	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	61	42	74	0	320	0	0
normalized size	1	1.45	1.	1.76	0.	7.62	0.	0.
time (sec)	N/A	0.218	0.041	0.053	0.	1.215	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	347	73	0
normalized size	1	1.	2.15	1.85	0.	8.68	1.82	0.
time (sec)	N/A	0.089	0.05	0.009	0.	1.335	1.156	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	348	66	0
normalized size	1	1.	2.15	1.85	0.	8.7	1.65	0.
time (sec)	N/A	0.089	0.048	0.009	0.	1.311	1.164	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	0	342	78	51
normalized size	1	1.	1.	1.	0.	8.14	1.86	1.21
time (sec)	N/A	0.059	0.02	0.002	0.	1.246	0.694	1.801

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	0	343	75	55
normalized size	1	1.	1.05	0.95	0.	7.8	1.7	1.25
time (sec)	N/A	0.062	0.022	0.003	0.	1.28	0.737	1.753

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	321	0	0
normalized size	1	1.	1.	1.86	0.	7.64	0.	0.
time (sec)	N/A	0.223	0.317	0.035	0.	1.276	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	74	0	320	0	0
normalized size	1	1.	1.	1.76	0.	7.62	0.	0.
time (sec)	N/A	0.216	0.045	0.033	0.	1.318	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	0	84	0	363	0	0
normalized size	1	1.	0.	2.	0.	8.64	0.	0.
time (sec)	N/A	0.249	0.48	0.066	0.	1.282	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	0	78	0	362	0	0
normalized size	1	1.	0.	1.86	0.	8.62	0.	0.
time (sec)	N/A	0.243	0.517	0.065	0.	1.327	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	126	4947	169	498	0	209
normalized size	1	1.	0.94	36.92	1.26	3.72	0.	1.56
time (sec)	N/A	0.373	0.218	0.077	1.418	1.485	0.	1.133

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	65	3410	84	340	88	97
normalized size	1	1.	0.94	49.42	1.22	4.93	1.28	1.41
time (sec)	N/A	0.215	0.089	0.019	1.298	1.443	4.081	1.122

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	1931	28	227	29	30
normalized size	1	1.	1.	83.96	1.22	9.87	1.26	1.3
time (sec)	N/A	0.086	0.022	0.015	1.578	1.241	2.389	1.154

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	107	2175	0	721	88	127
normalized size	1	1.	1.22	24.72	0.	8.19	1.	1.44
time (sec)	N/A	0.247	0.129	0.027	0.	1.904	4.75	1.166

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	291	2459	0	1131	0	275
normalized size	1	1.	1.93	16.28	0.	7.49	0.	1.82
time (sec)	N/A	0.352	1.054	0.047	0.	5.323	0.	1.188

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	157	3485	0	2371	0	244
normalized size	1	1.	1.07	23.71	0.	16.13	0.	1.66
time (sec)	N/A	0.239	0.258	0.029	0.	2.308	0.	1.185

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	83	1995	0	1017	0	144
normalized size	1	1.	0.81	19.37	0.	9.87	0.	1.4
time (sec)	N/A	0.067	0.101	0.019	0.	1.355	0.	1.108

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	139	2289	0	1149	0	285
normalized size	1	1.	0.87	14.31	0.	7.18	0.	1.78
time (sec)	N/A	0.24	0.276	0.027	0.	1.848	0.	1.163

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	126	1473	169	409	0	211
normalized size	1	1.	0.9	10.52	1.21	2.92	0.	1.51
time (sec)	N/A	0.299	0.181	0.099	1.316	1.281	0.	1.131

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	943	84	246	95	97
normalized size	1	1.	0.86	12.92	1.15	3.37	1.3	1.33
time (sec)	N/A	0.202	0.069	0.015	1.178	1.241	4.276	1.114

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	455	30	135	32	31
normalized size	1	1.	1.	17.5	1.15	5.19	1.23	1.19
time (sec)	N/A	0.111	0.028	0.013	1.193	1.269	2.51	1.126

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	107	636	0	536	97	127
normalized size	1	1.	1.15	6.84	0.	5.76	1.04	1.37
time (sec)	N/A	0.223	0.127	0.033	0.	1.555	5.011	1.133

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	307	863	0	946	0	275
normalized size	1	1.	1.99	5.6	0.	6.14	0.	1.79
time (sec)	N/A	0.298	0.684	0.036	0.	2.174	0.	1.112

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	296	1544	0	0	0	0
normalized size	1	1.	0.95	4.96	0.	0.	0.	0.
time (sec)	N/A	0.504	0.561	0.045	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	0	619	0	0	0	0
normalized size	1	1.	0.	2.04	0.	0.	0.	0.
time (sec)	N/A	0.299	0.166	0.046	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	300	300	0	619	0	0	0	0
normalized size	1	1.	0.	2.06	0.	0.	0.	0.
time (sec)	N/A	0.259	0.144	0.012	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	496	3560	0	0	0	0
normalized size	1	1.	1.55	11.16	0.	0.	0.	0.
time (sec)	N/A	0.408	0.701	0.034	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	604	1789	0	0	0	0
normalized size	1	1.	1.86	5.52	0.	0.	0.	0.
time (sec)	N/A	0.413	6.306	0.033	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	320	0	0	0	0	0
normalized size	1	1.	2.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.633	0.009	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	156	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.293	0.011	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	82	51	32	55
normalized size	1	1.	1.	0.	3.04	1.89	1.19	2.04
time (sec)	N/A	0.11	0.061	0.013	1.326	1.867	18.495	1.124

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	26	7	8
normalized size	1	1.	1.	0.88	1.	3.25	0.88	1.
time (sec)	N/A	0.006	0.002	0.004	1.843	1.239	0.203	1.091

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	28	85	26	30
normalized size	1	1.	1.	0.77	2.15	6.54	2.	2.31
time (sec)	N/A	0.008	0.004	0.004	2.493	1.336	0.397	1.098

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	59	22	27
normalized size	1	1.	1.	0.74	0.96	2.19	0.81	1.
time (sec)	N/A	0.012	0.01	0.007	1.316	1.177	0.213	1.13

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	76	0	32
normalized size	1	1.	1.	2.88	1.	2.38	0.	1.
time (sec)	N/A	0.014	0.012	0.023	1.261	1.248	0.	1.12

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	59	22	26
normalized size	1	1.	1.	0.8	1.04	2.36	0.88	1.04
time (sec)	N/A	0.011	0.009	0.004	1.786	1.166	0.21	1.099

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	19	43	15	20
normalized size	1	1.	0.9	0.75	0.95	2.15	0.75	1.
time (sec)	N/A	0.009	0.006	0.003	1.274	1.289	0.137	1.106

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	24	46	61	167	68	61
normalized size	1	1.	0.39	0.74	0.98	2.69	1.1	0.98
time (sec)	N/A	0.037	0.007	0.003	1.774	1.37	0.724	1.091

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	173	66	176	0	66
normalized size	1	1.	1.	2.37	0.9	2.41	0.	0.9
time (sec)	N/A	0.028	0.024	0.087	1.3	1.185	0.	1.122

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	83	111	286	121	111
normalized size	1	1.	1.	0.64	0.85	2.2	0.93	0.85
time (sec)	N/A	0.046	0.036	0.003	1.403	1.276	1.982	1.213

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	22	175	367	2130	0	188
normalized size	1	1.	0.11	0.88	1.84	10.65	0.	0.94
time (sec)	N/A	0.396	0.007	0.031	2.109	8.597	0.	1.467

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	26	7	8
normalized size	1	1.	1.	0.88	1.	3.25	0.88	1.
time (sec)	N/A	0.005	0.002	0.003	1.999	1.223	0.292	1.117

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	50	17	20
normalized size	1	1.	1.	0.84	1.05	2.63	0.89	1.05
time (sec)	N/A	0.014	0.009	0.001	1.295	1.285	0.143	1.088

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	24	71	112	404	110	112
normalized size	1	1.	0.22	0.66	1.04	3.74	1.02	1.04
time (sec)	N/A	0.083	0.007	0.005	1.701	1.271	1.932	1.108

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	61	81	221	0	81
normalized size	1	1.	1.09	0.8	1.07	2.91	0.	1.07
time (sec)	N/A	0.034	0.018	0.003	1.891	1.321	0.	1.124

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	76	101	273	0	101
normalized size	1	1.	1.	0.64	0.85	2.29	0.	0.85
time (sec)	N/A	0.049	0.033	0.004	1.171	1.232	0.	1.117

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	29	242	396	1858	561	189
normalized size	1	1.	0.14	1.2	1.97	9.24	2.79	0.94
time (sec)	N/A	0.224	0.006	0.014	2.268	8.518	35.108	1.751

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	20	95	0	212
normalized size	1	1.	0.97	1.	0.56	2.64	0.	5.89
time (sec)	N/A	0.037	0.021	0.003	1.757	1.562	0.	1.242

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	7	72	0	170
normalized size	1	1.	1.	0.93	0.24	2.48	0.	5.86
time (sec)	N/A	0.034	0.015	0.003	1.446	1.408	0.	1.171

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	7	72	0	76
normalized size	1	1.	1.	0.93	0.24	2.48	0.	2.62
time (sec)	N/A	0.024	0.012	0.002	1.183	1.476	0.	1.136

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	7	66	0	69
normalized size	1	1.	1.	1.	0.28	2.64	0.	2.76
time (sec)	N/A	0.014	0.01	0.003	1.39	1.463	0.	1.118

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	7	62	0	57
normalized size	1	1.	1.	1.	0.29	2.58	0.	2.38
time (sec)	N/A	0.034	0.012	0.003	2.053	1.488	0.	1.123

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	7	70	0	81
normalized size	1	1.	1.	0.93	0.24	2.41	0.	2.79
time (sec)	N/A	0.033	0.011	0.002	1.613	1.409	0.	1.141

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.065	0.091	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	112	0	0	0	121	0
normalized size	1	1.	0.81	0.	0.	0.	0.88	0.
time (sec)	N/A	0.117	0.075	0.029	0.	0.	5.365	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	75	0
normalized size	1	1.	0.92	0.	0.	0.	0.95	0.
time (sec)	N/A	0.038	0.031	0.018	0.	0.	3.429	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0
normalized size	1	1.	1.25	0.	0.	0.	0.85	0.
time (sec)	N/A	0.01	0.017	0.012	0.	0.	1.362	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.044	0.029	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.024	0.037	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.068	0.052	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	155	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.144	0.088	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	11	85	0	0
normalized size	1	1.	1.	1.06	0.33	2.58	0.	0.
time (sec)	N/A	0.031	0.017	0.001	1.156	1.512	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	7	82	0	53
normalized size	1	1.	1.	1.	0.23	2.65	0.	1.71
time (sec)	N/A	0.035	0.01	0.003	1.183	1.462	0.	1.11

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	42	9	77	0	0
normalized size	1	1.	1.	1.5	0.32	2.75	0.	0.
time (sec)	N/A	0.026	0.008	0.01	1.159	1.442	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	42	5	115	0	42
normalized size	1	1.	1.	1.5	0.18	4.11	0.	1.5
time (sec)	N/A	0.019	0.008	0.007	1.062	1.616	0.	1.102

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	9	82	0	0
normalized size	1	1.	1.	1.08	0.35	3.15	0.	0.
time (sec)	N/A	0.032	0.008	0.003	1.075	1.482	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	7	93	0	27
normalized size	1	1.	1.	1.	0.23	3.	0.	0.87
time (sec)	N/A	0.034	0.007	0.001	1.167	1.478	0.	1.078

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	540	1145	0	0	0	0
normalized size	1	1.	1.33	2.82	0.	0.	0.	0.
time (sec)	N/A	0.464	3.033	0.108	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	31	12	15
normalized size	1	1.	1.	1.2	1.	2.07	0.8	1.
time (sec)	N/A	0.009	0.026	0.006	1.105	1.365	0.185	1.106

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	72	20	18	51	31	18
normalized size	1	1.	4.24	1.18	1.06	3.	1.82	1.06
time (sec)	N/A	0.009	0.046	0.004	1.099	1.429	0.252	1.123

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	32	31	15
normalized size	1	1.	1.	1.2	1.	2.13	2.07	1.
time (sec)	N/A	0.008	0.028	0.004	1.054	1.448	0.247	1.102

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	31	31	15
normalized size	1	1.	1.	1.2	1.	2.07	2.07	1.
time (sec)	N/A	0.008	0.034	0.003	1.089	1.448	0.243	1.146

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	15	26	10	15
normalized size	1	1.	1.	1.62	1.15	2.	0.77	1.15
time (sec)	N/A	0.01	0.009	0.002	1.089	1.408	0.139	1.14

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	34	45	112	39	45
normalized size	1	1.	1.	0.77	1.02	2.55	0.89	1.02
time (sec)	N/A	0.024	0.021	0.003	1.111	1.598	0.176	1.127

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	20	16	20	49	17	22
normalized size	1	1.	0.95	0.76	0.95	2.33	0.81	1.05
time (sec)	N/A	0.013	0.009	0.003	1.057	1.662	0.132	1.134

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	72	27	35
normalized size	1	1.	1.	0.82	1.06	2.18	0.82	1.06
time (sec)	N/A	0.022	0.014	0.004	1.027	1.589	0.145	1.109

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	41	73	42	51
normalized size	1	1.	1.	1.06	1.24	2.21	1.27	1.55
time (sec)	N/A	0.021	0.015	0.001	1.1	1.681	0.417	1.143

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	17	17	13	14	24	32	0
normalized size	1	1.7	1.7	1.3	1.4	2.4	3.2	0.
time (sec)	N/A	0.052	0.027	0.016	1.108	1.833	8.937	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	116	24	0	0
normalized size	1	1.	1.	0.76	6.82	1.41	0.	0.
time (sec)	N/A	0.038	0.016	0.021	1.119	1.598	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	131	0	0	0
normalized size	1	1.	0.92	1.03	3.54	0.	0.	0.
time (sec)	N/A	0.092	0.363	0.023	1.56	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	128	0	0	0
normalized size	1	1.	0.91	1.03	3.66	0.	0.	0.
time (sec)	N/A	0.097	0.332	0.022	1.483	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	35	124	0	0	0
normalized size	1	1.	0.91	1.03	3.65	0.	0.	0.
time (sec)	N/A	0.117	0.321	0.02	1.588	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F(-1)	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	128	0	0	0
normalized size	1	0.	0.92	1.03	3.46	0.	0.	0.
time (sec)	N/A	3.374	0.95	0.021	1.486	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F(-1)	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	128	0	0	0
normalized size	1	0.	0.92	1.03	3.46	0.	0.	0.
time (sec)	N/A	2.998	1.398	0.023	1.614	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	88	78	204	217	139	173
normalized size	1	1.	0.48	0.42	1.1	1.17	0.75	0.94
time (sec)	N/A	0.255	0.313	0.003	1.108	1.984	5.395	1.242

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	77	66	151	188	110	139
normalized size	1	1.	0.56	0.48	1.09	1.36	0.8	1.01
time (sec)	N/A	0.166	0.189	0.003	1.149	1.922	4.494	1.288

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	54	97	151	94	115
normalized size	1	1.	0.71	0.61	1.09	1.7	1.06	1.29
time (sec)	N/A	0.09	0.103	0.002	1.109	1.919	3.6	1.213

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	47	109	68	53
normalized size	1	1.	0.98	0.85	1.15	2.66	1.66	1.29
time (sec)	N/A	0.03	0.027	0.002	1.105	1.935	0.191	1.238

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	79	51	0	301	65	105
normalized size	1	1.	1.39	0.89	0.	5.28	1.14	1.84
time (sec)	N/A	0.066	0.167	0.006	0.	1.983	18.625	1.188

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	161	60	0	347	139	153
normalized size	1	1.	2.98	1.11	0.	6.43	2.57	2.83
time (sec)	N/A	0.066	0.198	0.01	0.	1.86	41.032	1.267

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	221	81	0	427	292	170
normalized size	1	1.	2.76	1.01	0.	5.34	3.65	2.12
time (sec)	N/A	0.074	0.357	0.012	0.	1.722	119.286	1.167

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	232	383	362	697	0	1548
normalized size	1	1.	0.71	1.17	1.11	2.14	0.	4.75
time (sec)	N/A	0.243	0.42	0.004	1.176	2.376	0.	1.348

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	147	183	225	441	0	923
normalized size	1	1.	0.66	0.82	1.	1.97	0.	4.12
time (sec)	N/A	0.156	0.198	0.003	1.096	2.349	0.	1.216

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	84	94	126	240	0	460
normalized size	1	1.	0.63	0.71	0.95	1.8	0.	3.46
time (sec)	N/A	0.096	0.087	0.002	1.101	2.393	0.	1.16

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	41	58	122	0	159
normalized size	1	1.	0.77	0.73	1.04	2.18	0.	2.84
time (sec)	N/A	0.033	0.024	0.004	1.11	2.389	0.	1.12

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	116	221	0	489	0	0
normalized size	1	1.	1.	1.91	0.	4.22	0.	0.
time (sec)	N/A	0.156	0.136	0.035	0.	2.519	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	181	151	0	1871	0	0
normalized size	1	1.	1.32	1.1	0.	13.66	0.	0.
time (sec)	N/A	0.169	0.19	0.017	0.	2.57	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	258	784	0	5759	0	0
normalized size	1	1.	1.15	3.5	0.	25.71	0.	0.
time (sec)	N/A	0.429	0.437	0.029	0.	3.441	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	213	394	328	501	0	533
normalized size	1	1.	0.93	1.71	1.43	2.18	0.	2.32
time (sec)	N/A	0.259	0.204	0.006	1.394	1.861	0.	1.373

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	138	235	200	306	0	320
normalized size	1	1.	0.91	1.56	1.32	2.03	0.	2.12
time (sec)	N/A	0.157	0.135	0.005	1.222	1.748	0.	1.24

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	116	109	166	109	177
normalized size	1	1.	0.91	1.29	1.21	1.84	1.21	1.97
time (sec)	N/A	0.08	0.065	0.004	1.111	1.644	3.07	1.195

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	87	47	80	49	68
normalized size	1	1.	0.95	2.12	1.15	1.95	1.2	1.66
time (sec)	N/A	0.024	0.019	0.007	0.976	1.739	0.496	1.179

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	61	77	0	301	85	155
normalized size	1	1.	0.74	0.94	0.	3.67	1.04	1.89
time (sec)	N/A	0.079	0.081	0.007	0.	1.892	8.244	1.206

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	144	216	0	636	0	342
normalized size	1	1.	1.11	1.66	0.	4.89	0.	2.63
time (sec)	N/A	0.178	0.195	0.017	0.	2.315	0.	1.235

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	228	459	0	1112	0	649
normalized size	1	1.	1.12	2.25	0.	5.45	0.	3.18
time (sec)	N/A	0.278	0.433	0.014	0.	5.623	0.	1.231

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	273	416	339	838	0	0
normalized size	1	1.	1.14	1.73	1.41	3.49	0.	0.
time (sec)	N/A	0.28	0.281	0.008	1.063	1.619	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	185	253	213	564	0	0
normalized size	1	1.	1.11	1.52	1.28	3.4	0.	0.
time (sec)	N/A	0.172	0.173	0.009	1.039	1.741	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	112	125	122	335	131	0
normalized size	1	1.	1.18	1.32	1.28	3.53	1.38	0.
time (sec)	N/A	0.09	0.088	0.007	1.114	1.731	21.915	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	142	58	154	124	0
normalized size	1	1.	0.85	3.02	1.23	3.28	2.64	0.
time (sec)	N/A	0.033	0.034	0.016	1.016	1.711	0.919	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	164	161	0	936	153	0
normalized size	1	1.	1.27	1.25	0.	7.26	1.19	0.
time (sec)	N/A	0.119	0.283	0.008	0.	1.898	26.97	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	230	312	0	1724	0	0
normalized size	1	1.	1.14	1.54	0.	8.53	0.	0.
time (sec)	N/A	0.245	0.799	0.016	0.	3.915	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	401	610	0	2507	0	0
normalized size	1	1.	1.31	1.99	0.	8.19	0.	0.
time (sec)	N/A	0.403	0.91	0.018	0.	10.669	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	232	383	362	568	0	1449
normalized size	1	1.	0.72	1.18	1.12	1.75	0.	4.47
time (sec)	N/A	0.23	0.355	0.003	1.023	2.28	0.	1.339

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	147	183	225	342	0	849
normalized size	1	1.	0.66	0.82	1.01	1.54	0.	3.82
time (sec)	N/A	0.158	0.174	0.003	1.077	2.244	0.	1.292

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	84	94	126	178	0	412
normalized size	1	1.	0.64	0.72	0.96	1.36	0.	3.15
time (sec)	N/A	0.094	0.101	0.002	1.104	2.346	0.	1.222

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	41	57	85	0	135
normalized size	1	1.	0.78	0.76	1.06	1.57	0.	2.5
time (sec)	N/A	0.031	0.023	0.005	0.997	2.294	0.	1.222

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	92	0	1438	0	0
normalized size	1	1.	1.	0.95	0.	14.82	0.	0.
time (sec)	N/A	0.085	0.11	0.012	0.	2.444	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	216	265	0	4913	0	0
normalized size	1	1.	1.33	1.63	0.	30.14	0.	0.
time (sec)	N/A	0.203	0.258	0.022	0.	2.972	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	281	840	0	9651	0	0
normalized size	1	1.	1.08	3.22	0.	36.98	0.	0.
time (sec)	N/A	0.481	0.696	0.069	0.	6.742	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	555	0	983	3067	0	0
normalized size	1	1.	1.59	0.	2.81	8.76	0.	0.
time (sec)	N/A	0.279	0.871	0.004	1.112	2.927	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	284	0	543	1492	0	0
normalized size	1	1.	1.17	0.	2.24	6.17	0.	0.
time (sec)	N/A	0.182	0.389	0.004	1.145	2.454	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	128	0	252	599	0	5399
normalized size	1	1.	0.88	0.	1.74	4.13	0.	37.23
time (sec)	N/A	0.108	0.169	0.004	1.148	2.097	0.	2.576

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	53	0	81	174	0	817
normalized size	1	1.	0.85	0.	1.31	2.81	0.	13.18
time (sec)	N/A	0.04	0.036	0.003	1.138	2.02	0.	1.45

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	136	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.1	0.005	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	77	70	0	393	124	0
normalized size	1	1.	0.83	0.75	0.	4.23	1.33	0.
time (sec)	N/A	0.075	0.078	0.006	0.	1.85	73.941	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	54	0	309	102	0
normalized size	1	1.	0.87	0.77	0.	4.41	1.46	0.
time (sec)	N/A	0.057	0.043	0.005	0.	1.842	49.312	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	40	0	244	0	0
normalized size	1	1.	0.96	0.82	0.	4.98	0.	0.
time (sec)	N/A	0.042	0.021	0.003	0.	1.76	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	185	0	0
normalized size	1	1.	1.	0.83	0.	6.17	0.	0.
time (sec)	N/A	0.032	0.019	0.006	0.	1.835	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	43	0	378	48	0
normalized size	1	1.	0.79	0.83	0.	7.27	0.92	0.
time (sec)	N/A	0.046	0.029	0.005	0.	1.788	6.01	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	43	59	0	603	70	0
normalized size	1	1.	0.57	0.79	0.	8.04	0.93	0.
time (sec)	N/A	0.064	0.034	0.007	0.	1.98	11.85	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	81	86	0	392	117	0
normalized size	1	1.	0.8	0.85	0.	3.88	1.16	0.
time (sec)	N/A	0.073	0.08	0.009	0.	1.987	69.661	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	65	0	306	97	0
normalized size	1	1.	0.87	0.86	0.	4.03	1.28	0.
time (sec)	N/A	0.057	0.048	0.004	0.	1.57	44.408	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	46	0	243	0	0
normalized size	1	1.	0.96	0.87	0.	4.58	0.	0.
time (sec)	N/A	0.041	0.022	0.003	0.	1.601	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	184	0	0
normalized size	1	1.	1.	0.84	0.	5.75	0.	0.
time (sec)	N/A	0.031	0.02	0.004	0.	1.59	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	44	49	0	378	44	0
normalized size	1	1.	0.79	0.88	0.	6.75	0.79	0.
time (sec)	N/A	0.044	0.031	0.005	0.	1.559	7.755	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	46	70	0	602	63	0
normalized size	1	1.	0.57	0.86	0.	7.43	0.78	0.
time (sec)	N/A	0.061	0.038	0.006	0.	1.551	10.965	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	0	142	24	28
normalized size	1	1.	1.	0.78	0.	6.17	1.04	1.22
time (sec)	N/A	0.007	0.005	0.003	0.	1.529	1.02	1.159

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	185	0	0
normalized size	1	1.	1.	0.83	0.	6.17	0.	0.
time (sec)	N/A	0.031	0.032	0.006	0.	1.501	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	300	0	0
normalized size	1	1.	1.	0.86	0.	8.11	0.	0.
time (sec)	N/A	0.167	0.066	0.006	0.	1.589	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	386	0	0
normalized size	1	1.	1.	0.89	0.	8.77	0.	0.
time (sec)	N/A	0.373	0.146	0.015	0.	1.568	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	0	483	0	0
normalized size	1	1.	1.	0.9	0.	9.47	0.	0.
time (sec)	N/A	0.656	0.265	0.02	0.	1.548	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	34	83	162	140	0	104
normalized size	1	1.	0.45	1.09	2.13	1.84	0.	1.37
time (sec)	N/A	0.026	0.01	0.011	2.136	1.487	0.	1.192

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	35	69	90	124	60	90
normalized size	1	1.	0.58	1.15	1.5	2.07	1.	1.5
time (sec)	N/A	0.02	0.007	0.007	2.028	1.542	73.426	1.115

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	34	55	41	107	39	77
normalized size	1	1.	0.77	1.25	0.93	2.43	0.89	1.75
time (sec)	N/A	0.014	0.007	0.006	2.051	1.449	27.129	1.164

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	22	30	8	50
normalized size	1	1.	1.	1.44	2.44	3.33	0.89	5.56
time (sec)	N/A	0.004	0.003	0.004	1.503	1.428	1.883	1.161

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	29	41	61	20	78
normalized size	1	1.	1.14	1.38	1.95	2.9	0.95	3.71
time (sec)	N/A	0.011	0.006	0.003	1.042	1.459	3.162	1.176

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	34	51	88	34	92
normalized size	1	1.	0.94	1.	1.5	2.59	1.	2.71
time (sec)	N/A	0.014	0.007	0.003	1.03	1.412	4.489	1.159

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	23	15	63	8	15
normalized size	1	1.	2.22	2.56	1.67	7.	0.89	1.67
time (sec)	N/A	0.004	0.006	0.003	1.55	1.442	2.405	1.129

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	12	0	63	10	0
normalized size	1	1.	2.22	1.33	0.	7.	1.11	0.
time (sec)	N/A	0.004	0.004	0.002	0.	1.582	2.042	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	1059	22	167	14	22
normalized size	1	1.	1.	58.83	1.22	9.28	0.78	1.22
time (sec)	N/A	0.072	0.041	0.039	1.125	1.783	1.98	1.16

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	70	28	80	19	22
normalized size	1	1.	1.	4.38	1.75	5.	1.19	1.38
time (sec)	N/A	0.061	0.027	0.033	1.125	1.649	0.204	1.144

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	30	27	66	90	94	51
normalized size	1	1.	0.68	0.61	1.5	2.05	2.14	1.16
time (sec)	N/A	0.031	0.014	0.006	1.699	1.626	0.634	1.165

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	57	58	120	184	133	186
normalized size	1	1.	0.47	0.48	0.99	1.52	1.1	1.54
time (sec)	N/A	0.111	0.043	0.012	1.856	1.859	14.736	1.142

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	42	0	571	49	55
normalized size	1	1.	1.42	0.84	0.	11.42	0.98	1.1
time (sec)	N/A	0.043	0.031	0.018	0.	1.796	2.497	1.152

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	133	0	571	37	55
normalized size	1	1.	1.42	2.66	0.	11.42	0.74	1.1
time (sec)	N/A	0.064	0.013	0.013	0.	1.612	38.997	1.186

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	56	0	799	97	74
normalized size	1	1.	0.93	0.82	0.	11.75	1.43	1.09
time (sec)	N/A	0.042	0.19	0.013	0.	1.647	3.151	1.163

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	314	0	799	0	70
normalized size	1	1.	0.93	4.62	0.	11.75	0.	1.03
time (sec)	N/A	0.509	0.073	0.02	0.	1.605	0.	1.186

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	45	0	122	0	0
normalized size	1	1.	1.15	1.32	0.	3.59	0.	0.
time (sec)	N/A	0.031	0.033	0.009	0.	3.889	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	51	42	0	157	0	0
normalized size	1	1.	0.69	0.57	0.	2.12	0.	0.
time (sec)	N/A	0.045	0.043	0.003	0.	4.051	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	38	14	22
normalized size	1	1.	1.	0.74	0.95	2.	0.74	1.16
time (sec)	N/A	0.005	0.008	0.002	1.085	1.393	0.184	1.11

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	107	52	90	281	100	74
normalized size	1	1.	1.98	0.96	1.67	5.2	1.85	1.37
time (sec)	N/A	0.083	0.077	0.004	1.486	1.588	2.489	1.179

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	16	35	32	16
normalized size	1	1.	1.	1.29	1.14	2.5	2.29	1.14
time (sec)	N/A	0.019	0.005	0.01	0.951	1.401	2.108	1.147

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	91	62	180	94	68
normalized size	1	1.	0.95	1.49	1.02	2.95	1.54	1.11
time (sec)	N/A	0.044	0.043	0.009	1.439	1.476	1.255	1.15

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	41	126	39	41
normalized size	1	1.	1.	2.51	1.11	3.41	1.05	1.11
time (sec)	N/A	0.038	0.015	0.01	1.441	1.53	1.198	1.136

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	91	61	174	92	66
normalized size	1	1.	0.95	1.49	1.	2.85	1.51	1.08
time (sec)	N/A	0.031	0.025	0.006	1.465	1.486	1.097	1.152

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	54	28	72	36	30
normalized size	1	1.	1.	1.74	0.9	2.32	1.16	0.97
time (sec)	N/A	0.021	0.011	0.013	0.961	1.425	1.289	1.158

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	101	69	178	92	73
normalized size	1	1.	0.95	1.55	1.06	2.74	1.42	1.12
time (sec)	N/A	0.037	0.027	0.004	1.476	1.44	1.152	1.139

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	42	0	167	0	61
normalized size	1	1.	0.79	0.68	0.	2.69	0.	0.98
time (sec)	N/A	0.025	0.019	0.006	0.	4.142	0.	1.145

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	55	0	196	0	0
normalized size	1	1.	0.83	0.73	0.	2.61	0.	0.
time (sec)	N/A	0.044	0.057	0.004	0.	4.15	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	54	48	0	185	0	72
normalized size	1	1.	0.79	0.71	0.	2.72	0.	1.06
time (sec)	N/A	0.042	0.025	0.006	0.	4.196	0.	1.179

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	62	60	0	207	0	92
normalized size	1	1.	0.78	0.75	0.	2.59	0.	1.15
time (sec)	N/A	0.039	0.03	0.006	0.	4.113	0.	1.23

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	65	67	0	320	0	174
normalized size	1	1.	0.6	0.61	0.	2.94	0.	1.6
time (sec)	N/A	0.07	0.052	0.007	0.	8.643	0.	1.191

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	32	0	143	0	0
normalized size	1	1.	1.	0.68	0.	3.04	0.	0.
time (sec)	N/A	0.03	0.013	0.009	0.	3.674	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	52	66	153	58	113
normalized size	1	1.	0.84	0.78	0.99	2.28	0.87	1.69
time (sec)	N/A	0.13	0.071	0.003	1.485	1.462	24.838	1.148

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	60	54	69	192	60	117
normalized size	1	1.	0.85	0.76	0.97	2.7	0.85	1.65
time (sec)	N/A	0.11	0.038	0.004	1.48	1.461	33.642	1.175

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	97	62	80	44	62
normalized size	1	1.	1.	1.87	1.19	1.54	0.85	1.19
time (sec)	N/A	0.053	0.024	0.008	1.503	1.479	0.896	1.198

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	14	0	108	0	34
normalized size	1	1.	0.9	0.7	0.	5.4	0.	1.7
time (sec)	N/A	0.105	0.038	0.006	0.	3.682	0.	1.132

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	31	67	34	39	105	36	41
normalized size	1	0.72	1.56	0.79	0.91	2.44	0.84	0.95
time (sec)	N/A	0.068	0.029	0.006	1.55	1.484	2.91	1.18

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	127	90	120	235	0	120
normalized size	1	1.	1.09	0.78	1.03	2.03	0.	1.03
time (sec)	N/A	0.138	0.08	0.004	1.024	1.462	0.	1.222

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	72	124	0	0
normalized size	1	1.	0.7	0.65	0.87	1.49	0.	0.
time (sec)	N/A	0.059	0.06	0.01	1.002	1.505	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	43	41	54	134	216	0
normalized size	1	1.	0.67	0.64	0.84	2.09	3.38	0.
time (sec)	N/A	0.049	0.029	0.01	1.041	1.462	2.264	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	57	59	78	171	0	0
normalized size	1	1.	0.7	0.72	0.95	2.09	0.	0.
time (sec)	N/A	0.08	0.048	0.012	0.997	1.435	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	72	124	0	0
normalized size	1	1.	0.7	0.65	0.87	1.49	0.	0.
time (sec)	N/A	0.047	0.02	0.	1.011	1.526	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	135	121	162	293	0	0
normalized size	1	1.	0.71	0.64	0.85	1.54	0.	0.
time (sec)	N/A	0.366	0.104	0.013	1.002	1.546	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	183	154	207	309	0	0
normalized size	1	1.	0.79	0.66	0.89	1.33	0.	0.
time (sec)	N/A	0.381	0.13	0.017	1.012	1.499	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	103	107	143	228	0	0
normalized size	1	1.	0.64	0.67	0.89	1.42	0.	0.
time (sec)	N/A	0.276	0.09	0.006	1.009	1.512	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	103	0	34
normalized size	1	1.	1.	0.8	0.	5.15	0.	1.7
time (sec)	N/A	0.078	0.021	0.001	0.	3.702	0.	1.223

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	52	44	38	0	246	0	92
normalized size	1	1.18	1.	0.86	0.	5.59	0.	2.09
time (sec)	N/A	0.035	0.018	0.009	0.	5.814	0.	1.222

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	80	78	111	99	119
normalized size	1	1.	0.93	1.48	1.44	2.06	1.83	2.2
time (sec)	N/A	0.381	0.083	0.005	0.973	1.483	21.229	1.211

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	53	50	0	231	0	59
normalized size	1	1.	0.76	0.71	0.	3.3	0.	0.84
time (sec)	N/A	0.035	0.028	0.006	0.	7.01	0.	1.23

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	49	17	20
normalized size	1	1.	1.	0.84	1.05	2.58	0.89	1.05
time (sec)	N/A	0.021	0.011	0.003	1.037	1.435	0.148	1.206

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	66	50	66	236	71	68
normalized size	1	1.	0.86	0.65	0.86	3.06	0.92	0.88
time (sec)	N/A	0.066	0.056	0.005	1.015	1.455	27.686	1.159

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	B	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	80	224	68	443	0	986	56	0
normalized size	1	2.8	0.85	5.54	0.	12.32	0.7	0.
time (sec)	N/A	0.286	0.104	0.033	0.	1.601	7.367	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	130	0	248	0	0
normalized size	1	1.	1.	1.46	0.	2.79	0.	0.
time (sec)	N/A	0.263	0.062	0.013	0.	23.53	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	68	0	189	0	0
normalized size	1	1.	1.	1.11	0.	3.1	0.	0.
time (sec)	N/A	0.513	0.051	0.015	0.	15.355	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	28	36	53	26	20
normalized size	1	1.	1.	3.5	4.5	6.62	3.25	2.5
time (sec)	N/A	0.002	0.002	0.003	0.98	1.58	0.983	1.217

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	32	36	72	0	30
normalized size	1	1.	1.	4.	4.5	9.	0.	3.75
time (sec)	N/A	0.011	0.006	0.01	1.018	1.595	0.	1.137

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	39	66	86	60	31
normalized size	1	1.	1.91	1.77	3.	3.91	2.73	1.41
time (sec)	N/A	0.003	0.016	0.003	0.984	1.784	1.542	1.239

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	45	69	117	0	47
normalized size	1	1.	1.91	2.05	3.14	5.32	0.	2.14
time (sec)	N/A	0.005	0.002	0.003	0.976	1.706	0.	1.211

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	50	43	27	101	0	57
normalized size	1	1.	1.39	1.19	0.75	2.81	0.	1.58
time (sec)	N/A	0.004	0.018	0.014	1.492	1.691	0.	1.146

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	50	60	55	96	0	69
normalized size	1	1.	1.39	1.67	1.53	2.67	0.	1.92
time (sec)	N/A	0.014	0.004	0.013	1.474	1.78	0.	1.157

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	76	76	74	130	83	65
normalized size	1	1.	1.1	1.1	1.07	1.88	1.2	0.94
time (sec)	N/A	0.013	0.074	0.01	1.005	1.712	14.268	1.187

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	76	79	186	182	0	84
normalized size	1	1.	1.1	1.14	2.7	2.64	0.	1.22
time (sec)	N/A	0.025	0.038	0.009	1.022	1.676	0.	1.12

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	32	33	18	38	0	27
normalized size	1	1.	2.13	2.2	1.2	2.53	0.	1.8
time (sec)	N/A	0.012	0.013	0.004	1.477	1.682	0.	1.138

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	34	30	20	46	0	22
normalized size	1	1.	1.89	1.67	1.11	2.56	0.	1.22
time (sec)	N/A	0.021	0.014	0.01	1.446	1.691	0.	1.182

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	93	85	32	54	0	55
normalized size	1	1.	3.88	3.54	1.33	2.25	0.	2.29
time (sec)	N/A	0.06	0.2	0.029	1.459	1.715	0.	1.208

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	97	80	0	251	0	100
normalized size	1	1.	2.37	1.95	0.	6.12	0.	2.44
time (sec)	N/A	0.067	0.073	0.02	0.	1.727	0.	1.258

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	46	50	72	0	49
normalized size	1	1.	1.34	1.44	1.56	2.25	0.	1.53
time (sec)	N/A	0.012	0.014	0.003	1.509	1.712	0.	1.199

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	67	39	58	90	0	39
normalized size	1	1.	1.76	1.03	1.53	2.37	0.	1.03
time (sec)	N/A	0.014	0.022	0.004	1.492	1.774	0.	1.127

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	78	64	66	90	0	49
normalized size	1	1.	1.86	1.52	1.57	2.14	0.	1.17
time (sec)	N/A	0.016	0.042	0.014	1.462	1.659	0.	1.144

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	78	60	95	142	0	54
normalized size	1	1.	1.9	1.46	2.32	3.46	0.	1.32
time (sec)	N/A	0.019	0.075	0.01	0.989	1.802	0.	1.159

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	123	152	0	424	0	161
normalized size	1	1.	1.62	2.	0.	5.58	0.	2.12
time (sec)	N/A	0.039	0.274	0.004	0.	1.781	0.	1.212

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	76	76	108	149	0	100
normalized size	1	1.	1.55	1.55	2.2	3.04	0.	2.04
time (sec)	N/A	0.012	0.046	0.01	1.471	1.851	0.	1.17

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	79	106	72	111	0	0
normalized size	1	1.	1.72	2.3	1.57	2.41	0.	0.
time (sec)	N/A	0.023	0.036	0.02	1.512	1.872	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	17	36	45	0	39
normalized size	1	1.	0.95	0.85	1.8	2.25	0.	1.95
time (sec)	N/A	0.056	0.009	0.003	0.972	1.682	0.	1.158

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	22	45	0	39
normalized size	1	1.	0.94	0.94	1.22	2.5	0.	2.17
time (sec)	N/A	0.031	0.007	0.002	1.023	1.745	0.	1.237

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	106	79	139	243	0	144
normalized size	1	1.	1.96	1.46	2.57	4.5	0.	2.67
time (sec)	N/A	0.057	0.075	0.015	1.465	1.753	0.	1.219

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	18	0	39	0	31
normalized size	1	1.	1.	1.64	0.	3.55	0.	2.82
time (sec)	N/A	0.003	0.005	0.003	0.	1.623	0.	1.109

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	0	82	0	0
normalized size	1	1.	1.41	1.38	0.	2.83	0.	0.
time (sec)	N/A	0.012	0.229	0.018	0.	1.848	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	197	551	0	972	0	387
normalized size	1	1.	1.09	3.06	0.	5.4	0.	2.15
time (sec)	N/A	0.195	0.389	0.059	0.	1.847	0.	1.296

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	306	1066	0	440	0	473
normalized size	1	1.	1.78	6.2	0.	2.56	0.	2.75
time (sec)	N/A	0.144	0.465	0.027	0.	1.784	0.	1.232

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	333	5984	0	576	0	610
normalized size	1	1.	1.08	19.49	0.	1.88	0.	1.99
time (sec)	N/A	0.251	1.158	0.038	0.	1.782	0.	1.232

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	71	0	227	0	109
normalized size	1	1.	0.91	1.09	0.	3.49	0.	1.68
time (sec)	N/A	0.03	0.027	0.007	0.	1.772	0.	1.171

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	79	118	0	262	0	193
normalized size	1	1.	0.95	1.42	0.	3.16	0.	2.33
time (sec)	N/A	0.033	0.027	0.015	0.	1.766	0.	1.212

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	146	0	347	0	248
normalized size	1	1.	0.96	1.45	0.	3.44	0.	2.46
time (sec)	N/A	0.037	0.038	0.022	0.	1.719	0.	1.145

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	187	370	0	529	0	266
normalized size	1	1.	1.73	3.43	0.	4.9	0.	2.46
time (sec)	N/A	0.102	0.414	0.026	0.	1.821	0.	1.15

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	881	2407	0	324	0	355
normalized size	1	1.	10.13	27.67	0.	3.72	0.	4.08
time (sec)	N/A	0.065	1.621	0.079	0.	1.716	0.	1.137

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	914	14529	0	458	0	495
normalized size	1	1.	6.13	97.51	0.	3.07	0.	3.32
time (sec)	N/A	0.095	2.409	0.233	0.	1.808	0.	1.291

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	59	62	51	80	130	182	69
normalized size	1	1.4	1.48	1.21	1.9	3.1	4.33	1.64
time (sec)	N/A	0.215	0.345	0.007	1.155	1.642	0.775	1.171

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	59	62	51	80	130	182	69
normalized size	1	1.4	1.48	1.21	1.9	3.1	4.33	1.64
time (sec)	N/A	0.239	0.302	0.006	1.178	1.789	12.365	1.18

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	0	0	0
normalized size	1	1.	2.73	10.29	0.	0.	0.	0.
time (sec)	N/A	0.071	0.68	0.109	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	0	0	0
normalized size	1	1.	4.13	15.26	0.	0.	0.	0.
time (sec)	N/A	0.049	0.598	0.024	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	0	0	0
normalized size	1	1.	9.18	11.76	0.	0.	0.	0.
time (sec)	N/A	0.014	0.15	0.026	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	261	963	0	0	0	0
normalized size	1	1.	3.58	13.19	0.	0.	0.	0.
time (sec)	N/A	0.051	0.869	0.032	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	298	1039	0	0	0	0
normalized size	1	1.	2.73	9.53	0.	0.	0.	0.
time (sec)	N/A	0.073	1.047	0.033	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	0	0	0
normalized size	1	1.	2.73	10.29	0.	0.	0.	0.
time (sec)	N/A	0.069	0.989	0.032	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	0	0	0
normalized size	1	1.	4.13	15.26	0.	0.	0.	0.
time (sec)	N/A	0.049	0.746	0.026	0.	0.	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	0	0	0
normalized size	1	1.	5.88	11.76	0.	0.	0.	0.
time (sec)	N/A	0.012	0.262	0.025	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	298	963	0	0	0	0
normalized size	1	1.	4.08	13.19	0.	0.	0.	0.
time (sec)	N/A	0.051	0.971	0.03	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	327	1039	0	0	0	0
normalized size	1	1.	3.	9.53	0.	0.	0.	0.
time (sec)	N/A	0.07	1.061	0.032	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	730	730	10468	5229	0	0	0	0
normalized size	1	1.	14.34	7.16	0.	0.	0.	0.
time (sec)	N/A	0.903	6.198	0.181	0.	0.	0.	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	622	622	5218	4890	0	0	0	0
normalized size	1	1.	8.39	7.86	0.	0.	0.	0.
time (sec)	N/A	0.66	6.095	0.03	0.	0.	0.	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	822	1056	0	0	0	0
normalized size	1	1.	3.62	4.65	0.	0.	0.	0.
time (sec)	N/A	0.173	2.048	0.026	0.	0.	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	674	674	5276	5024	0	0	0	0
normalized size	1	1.	7.83	7.45	0.	0.	0.	0.
time (sec)	N/A	0.683	6.133	0.04	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	663	663	7543	7887	0	0	0	0
normalized size	1	1.	11.38	11.9	0.	0.	0.	0.
time (sec)	N/A	0.81	6.124	0.24	0.	0.	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	1065	1704	0	0	0	0
normalized size	1	1.	4.53	7.25	0.	0.	0.	0.
time (sec)	N/A	0.177	2.235	0.024	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	748	748	7629	8103	0	0	0	0
normalized size	1	1.	10.2	10.83	0.	0.	0.	0.
time (sec)	N/A	0.786	6.151	0.04	0.	0.	0.	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	6287	2655	0	0	0	0
normalized size	1	1.	13.91	5.87	0.	0.	0.	0.
time (sec)	N/A	0.579	6.122	0.063	0.	0.	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	3470	2519	0	0	0	0
normalized size	1	1.	8.74	6.35	0.	0.	0.	0.
time (sec)	N/A	0.401	6.058	0.016	0.	0.	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	540	530	0	0	0	0
normalized size	1	1.	3.75	3.68	0.	0.	0.	0.
time (sec)	N/A	0.104	1.401	0.015	0.	0.	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	3526	2601	0	0	0	0
normalized size	1	1.	8.07	5.95	0.	0.	0.	0.
time (sec)	N/A	0.416	6.081	0.018	0.	0.	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	517	517	6386	2757	0	0	0	0
normalized size	1	1.	12.35	5.33	0.	0.	0.	0.
time (sec)	N/A	0.56	6.211	0.025	0.	0.	0.	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	558	558	7235	2694	0	0	0	0
normalized size	1	1.	12.97	4.83	0.	0.	0.	0.
time (sec)	N/A	0.565	6.129	0.018	0.	0.	0.	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	4389	2551	0	0	0	0
normalized size	1	1.	9.42	5.47	0.	0.	0.	0.
time (sec)	N/A	0.455	6.088	0.016	0.	0.	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	813	788	0	0	0	0
normalized size	1	1.	4.54	4.4	0.	0.	0.	0.
time (sec)	N/A	0.152	2.734	0.016	0.	0.	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	3593	2616	0	0	0	0
normalized size	1	1.	7.58	5.52	0.	0.	0.	0.
time (sec)	N/A	0.435	6.08	0.016	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	591	591	6452	2777	0	0	0	0
normalized size	1	1.	10.92	4.7	0.	0.	0.	0.
time (sec)	N/A	0.584	6.12	0.023	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	585	585	8500	2733	0	0	0	0
normalized size	1	1.	14.53	4.67	0.	0.	0.	0.
time (sec)	N/A	0.679	6.164	0.019	0.	0.	0.	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	5647	2582	0	0	0	0
normalized size	1	1.	11.64	5.32	0.	0.	0.	0.
time (sec)	N/A	0.53	6.12	0.019	0.	0.	0.	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	1145	1147	0	0	0	0
normalized size	1	1.	2.95	2.96	0.	0.	0.	0.
time (sec)	N/A	0.387	5.817	0.019	0.	0.	0.	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	2941	2607	0	0	0	0
normalized size	1	1.	9.46	8.38	0.	0.	0.	0.
time (sec)	N/A	0.327	6.111	0.022	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	582	582	5812	2780	0	0	0	0
normalized size	1	1.	9.99	4.78	0.	0.	0.	0.
time (sec)	N/A	0.675	6.163	0.029	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	927	965	0	0	0	0
normalized size	1	1.	7.19	7.48	0.	0.	0.	0.
time (sec)	N/A	0.302	0.344	1.415	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	4865	4426	0	0	0	0
normalized size	1	1.	11.29	10.27	0.	0.	0.	0.
time (sec)	N/A	0.528	6.047	0.019	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	249	961	0	0	0	0
normalized size	1	1.	2.31	8.9	0.	0.	0.	0.
time (sec)	N/A	0.217	0.525	0.654	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	602	2564	0	0	0	0
normalized size	1	1.	1.64	6.99	0.	0.	0.	0.
time (sec)	N/A	0.376	4.318	0.016	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	1148	1180	0	0	0	0
normalized size	1	1.	9.11	9.37	0.	0.	0.	0.
time (sec)	N/A	0.326	0.355	0.76	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	434	434	6019	5421	0	0	0	0
normalized size	1	1.	13.87	12.49	0.	0.	0.	0.
time (sec)	N/A	0.528	6.049	0.017	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	577	577	6084	5477	0	0	0	0
normalized size	1	1.	10.54	9.49	0.	0.	0.	0.
time (sec)	N/A	0.688	6.053	0.018	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	826	1182	0	0	0	0
normalized size	1	1.	6.35	9.09	0.	0.	0.	0.
time (sec)	N/A	0.26	0.122	0.339	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	5428	5427	0	0	0	0
normalized size	1	1.	12.23	12.22	0.	0.	0.	0.
time (sec)	N/A	0.468	6.048	0.016	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	76	77	236	0	0
normalized size	1	1.	1.02	1.36	1.38	4.21	0.	0.
time (sec)	N/A	0.211	0.103	0.019	1.609	1.84	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	0	225	63	120
normalized size	1	1.	1.	0.86	0.	3.46	0.97	1.85
time (sec)	N/A	0.159	0.057	0.006	0.	1.738	3.751	1.152

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	101	88	58	0	228	0	108
normalized size	1	1.91	1.66	1.09	0.	4.3	0.	2.04
time (sec)	N/A	0.215	0.143	0.005	0.	1.716	0.	1.124

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	35	12	16
normalized size	1	1.	1.	0.93	1.14	2.5	0.86	1.14
time (sec)	N/A	0.117	0.006	0.002	1.719	1.741	0.138	1.189

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	46	61	130	60	58
normalized size	1	1.	1.	0.75	1.	2.13	0.98	0.95
time (sec)	N/A	0.166	0.046	0.003	1.117	1.702	1.907	1.142

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	86	115	243	116	113
normalized size	1	1.	1.	0.75	1.01	2.13	1.02	0.99
time (sec)	N/A	0.193	0.076	0.003	1.105	1.704	2.754	1.116

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	100	102	0	390	70	0
normalized size	1	1.	1.72	1.76	0.	6.72	1.21	0.
time (sec)	N/A	0.057	0.061	0.016	0.	1.707	2.44	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	100	102	0	390	70	0
normalized size	1	1.	1.72	1.76	0.	6.72	1.21	0.
time (sec)	N/A	0.137	0.007	0.008	0.	1.817	7.916	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	64	39	0	574	0	528
normalized size	1	1.	1.21	0.74	0.	10.83	0.	9.96
time (sec)	N/A	0.031	0.046	0.013	0.	1.769	0.	1.26

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	54	27	24
normalized size	1	1.	1.	0.78	1.	2.35	1.17	1.04
time (sec)	N/A	0.007	0.012	0.002	1.12	1.695	0.181	1.104

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	23	68	105	39
normalized size	1	1.	1.	1.13	1.	2.96	4.57	1.7
time (sec)	N/A	0.008	0.004	0.003	1.541	1.639	71.253	1.156

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	58	49	111	0	54
normalized size	1	1.	1.	1.76	1.48	3.36	0.	1.64
time (sec)	N/A	0.015	0.012	0.012	1.045	1.766	0.	1.164

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	27	35	3	15	2	42
normalized size	1	1.	0.6	0.78	0.07	0.33	0.04	0.93
time (sec)	N/A	0.158	0.067	0.005	1.671	1.657	0.096	1.094

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	27	35	3	15	2	42
normalized size	1	1.	0.6	0.78	0.07	0.33	0.04	0.93
time (sec)	N/A	0.055	0.013	0.003	1.679	1.658	0.094	1.126

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	27	35	3	15	2	42
normalized size	1	1.	0.6	0.78	0.07	0.33	0.04	0.93
time (sec)	N/A	0.098	0.013	0.003	1.723	1.83	0.095	1.127

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	27	27	3	15	2	16
normalized size	1	1.	0.6	0.6	0.07	0.33	0.04	0.36
time (sec)	N/A	0.133	0.034	0.007	1.701	1.685	0.092	1.114

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	41	0	26	0	8
normalized size	1	1.	0.94	0.79	0.	0.5	0.	0.15
time (sec)	N/A	0.183	0.06	0.006	0.	1.968	0.	1.107

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	30	8	26	0	8
normalized size	1	1.	0.94	0.58	0.15	0.5	0.	0.15
time (sec)	N/A	0.126	0.027	0.007	1.713	1.994	0.	1.095

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	64	40	0	31	0	11
normalized size	1	1.	0.94	0.59	0.	0.46	0.	0.16
time (sec)	N/A	0.155	0.158	0.01	0.	1.941	0.	1.122

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	36	65	0	0
normalized size	1	1.	0.74	1.03	1.16	2.1	0.	0.
time (sec)	N/A	0.015	0.189	0.009	1.571	1.677	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	34	65	0	74
normalized size	1	1.	0.74	1.03	1.1	2.1	0.	2.39
time (sec)	N/A	0.288	0.146	0.001	1.278	1.742	0.	1.14

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	65	0	92
normalized size	1	1.	0.74	2.61	0.	2.1	0.	2.97
time (sec)	N/A	0.14	0.09	0.032	0.	1.622	0.	1.114

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	65	0	92
normalized size	1	1.	0.74	2.61	0.	2.1	0.	2.97
time (sec)	N/A	0.125	0.058	0.002	0.	1.612	0.	1.124

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	0	65	0	92
normalized size	1	1.	0.74	1.03	0.	2.1	0.	2.97
time (sec)	N/A	0.654	0.18	0.004	0.	1.627	0.	1.238

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	73	115	93	212	0
normalized size	1	1.	0.88	1.7	2.67	2.16	4.93	0.
time (sec)	N/A	0.067	0.041	0.015	1.015	1.747	9.885	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	45	0	111	0	77
normalized size	1	1.	1.	1.07	0.	2.64	0.	1.83
time (sec)	N/A	0.13	0.038	0.004	0.	1.679	0.	1.136

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	262	0	212	0	180
normalized size	1	1.	0.83	4.03	0.	3.26	0.	2.77
time (sec)	N/A	0.133	0.047	0.032	0.	1.654	0.	1.154

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	240	1407	0	2431	0	724
normalized size	1	1.	1.23	7.22	0.	12.47	0.	3.71
time (sec)	N/A	0.207	0.324	0.028	0.	1.991	0.	1.164

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	46	140	37	46
normalized size	1	1.	1.	1.04	1.64	5.	1.32	1.64
time (sec)	N/A	0.014	0.014	0.025	1.531	1.764	12.237	1.126

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	0	140	0	46
normalized size	1	1.	1.	1.04	0.	5.	0.	1.64
time (sec)	N/A	0.058	0.006	0.02	0.	1.788	0.	1.094

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	47	17	24
normalized size	1	1.	1.	0.86	1.09	2.14	0.77	1.09
time (sec)	N/A	0.005	0.007	0.002	1.144	1.752	0.134	1.101

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	47	17	24
normalized size	1	1.	1.	0.86	1.09	2.14	0.77	1.09
time (sec)	N/A	0.082	0.002	0.003	1.109	1.805	1.15	1.138

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	0	36	36	15
normalized size	1	1.	1.	0.71	0.	2.12	2.12	0.88
time (sec)	N/A	0.061	0.016	0.006	0.	2.074	1.237	1.098

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	125	0	36	0	0
normalized size	1	1.	1.	7.35	0.	2.12	0.	0.
time (sec)	N/A	0.047	0.025	0.059	0.	2.369	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	58	7	8
normalized size	1	1.	1.	0.7	0.8	5.8	0.7	0.8
time (sec)	N/A	0.001	0.004	0.003	1.652	1.497	0.134	1.124

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	34	8	74	51	16
normalized size	1	1.	1.	3.4	0.8	7.4	5.1	1.6
time (sec)	N/A	0.002	0.004	0.007	1.622	1.51	1.042	1.136

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	58	7	8
normalized size	1	1.	1.	0.7	0.8	5.8	0.7	0.8
time (sec)	N/A	0.004	0.004	0.006	1.676	1.428	1.25	1.181

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	11	77	0	8
normalized size	1	1.	1.	0.58	0.92	6.42	0.	0.67
time (sec)	N/A	0.007	0.006	0.002	1.721	1.478	0.	1.195

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	31	11	81	41	18
normalized size	1	1.	1.75	2.58	0.92	6.75	3.42	1.5
time (sec)	N/A	0.007	0.012	0.006	1.606	1.441	1.015	1.141

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	7	11	77	0	8
normalized size	1	1.	1.75	0.58	0.92	6.42	0.	0.67
time (sec)	N/A	0.009	0.003	0.005	2.101	1.457	0.	1.325

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	11	77	0	5
normalized size	1	1.	1.	1.25	2.75	19.25	0.	1.25
time (sec)	N/A	0.005	0.006	0.003	1.899	1.501	0.	1.132

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	18	29	11	81	41	18
normalized size	1	1.	4.5	7.25	2.75	20.25	10.25	4.5
time (sec)	N/A	0.006	0.01	0.006	2.033	1.515	1.013	1.154

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	18	5	11	77	0	5
normalized size	1	1.	4.5	1.25	2.75	19.25	0.	1.25
time (sec)	N/A	0.007	0.003	0.004	2.183	1.482	0.	1.135

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	24	8	9
normalized size	1	1.	1.	0.73	0.82	2.18	0.73	0.82
time (sec)	N/A	0.001	0.001	0.001	1.409	1.405	0.053	1.126

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	24	8	9
normalized size	1	1.	1.	0.73	0.82	2.18	0.73	0.82
time (sec)	N/A	0.008	0.	0.003	1.289	1.453	0.132	1.161

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	0	65	7	14
normalized size	1	1.	1.	1.11	0.	2.41	0.26	0.52
time (sec)	N/A	0.014	0.008	0.004	0.	1.469	0.854	1.157

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	0	65	0	14
normalized size	1	1.	1.	1.11	0.	2.41	0.	0.52
time (sec)	N/A	0.021	0.005	0.003	0.	1.475	0.	1.13

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	33	56	28	19	35	15	20
normalized size	1	1.32	2.24	1.12	0.76	1.4	0.6	0.8
time (sec)	N/A	0.014	0.02	0.003	1.226	1.427	1.189	1.152

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	33	56	28	0	35	0	28
normalized size	1	1.32	2.24	1.12	0.	1.4	0.	1.12
time (sec)	N/A	0.022	0.003	0.003	0.	1.432	0.	1.166

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	23	8	12
normalized size	1	1.	1.	0.91	1.09	2.09	0.73	1.09
time (sec)	N/A	0.001	0.003	0.	1.408	1.411	0.052	1.145

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	20	16	42	0	20
normalized size	1	1.	2.09	1.82	1.45	3.82	0.	1.82
time (sec)	N/A	0.001	0.021	0.002	1.422	1.454	0.	1.087

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	20	7	9
normalized size	1	1.	1.	0.89	1.	2.22	0.78	1.
time (sec)	N/A	0.001	0.002	0.001	1.574	1.444	0.051	1.133

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	25	22	9	54	0	18
normalized size	1	1.	2.78	2.44	1.	6.	0.	2.
time (sec)	N/A	0.001	0.023	0.002	1.47	1.438	0.	1.126

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	35	10	12
normalized size	1	1.	1.	0.77	0.92	2.69	0.77	0.92
time (sec)	N/A	0.001	0.003	0.003	1.151	1.418	0.053	1.103

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	20	16	54	0	20
normalized size	1	1.	1.92	1.54	1.23	4.15	0.	1.54
time (sec)	N/A	0.001	0.02	0.003	1.113	1.464	0.	1.106

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	26	8	9
normalized size	1	1.	1.	0.73	0.82	2.36	0.73	0.82
time (sec)	N/A	0.001	0.002	0.001	0.992	1.413	0.051	1.105

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	27	22	9	68	0	18
normalized size	1	1.	2.45	2.	0.82	6.18	0.	1.64
time (sec)	N/A	0.001	0.022	0.003	0.987	1.465	0.	1.104

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	49	67	55	157	97	53
normalized size	1	1.	1.4	1.91	1.57	4.49	2.77	1.51
time (sec)	N/A	0.007	0.019	0.005	1.612	1.481	1.614	1.087

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	49	86	0	252	0	0
normalized size	1	1.	1.4	2.46	0.	7.2	0.	0.
time (sec)	N/A	0.007	0.008	0.012	0.	1.588	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	70	72	201	0	0
normalized size	1	1.	1.42	1.63	1.67	4.67	0.	0.
time (sec)	N/A	0.013	0.038	0.015	1.562	1.506	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	47	41	72	161	0	59
normalized size	1	1.	1.34	1.17	2.06	4.6	0.	1.69
time (sec)	N/A	0.061	0.021	0.008	1.534	1.49	0.	1.128

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	72	134	105	234	0	0
normalized size	1	1.	1.41	2.63	2.06	4.59	0.	0.
time (sec)	N/A	0.024	0.06	0.036	1.499	1.511	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	59	75	105	193	0	117
normalized size	1	1.	1.31	1.67	2.33	4.29	0.	2.6
time (sec)	N/A	0.091	0.038	0.012	1.708	1.539	0.	1.185

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	47	2	3
normalized size	1	1.	1.	1.5	1.5	23.5	1.	1.5
time (sec)	N/A	0.001	0.004	0.003	1.858	1.408	0.124	1.125

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	0	66	0	0
normalized size	1	1.	16.	14.5	0.	33.	0.	0.
time (sec)	N/A	0.001	0.025	0.013	0.	1.477	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	35	2	19
normalized size	1	1.	1.	1.5	1.5	17.5	1.	9.5
time (sec)	N/A	0.001	0.003	0.002	1.491	1.435	0.124	1.122

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	0	171	0	0
normalized size	1	1.	21.	14.5	0.	85.5	0.	0.
time (sec)	N/A	0.001	0.023	0.009	0.	1.461	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	18	23	74	15	23
normalized size	1	1.	0.87	0.78	1.	3.22	0.65	1.
time (sec)	N/A	0.003	0.005	0.003	1.675	1.47	0.184	1.124

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	50	42	0	144	0	0
normalized size	1	1.	2.17	1.83	0.	6.26	0.	0.
time (sec)	N/A	0.003	0.041	0.008	0.	1.451	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	16	20	69	15	34
normalized size	1	1.	0.86	0.76	0.95	3.29	0.71	1.62
time (sec)	N/A	0.002	0.004	0.003	1.487	1.416	0.181	1.148

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	70	47	0	258	0	0
normalized size	1	1.	3.33	2.24	0.	12.29	0.	0.
time (sec)	N/A	0.003	0.053	0.007	0.	1.387	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	57	53	0	0	0	0	0
normalized size	1	1.16	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.012	0.036	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	21	0	84	58	41
normalized size	1	1.	1.36	0.75	0.	3.	2.07	1.46
time (sec)	N/A	0.009	0.011	0.003	0.	1.438	0.356	1.132

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	175	0	220	17	189
normalized size	1	1.	1.	4.73	0.	5.95	0.46	5.11
time (sec)	N/A	0.042	0.022	0.031	0.	1.483	0.152	1.132

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	74	33	0	236	27	119
normalized size	1	1.	1.85	0.82	0.	5.9	0.68	2.98
time (sec)	N/A	0.043	0.039	0.009	0.	1.434	0.197	1.094

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	77	111	127	174	0	158
normalized size	1	1.	1.43	2.06	2.35	3.22	0.	2.93
time (sec)	N/A	0.128	0.056	0.012	1.532	1.442	0.	1.131

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	77	111	0	174	0	158
normalized size	1	1.	1.28	1.85	0.	2.9	0.	2.63
time (sec)	N/A	0.3	0.085	0.005	0.	1.438	0.	1.241

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	38	0	163	0	68
normalized size	1	1.	0.76	0.75	0.	3.2	0.	1.33
time (sec)	N/A	0.111	0.05	0.005	0.	1.468	0.	1.139

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	38	73	163	0	68
normalized size	1	1.	0.76	0.75	1.43	3.2	0.	1.33
time (sec)	N/A	0.097	0.046	0.004	1.579	1.459	0.	1.123

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	51	73	180	0	0
normalized size	1	1.	0.76	1.	1.43	3.53	0.	0.
time (sec)	N/A	0.148	0.024	0.008	1.61	1.465	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	82	51	0	180	0	0
normalized size	1	1.	1.52	0.94	0.	3.33	0.	0.
time (sec)	N/A	0.049	0.14	0.004	0.	1.441	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	0	53	0	0
normalized size	1	1.	0.85	0.74	0.	1.96	0.	0.
time (sec)	N/A	0.03	0.01	0.001	0.	1.394	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	0	38	0	0
normalized size	1	1.	0.86	0.93	0.	2.71	0.	0.
time (sec)	N/A	0.149	0.026	0.006	0.	1.458	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	38	0	0
normalized size	1	1.	1.	0.92	0.	3.17	0.	0.
time (sec)	N/A	0.07	0.01	0.004	0.	1.505	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	34	0	68	0	0
normalized size	1	1.	0.81	0.94	0.	1.89	0.	0.
time (sec)	N/A	0.14	0.02	0.005	0.	1.452	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	0	68	0	0
normalized size	1	1.	0.88	1.03	0.	2.06	0.	0.
time (sec)	N/A	0.193	0.012	0.003	0.	1.465	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	105	117	0	467	0	0
normalized size	1	1.	1.5	1.67	0.	6.67	0.	0.
time (sec)	N/A	0.068	0.125	0.042	0.	1.614	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	52	63	0	97	0	65
normalized size	1	1.	0.63	0.76	0.	1.17	0.	0.78
time (sec)	N/A	0.154	0.062	0.023	0.	1.485	0.	1.16

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	73	91	63	0	81	0	24
normalized size	1	1.55	1.94	1.34	0.	1.72	0.	0.51
time (sec)	N/A	0.46	0.117	0.016	0.	1.462	0.	1.115

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	70	75	0	178	0	45
normalized size	1	1.	0.57	0.61	0.	1.45	0.	0.37
time (sec)	N/A	0.286	0.02	0.007	0.	1.53	0.	1.113

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	64	62	0	292	0	116
normalized size	1	1.	0.48	0.47	0.	2.2	0.	0.87
time (sec)	N/A	0.074	0.073	0.015	0.	1.436	0.	1.138

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	44	49	0	203	0	90
normalized size	1	1.	0.49	0.54	0.	2.26	0.	1.
time (sec)	N/A	0.048	0.034	0.01	0.	1.486	0.	1.195

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	40	42	0	178	0	66
normalized size	1	1.	0.66	0.69	0.	2.92	0.	1.08
time (sec)	N/A	0.03	0.016	0.005	0.	1.498	0.	1.093

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	72	79	0	360	0	119
normalized size	1	1.	0.66	0.72	0.	3.3	0.	1.09
time (sec)	N/A	0.065	0.028	0.03	0.	1.491	0.	1.18

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	95	218	0	513	0	0
normalized size	1	1.	0.66	1.51	0.	3.56	0.	0.
time (sec)	N/A	0.082	0.09	0.01	0.	1.532	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	28	0	80	0	41
normalized size	1	1.	0.93	1.	0.	2.86	0.	1.46
time (sec)	N/A	0.116	0.011	0.003	0.	1.44	0.	1.106

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.095	0.017	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.164	0.018	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.025	0.016	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.02	0.015	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.005	0.013	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.005	0.007	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.006	0.007	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.006	0.005	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.009	0.007	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.009	0.008	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	348	0	0
normalized size	1	1.	1.	0.	0.	7.4	0.	0.
time (sec)	N/A	0.109	0.023	0.016	0.	12.045	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	366	0	0
normalized size	1	1.	1.	0.	0.	7.62	0.	0.
time (sec)	N/A	0.108	0.019	0.013	0.	13.006	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	0.109	0.033	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	0.098	0.024	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	36	93	37	36
normalized size	1	1.	1.	0.68	0.88	2.27	0.9	0.88
time (sec)	N/A	0.045	0.026	0.003	1.733	1.648	8.361	1.165

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	38	21	27	70	24	27
normalized size	1	1.	1.46	0.81	1.04	2.69	0.92	1.04
time (sec)	N/A	0.04	0.025	0.003	1.788	1.75	4.89	1.097

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	48	41	105	39	41
normalized size	1	1.	1.29	1.14	0.98	2.5	0.93	0.98
time (sec)	N/A	0.154	0.022	0.012	1.657	1.657	16.681	1.08

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	48	50	0	184	0	59
normalized size	1	1.	2.4	2.5	0.	9.2	0.	2.95
time (sec)	N/A	0.008	0.013	0.01	0.	1.781	0.	1.09

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	52	62	0	186	0	54
normalized size	1	1.	2.6	3.1	0.	9.3	0.	2.7
time (sec)	N/A	0.009	0.013	0.008	0.	1.55	0.	1.104

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	136	244	0	3367	0	0
normalized size	1	1.	1.12	2.02	0.	27.83	0.	0.
time (sec)	N/A	0.165	0.099	0.025	0.	6.003	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	189	397	0	0	0	0
normalized size	1	1.	1.04	2.19	0.	0.	0.	0.
time (sec)	N/A	0.273	0.275	0.03	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	22	55	0	15
normalized size	1	1.	1.	0.65	0.85	2.12	0.	0.58
time (sec)	N/A	0.006	0.012	0.002	1.054	1.899	0.	1.066

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	23	30	68	0	30
normalized size	1	1.	1.04	0.88	1.15	2.62	0.	1.15
time (sec)	N/A	0.011	0.03	0.006	1.666	1.595	0.	1.122

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	15	31	12	15
normalized size	1	1.	0.87	1.07	1.	2.07	0.8	1.
time (sec)	N/A	0.003	0.007	0.005	1.048	1.416	0.147	1.092

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	19	23	38	62	0	36
normalized size	1	1.	0.86	1.05	1.73	2.82	0.	1.64
time (sec)	N/A	0.005	0.01	0.004	1.034	1.444	0.	1.12

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	37	13	12	49	0	23
normalized size	1	1.	3.08	1.08	1.	4.08	0.	1.92
time (sec)	N/A	0.007	0.011	0.005	1.637	1.464	0.	1.111

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	37	15	15	51	0	23
normalized size	1	1.	3.08	1.25	1.25	4.25	0.	1.92
time (sec)	N/A	0.008	0.012	0.006	1.681	1.482	0.	1.114

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	17	18	26	10	18
normalized size	1	1.	0.8	1.13	1.2	1.73	0.67	1.2
time (sec)	N/A	0.004	0.006	0.001	1.104	1.418	0.122	1.096

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	95	54	57	127	0	72
normalized size	1	1.	1.76	1.	1.06	2.35	0.	1.33
time (sec)	N/A	0.04	0.058	0.007	1.607	1.501	0.	1.139

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	342	0	0	0	0
normalized size	1	1.	0.97	5.8	0.	0.	0.	0.
time (sec)	N/A	0.069	0.035	0.067	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	28	0	84	0	45
normalized size	1	1.	0.66	0.47	0.	1.42	0.	0.76
time (sec)	N/A	0.052	0.019	0.007	0.	1.837	0.	1.084

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	51	38	0	116	0	69
normalized size	1	1.	0.54	0.4	0.	1.23	0.	0.73
time (sec)	N/A	0.091	0.03	0.003	0.	1.922	0.	1.102

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	0	50	26	24
normalized size	1	1.	1.	1.11	0.	2.78	1.44	1.33
time (sec)	N/A	0.049	0.009	0.005	0.	1.675	0.505	1.13

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	84	81	0	147	0	81
normalized size	1	1.	0.79	0.76	0.	1.37	0.	0.76
time (sec)	N/A	0.029	0.028	0.005	0.	1.632	0.	1.122

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	19	131	0	19
normalized size	1	1.	1.	0.96	0.76	5.24	0.	0.76
time (sec)	N/A	0.016	0.019	0.003	1.071	1.657	0.	1.119

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	45	123	49	45
normalized size	1	1.	1.	0.81	1.07	2.93	1.17	1.07
time (sec)	N/A	0.029	0.014	0.005	1.692	1.458	0.247	1.13

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	34	96	37	34
normalized size	1	1.	1.	0.81	1.06	3.	1.16	1.06
time (sec)	N/A	0.023	0.01	0.003	1.567	1.449	0.229	1.076

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	49	53	0	267	0	61
normalized size	1	1.	0.64	0.7	0.	3.51	0.	0.8
time (sec)	N/A	0.023	0.016	0.001	0.	1.683	0.	1.106

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	53	0	171	48	107
normalized size	1	1.	1.	1.15	0.	3.72	1.04	2.33
time (sec)	N/A	0.086	0.035	0.003	0.	1.714	2.229	1.097

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	18	0	47	0	0
normalized size	1	1.	1.15	0.9	0.	2.35	0.	0.
time (sec)	N/A	0.006	0.03	0.003	0.	1.682	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	36	18	35	51	49	68	0
normalized size	1	1.03	0.51	1.	1.46	1.4	1.94	0.
time (sec)	N/A	0.011	0.008	0.008	1.689	1.648	1.151	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	111	0	18	39	0	0
normalized size	1	1.	7.4	0.	1.2	2.6	0.	0.
time (sec)	N/A	0.018	0.178	0.046	1.247	1.535	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	28	25	50	77	46	50
normalized size	1	1.	0.53	0.47	0.94	1.45	0.87	0.94
time (sec)	N/A	0.021	0.025	0.005	1.001	1.441	24.669	1.102

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	81	27	34
normalized size	1	1.	1.	0.84	1.1	2.61	0.87	1.1
time (sec)	N/A	0.015	0.012	0.008	1.107	1.474	0.22	1.115

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	12	23	8	12
normalized size	1	1.	1.	1.27	1.09	2.09	0.73	1.09
time (sec)	N/A	0.003	0.005	0.003	0.981	1.429	0.124	1.098

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	23	5	5
normalized size	1	1.	1.	0.83	0.83	3.83	0.83	0.83
time (sec)	N/A	0.002	0.003	0.003	1.527	1.502	0.205	1.111

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	22	34	0	34
normalized size	1	1.	0.85	0.85	1.1	1.7	0.	1.7
time (sec)	N/A	0.005	0.006	0.002	1.494	1.458	0.	1.119

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	35	31	12	15
normalized size	1	1.	1.	0.71	2.06	1.82	0.71	0.88
time (sec)	N/A	0.003	0.002	0.	1.005	1.495	0.124	1.124

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	15	38	15	15
normalized size	1	1.	1.	0.63	0.79	2.	0.79	0.79
time (sec)	N/A	0.003	0.003	0.	1.119	1.395	1.776	1.132

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	36	90	37	36
normalized size	1	1.	1.	0.68	0.88	2.2	0.9	0.88
time (sec)	N/A	0.01	0.007	0.001	1.49	1.482	2.899	1.12

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	88	64	85	238	39	0
normalized size	1	1.	1.31	0.96	1.27	3.55	0.58	0.
time (sec)	N/A	0.03	0.019	0.009	1.476	1.798	1.121	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	24	8	9
normalized size	1	1.	1.	0.73	0.82	2.18	0.73	0.82
time (sec)	N/A	0.001	0.001	0.	0.993	1.693	0.054	1.224

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	24	8	9
normalized size	1	1.	1.	0.73	0.82	2.18	0.73	0.82
time (sec)	N/A	0.001	0.002	0.	1.004	1.487	0.054	1.126

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	44	9	24	8	9
normalized size	1	1.	1.	4.	0.82	2.18	0.73	0.82
time (sec)	N/A	0.002	0.	0.009	1.	1.422	5.643	1.204

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	95	49	0	435	0	92
normalized size	1	1.	1.56	0.8	0.	7.13	0.	1.51
time (sec)	N/A	0.025	0.063	0.006	0.	1.547	0.	1.274

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	94	55	0	439	0	80
normalized size	1	1.	1.45	0.85	0.	6.75	0.	1.23
time (sec)	N/A	0.025	0.074	0.01	0.	1.517	0.	1.239

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	28	85	26	30
normalized size	1	1.	1.	0.77	2.15	6.54	2.	2.31
time (sec)	N/A	0.007	0.004	0.004	1.663	1.493	0.394	1.131

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	28	85	26	30
normalized size	1	1.	1.	0.77	2.15	6.54	2.	2.31
time (sec)	N/A	0.011	0.003	0.005	1.663	1.469	0.574	1.165

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	82	104	320	168	108
normalized size	1	1.	1.	1.14	1.44	4.44	2.33	1.5
time (sec)	N/A	0.104	0.093	0.013	1.835	1.582	1.081	1.195

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	23	20	54	0	32
normalized size	1	1.	0.62	0.62	0.54	1.46	0.	0.86
time (sec)	N/A	0.027	0.008	0.002	1.036	1.425	0.	1.155

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	37	13	12	49	0	23
normalized size	1	1.	3.08	1.08	1.	4.08	0.	1.92
time (sec)	N/A	0.007	0.01	0.	1.506	1.482	0.	1.141

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	60	67	0	250	0	69
normalized size	1	1.	0.63	0.71	0.	2.63	0.	0.73
time (sec)	N/A	0.054	0.035	0.003	0.	8.251	0.	1.156

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	24	31	74	184	31
normalized size	1	1.	0.8	0.69	0.89	2.11	5.26	0.89
time (sec)	N/A	0.011	0.01	0.002	0.996	1.493	1.025	1.24

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	28	36	66	265	43
normalized size	1	1.	0.81	0.76	0.97	1.78	7.16	1.16
time (sec)	N/A	0.013	0.01	0.004	1.079	1.432	0.961	1.205

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	0	95	32	39
normalized size	1	1.	0.9	1.66	0.	3.28	1.1	1.34
time (sec)	N/A	0.026	0.027	0.005	0.	1.45	1.711	1.151

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	48	0	96	87	43
normalized size	1	1.	1.04	1.92	0.	3.84	3.48	1.72
time (sec)	N/A	0.025	0.029	0.003	0.	1.454	2.719	1.175

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	0	43	56	20
normalized size	1	1.	1.	0.76	0.	2.05	2.67	0.95
time (sec)	N/A	0.023	0.024	0.003	0.	1.472	0.359	1.203

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	175	0	252	0	142
normalized size	1	1.	0.83	2.69	0.	3.88	0.	2.18
time (sec)	N/A	0.054	0.053	0.01	0.	1.469	0.	1.239

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	0	104	0	85
normalized size	1	1.	1.	0.9	0.	3.35	0.	2.74
time (sec)	N/A	0.042	0.032	0.007	0.	1.449	0.	1.162

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	58	54	0	174	0	0
normalized size	1	1.	0.71	0.66	0.	2.12	0.	0.
time (sec)	N/A	0.044	0.065	0.003	0.	4.617	0.	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	56	74	190	155	74
normalized size	1	1.	1.	0.76	1.	2.57	2.09	1.
time (sec)	N/A	0.111	0.037	0.007	1.49	1.49	2.785	1.151

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	81	108	290	221	108
normalized size	1	1.	1.07	0.7	0.94	2.52	1.92	0.94
time (sec)	N/A	0.15	0.081	0.006	1.677	1.463	4.04	1.176

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	51	0	51	0	5
normalized size	1	1.	1.	12.75	0.	12.75	0.	1.25
time (sec)	N/A	0.043	0.059	0.006	0.	1.434	0.	1.216

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	68	109	0	42
normalized size	1	1.	1.	1.86	3.09	4.95	0.	1.91
time (sec)	N/A	0.01	0.007	0.005	1.107	1.471	0.	1.22

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	50	63	0	38
normalized size	1	1.	1.	1.67	2.08	2.62	0.	1.58
time (sec)	N/A	0.009	0.007	0.004	1.637	1.479	0.	1.175

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	38	47	69	109	0	47
normalized size	1	1.17	1.58	1.96	2.88	4.54	0.	1.96
time (sec)	N/A	0.011	0.014	0.005	1.05	1.434	0.	1.142

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	60	51	100	32	51
normalized size	1	1.	1.	2.5	2.12	4.17	1.33	2.12
time (sec)	N/A	0.017	0.006	0.004	1.147	1.478	3.054	1.327

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	45	69	117	0	47
normalized size	1	1.	1.91	2.05	3.14	5.32	0.	2.14
time (sec)	N/A	0.005	0.015	0.	1.12	1.432	0.	1.256

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	44	47	65	0	47
normalized size	1	1.	1.48	1.52	1.62	2.24	0.	1.62
time (sec)	N/A	0.012	0.013	0.005	1.617	1.465	0.	1.19

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	49	82	0	34
normalized size	1	1.	0.97	0.85	1.48	2.48	0.	1.03
time (sec)	N/A	0.012	0.037	0.003	1.533	1.442	0.	1.155

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	8	39	0	8
normalized size	1	1.	1.5	0.88	1.	4.88	0.	1.
time (sec)	N/A	0.005	0.008	0.003	1.545	1.449	0.	1.21

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	15	15	47	0	22
normalized size	1	1.	0.85	1.15	1.15	3.62	0.	1.69
time (sec)	N/A	0.013	0.003	0.005	1.085	1.427	0.	1.159

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	0	90	0	99
normalized size	1	1.	1.	1.86	0.	4.09	0.	4.5
time (sec)	N/A	0.032	0.021	0.012	0.	1.502	0.	1.233

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	32	15	45	14	18
normalized size	1	1.	1.62	1.33	0.62	1.88	0.58	0.75
time (sec)	N/A	0.027	0.024	0.014	1.58	1.432	0.203	1.156

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.01	0.02	0.002	0.	1.472	0.	1.172

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.011	0.003	0.006	0.	1.478	0.	1.205

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.012	0.004	0.003	0.	1.505	0.	1.24

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.013	0.004	0.003	0.	1.486	0.	1.219

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.012	0.004	0.003	0.	1.475	0.	1.264

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	0	154	0	47
normalized size	1	1.	2.04	1.04	0.	5.5	0.	1.68
time (sec)	N/A	0.013	0.004	0.004	0.	1.495	0.	1.205

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	0	197	0	68
normalized size	1	1.	1.45	0.92	0.	4.92	0.	1.7
time (sec)	N/A	0.017	0.016	0.003	0.	1.505	0.	1.251

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	0	197	0	68
normalized size	1	1.	1.45	0.92	0.	4.92	0.	1.7
time (sec)	N/A	0.017	0.004	0.005	0.	1.469	0.	1.215

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	0	197	0	68
normalized size	1	1.	1.45	0.92	0.	4.92	0.	1.7
time (sec)	N/A	0.017	0.004	0.004	0.	1.533	0.	1.224

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	0	197	0	68
normalized size	1	1.	1.45	0.92	0.	4.92	0.	1.7
time (sec)	N/A	0.019	0.004	0.003	0.	1.463	0.	1.25

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	A	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	190	0	0
normalized size	1	0.	0.	0.	0.	3.02	0.	0.
time (sec)	N/A	0.027	0.026	0.009	0.	5.359	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	66	0	180	0	0	0	0	0
normalized size	1	0.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.522	0.019	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	F	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	319	34	438	0	1173	0	294
normalized size	1	4.09	0.44	5.62	0.	15.04	0.	3.77
time (sec)	N/A	0.568	0.417	0.147	0.	1.943	0.	1.35

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	87	753	0	2399	0	369
normalized size	1	1.	0.69	5.98	0.	19.04	0.	2.93
time (sec)	N/A	0.157	0.036	0.075	0.	2.024	0.	1.35

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	0	0	149	0	0
normalized size	1	1.	1.09	0.	0.	6.77	0.	0.
time (sec)	N/A	0.062	1.004	0.025	0.	6.059	0.	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.469	0.023	0.	0.	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	47	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.448	0.023	0.	0.	0.	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	330	1528	0	0	0	0
normalized size	1	1.	1.79	8.3	0.	0.	0.	0.
time (sec)	N/A	0.233	0.57	0.382	0.	0.	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	90	1036	0	0	0	0
normalized size	1	1.	0.69	7.91	0.	0.	0.	0.
time (sec)	N/A	0.1	0.057	0.019	0.	0.	0.	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	419	514	0	647	0	0
normalized size	1	1.	7.76	9.52	0.	11.98	0.	0.
time (sec)	N/A	0.253	1.824	0.034	0.	111.351	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	416	517	0	647	0	0
normalized size	1	1.	7.85	9.75	0.	12.21	0.	0.
time (sec)	N/A	0.26	1.427	0.053	0.	118.877	0.	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	160	69	0	481	0	221
normalized size	1	1.	1.9	0.82	0.	5.73	0.	2.63
time (sec)	N/A	0.122	0.31	0.018	0.	1.755	0.	1.347

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	9	45	36	20
normalized size	1	1.	0.85	1.15	0.45	2.25	1.8	1.
time (sec)	N/A	0.004	0.005	0.002	1.875	1.613	0.441	1.237

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.012	0.017	0.	0.	0.	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	13884	242984	0	698	0	0
normalized size	1	1.	157.77	2761.18	0.	7.93	0.	0.
time (sec)	N/A	0.249	6.505	0.125	0.	31.048	0.	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	15147	269221	0	699	0	0
normalized size	1	1.	172.12	3059.33	0.	7.94	0.	0.
time (sec)	N/A	0.329	6.529	0.135	0.	30.33	0.	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	148	0	0	385	0	0
normalized size	1	1.	3.22	0.	0.	8.37	0.	0.
time (sec)	N/A	0.622	1.076	0.042	0.	71.298	0.	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	161	0	0	396	0	0
normalized size	1	1.	3.5	0.	0.	8.61	0.	0.
time (sec)	N/A	0.624	1.162	0.036	0.	69.054	0.	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	148	0	0	385	0	0
normalized size	1	1.	3.22	0.	0.	8.37	0.	0.
time (sec)	N/A	1.171	0.161	0.02	0.	71.951	0.	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	161	0	0	396	0	0
normalized size	1	1.	3.5	0.	0.	8.61	0.	0.
time (sec)	N/A	1.168	0.185	0.023	0.	68.296	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	75	147	127	317	17	81
normalized size	1	1.	3.95	7.74	6.68	16.68	0.89	4.26
time (sec)	N/A	0.557	1.345	0.049	1.239	1.874	171.598	1.407

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	123	120	0	0	1281	0	0
normalized size	1	1.37	1.33	0.	0.	14.23	0.	0.
time (sec)	N/A	0.114	0.147	0.165	0.	79.788	0.	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	738	0	0
normalized size	1	0.	0.	0.	0.	7.17	0.	0.
time (sec)	N/A	0.534	0.178	0.16	0.	47.264	0.	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	47	46	88	0	132	0	0
normalized size	1	0.96	0.94	1.8	0.	2.69	0.	0.
time (sec)	N/A	0.12	0.02	0.016	0.	2.201	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	383	555	0	0	0	0
normalized size	1	1.	4.79	6.94	0.	0.	0.	0.
time (sec)	N/A	0.449	0.798	0.06	0.	0.	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	0	1
normalized size	1	1.	1.	2.	1.	4.	0.	1.
time (sec)	N/A	0.001	0.	0.	1.011	1.624	0.053	1.13

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	1932	234	0	204	0	261
normalized size	1	1.	15.84	1.92	0.	1.67	0.	2.14
time (sec)	N/A	0.154	4.14	0.032	0.	1.73	0.	1.174

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	149	1910	234	0	204	0	261
normalized size	1	1.22	15.66	1.92	0.	1.67	0.	2.14
time (sec)	N/A	0.388	1.587	0.052	0.	1.799	0.	1.238

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	34	0	31	32	22	73	73	109
normalized size	1	0.	0.91	0.94	0.65	2.15	2.15	3.21
time (sec)	N/A	0.065	0.034	0.	1.514	1.759	169.789	1.129

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	1671	1597	0	537	0	0
normalized size	1	1.	9.44	9.02	0.	3.03	0.	0.
time (sec)	N/A	0.089	6.193	0.568	0.	2.576	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	243	2787	2992	0	292	0	0
normalized size	1	2.43	27.87	29.92	0.	2.92	0.	0.
time (sec)	N/A	0.139	6.103	0.816	0.	1.894	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [23] had the largest ratio of [0.7895]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.	19	0.21
2	A	4	4	1.	23	0.174
3	A	4	4	1.	21	0.19
4	A	4	4	1.	21	0.19
5	A	4	4	1.	33	0.121
6	A	4	4	1.	35	0.114
7	A	4	4	1.	36	0.111
8	A	4	4	1.	36	0.111
9	A	4	4	1.	24	0.167
10	A	4	4	1.	20	0.2
11	A	4	4	1.	24	0.167
12	A	4	4	1.	22	0.182
13	A	4	4	1.	22	0.182
14	A	8	8	1.	15	0.533
15	A	8	8	1.	17	0.471
16	A	8	8	1.	15	0.533
17	A	8	8	1.	17	0.471
18	A	1	1	1.	25	0.04
19	A	3	3	1.	25	0.12
20	F	0	0	N/A	0	N/A
21	F	0	0	N/A	0	N/A
22	F	0	0	N/A	0	N/A
23	A	23	15	1.29	19	0.79
24	A	15	14	1.23	19	0.737
25	A	14	13	1.28	19	0.684
26	A	12	11	1.34	17	0.647
27	F	0	0	N/A	0	N/A
28	F	0	0	N/A	0	N/A
29	A	10	6	1.	19	0.316
30	A	8	6	1.	19	0.316
31	A	7	6	1.	19	0.316
32	A	5	4	1.	17	0.235
33	F	0	0	N/A	0	N/A
34	F	0	0	N/A	0	N/A
35	F	0	0	N/A	0	N/A
36	A	22	14	1.36	19	0.737
37	A	14	13	1.28	19	0.684
38	A	13	12	1.38	19	0.632
39	A	11	10	1.42	17	0.588
40	F	0	0	N/A	0	N/A
41	F	0	0	N/A	0	N/A
42	F	0	0	N/A	0	N/A
43	A	2	2	1.	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	2	2	1.	32	0.062
45	A	2	2	1.	30	0.067
46	A	2	2	1.	30	0.067
47	A	2	2	1.	53	0.038
48	A	2	2	1.	55	0.036
49	A	2	2	1.	56	0.036
50	A	2	2	1.	56	0.036
51	A	2	2	1.	30	0.067
52	A	4	4	1.	24	0.167
53	A	4	4	1.	28	0.143
54	A	4	4	1.	26	0.154
55	A	4	4	1.	26	0.154
56	A	4	4	1.	24	0.167
57	A	4	4	1.	28	0.143
58	A	4	4	1.	26	0.154
59	A	4	4	1.	26	0.154
60	A	4	4	1.	38	0.105
61	A	4	4	1.	40	0.1
62	A	4	4	1.	41	0.098
63	A	4	4	1.	41	0.098
64	A	4	4	1.	29	0.138
65	A	4	4	1.	20	0.2
66	A	4	4	1.	24	0.167
67	A	4	4	1.	22	0.182
68	A	4	4	1.	22	0.182
69	A	4	4	1.	34	0.118
70	A	4	4	1.	36	0.111
71	A	4	4	1.	37	0.108
72	A	4	4	1.	37	0.108
73	A	4	4	1.	25	0.16
74	A	2	2	1.	20	0.1
75	A	2	2	1.	22	0.091
76	A	2	2	1.	20	0.1
77	A	2	2	1.	22	0.091
78	A	2	2	1.	43	0.047
79	A	2	2	1.	44	0.045
80	A	2	2	1.	45	0.044
81	A	2	2	1.	46	0.043
82	A	2	2	1.	30	0.067
83	A	4	4	1.	22	0.182
84	A	4	4	1.	22	0.182
85	A	4	4	1.	20	0.2
86	A	4	4	1.	24	0.167
87	A	4	4	1.	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	4	4	1.	35	0.114
89	A	4	4	1.	36	0.111
90	A	4	4	1.	38	0.105
91	A	4	4	1.	29	0.138
92	A	4	4	1.	18	0.222
93	A	4	4	1.	18	0.222
94	A	4	4	1.	16	0.25
95	A	4	4	1.	20	0.2
96	A	4	4	1.	31	0.129
97	A	4	4	1.	31	0.129
98	A	4	4	1.	32	0.125
99	A	4	4	1.	34	0.118
100	A	4	4	1.	25	0.16
101	A	2	2	1.	30	0.067
102	A	2	2	1.	36	0.056
103	A	2	2	1.	34	0.059
104	A	2	2	1.	32	0.062
105	A	2	2	1.	58	0.034
106	A	2	2	1.	61	0.033
107	A	2	2	1.	62	0.032
108	A	2	2	1.	61	0.033
109	A	2	2	1.	52	0.038
110	A	2	2	1.	55	0.036
111	A	2	2	1.	56	0.036
112	A	2	2	1.	55	0.036
113	A	2	2	1.	30	0.067
114	A	2	2	1.	36	0.056
115	A	2	2	1.	34	0.059
116	A	2	2	1.	32	0.062
117	A	2	2	1.	58	0.034
118	A	2	2	1.	61	0.033
119	A	2	2	1.	62	0.032
120	A	2	2	1.	61	0.033
121	A	2	2	1.	52	0.038
122	A	2	2	1.	55	0.036
123	A	2	2	1.	56	0.036
124	A	2	2	1.	55	0.036
125	A	4	4	1.	23	0.174
126	A	4	4	1.	25	0.16
127	A	4	4	1.	25	0.16
128	A	4	4	1.	29	0.138
129	A	4	4	1.	27	0.148
130	A	4	4	1.	27	0.148
131	A	4	4	1.	42	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	4	4	1.	44	0.091
133	A	4	4	1.	45	0.089
134	A	4	4	1.	45	0.089
135	A	4	4	1.	21	0.19
136	A	4	4	1.	25	0.16
137	A	4	4	1.	23	0.174
138	A	4	4	1.	23	0.174
139	A	4	4	1.	23	0.174
140	A	4	4	1.	38	0.105
141	A	4	4	1.	40	0.1
142	A	4	4	1.	41	0.098
143	A	4	4	1.	41	0.098
144	A	6	6	1.	25	0.24
145	A	6	6	1.	29	0.207
146	A	6	6	1.	27	0.222
147	A	6	6	1.	27	0.222
148	A	6	6	1.	27	0.222
149	A	6	6	1.	31	0.194
150	A	6	6	1.	29	0.207
151	A	6	6	1.	29	0.207
152	A	5	5	1.	21	0.238
153	A	5	5	1.	25	0.2
154	A	5	5	1.	23	0.217
155	A	5	5	1.	23	0.217
156	A	5	5	1.	23	0.217
157	A	5	5	1.	27	0.185
158	A	5	5	1.	25	0.2
159	A	5	5	1.	25	0.2
160	A	8	8	1.	16	0.5
161	A	8	8	1.	18	0.444
162	A	8	8	1.	16	0.5
163	A	8	8	1.	18	0.444
164	A	8	8	1.	22	0.364
165	A	8	8	1.	24	0.333
166	A	8	8	1.	22	0.364
167	A	8	8	1.	24	0.333
168	A	6	6	1.	18	0.333
169	A	6	6	1.	20	0.3
170	A	6	6	1.	18	0.333
171	A	6	6	1.	20	0.3
172	A	1	1	1.	31	0.032
173	A	3	3	1.	30	0.1
174	A	2	1	1.	18	0.056
175	A	2	1	1.	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	2	1	1.	15	0.067
177	A	3	2	1.	18	0.111
178	A	2	1	1.	20	0.05
179	A	2	1	1.	18	0.056
180	A	2	1	1.	17	0.059
181	A	3	2	1.	20	0.1
182	A	2	1	1.	20	0.05
183	A	2	1	1.	18	0.056
184	A	2	1	1.	17	0.059
185	A	3	2	1.	20	0.1
186	A	7	2	1.	20	0.1
187	A	7	2	1.	20	0.1
188	A	7	2	1.	20	0.1
189	A	5	2	1.	20	0.1
190	A	5	2	1.	18	0.111
191	A	5	2	1.	17	0.118
192	A	8	3	1.	20	0.15
193	A	8	3	1.	20	0.15
194	A	5	2	1.	22	0.091
195	A	8	3	1.	20	0.15
196	A	13	12	1.	19	0.632
197	F	0	0	N/A	0	N/A
198	A	2	2	1.	27	0.074
199	A	2	2	1.	29	0.069
200	A	2	2	1.	27	0.074
201	A	2	2	1.	29	0.069
202	A	2	2	1.	31	0.065
203	A	2	2	1.	35	0.057
204	A	2	2	1.	33	0.061
205	A	2	2	1.	33	0.061
206	A	11	10	1.	19	0.526
207	A	10	9	1.	19	0.474
208	A	8	6	1.	17	0.353
209	A	2	2	1.	11	0.182
210	A	15	13	1.	19	0.684
211	A	32	15	1.	19	0.79
212	A	9	8	1.	19	0.421
213	A	8	7	1.	19	0.368
214	A	6	5	1.	17	0.294
215	A	1	1	1.	11	0.091
216	A	7	7	1.	19	0.368
217	A	11	11	1.	19	0.579
218	A	12	12	1.	19	0.632
219	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	4	4	1.	19	0.21
221	A	3	3	1.	17	0.176
222	A	2	2	1.	11	0.182
223	A	14	13	1.	19	0.684
224	A	10	3	1.	20	0.15
225	A	10	3	1.	22	0.136
226	A	16	8	1.	24	0.333
227	A	4	3	1.27	19	0.158
228	A	4	3	0.94	19	0.158
229	A	4	3	1.31	19	0.158
230	A	4	4	1.18	19	0.21
231	A	4	3	1.31	19	0.158
232	A	3	3	1.	17	0.176
233	A	4	3	1.3	15	0.2
234	A	5	4	1.32	19	0.21
235	A	4	3	1.33	19	0.158
236	A	5	4	1.31	19	0.21
237	A	9	5	1.	19	0.263
238	A	3	3	1.	17	0.176
239	A	8	4	1.	15	0.267
240	A	9	5	1.01	19	0.263
241	A	8	5	1.	19	0.263
242	A	9	6	1.01	19	0.316
243	A	4	4	0.97	19	0.21
244	A	3	3	1.	17	0.176
245	A	2	2	1.	15	0.133
246	A	5	5	1.03	19	0.263
247	A	3	3	1.	19	0.158
248	A	6	5	1.08	19	0.263
249	A	4	3	1.1	21	0.143
250	A	4	3	1.11	21	0.143
251	A	4	3	1.12	21	0.143
252	A	3	3	1.	19	0.158
253	A	7	7	1.21	21	0.333
254	A	7	7	1.14	21	0.333
255	A	6	6	1.26	21	0.286
256	A	5	5	1.23	17	0.294
257	A	5	5	1.23	21	0.238
258	A	6	6	1.25	21	0.286
259	A	4	4	1.	15	0.267
260	A	6	6	1.	17	0.353
261	A	3	3	1.	15	0.2
262	A	3	3	1.	17	0.176
263	A	5	5	1.	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
264	A	4	4	1.	26	0.154
265	A	3	3	1.	24	0.125
266	A	4	3	1.	26	0.115
267	A	3	3	1.	26	0.115
268	A	4	4	1.	26	0.154
269	A	5	5	1.	26	0.192
270	A	7	7	1.	26	0.269
271	A	6	6	1.	26	0.231
272	A	5	5	1.	22	0.227
273	A	7	7	1.	26	0.269
274	A	7	7	1.	26	0.269
275	A	8	7	1.	26	0.269
276	A	6	6	1.	26	0.231
277	A	5	5	1.	26	0.192
278	A	4	4	1.	24	0.167
279	A	5	4	1.	26	0.154
280	A	4	4	1.	26	0.154
281	A	5	5	1.	26	0.192
282	A	6	6	1.	26	0.231
283	A	8	8	1.	26	0.308
284	A	7	7	1.	26	0.269
285	A	6	6	1.	22	0.273
286	A	7	7	1.	26	0.269
287	A	8	7	1.	26	0.269
288	A	9	7	1.	26	0.269
289	A	3	3	1.	21	0.143
290	A	3	3	1.	23	0.13
291	A	3	3	1.	23	0.13
292	A	5	5	1.	23	0.217
293	A	4	4	1.	25	0.16
294	A	4	4	1.	23	0.174
295	A	4	4	1.	28	0.143
296	A	5	5	1.	26	0.192
297	A	4	4	1.	26	0.154
298	A	3	3	1.	24	0.125
299	A	4	3	1.	26	0.115
300	A	3	3	1.	26	0.115
301	A	4	4	1.	26	0.154
302	A	7	7	1.	26	0.269
303	A	6	6	1.	26	0.231
304	A	5	5	1.	22	0.227
305	A	7	7	1.	26	0.269
306	A	7	7	1.	26	0.269
307	A	6	5	0.98	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	5	4	1.	26	0.154
309	A	4	4	1.	24	0.167
310	A	5	4	1.	26	0.154
311	A	4	4	1.	26	0.154
312	A	5	4	1.	26	0.154
313	A	8	8	1.	26	0.308
314	A	7	7	1.	26	0.269
315	A	6	6	1.	22	0.273
316	A	7	7	1.	26	0.269
317	A	8	7	1.	26	0.269
318	A	9	9	1.2	21	0.429
319	A	8	8	1.28	21	0.381
320	A	5	5	1.	19	0.263
321	A	9	9	1.92	21	0.429
322	A	6	6	1.35	21	0.286
323	A	7	7	1.25	21	0.333
324	A	9	8	1.02	21	0.381
325	A	8	8	1.3	21	0.381
326	A	7	7	1.31	21	0.333
327	A	6	6	1.31	17	0.353
328	A	8	8	1.33	21	0.381
329	A	8	8	1.3	21	0.381
330	A	9	8	1.28	21	0.381
331	A	10	9	1.25	21	0.429
332	A	9	8	1.29	21	0.381
333	A	6	6	1.	19	0.316
334	A	10	10	1.63	21	0.476
335	A	7	6	1.23	21	0.286
336	A	8	7	1.27	21	0.333
337	A	10	9	0.98	21	0.429
338	A	9	9	1.3	21	0.429
339	A	8	8	1.3	21	0.381
340	A	7	7	1.34	17	0.412
341	A	8	8	1.35	21	0.381
342	A	9	8	1.34	21	0.381
343	A	10	8	1.31	21	0.381
344	A	9	9	1.19	21	0.429
345	A	8	8	1.28	21	0.381
346	A	5	5	1.	19	0.263
347	A	9	9	1.92	21	0.429
348	A	6	6	1.37	21	0.286
349	A	7	7	1.23	21	0.333
350	A	8	8	1.12	21	0.381
351	A	7	7	1.12	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
352	A	6	6	1.12	17	0.353
353	A	8	8	1.13	21	0.381
354	A	8	8	1.13	21	0.381
355	A	10	9	1.04	21	0.429
356	A	9	8	1.29	21	0.381
357	A	6	5	1.04	19	0.263
358	A	10	10	1.6	21	0.476
359	A	7	6	1.19	21	0.286
360	A	8	7	1.16	21	0.333
361	A	9	9	1.16	21	0.429
362	A	8	8	1.16	21	0.381
363	A	7	7	1.15	17	0.412
364	A	8	8	1.16	21	0.381
365	A	9	8	1.16	21	0.381
366	A	6	5	1.	19	0.263
367	A	5	5	1.	19	0.263
368	A	4	4	1.	19	0.21
369	A	2	2	1.	19	0.105
370	A	3	3	1.	19	0.158
371	A	4	4	1.	22	0.182
372	A	5	5	1.	19	0.263
373	A	6	5	1.	22	0.227
374	A	8	5	1.	33	0.152
375	A	9	6	1.	30	0.2
376	A	6	6	1.	19	0.316
377	A	3	3	1.	19	0.158
378	A	4	4	1.	19	0.21
379	A	2	2	1.	19	0.105
380	A	4	4	1.	17	0.235
381	A	3	3	1.	19	0.158
382	A	4	4	1.	19	0.21
383	A	6	6	1.	19	0.316
384	A	2	2	1.	19	0.105
385	A	2	2	1.	19	0.105
386	A	5	5	1.	19	0.263
387	A	4	4	1.	19	0.21
388	A	3	3	1.	17	0.176
389	A	3	3	1.	19	0.158
390	A	4	4	1.	19	0.21
391	A	6	6	1.	19	0.316
392	A	5	5	1.	19	0.263
393	A	2	2	1.	21	0.095
394	A	2	2	1.	19	0.105
395	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
396	C	5	3	2.35	54	0.056
397	A	2	2	1.	26	0.077
398	A	2	2	1.	7	0.286
399	A	3	2	1.	15	0.133
400	A	4	2	1.	22	0.091
401	A	5	2	1.	25	0.08
402	A	5	2	1.	23	0.087
403	A	2	1	1.	21	0.048
404	A	7	4	1.	25	0.16
405	A	7	4	1.	25	0.16
406	A	9	7	1.	25	0.28
407	A	8	6	1.	23	0.261
408	A	7	5	1.81	21	0.238
409	A	9	8	1.	25	0.32
410	A	9	8	1.	25	0.32
411	A	10	2	1.	25	0.08
412	A	10	2	1.	23	0.087
413	B	6	2	2.36	21	0.095
414	A	8	4	1.	25	0.16
415	A	14	5	1.38	25	0.2
416	A	3	3	1.	15	0.2
417	A	3	3	1.	15	0.2
418	A	2	1	1.	17	0.059
419	A	5	3	1.	23	0.13
420	A	5	4	1.	23	0.174
421	A	3	2	1.	21	0.095
422	A	4	3	1.	19	0.158
423	A	6	5	1.	23	0.217
424	A	4	3	1.	23	0.13
425	A	6	5	1.	23	0.217
426	A	5	2	1.	25	0.08
427	A	5	2	1.	25	0.08
428	A	3	2	1.	23	0.087
429	A	8	4	1.	21	0.19
430	A	7	4	1.	25	0.16
431	A	9	5	1.	25	0.2
432	A	8	6	1.	25	0.24
433	A	7	5	1.	25	0.2
434	A	9	8	1.	23	0.348
435	A	9	8	1.	21	0.381
436	A	6	4	1.	25	0.16
437	A	7	5	1.	25	0.2
438	A	10	2	1.	25	0.08
439	A	6	2	1.	25	0.08

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
440	A	8	4	1.	25	0.16
441	A	14	5	1.42	23	0.217
442	A	16	5	1.68	21	0.238
443	A	4	3	1.	27	0.111
444	A	6	4	1.	42	0.095
445	A	6	5	1.	42	0.119
446	A	4	3	1.	40	0.075
447	A	5	4	1.	39	0.103
448	A	7	6	1.	42	0.143
449	A	5	4	1.	42	0.095
450	A	7	6	1.	42	0.143
451	A	15	7	1.	39	0.18
452	A	9	4	1.	35	0.114
453	A	4	3	1.	25	0.12
454	A	3	2	1.	25	0.08
455	A	3	2	1.	25	0.08
456	A	4	3	1.	23	0.13
457	A	3	2	1.	25	0.08
458	A	3	2	1.	25	0.08
459	A	3	2	1.	25	0.08
460	A	6	5	1.	27	0.185
461	A	6	5	1.	27	0.185
462	A	6	5	1.	27	0.185
463	A	5	5	1.	27	0.185
464	A	5	5	1.	27	0.185
465	A	6	5	1.	27	0.185
466	A	3	2	1.	17	0.118
467	A	3	2	1.	26	0.077
468	A	1	1	1.	17	0.059
469	A	1	1	1.	15	0.067
470	A	1	1	1.	15	0.067
471	A	1	1	1.	25	0.04
472	A	4	3	1.	28	0.107
473	A	3	2	1.	28	0.071
474	A	3	2	1.	28	0.071
475	A	4	3	1.	26	0.115
476	A	3	2	1.	28	0.071
477	A	3	2	1.	28	0.071
478	A	3	2	1.	28	0.071
479	A	6	5	1.	30	0.167
480	A	6	5	1.	30	0.167
481	A	6	5	1.	30	0.167
482	A	5	5	1.	30	0.167
483	A	5	5	1.	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
484	A	6	5	1.	30	0.167
485	A	3	2	1.	21	0.095
486	A	3	2	1.	19	0.105
487	A	3	2	1.	13	0.154
488	A	2	2	1.	21	0.095
489	A	2	2	1.	21	0.095
490	A	3	2	1.	23	0.087
491	A	3	2	1.	21	0.095
492	A	3	2	1.	15	0.133
493	A	2	2	1.	23	0.087
494	A	2	2	1.	23	0.087
495	A	3	2	1.	23	0.087
496	A	3	2	1.	23	0.087
497	A	3	2	1.	23	0.087
498	A	2	2	1.	23	0.087
499	A	2	2	1.	23	0.087
500	A	2	2	1.	23	0.087
501	A	3	2	1.	25	0.08
502	A	3	2	1.	25	0.08
503	A	3	2	1.	25	0.08
504	A	2	2	1.	25	0.08
505	A	2	2	1.	25	0.08
506	A	2	2	1.	25	0.08
507	A	4	3	1.	56	0.054
508	A	4	3	1.	54	0.056
509	A	4	3	1.	33	0.091
510	A	2	2	1.	56	0.036
511	A	3	3	1.	56	0.054
512	A	5	4	1.	33	0.121
513	A	3	3	1.	56	0.054
514	A	4	3	1.	58	0.052
515	A	4	3	1.	58	0.052
516	A	3	3	1.	58	0.052
517	A	3	3	1.	58	0.052
518	A	4	4	1.	58	0.069
519	A	5	4	1.	62	0.065
520	A	4	4	1.	62	0.065
521	A	4	4	1.	62	0.065
522	A	5	5	1.	60	0.083
523	A	7	6	1.	30	0.2
524	A	4	3	1.	37	0.081
525	A	4	3	1.	37	0.081
526	A	2	2	1.	37	0.054
527	A	2	2	1.	37	0.054

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
528	A	2	2	1.	42	0.048
529	A	2	2	1.	42	0.048
530	A	4	4	1.	30	0.133
531	A	4	4	1.	30	0.133
532	A	2	2	1.	42	0.048
533	A	2	2	1.	42	0.048
534	A	2	2	1.45	51	0.039
535	A	2	2	1.45	51	0.039
536	A	2	2	1.	40	0.05
537	A	2	2	1.	40	0.05
538	A	4	4	1.	32	0.125
539	A	4	4	1.	32	0.125
540	A	2	2	1.	51	0.039
541	A	2	2	1.	51	0.039
542	A	2	2	1.	56	0.036
543	A	2	2	1.	56	0.036
544	A	4	2	1.	29	0.069
545	A	4	2	1.	29	0.069
546	A	3	2	1.	27	0.074
547	A	7	6	1.	29	0.207
548	A	8	6	1.	29	0.207
549	A	8	7	1.	29	0.241
550	A	4	3	1.	25	0.12
551	A	7	6	1.	29	0.207
552	A	4	2	1.	29	0.069
553	A	4	2	1.	29	0.069
554	A	3	2	1.	29	0.069
555	A	7	6	1.	29	0.207
556	A	8	6	1.	29	0.207
557	A	10	10	1.	29	0.345
558	A	9	9	1.	27	0.333
559	A	9	9	1.	25	0.36
560	A	10	10	1.	29	0.345
561	A	10	10	1.	29	0.345
562	A	4	4	1.	25	0.16
563	A	4	4	1.	29	0.138
564	A	3	2	1.	31	0.065
565	A	3	3	1.	15	0.2
566	A	5	5	1.	15	0.333
567	A	4	3	1.	15	0.2
568	A	4	3	1.	13	0.231
569	A	4	3	1.	13	0.231
570	A	4	3	1.	15	0.2
571	A	9	9	1.	13	0.692

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
572	A	4	3	1.	13	0.231
573	A	4	3	1.	13	0.231
574	A	9	9	1.	15	0.6
575	A	3	3	1.	13	0.231
576	A	4	3	1.	13	0.231
577	A	13	10	1.	15	0.667
578	A	10	7	1.	19	0.368
579	A	4	3	1.	19	0.158
580	A	10	9	1.	21	0.429
581	A	3	3	1.	26	0.115
582	A	3	3	1.	26	0.115
583	A	3	3	1.	24	0.125
584	A	3	3	1.	23	0.13
585	A	3	3	1.	26	0.115
586	A	3	3	1.	26	0.115
587	A	4	3	1.	17	0.176
588	A	5	5	1.	17	0.294
589	A	4	4	1.	15	0.267
590	A	2	2	1.	9	0.222
591	A	5	5	1.	17	0.294
592	A	3	3	1.	17	0.176
593	A	4	4	1.	17	0.235
594	A	5	5	1.	17	0.294
595	A	3	3	1.	28	0.107
596	A	3	3	1.	28	0.107
597	A	3	3	1.	26	0.115
598	A	3	3	1.	25	0.12
599	A	3	3	1.	28	0.107
600	A	3	3	1.	28	0.107
601	A	8	6	1.	21	0.286
602	A	1	1	1.	17	0.059
603	A	1	1	1.	21	0.048
604	A	1	1	1.	17	0.059
605	A	1	1	1.	19	0.053
606	A	1	1	1.	21	0.048
607	A	4	3	1.	25	0.12
608	A	3	2	1.	17	0.118
609	A	4	3	1.	27	0.111
610	A	4	3	1.	25	0.12
611	A	5	4	1.7	22	0.182
612	A	4	3	1.	29	0.103
613	A	1	1	1.	176	0.006
614	A	1	1	1.	174	0.006
615	A	1	1	1.	164	0.006

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
616	F	0	0	N/A	0	N/A
617	F	0	0	N/A	0	N/A
618	A	4	3	1.	19	0.158
619	A	4	3	1.	19	0.158
620	A	4	3	1.	17	0.176
621	A	4	3	1.	15	0.2
622	A	7	6	1.	19	0.316
623	A	6	6	1.	19	0.316
624	A	6	6	1.	19	0.316
625	A	4	3	1.	21	0.143
626	A	4	3	1.	21	0.143
627	A	4	3	1.	19	0.158
628	A	4	3	1.	17	0.176
629	A	7	6	1.	21	0.286
630	A	8	7	1.	21	0.333
631	A	9	8	1.	21	0.381
632	A	4	3	1.	19	0.158
633	A	4	3	1.	19	0.158
634	A	4	3	1.	17	0.176
635	A	4	3	1.	15	0.2
636	A	7	6	1.	19	0.316
637	A	8	7	1.	19	0.368
638	A	9	7	1.	19	0.368
639	A	4	3	1.	19	0.158
640	A	4	3	1.	19	0.158
641	A	4	3	1.	17	0.176
642	A	4	3	1.	15	0.2
643	A	7	6	1.	19	0.316
644	A	8	7	1.	19	0.368
645	A	9	7	1.	19	0.368
646	A	4	3	1.	21	0.143
647	A	4	3	1.	21	0.143
648	A	4	3	1.	19	0.158
649	A	4	3	1.	17	0.176
650	A	6	5	1.	21	0.238
651	A	7	6	1.	21	0.286
652	A	8	6	1.	21	0.286
653	A	4	3	1.	19	0.158
654	A	4	3	1.	19	0.158
655	A	4	3	1.	17	0.176
656	A	4	3	1.	15	0.2
657	A	6	4	1.	19	0.21
658	A	8	6	1.	17	0.353
659	A	7	6	1.	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
660	A	6	6	1.	17	0.353
661	A	5	5	1.	17	0.294
662	A	6	6	1.	17	0.353
663	A	7	6	1.	17	0.353
664	A	8	6	1.	19	0.316
665	A	7	6	1.	19	0.316
666	A	6	6	1.	19	0.316
667	A	5	5	1.	19	0.263
668	A	6	6	1.	19	0.316
669	A	7	6	1.	19	0.316
670	A	2	2	1.	13	0.154
671	A	5	5	1.	17	0.294
672	A	6	5	1.	21	0.238
673	A	7	5	1.	25	0.2
674	A	8	5	1.	29	0.172
675	A	8	6	1.	20	0.3
676	A	7	6	1.	20	0.3
677	A	6	6	1.	18	0.333
678	A	2	2	1.	20	0.1
679	A	4	3	1.	20	0.15
680	A	4	3	1.	20	0.15
681	A	2	2	1.	18	0.111
682	A	2	2	1.	20	0.1
683	A	3	2	1.	22	0.091
684	A	3	3	1.	17	0.176
685	A	3	3	1.	23	0.13
686	A	3	2	1.	22	0.091
687	A	5	4	1.	34	0.118
688	A	5	5	1.	31	0.161
689	A	6	4	1.	47	0.085
690	A	9	5	1.	58	0.086
691	A	4	4	1.	11	0.364
692	A	6	6	1.	11	0.546
693	A	2	1	1.	17	0.059
694	A	8	6	1.	13	0.462
695	A	2	1	1.	16	0.062
696	A	4	2	1.	14	0.143
697	A	5	4	1.	12	0.333
698	A	4	2	1.	13	0.154
699	A	4	2	1.	13	0.154
700	A	4	2	1.	15	0.133
701	A	5	5	1.	12	0.417
702	A	6	5	1.	14	0.357
703	A	5	4	1.	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
704	A	5	4	1.	17	0.235
705	A	5	4	1.	17	0.235
706	A	4	3	1.	13	0.231
707	A	7	5	1.	18	0.278
708	A	7	5	1.	20	0.25
709	A	8	5	1.	18	0.278
710	A	4	3	1.	26	0.115
711	A	10	6	0.72	17	0.353
712	A	3	1	1.	27	0.037
713	A	5	3	1.	17	0.176
714	A	5	3	1.	17	0.176
715	A	5	3	1.	23	0.13
716	A	5	3	1.	17	0.176
717	A	6	2	1.	23	0.087
718	A	5	1	1.	25	0.04
719	A	5	2	1.	21	0.095
720	A	3	2	1.	23	0.087
721	A	4	3	1.18	16	0.188
722	A	3	1	1.	28	0.036
723	A	5	5	1.	16	0.312
724	A	4	3	1.	22	0.136
725	A	8	4	1.	21	0.19
726	B	16	10	2.8	20	0.5
727	A	9	6	1.	25	0.24
728	A	6	4	1.	35	0.114
729	A	2	2	1.	13	0.154
730	A	3	3	1.	15	0.2
731	A	3	3	1.	13	0.231
732	A	4	4	1.	11	0.364
733	A	3	3	1.	18	0.167
734	A	4	4	1.	17	0.235
735	A	4	4	1.	18	0.222
736	A	5	5	1.	17	0.294
737	A	2	2	1.	16	0.125
738	A	2	2	1.	21	0.095
739	A	3	3	1.	26	0.115
740	A	3	3	1.	25	0.12
741	A	3	3	1.	12	0.25
742	A	3	3	1.	15	0.2
743	A	3	3	1.	15	0.2
744	A	3	3	1.	15	0.2
745	A	3	3	1.	17	0.176
746	A	4	4	1.	15	0.267
747	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
748	A	3	3	1.	22	0.136
749	A	4	4	1.	20	0.2
750	A	7	7	1.	20	0.35
751	A	2	2	1.	15	0.133
752	A	5	5	1.	21	0.238
753	A	8	6	1.	18	0.333
754	A	5	4	1.	18	0.222
755	A	6	4	1.	18	0.222
756	A	3	2	1.	16	0.125
757	A	3	2	1.	16	0.125
758	A	3	2	1.	16	0.125
759	A	10	9	1.	18	0.5
760	A	5	4	1.	18	0.222
761	A	6	5	1.	18	0.278
762	A	5	5	1.4	35	0.143
763	A	6	6	1.4	28	0.214
764	A	7	7	1.	23	0.304
765	A	6	6	1.	23	0.261
766	A	3	3	1.	23	0.13
767	A	6	6	1.	23	0.261
768	A	7	7	1.	23	0.304
769	A	7	7	1.	19	0.368
770	A	6	6	1.	19	0.316
771	A	3	3	1.	19	0.158
772	A	6	6	1.	19	0.316
773	A	7	7	1.	19	0.368
774	A	6	6	1.	31	0.194
775	A	5	5	1.	31	0.161
776	A	2	2	1.	31	0.065
777	A	5	5	1.	31	0.161
778	A	5	5	1.	34	0.147
779	A	2	2	1.	34	0.059
780	A	5	5	1.	34	0.147
781	A	8	8	1.	24	0.333
782	A	7	7	1.	24	0.292
783	A	3	3	1.	24	0.125
784	A	7	7	1.	24	0.292
785	A	8	8	1.	24	0.333
786	A	14	13	1.	26	0.5
787	A	12	12	1.	26	0.462
788	A	7	7	1.	26	0.269
789	A	10	10	1.	26	0.385
790	A	12	12	1.	26	0.462
791	A	15	13	1.	28	0.464

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
792	A	13	13	1.	28	0.464
793	A	11	11	1.	28	0.393
794	A	10	9	1.	28	0.321
795	A	13	12	1.	28	0.429
796	A	4	4	1.	19	0.21
797	A	10	10	1.	19	0.526
798	A	3	3	1.	19	0.158
799	A	9	9	1.	19	0.474
800	A	4	4	1.	24	0.167
801	A	10	10	1.	24	0.417
802	A	12	11	1.	24	0.458
803	A	4	4	1.	24	0.167
804	A	10	10	1.	24	0.417
805	A	11	10	1.	27	0.37
806	A	14	7	1.	20	0.35
807	A	12	6	1.91	21	0.286
808	A	2	1	1.	22	0.045
809	A	4	2	1.	18	0.111
810	A	4	2	1.	18	0.111
811	A	5	5	1.	23	0.217
812	A	6	6	1.	22	0.273
813	A	6	6	1.	20	0.3
814	A	3	2	1.	15	0.133
815	A	3	2	1.	21	0.095
816	A	5	4	1.	19	0.21
817	A	9	6	1.	24	0.25
818	A	9	6	1.	24	0.25
819	A	10	7	1.	21	0.333
820	A	11	8	1.	13	0.615
821	A	12	8	1.	25	0.32
822	A	13	9	1.	15	0.6
823	A	14	9	1.	19	0.474
824	A	2	1	1.	35	0.029
825	A	8	5	1.	37	0.135
826	A	16	8	1.	29	0.276
827	A	16	8	1.	36	0.222
828	A	31	13	1.	43	0.302
829	A	5	4	1.	21	0.19
830	A	6	4	1.	27	0.148
831	A	8	6	1.	27	0.222
832	A	4	4	1.	27	0.148
833	A	4	4	1.	20	0.2
834	A	6	6	1.	29	0.207
835	A	2	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
836	A	3	2	1.	25	0.08
837	A	2	2	1.	28	0.071
838	A	3	3	1.	26	0.115
839	A	1	1	1.	11	0.091
840	A	2	2	1.	19	0.105
841	A	2	2	1.	15	0.133
842	A	2	2	1.	14	0.143
843	A	3	3	1.	17	0.176
844	A	3	3	1.	13	0.231
845	A	2	2	1.	14	0.143
846	A	3	3	1.	17	0.176
847	A	3	3	1.	13	0.231
848	A	1	0	1.	9	0.
849	A	4	3	1.	15	0.2
850	A	2	2	1.	13	0.154
851	A	3	3	1.	19	0.158
852	A	3	3	1.32	11	0.273
853	A	4	4	1.32	17	0.235
854	A	1	1	1.	9	0.111
855	A	2	2	1.	19	0.105
856	A	1	1	1.	7	0.143
857	A	2	2	1.	21	0.095
858	A	1	1	1.	9	0.111
859	A	2	2	1.	19	0.105
860	A	1	1	1.	7	0.143
861	A	2	2	1.	21	0.095
862	A	3	3	1.	17	0.176
863	A	4	4	1.	30	0.133
864	A	7	7	1.	20	0.35
865	A	6	6	1.	20	0.3
866	A	7	7	1.	23	0.304
867	A	6	6	1.	25	0.24
868	A	1	1	1.	11	0.091
869	A	2	2	1.	21	0.095
870	A	1	1	1.	9	0.111
871	A	2	2	1.	23	0.087
872	A	2	2	1.	11	0.182
873	A	3	3	1.	21	0.143
874	A	2	2	1.	9	0.222
875	A	3	3	1.	23	0.13
876	A	3	3	1.16	14	0.214
877	A	4	4	1.	15	0.267
878	A	7	6	1.	17	0.353
879	A	7	6	1.	17	0.353

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
880	A	10	8	1.	34	0.235
881	A	12	7	1.	30	0.233
882	A	5	4	1.	21	0.19
883	A	7	5	1.	23	0.217
884	A	9	7	1.	25	0.28
885	A	7	6	1.	25	0.24
886	A	3	3	1.	13	0.231
887	A	2	2	1.	24	0.083
888	A	2	2	1.	22	0.091
889	A	2	2	1.	30	0.067
890	A	3	3	1.	27	0.111
891	A	5	5	1.	23	0.217
892	A	7	5	1.	28	0.179
893	A	13	10	1.55	25	0.4
894	A	8	6	1.	29	0.207
895	A	6	6	1.	16	0.375
896	A	6	6	1.	16	0.375
897	A	4	4	1.	16	0.25
898	A	7	7	1.	16	0.438
899	A	8	8	1.	16	0.5
900	A	3	3	1.	24	0.125
901	A	0	0	0.	0	0.
902	A	0	0	0.	0	0.
903	A	0	0	0.	0	0.
904	A	0	0	0.	0	0.
905	A	0	0	0.	0	0.
906	A	0	0	0.	0	0.
907	A	0	0	0.	0	0.
908	A	0	0	0.	0	0.
909	A	0	0	0.	0	0.
910	A	0	0	0.	0	0.
911	A	2	2	1.	37	0.054
912	A	2	2	1.	38	0.053
913	A	5	4	1.	40	0.1
914	A	7	5	1.	40	0.125
915	A	6	5	1.	18	0.278
916	A	7	6	1.	21	0.286
917	A	8	6	1.	22	0.273
918	A	3	3	1.	21	0.143
919	A	3	3	1.	24	0.125
920	A	11	10	1.	19	0.526
921	A	10	7	1.	24	0.292
922	A	4	3	1.	19	0.158
923	A	3	3	1.	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
924	A	1	1	1.	15	0.067
925	A	1	1	1.	17	0.059
926	A	2	2	1.	17	0.118
927	A	2	2	1.	17	0.118
928	A	1	1	1.	17	0.059
929	A	6	6	1.	17	0.353
930	A	5	5	1.	11	0.454
931	A	3	3	1.	11	0.273
932	A	5	3	1.	13	0.231
933	A	2	2	1.	21	0.095
934	A	5	5	1.	16	0.312
935	A	2	2	1.	13	0.154
936	A	6	6	1.	16	0.375
937	A	5	4	1.	12	0.333
938	A	4	3	1.	18	0.167
939	A	10	6	1.	17	0.353
940	A	1	1	1.	17	0.059
941	C	3	2	1.03	27	0.074
942	A	2	2	1.	37	0.054
943	A	3	2	1.	17	0.118
944	A	5	4	1.	17	0.235
945	A	1	1	1.	15	0.067
946	A	3	3	1.	14	0.214
947	A	1	1	1.	17	0.059
948	A	2	1	1.	13	0.077
949	A	2	1	1.	15	0.067
950	A	7	4	1.	15	0.267
951	A	6	6	1.	15	0.4
952	A	1	0	1.	9	0.
953	A	1	0	1.	9	0.
954	A	2	1	1.	19	0.053
955	A	3	3	1.	15	0.2
956	A	3	3	1.	16	0.188
957	A	4	4	1.	15	0.267
958	A	5	5	1.	15	0.333
959	A	4	4	1.	25	0.16
960	A	2	2	1.	11	0.182
961	A	2	2	1.	17	0.118
962	A	6	6	1.	22	0.273
963	A	4	3	1.	13	0.231
964	A	4	3	1.	15	0.2
965	A	4	4	1.	19	0.21
966	A	4	4	1.	21	0.19
967	A	3	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
968	A	7	7	1.	19	0.368
969	A	7	6	1.	19	0.316
970	A	6	6	1.	17	0.353
971	A	10	8	1.	17	0.471
972	A	11	9	1.	17	0.529
973	A	3	3	1.	22	0.136
974	A	5	5	1.	11	0.454
975	A	5	5	1.	13	0.385
976	A	5	5	1.17	11	0.454
977	A	5	5	1.	15	0.333
978	A	4	4	1.	11	0.364
979	A	5	5	1.	13	0.385
980	A	4	4	1.	11	0.364
981	A	3	3	1.	11	0.273
982	A	2	2	1.	11	0.182
983	A	5	5	1.	19	0.263
984	A	2	2	1.	23	0.087
985	A	2	2	1.	13	0.154
986	A	3	3	1.	11	0.273
987	A	3	3	1.	15	0.2
988	A	3	3	1.	19	0.158
989	A	3	3	1.	19	0.158
990	A	3	3	1.	19	0.158
991	A	2	2	1.	15	0.133
992	A	3	3	1.	15	0.2
993	A	3	3	1.	12	0.25
994	A	3	3	1.	16	0.188
995	F	0	0	N/A	0	N/A
996	F	0	0	N/A	0	N/A
997	B	25	12	4.09	31	0.387
998	A	5	4	1.	25	0.16
999	A	2	2	1.	27	0.074
1000	A	2	2	1.	33	0.061
1001	A	2	2	1.	34	0.059
1002	A	7	6	1.	51	0.118
1003	A	2	2	1.	49	0.041
1004	A	2	2	1.	43	0.047
1005	A	2	2	1.	44	0.045
1006	A	9	8	1.	20	0.4
1007	A	3	3	1.	15	0.2
1008	A	3	3	1.	15	0.2
1009	A	1	1	1.	52	0.019
1010	A	1	1	1.	57	0.018
1011	A	2	2	1.	59	0.034

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1012	A	2	2	1.	58	0.034
1013	A	3	3	1.	58	0.052
1014	A	3	3	1.	57	0.053
1015	A	3	3	1.	66	0.045
1016	A	9	9	1.37	31	0.29
1017	F	0	0	N/A	0	N/A
1018	C	9	6	0.96	20	0.3
1019	A	2	2	1.	46	0.043
1020	A	1	0	1.	15	0.
1021	A	12	9	1.	17	0.529
1022	A	13	10	1.22	33	0.303
1023	F	0	0	N/A	0	N/A
1024	A	1	1	1.	38	0.026
1025	B	2	2	2.43	30	0.067

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/3
)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipti
cF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.177738, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2134, 218, 2137, 203}

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/3
)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipti
cF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x,\right.$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.191407, size = 148, normalized size = 1.02

$$\frac{4i\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] time = 0.117, size = 139, normalized size = 1.

$$2\frac{3/2 - i/2\sqrt{3}}{\sqrt{x^3+1}\left(2^{2/3}-1\right)}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticPi}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\frac{-3/2+i/2\sqrt{3}}{2^{2/3}-1},\sqrt{\frac{-}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x^2-2^{\frac{2}{3}}x+2\cdot 2^{\frac{1}{3}}\right)}{x^6+5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x^2 - 2^(2/3)*x + 2*2^(1/3))/(x^6 + 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.2 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.190268, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2134, 218, 2137, 203}

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```


```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```


Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \right.$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.123902, size = 148, normalized size = 0.92

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] time = 0.085, size = 143, normalized size = 0.9

$$\frac{\frac{2i}{3}\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}-2^{\frac{2}{3}}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \frac{}{-\frac{1}{2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2
+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3
^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-x^3+1}\left(x^2+2^{\frac{2}{3}}x+2\cdot 2^{\frac{1}{3}}\right)}{x^6-5x^3+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x^2 + 2^(2/3)*x + 2*2^(1/3))/(x^6 - 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.3 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.18896, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2134, 219, 2137, 206}

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-3x^2}\right.$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.142751, size = 146, normalized size = 0.9

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.053, size = 143, normalized size = 0.9

$$-2\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}\left(-2^{2/3}+1\right)}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticPi}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\frac{3/2+i/2\sqrt{3}}{-2^{2/3}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.4 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(1
/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.178302, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2134, 219, 2137, 206}

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(1
/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```


Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.109399, size = 150, normalized size = 0.96

$$\frac{4i\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] time = 0.043, size = 139, normalized size = 0.9

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2^{\frac{2}{3}}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \frac{i\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+
1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.5 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=280

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])
/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.334438, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2134, 218, 2137, 203}

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])
/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\ldots\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ldots}$$

Mathematica [C] time = 0.205409, size = 164, normalized size = 0.59

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError
```

3.6 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=288

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.332459, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2134, 218, 2137, 203}

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2]*Sqrt[a - b*x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} -$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a}}$$

Mathematica [C] time = 0.207024, size = 166, normalized size = 0.58

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError
```

3.7 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{bx}\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{bx}\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*
b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.343623, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2134, 219, 2137, 206}

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{bx}\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*
b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] time = 0.123334, size = 167, normalized size = 0.56

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError
```

3.8 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.329029, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2134, 219, 2137, 206}

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$=\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] time = 0.145152, size = 167, normalized size = 0.57

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*x^3]
)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a-bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError
```

3.9 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=249

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]*
c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(
2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
-7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c
 + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] time = 0.286599, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2134, 218, 2137, 203}

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]*
c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(
2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
-7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c
 + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3c} + \frac{2\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3c}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}} + \frac{2}{\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+\ldots}{\left(1+\sqrt{3}\right)c+\ldots}\right.\right.}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Mathematica [C] time = 0.206626, size = 169, normalized size = 0.68

$$-\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\frac{4d^2x^2}{c^2}-\frac{2\sqrt[3]{2}dx}{c}+2^{2/3}}\Pi\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] ((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) -
 (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (
-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)
]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [B] time = 0.157, size = 495, normalized size = 2.

$$2\frac{1}{d\sqrt{4d^3x^3+c^3}}\left(\frac{\left(1/4\sqrt[3]{2}-i/4\sqrt{3}\sqrt[3]{2}\right)c}{d}-\frac{\left(1/4\sqrt[3]{2}+i/4\sqrt{3}\sqrt[3]{2}\right)c}{d}\right)\sqrt{\left(x-\frac{\left(1/4\sqrt[3]{2}+i/4\sqrt{3}\sqrt[3]{2}\right)c}{d}\right)\left(\frac{\left(1/4\sqrt[3]{2}-i/4\sqrt{3}\sqrt[3]{2}\right.}{d}\right.}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))
*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1
/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,d^3x^3 + c^3}}{4\,d^4x^4 + 4\,cd^3x^3 + c^3dx + c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*d^3*x^3 + c^3)/(4*d^4*x^4 + 4*c*d^3*x^3 + c^3*d*x + c^4), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c + dx)\sqrt{c^3 + 4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.10 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])]
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.217381, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2135, 218, 2140, 203}

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])]
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
```

```
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\int \frac{6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{12\sqrt{3}}+\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}}$$
$$=\frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx,\right.}{\sqrt{3}}$$
$$=\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.209262, size = 136, normalized size = 0.93

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[1 + x^3])
```

Maple [A] time = 0.046, size = 132, normalized size = 0.9

$$\frac{\left(3-i\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\text{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{\left(-\frac{3}{2}+\frac{i}{2}\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
```

```
)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2
),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2*x -
 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.11 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.178896, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2135, 218, 2140, 203}

$$-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
```

```
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{-6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x,\right.}{\sqrt{3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.126969, size = 136, normalized size = 0.83

$$\frac{4\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[1 - x^3])
```

Maple [A] time = 0.047, size = 143, normalized size = 0.9

$$\frac{\frac{2i}{3}\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \frac{}{-\frac{3}{2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2
```

```
+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3
^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^5 - 2*x^4 - 2*x^3 - x^2 + 2*x
+ 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.12 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=167

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[
3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.167171, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2135, 219, 2140, 206}

$$-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[
3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
```

```
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\int \frac{6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,d\right.}{\sqrt{3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.16414, size = 134, normalized size = 0.8

$$\frac{4\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.035, size = 132, normalized size = 0.8

$$\frac{\left(-3-i\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\text{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\frac{\left(\frac{3}{2}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
```

```
)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/
2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x + sqrt(3) - 1)/(x^5 - 2*x^4 - 2*x^3 - x^2 + 2*x
+ 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.13 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=157

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.158993, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2135, 219, 2140, 206}

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```


Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
```

```
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\int \frac{-6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x,$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.106153, size = 138, normalized size = 0.88

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[-1 - x^3])
```

Maple [A] time = 0.04, size = 139, normalized size = 0.9

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \frac{i\sqrt{}}{\frac{3}{2}+\frac{i}{2}\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+
```

```
1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2*x
 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.14 $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=331

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{26+15\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{4\sqrt[4]{3}(x+\ldots}{\ldots}$$

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[26 + 15*Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.663372, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2136, 218, 2142, 2113, 537, 571, 93, 204}

$$\frac{2\sqrt{26+15\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{4\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[26 + 15*Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+\cdots}{(1+\sqrt{3}\cdots}}\right.\cdots}{\cdots}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+\cdots}{(1+\sqrt{3}\cdots}}\right.\cdots}{\cdots}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x\cdots}{(1+\sqrt{3}+\cdots}}\cdots}{\cdots}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x\cdots}{(1+\sqrt{3}+\cdots}}\cdots}{\cdots}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|\cdots\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.0692864, size = 128, normalized size = 0.39

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(\sqrt{3}+7i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] time = 0.019, size = 123, normalized size = 0.4

$$\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\frac{3}{4}+\frac{i}{4}\sqrt{3},\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(x^3+1)^(1/2),x)
```

```
[Out] (3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I
*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^3+1}}{x^4+3\,x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.15 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=382

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{4\sqrt[4]{3}\sqrt{2}\cdots}{}$$

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(2
*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*S
qrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*S
qrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])
/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.723202, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2136, 218, 2142, 2113, 537, 571, 93, 206}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{4\sqrt[4]{3}\sqrt{2}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(2
*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*S
qrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*S
qrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])
/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \cdots\right.}{\left(4 \cdots\right.}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \cdots}{\cdots}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \cdots}{\cdots}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-\cdots\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0795057, size = 128, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(\sqrt{3}+5i\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] time = 0.023, size = 133, normalized size = 0.4

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{5}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i}{2}\sqrt{3}},\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(5/2
+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^4+3\,x^3-x-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)\left(x^2+x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.16 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=376

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(2
*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[62 - 3
5*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqr
t[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.576969, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2136, 219, 2142, 2113, 537, 571, 93, 206}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(2
*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[62 - 3
5*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqr
t[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+\cdots}{(1+\cdots}}\right.}{}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+\cdots}{(1+\cdots}}\right.}{}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+\cdots}{(1+\cdots}}}{}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+\cdots}{(1+\cdots}}}{}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right.\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.0557055, size = 126, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(\sqrt{3}+5i\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.018, size = 124, normalized size = 0.3

$$\frac{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}{2}\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{3}{8}+\frac{i}{8}\sqrt{3},\sqrt{\frac{\frac{3}{2}\cdots}{\frac{3}{2}\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(x^3-1)^(1/2),x)
```

```
[Out] 1/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1
/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^3-1}}{x^4+3\,x^3-x-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.17 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=342

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{2}}$$

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sq
rt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + S
qrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.575111, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2136, 219, 2142, 2113, 537, 571, 93, 204}

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sq
rt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + S
qrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}}{\left(-2+\sqrt{3}\right)}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}}{\sqrt{-(1}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.0662472, size = 130, normalized size = 0.38

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(\sqrt{3}+7i\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] time = 0.023, size = 133, normalized size = 0.4

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{7}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i}{2}\sqrt{3}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/2+
1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/
2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^4+3\,x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)\left(x^2-x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.18 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=139

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c
- d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Rubi [A] time = 0.0704251, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2148}

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```


```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c
- d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```


Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd}$$

Mathematica [F] time = 0.0735107, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\frac{1}{\sqrt[3]{d^3x^3-c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-c+dx)\left(c^2+cdx+d^2x^2\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^3x^3 - c^3\right)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.19 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=186

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\frac{\log}{}$$

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sq
rt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d
) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) +
(3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)
```

Rubi [A] time = 0.203521, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2149, 239, 2151}

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sq
rt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d
) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) +
(3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)
```

Rule 2149

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c} + \frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} +$$

Mathematica [F] time = 0.0633519, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\frac{1}{\sqrt[3]{d^3x^3+2c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^3x^3+2c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c + dx)\sqrt[3]{2c^3 + d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^3x^3 + 2\,c^3)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.20 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}-\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}c^2d}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2c^2d}-\frac{\log(c}{2c}$$

```
[Out] -ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c^2*d) +
(Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*
c^2*d) - Log[c + d*x]/(2*c^2*d) - Log[d*x - (2*c^3 + d^3*x^3)^(1/3)]/(4*c^2
*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c^2*d)
```

Rubi [F] time = 0.105546, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.0625508, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\left(d^3x^3+2\,c^3\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)

[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)

[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")

[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

3.21 $\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=147

$$-\frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\ 2^{2/3}}+\frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}x+2\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\sqrt[3]{2}x+1\right)}{2^{2/3}}$$

```
[Out] -(ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + (Sqrt[3]
*ArcTan[(1 + (2*(2^(2/3) + x))/(1 + x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[1 +
 2^(1/3)*x]/2^(2/3) - Log[x - (1 + x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[2 + 2^(
1/3)*x - 2^(1/3)*(1 + x^3)^(1/3)])/(2*2^(2/3))
```

Rubi [F] time = 0.0854677, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]

[Out] Defer[Int][1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.0724752, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]

[Out] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{1+\sqrt[3]{2}x}\left(x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x)
```

```
[Out] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Fricas [B] time = 20.1788, size = 7992, normalized size = 54.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(442971093109
3017274143382940539963665445172591640340075959634542018 3*x^16 + 46991175387
757729726668749336126627429821975172615651174879678821030 4*x^13 - 168603219
036433260440647021325346295645242325246375460547582960409424*x^10 - 1978806
30118237657393829295422779262737333028339787658261155833289344 0*x^7 - 14400
9089168717758142291876308930196860258103687221308438991237030187 2*x^4 - 2^(
2/3)*(5227107745312510761299592397765475834939487692288555281920999986641 3*
x^15 + 590674547854548577293285820788340778493299281255213360593997994805 17
2*x^12 + 306314261222931431619887382966630423064822217690279625339197857781
7900*x^9 + 733104955869757780900835257159703940345796885706673027778611495 9
327080*x^6 + 772324480675629044375977054678087297173944475017351963554418 61
14816064*x^3 + 291168089878390092195634857418355141558919044601510645260807
0501424800) + 6*2^(1/3)*(1260135599621632209331474867914912054330214068567 7
058235520929344665*x^14 - 555869063001966513924627194919212678478207988900 1
985022711593808971 8*x^11 - 450398920105320599307639536027883986131793624729
303407436233610788504*x^8 - 721888705880948261432517052670394106238338943 84
437355390651087986658 4*x^5 - 33866815806868437343630927306784946440569136 07
513785074424729217745 44*x^2) - 623676430454539792290217012355944404253806 60
14097629243324078051968 0*x)*(x^3 + 1)^(2/3) - 13910019318573948542*sqrt(3)*
(202441513867627285828731764409166422760369138467219643425703198742 72*x^17
+ 74114613707883499096895869495695352578696821616279136914156107923134 2*x^1
4 + 217984319727177540114743839610166687553704366334519910306529071835066 0*
x^11 + 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 6905839793022126495418466717523235786717623615649871985323720776 17
072*x^5 + 425604467193959940435036909294930892503769478498985960943870691 96
992*x^2 + 2^(2/3)*(581759530164412505528941290287858488953431467069124527 80
410096144857*x^16 + 603329123440225928459512442880846367498086340467210508 4
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
```

```
7482692017634440*x^10 - 315373668616978600368729679828820826067145203897860
799345951918357208*x^7 - 153598978117589845490400976408047769812343914000095
2325783379529417102 4*x^4 - 774581653994506522185065060515457999562469670838
03571070027910096 0480*x) - 2*2^(1/3)*(442503373958626236413084321461052655 9
15849816922169442468726224375 86*x^15 + 937303319945530879145881930294041650
157381457193700122532562371428 33*x^12 + 1321854131659545520395638093435834 8
619932882852548406311430877544538 16*x^9 + 4247705767701746889589213825725 27
81622024317737600109081215316558582 40*x^6 + 4593245463688643634993735851341
62183835983817018828550015173318585504 0*x^3 + 161588373761478929714291077 07
869228809509709698905305411015386387388 00))*(x^3 + 1)^(1/3) + sqrt(3)*(5808
458566248141380585366589250357524223410237450426571441100184341339711713923
78653765*x^18 + 851285021120165859632032242350797943674503706160466252288 1
061739848890113983919394938 44*x^15 + 460376746342993998764649333533333936 51
79871449886195969768495285918127951444917234880113 2*x^12 + 1000163483533668
1235799972394854096695243561183658042029483382705876658545646361121556291 2*
x^9 + 913977586253668076790534210688867294404951076896021210254557365342556
4237012293570062811 2*x^6 + 2767920647122214781893234891470727140655412121 61
417347858639664511393385455690463968429 44*x^3 - 13910019318573948542*2^(2/3
)*(3844366680114123938578119587438413410802428820066154040455085354797*x^17
 - 4931319711549190780631731959832802785947037704060043883265521247935 91*x^
14 - 226365632973375052657523978839334180427226832840407837738697965541162 8
*x^11 - 36032960889596430400658826061569773329427783689708679588412662754 05
688*x^8 - 2375143924145462474790789297643082581023352457583644433698318090 2
72160*x^5 - 538527827084536759298395164308728360347336217790784309877024260
129712*x^2) + 16692023182288738250 4*2^(1/3)*(13595892044042828366275982006 7
0804939503290969888000412994951133922 6*x^16 + 1351333848851582503771790485 9
59913464507711993272362079564211134619 03*x^13 + 4022458990280584368230681 09
52188584025877561061471182634365786887935 9*x^10 + 547258710149879334691832 9
99834525308297790387563356879645468036532966*x^7 + 36367419970364096388496 0
0121243872631062549095216406631543023021164 04*x^4 + 971238957407046440052 92
0552224644980115018429446394060260205323401 20*x) - 180077408083879446119265
390325980259185018839401686617070765560907623616768789393655840 0))/(4912705
74577547337465577862499678580919468289682240641599400002541818630173299555 5
53387*x^18 + 10277776658535231887928963830517649364075160462302952752368573
52973860457707512834583049 6*x^15 + 38053074604041164955598613382506657588 71
803380001542884868735451581940811327528082006722 8*x^12 + 104552977375786496
05615651522868639336063425038920613481665234759510520099015608943001368 0*x^
9 + 1937897787862171089238325621006741761898811317322806945023580588310752 3
14615088176603874 40*x^6 + 176250773615214113270216364768545940531543731282 5
773389899739161344093499455872519557015 68*x^3 + 58729517358150193708087322 4
842839507067739341828673494493229041410702015901853308890480 00)) + 1/12*2^(
1/3)*log((6048*x^16 + 6048*x^13 - 9072*x^10 - 12204*x^7 - 2808*x^4 + 2^(2/3
)*(352*x^18 - 5136*x^15 - 10632*x^12 - 3224*x^9 + 3390*x^6 + 1434*x^3 - 35)
 + 3*(2032*x^14 + 752*x^11 - 3000*x^8 - 1576*x^5 + 172*x^2 + 2^(2/3)*(112*x
^16 - 1760*x^13 - 2228*x^10 + 356*x^7 + 707*x^4 - 22*x) - 2*2^(1/3)*(352*x^
15 - 728*x^12 - 1736*x^9 - 451*x^6 + 215*x^3 - 1))*(x^3 + 1)^(2/3) - 18*2^(
1/3)*(112*x^17 - 192*x^14 - 820*x^11 - 586*x^8 - 21*x^5 + 49*x^2) + 3*(2096
*x^15 + 1664*x^12 - 2680*x^9 - 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 - 176
0*x^14 - 2996*x^11 - 472*x^8 + 779*x^5 + 125*x^2) - 2*2^(1/3)*(336*x^16 - 6
64*x^13 - 2132*x^10 - 1107*x^7 + 55*x^4 + 29*x) + 16)*(x^3 + 1)^(1/3) + 324
*x)/(64*x^18 + 192*x^15 + 240*x^12 + 160*x^9 + 60*x^6 + 12*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(\sqrt[3]{2}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
```

```
[Out] Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

3.22 $\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=159

$$\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}x-2\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*2^(2/3) - 2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 -
 2^(1/3)*x]/2^(2/3) + Log[-x - (1 - x^3)^(1/3)]/(2*2^(2/3)) - (3*Log[-2 + 2
^(1/3)*x + 2^(1/3)*(1 - x^3)^(1/3)])/(2*2^(2/3))
```

Rubi [F] time = 0.102283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.0851748, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{1-\sqrt[3]{2}x}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x)

[Out] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")

[Out] -integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Fricas [B] time = 14.2904, size = 7996, normalized size = 50.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(4429710931093
0172741433829405399636654451725916403400759596345420183*x^16 - 469911753877
577297266687493361266274298219751726156511748796788210304*x^13 - 1686032190
364332604406470213253462956452423252463754605475829604094 24*x^10 + 19788063
011823765739382929542277926273733302833978765826115583328934 40*x^7 - 144009
089168717758142291876308930196860258103687221308438991237030 1872*x^4 + 2^(2
/3)*(522710774531251076129959239776547583493948769228855528192099 99866413*x
^15 - 5906745478545485772932858207883407784932992812552133605939979 94805172
*x^12 + 30631426122293143161988738296663042306482221769027962533919785 77817
900*x^9 - 733104955869757780900835257159703940345796885706673027778611 49593
27080*x^6 + 7723244806756290443759770546780872971739444750173519635544186 11
4816064*x^3 - 2911680898783900921956348574183551415589190446015106452608070
501424800) + 6*2^(1/3)*(1260135599621632209331474867914912054330214068567 70
58235520929344665*x^14 + 55586906300196651392462719491921267847820798890019
85022711593808971 8*x^11 - 4503989201053205993076395360278839861317936247293
0340743623361078850 4*x^8 + 72188870588094826143251705267039410623833894384 4
373553906510879866584*x^5 - 338668158068684373436309273067849464405691360 75
1378507442472921774544*x^2) + 623676430454539792290217012355944404253806601
409762924332407805196 80*x)*(-x^3 + 1)^(2/3) + 13910019318573948542*sqrt(3)*
(20244151386762728582873176440916642276036913846721964342570319874272*x^17
- 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 217984319727177540114743839610166687553704366334519910306529071835066 0*
x^11 - 2111024935028444803027635033172373996998638870275081528835019029426 8
08*x^8 + 69058397930221264954184667175232357867176236156498719853237207761 7
072*x^5 - 42560446719395994043503690929493089250376947849898596094387069196
992*x^2 - 2^(2/3)*(58175953016441250552894129028785848895343146706912452780
410096144857*x^16 - 60332912344022592845951244288084636749808634046721050 84
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 + 315373668616978600368729679828820826067145203897860
799345951918357208*x^7 - 1535989781175898454904009764080477698123439140009 5
```

```
23257833795294171024*x^4 + 774581653994506522185065060515457999562469670838
035710700279100960480*x) - 2*2^(1/3)*(4425033739586262364130843214610526559
1584981692216944246872622437586*x^15 - 937303319945530879145881930294041650
15738145719370012253256237142833*x^12 + 13218541316595455203956380934358348
61993288285254840631143087754453816*x^9 - 424770576770174688958921382572527
8162202431773760010908121531655858240*x^6 + 4593245463688643634993735851341
621838359838170188285500151733185855040*x^3 - 16158837376147892971429107707
86922880950970969890530541101538638738800))*(-x^3 + 1)^(1/3) + sqrt(3)*(580
845856624814138058536658925035752422341023745042657144110018434133971171392
378653765*x^18 - 8512850211201658596320322423507979436745037061604662252288
106173984889011398391939493844*x^15 + 4603767463429939987646493335333339365
1798714498861959697684952859181279514449172348801132*x^12 - 100016348353366
812357999723948540966952435611836580420294833827058766585456463611215562912
*x^9 + 91397758625366807679053421068886729440495107689602121025455736534255
642370122935700628112*x^6 - 267679206471222147818932348914707271406554121216
141734785863966451139338545569046396842944*x^3 + 13910019318573948542*2^(2/
3)*(3844366680114123938578119587438413410802428820066154040455085354797*x^1
7 + 493131971154919078063173195983280278594703770406004388326552124793591*x
^14 - 226365632973375052657523978839334180427226832840407837738697965541162
8*x^11 + 360329608895964304006588260615697733294277836897086795884126627540
5688*x^8 - 2375143924145462474790789297643082581023352457583644433698318090
272160*x^5 + 53852782708453675929839516430872836034733621779078430987702426
0129712*x^2) + 166920231822887382504*2^(1/3)*(13595892044042828366275982006
708049395032909698880004129949511339226*x^16 - 1351333848851582503771790485
95991346450771199327236207956421113461903*x^13 + 40224589902805843682306810
9521885840258775610614711826343657868879359*x^10 - 547258710149879334691832
999834525308297790387563356879645468036532966*x^7 + 36367419970364096388496
0012124387263106254909521640663154302302116404*x^4 - 9712389574070464400529
2055222464498011501842944639406026020532340120*x) - 18007740808387944611926
53903259802591850188394016866170707655609076236167687893936558400))/(491270
574577547337465577862499678580919468289682240641599400002541818630173299555
553387*x^18 - 1027777665853523188792896383051764936407516046230295275236857
3529738604577075128345830496*x^15 + 380530746040411649555986133825066575887
18033800015428848687354515819408113275280820067228*x^12 - 10455297737578649
6056156515228686393360634250389206134816652347595105200990156089430013680*x
^9 + 1937897787862171089238325621006741761898811317322806945023580588310752
31461508817660387440*x^6 - 176250773615214113270216364768545940531543731282
577338989973916134409349945587251955701568*x^3 + 58729517358150193708087322
484283950706773934182867349449322904141070201590185330889048000)) - 1/12*2^
(1/3)*log((6048*x^16 - 6048*x^13 - 9072*x^10 + 12204*x^7 - 2808*x^4 + 2^(2/
3)*(352*x^18 + 5136*x^15 - 10632*x^12 + 3224*x^9 + 3390*x^6 - 1434*x^3 - 35
) + 3*(2032*x^14 - 752*x^11 - 3000*x^8 + 1576*x^5 + 172*x^2 + 2^(2/3)*(112*
x^16 + 1760*x^13 - 2228*x^10 - 356*x^7 + 707*x^4 + 22*x) + 2*2^(1/3)*(352*x
^15 + 728*x^12 - 1736*x^9 + 451*x^6 + 215*x^3 + 1))*(-x^3 + 1)^(2/3) + 18*2
^(1/3)*(112*x^17 + 192*x^14 - 820*x^11 + 586*x^8 - 21*x^5 - 49*x^2) - 3*(20
96*x^15 - 1664*x^12 - 2680*x^9 + 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 + 1
760*x^14 - 2996*x^11 + 472*x^8 + 779*x^5 - 125*x^2) + 2*2^(1/3)*(336*x^16 +
 664*x^13 - 2132*x^10 + 1107*x^7 + 55*x^4 - 29*x) - 16)*(-x^3 + 1)^(1/3) -
324*x)/(64*x^18 - 192*x^15 + 240*x^12 - 160*x^9 + 60*x^6 - 12*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt[3]{2}x\left(1-x^3\right)^{\frac{2}{3}}-\left(1-x^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

3.23 $\int (c+dx)^4 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=387

$$\frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}+\frac{a^2d^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}-\frac{4ac^3d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{3ac^2d^2\cdots}{2\cdots}$$

```
[Out] (3*a*c^2*d^2*(a + b*x^3)^(1/3))/(2*b) + (a*d^4*x^2*(a + b*x^3)^(1/3))/(18*b
) + ((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3
*x^4 + 5*d^4*x^5))/30 - (4*a*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a^2*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)) + (a*c^4*x*(1 + (b*x^3)/a)^(2
/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) +
 (a*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(5*(a + b*x^3)^(2/3)) - (2*a*c^3*d*Log[b^(1/3)*x - (a + b*x^3)^(1/
3)])/(3*b^(2/3)) + (a^2*d^4*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(5/3)
)
```

Rubi [A] time = 0.396989, antiderivative size = 498, normalized size of antiderivative = 1.29, number of steps used = 23, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.79, Rules used = {1853, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628, 261, 365, 364, 321}

$$\frac{a^2d^4\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{27b^{5/3}}-\frac{a^2d^4\log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{54b^{5/3}}+\frac{a^2d^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}-\frac{4ac^3d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{2ac^3d\log\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c^2*d^2*(a + b*x^3)^(1/3))/(2*b) + (a*d^4*x^2*(a + b*x^3)^(1/3))/(18*b
) + ((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3
*x^4 + 5*d^4*x^5))/30 - (4*a*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a^2*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)) + (a*c^4*x*(1 + (b*x^3)/a)^(2
/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) +
 (a*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(5*(a + b*x^3)^(2/3)) - (4*a*c^3*d*Log[1 - (b^(1/3)*x)/(a + b*x^3)
^(1/3)])/(9*b^(2/3)) + (a^2*d^4*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(27
*b^(5/3)) + (2*a*c^3*d*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x
)/(a + b*x^3)^(1/3)])/(9*b^(2/3)) - (a^2*d^4*Log[1 + (b^(2/3)*x^2)/(a + b*x
^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*b^(5/3))
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx = \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int \frac{\frac{c^4}{2}+\frac{4}{3}c^3dx+\frac{3}{2}c^2d^2x^2}{\left(a+bx^3\right.}$$

$$= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int\left(\frac{c^4}{2\left(a+bx^3\right)^{2/3}}+\frac{}{3(}\right.$$

$$= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+\frac{1}{2}\left(ac^4\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}dx$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4\right.$$

Mathematica [A] time = 0.200131, size = 163, normalized size = 0.42

$$\frac{\sqrt[3]{a+bx^3}\left(dx^2\left(12bc^3-ad^3\right){}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d^2\left(\left(a+bx^3\right)\sqrt[3]{\frac{bx^3}{a}+1}\left(9c^2+d^2x^2\right)+6bcdx^4\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right.\right.}{6b\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(6*b*c^4*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(12*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a
)] + d^2*((9*c^2 + d^2*x^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 6*b*c*d*x^4
*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])))/(6*b*(1 + (b*x^3)/a)^(1
/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^4 \sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}} (dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 4.41812, size = 212, normalized size = 0.55

$$\frac{\sqrt[3]{a}c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{4\sqrt[3]{a}c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{4\sqrt[3]{a}cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}d^4x^5\Gamma\left(\frac{5}{3}\right){}_2}{3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**4*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 4*a**(1/3)*c**3*d*x**2*gamma(2/3)*hyper((-1/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 4*a**(1/3)*c*d**3*x**4*g
amma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3
)) + a**(1/3)*d**4*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(8/3)) + 6*c**2*d**2*Piecewise((a**(1/3)*x**3/3, Eq(b,
 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{1}{3}} (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

3.24 $\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=242

$$-\frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{ac^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5bc^3-ad^3\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b\left(a+bx^3\right)^{2/3}}+\frac{1}{20}\sqrt[3]{a+bx^3}\,(2$$

```
[Out] (3*a*c*d^2*(a + b*x^3)^(1/3))/(4*b) + (a*d^3*x*(a + b*x^3)^(1/3))/(10*b) +
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/20
 - (a*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*b^(2/3)) + (a*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*b*(a + b*x^3)^(2/3)) - (a*c^2*d*Log[b^(1/
3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3))
```

Rubi [A] time = 0.305283, antiderivative size = 297, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {1853, 1888, 1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{ac^2d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{ac^2d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}}-\frac{ac^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5bc^3-ad^3\right){}_2F_1\left(\right.}{10b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c*d^2*(a + b*x^3)^(1/3))/(4*b) + (a*d^3*x*(a + b*x^3)^(1/3))/(10*b) +
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/20
 - (a*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*b^(2/3)) + (a*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*b*(a + b*x^3)^(2/3)) - (a*c^2*d*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) + (a*c^2*d*Log[1 + (b^(2/3)*x^2)
/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(2/3))
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+a\int\frac{\frac{c^3}{2}+c^2dx+\frac{3}{4}cd^2x^2+\frac{d^3x^3}{5}}{\left(a+bx^3\right)^{2/3}}dx$$

$$=\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a\int\frac{\frac{1}{5}\left(5bc^3-ad^3\right)+2bc^2dx}{\left(a+bx^3\right)^{2/3}}}{2b}$$

$$=\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a\int\frac{\frac{1}{5}\left(5bc^3-ad^3\right)+2bc^2dx}{\left(a+bx^3\right)^{2/3}}}{2b}$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a}{\ }$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\left(a\right.$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\left(a\right.$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a}{\ }$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a}{\ }$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a}{\ }$$

$$=\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)-\frac{ac}{\ }$$

Mathematica [A] time = 0.132894, size = 142, normalized size = 0.59

$$\frac{\sqrt[3]{a+bx^3}\left(d\left(6bc^2x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d\left(3c\left(a+bx^3\right)\sqrt[3]{\frac{bx^3}{a}+1}+bdx^4\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right)+4bc^3x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\right.\right.}{4b\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^3*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(6*b*c^2*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + d*(3*
c*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + b*d*x^4*Hypergeometric2F1[-1/3, 4/3,
7/3, -((b*x^3)/a)]))))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (dx+c)^3 \sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.29069, size = 160, normalized size = 0.66

$$\frac{\sqrt[3]{a}c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt[3]{a}d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+3cd^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3}\\\frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**3*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + a**(1/3)*c**2*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/gamma(5/3) + a**(1/3)*d**3*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 3*c*d
**2*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{1}{3}} (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

3.25 $\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=192

$$-\frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}-\frac{2acd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3}\right.}{2\left(a+bx^3\right)^{2/3}}$$

```
[Out] (a*d^2*(a + b*x^3)^(1/3))/(4*b) + ((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 +
 3*d^2*x^3))/12 - (2*a*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (a*c*d*Log[b^(1/3)
*x - (a + b*x^3)^(1/3)])/(3*b^(2/3))
```

Rubi [A] time = 0.204067, antiderivative size = 245, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1853, 1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{2acd\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{acd\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{9b^{2/3}}-\frac{2acd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (a*d^2*(a + b*x^3)^(1/3))/(4*b) + ((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 +
 3*d^2*x^3))/12 - (2*a*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (2*a*c*d*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(2/3)) + (a*c*d*Log[1 + (b^(2/3)*x^2)/(
a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(2/3))
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \frac{\frac{c^2}{2}+\frac{2cdx}{3}+\frac{d^2x^2}{4}}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \frac{\frac{c^2}{2}+\frac{2cdx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{4}\left(ad^2\right)\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \left(\frac{c^2}{2\left(a+bx^3\right)^{2/3}}+\frac{2cdx}{3\left(a+bx^3\right)^{2/3}}\right) d$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{1}{2}\left(ac^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{3}(2acd)\int$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{1}{3}(2acd)\,\mathrm{Subst}\left(\int \frac{x}{1-bx^3}\,dx,x,\frac{x}{\sqrt[3]{a+}}\right.$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}+$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)-\frac{2acd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{ac^2x\left(1+\frac{bx^3}{a}\right.}{2(}$$

Mathematica [A] time = 0.0783356, size = 111, normalized size = 0.58

$$\frac{\sqrt[3]{a+bx^3}\left(4bc^2x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(4bcx^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d\left(a+bx^3\right)\sqrt[3]{\frac{bx^3}{a}+1}\right)\right)}{4b\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^2*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 4*b*c*x^2*Hypergeometric2F1[-
1/3, 2/3, 5/3, -((b*x^3)/a)])))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (dx+c)^2 \sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.67772, size = 114, normalized size = 0.59

$$\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/
```

```
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + d**2*Piecewise((a**(1/3)*x*
*3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

3.26 $\int (c+dx)\sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=155

$$-\frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}-\frac{ad\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)+\frac{acx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

```
[Out] ((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 - (a*d*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c*x*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) -
(a*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(2/3))
```

Rubi [A] time = 0.150336, antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1853, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{ad\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{ad\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{2/3}}-\frac{ad\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)+\frac{acx\left(\frac{bx^3}{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 - (a*d*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c*x*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) -
(a*d*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(2/3)) + (a*d*Log[1 + (b^
(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(2/3))
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+a\int \frac{\frac{c}{2}+\frac{dx}{3}}{\left(a+bx^3\right)^{2/3}}dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+a\int \left(\frac{c}{2\left(a+bx^3\right)^{2/3}}+\frac{dx}{3\left(a+bx^3\right)^{2/3}}\right)dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+\frac{1}{2}(ac)\int \frac{1}{\left(a+bx^3\right)^{2/3}}dx+\frac{1}{3}(ad)\int \frac{x}{\left(a+bx^3\right)^{2/3}}dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+\frac{1}{3}(ad)\,\text{Subst}\left(\int \frac{x}{1-bx^3}dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)+\frac{\left(ac\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int}{2\left(a+bx^3\right.}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}+\frac{(ad)\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}dx,x,\right.}{9\sqrt[3]{b}}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{ad\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{(ad)}{}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{ad\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{ad\,\text{l}}{}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}-\frac{ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{ad\,\text{l}}{}$$

Mathematica [A] time = 0.0275182, size = 75, normalized size = 0.48

$$\frac{x\sqrt[3]{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(2*c*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)]))/(2*(1 + (b*x^3)/a)^(
1/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int (dx+c)\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)*(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^{\frac{1}{3}} (dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx^3 + a)^{\frac{1}{3}} (dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Sympy [C] time = 2.23732, size = 82, normalized size = 0.53

$$\frac{\sqrt[3]{a} c x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a} d x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^{\frac{1}{3}} (dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

3.27 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$

Optimal. Leaf size=435

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}-\frac{\sqrt[3]{bc^3}}{}$$

```
[Out] (a + b*x^3)^(1/3)/d + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b
*x^3)/a), -((d^3*x^3)/c^3)])/(c*(1 + (b*x^3)/a)^(1/3)) + (b^(1/3)*c*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c^3 - a
*d^3)^(1/3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/
Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)^(1/3)*ArcTan[(1 - (2*d*(a + b*x^
3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)
^(1/3)*Log[c^3 + d^3*x^3])/(3*d^2) + (b^(1/3)*c*Log[b^(1/3)*x - (a + b*x^3)
^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c -
 (a + b*x^3)^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[(b*c^3 - a*d^3)^(
1/3) + d*(a + b*x^3)^(1/3)])/(2*d^2)
```

Rubi [F] time = 0.0815726, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*x^3)^(1/3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Mathematica [F] time = 0.332267, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x), x]
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\sqrt[3]{b x^{3}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3}+a\right)^{\frac{1}{3}}}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+b x^{3}}}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3}+a\right)^{\frac{1}{3}}}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

3.28 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} dx$

Optimal. Leaf size=818

$$-\frac{d^3 \sqrt[3]{bx^3+a} F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right) x^4}{2c^5 \sqrt[3]{\frac{bx^3}{a}+1}} - \frac{d\sqrt[3]{bx^3+a}x^2}{c^3+d^3x^3} + \frac{\sqrt[3]{bx^3+a} F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right) x}{c^2 \sqrt[3]{\frac{bx^3}{a}+1}} - \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{bx^3+a}}}{\sqrt{3}}\right.}{\sqrt{3}d^2}$$

```
[Out] -((c^2*(a + b*x^3)^(1/3))/(d*(c^3 + d^3*x^3))) - (d*x^2*(a + b*x^3)^(1/3))/
(c^3 + d^3*x^3) + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3
)/a), -((d^3*x^3)/c^3)])/(c^2*(1 + (b*x^3)/a)^(1/3)) - (d^3*x^4*(a + b*x^3)
^(1/3)*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*
(1 + (b*x^3)/a)^(1/3)) - (b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/(Sqrt[3]*d^2) + (2*a*d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x
)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(2/3)) + ((
3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(
1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*ArcTan[(1
 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2*(b
*c^3 - a*d^3)^(2/3)) - (b*c^2*Log[c^3 + d^3*x^3])/(6*d^2*(b*c^3 - a*d^3)^(2
/3)) - (a*d*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3)^(2/3)) - ((3*b*c^3 - 2
*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b^(1/3)*Log
[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d^2) + (a*d*Log[((b*c^3 - a*d^3)^(1/3)*
x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(2/3)) + ((3*b*c^3 - 2*a*d^
3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6*c*d^2*(b*c^3 -
a*d^3)^(2/3)) + (b*c^2*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2
*d^2*(b*c^3 - a*d^3)^(2/3))
```

Rubi [F] time = 0.0794583, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} dx$$

Mathematica [F] time = 0.209719, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

3.29 $\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=310

$$\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{4acd^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}-\cdots$$

```
[Out] (3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^
4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/
3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*H
ypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3)) - (c^4
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```

Rubi [A] time = 0.17824, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1893, 239, 365, 364, 261, 321}

$$\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{4acd^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```


```
[Out] (3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^
4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/
3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*H
ypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3)) - (c^4
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```


Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{c^4}{\sqrt[3]{a+bx^3}}+\frac{4c^3dx}{\sqrt[3]{a+bx^3}}+\frac{6c^2d^2x^2}{\sqrt[3]{a+bx^3}}+\frac{4cd^3x^3}{\sqrt[3]{a+bx^3}}+\frac{d^4x^4}{\sqrt[3]{a+bx^3}}\right)dx$$

$$= c^4\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx+\left(4c^3d\right)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx+\left(6c^2d^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx+\left(4cd^3\right)\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx+d^4\int$$

$$= \frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c^4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}-\frac{\left(4acd^3\right)}{}$$

$$= \frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{4acd^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{2c^3dx^2\sqrt[3]{1}}{}$$

Mathematica [A] time = 0.464989, size = 392, normalized size = 1.26

$$\frac{5c\left(3bc^3\sqrt[3]{a+bx^3}\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-4ad^3\sqrt[3]{a+bx^3}\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)+2\sqrt[3]{a+bx^3}\left(4ad^3-3bc^3\right)\mathrm{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (180*b^(4/3)*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/
3, -((b*x^3)/a)] + 18*b^(4/3)*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2
F1[1/3, 5/3, 8/3, -((b*x^3)/a)] + 5*c*(54*a*b^(1/3)*c*d^2 + 24*a*b^(1/3)*d^
3*x + 54*b^(4/3)*c*d^2*x^3 + 24*b^(4/3)*d^3*x^4 + 2*Sqrt[3]*(3*b*c^3 - 4*a*
```

```
d^3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
] + 2*(-3*b*c^3 + 4*a*d^3)*(a + b*x^3)^(1/3)*Log[1 - (b^(1/3)*x)/(a + b*x^3
)^(1/3)] + 3*b*c^3*(a + b*x^3)^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3
) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - 4*a*d^3*(a + b*x^3)^(1/3)*Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(90*b^(4/3)*
(a + b*x^3)^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx+c)^4 \frac{1}{\sqrt[3]{bx^3+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 4.22527, size = 206, normalized size = 0.66

$$6c^2d^2\left(\begin{cases} \frac{x^3}{3\sqrt[3]{a}} & \text{for}\ b=0 \\ \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \right.}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(1/3),x)
```

```
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(
2*b), True)) + c**4*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((1/3, 2/
3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + 4*c*d**3*x*
*4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^4}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(1/3), x)
```

3.30 $\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=255

$$\frac{ad^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}-\frac{ad^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{c^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+$$

```
[Out] (3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3*
ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) -
(a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^
(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3
, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a + b*x^3
)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b^
(4/3))
```

Rubi [A] time = 0.137653, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1893, 239, 365, 364, 261, 321}

$$\frac{ad^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}-\frac{ad^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{c^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```


```
[Out] (3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3*
ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) -
(a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^
(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3
, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a + b*x^3
)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b^
(4/3))
```


Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx &= \int \left(\frac{c^3}{\sqrt[3]{a+bx^3}}+\frac{3c^2dx}{\sqrt[3]{a+bx^3}}+\frac{3cd^2x^2}{\sqrt[3]{a+bx^3}}+\frac{d^3x^3}{\sqrt[3]{a+bx^3}}\right)dx\\
&= c^3\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx+\left(3c^2d\right)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx+\left(3cd^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx+d^3\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{d^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}-\frac{\left(ad^3\right)\int \frac{}{\sqrt[3]{a}}}{3b}\\
&= \frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{d^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}}{2\sqrt[3]{a}}\end{aligned}$$

Mathematica [A] time = 0.377238, size = 287, normalized size = 1.13

$$\frac{1}{18}\left(\frac{3bc^3\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-ad^3\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)+\left(2ad^3-6bc^3\right)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\left(3bc^3-\right.}{b^{4/3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] ((27*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*
x^3)/a)])/(a + b*x^3)^(1/3) + (27*b^(1/3)*c*d^2*(a + b*x^3)^(2/3) + 6*b^(1/
3)*d^3*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*b^(
```

```
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + (-6*b*c^3 + 2*a*d^3)*Log[1 - (b^(1/3)
*x)/(a + b*x^3)^(1/3)] + 3*b*c^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*d^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2
/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3))/18
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (dx+c)^3 \frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.552, size = 155, normalized size = 0.61

$$3cd^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+\frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x**3+a)**(1/3),x)
```

```
[Out] 3*c*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b
), True)) + c**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*
```

```
pi)/a)/(3*a**(1/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (
5/3,), b*x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(5/3)) + d**3*x**4*gamma(4/
3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/
3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(1/3), x)
```

3.31 $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}} dx$

Optimal. Leaf size=147

$$-\frac{c^2 \log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{c^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

Rubi [A] time = 0.101936, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1886, 261, 1893, 239, 365, 364}

$$-\frac{c^2 \log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{c^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx &= d^2\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx+\int \frac{c^2+2cdx}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b}+\int \left(\frac{c^2}{\sqrt[3]{a+bx^3}}+\frac{2cdx}{\sqrt[3]{a+bx^3}}\right)dx\\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b}+c^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx+(2cd)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}+\frac{\left(2cd\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}\\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}-\frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [A] time = 0.165305, size = 201, normalized size = 1.37

$$
\frac{c^2\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{b}}-\frac{c^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{c^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2\left(a+\right.}{2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (c^2*Log[1 + (b^(2/3)*x^2)/(a
+ b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (dx+c)^2 \frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.60015, size = 110, normalized size = 0.75

$$d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b), T
rue)) + c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(1/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(1/3), x)
```

3.32 $\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=124

$$-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

```
[Out] (c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
 + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b
^(1/3))
```

Rubi [A] time = 0.0663397, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1893, 239, 365, 364}

$$-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] (c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
 + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b
^(1/3))
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx &= \int \left(\frac{c}{\sqrt[3]{a+bx^3}}+\frac{dx}{\sqrt[3]{a+bx^3}}\right)dx \\
&= c\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + d\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{\left(d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}} \\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [A] time = 0.114559, size = 163, normalized size = 1.31

$$
\frac{1}{6}\left(\frac{c\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{b}}+\frac{3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(a + b*x^3)^(1/3) + (c*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/6
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (dx+c)\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] time = 1.86781, size = 78, normalized size = 0.63

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\left(bx^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(1/3), x)
```

3.33 $\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}}+\frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

```
[Out] -(d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d
^3*x^3)/c^3)])/(2*c^2*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*(b*c^3 - a*d^3)^(
1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*(b*c^3 - a*d^3)^(1/3)) - A
rcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]]/(Sqrt[3]
*(b*c^3 - a*d^3)^(1/3)) + Log[c^3 + d^3*x^3]/(3*(b*c^3 - a*d^3)^(1/3)) - Lo
g[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^3)^(1/3)
) - Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^3)^(1/
3))
```

Rubi [F] time = 0.0475809, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F] time = 0.0616559, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

3.34 $\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}} \, dx$

Optimal. Leaf size=761

$$\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}}-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}-\frac{cd^3x\left(a+bx^3\right)^{2/3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)}+\frac{c^2d^2\left(a\right.}{\left(c^3+d^3x^3\right.}$$

```
[Out] (c^2*d^2*(a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (c*d^3*x*(a
 + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (d*x^2*(1 + (b*x^3)/a)
^(1/3)*AppellF1[2/3, 1/3, 2, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a
+ b*x^3)^(1/3)) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 2, 8/3,
 -((b*x^3)/a), -((d^3*x^3)/c^3)])/(5*c^6*(a + b*x^3)^(1/3)) + (2*a*d^3*ArcT
an[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqr
t[3]*c*(b*c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3
- a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*
d^3)^(4/3)) - (b*c^2*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(4/3)) + (b*c^2*Log[c^3 + d^3*x^3])
/(6*(b*c^3 - a*d^3)^(4/3)) + (a*d^3*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3
)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*(b*c^3 - a*d^3)^(
4/3)) - (a*d^3*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(
b*c^3 - a*d^3)^(4/3)) - ((3*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/
c - (a + b*x^3)^(1/3)])/(6*c*(b*c^3 - a*d^3)^(4/3)) - (b*c^2*Log[(b*c^3 - a
*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3)^(4/3))
```

Rubi [F] time = 0.0830621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}} \, dx = \int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}} \, dx$$

Mathematica [F] time = 0.336747, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2} \frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

3.35 $\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}} \, dx$

Optimal. Leaf size=1513

result too large to display

```
[Out] (3*c^4*d^2*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) - (3*c^
3*d^3*x*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (4*b*c^4
*d^2*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(b*c
^3 - 3*a*d^3)*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) + (d
^3*(3*b*c^3 - 7*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)^2*(c^3 + d^
3*x^3)) - (d^3*(9*b*c^3 - 5*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)
^2*(c^3 + d^3*x^3)) - (7*d^3*(3*b*c^3 + a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*
c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (3*d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2
/3, 1/3, 3, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^4*(a + b*x^3)^(1/3))
 + (6*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 3, 8/3, -((b*x^3)/a)
, -((d^3*x^3)/c^3)])/(5*c^7*(a + b*x^3)^(1/3)) + (2*a^2*d^6*ArcTan[(1 + (2*
(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b
*c^3 - a*d^3)^(7/3)) + (7*a*d^3*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a
*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b*c^3 - a*d
^3)^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*ArcTan[(1 + (2*(b*c^
3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b*c^3
- a*d^3)^(7/3)) - (4*b^2*c^4*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a
*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (b*c*(b*c^3 - 3*
a*d^3)*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])
/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (2*b^2*c^4*Log[c^3 + d^3*x^3])/(9*(b*c
^3 - a*d^3)^(7/3)) + (a^2*d^6*Log[c^3 + d^3*x^3])/(27*c^2*(b*c^3 - a*d^3)^(
7/3)) - (b*c*(b*c^3 - 3*a*d^3)*Log[c^3 + d^3*x^3])/(18*(b*c^3 - a*d^3)^(7/3
)) + (7*a*d^3*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(54*c^2*(b*c^3 - a*d^3)
^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*Log[c^3 + d^3*x^3])/(54
*c^2*(b*c^3 - a*d^3)^(7/3)) - (a^2*d^6*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a
 + b*x^3)^(1/3)])/(9*c^2*(b*c^3 - a*d^3)^(7/3)) - (7*a*d^3*(3*b*c^3 - a*d^3
)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(18*c^2*(b*c^3 - a*
d^3)^(7/3)) - ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*Log[((b*c^3 - a*d^3
)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(18*c^2*(b*c^3 - a*d^3)^(7/3)) - (2*b^2*
c^4*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(3*(b*c^3 - a*d^3)^(7
/3)) + (b*c*(b*c^3 - 3*a*d^3)*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/
3)])/(6*(b*c^3 - a*d^3)^(7/3))
```

Rubi [F] time = 0.0825825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c + dx)^3 \sqrt[3]{a + bx^3}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F] time = 0.475202, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx+c\right)^{3}}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

3.36 $\int \frac{(c+dx)^4}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=306

$$-\frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}-\frac{4c^3d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}+\frac{2ad^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}+\frac{6c^2d^2\sqrt[3]{a}}{b}$$

```
[Out] (6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c^
3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)
) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3
, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c^3*d*Log
[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3) + (a*d^4*Log[b^(1/3)*x - (a + b*x^
3)^(1/3)])/(3*b^(5/3))
```

Rubi [A] time = 0.267671, antiderivative size = 416, normalized size of antiderivative = 1.36, number of steps used = 22, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {1893, 246, 245, 331, 292, 31, 634, 617, 204, 628, 261, 365, 364, 321}

$$-\frac{4c^3d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{2c^3d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}}-\frac{4c^3d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2ad^4\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}}-\frac{ad^4\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c^
3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)
) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3
, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (4*c^3*d*Log
[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) + (2*a*d^4*Log[1 - (b^(1/3
)*x)/(a + b*x^3)^(1/3)])/(9*b^(5/3)) + (2*c^3*d*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) - (a*d^4*Log[1 +
 (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(5/
3))
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(c+dx)^4}{\left(a+bx^3\right)^{2/3}}\,dx = \int \left(\frac{c^4}{\left(a+bx^3\right)^{2/3}} + \frac{4c^3dx}{\left(a+bx^3\right)^{2/3}} + \frac{6c^2d^2x^2}{\left(a+bx^3\right)^{2/3}} + \frac{4cd^3x^3}{\left(a+bx^3\right)^{2/3}} + \frac{d^4x^4}{\left(a+bx^3\right)^{2/3}}\right) dx$$

$$= c^4 \int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + \left(4c^3d\right) \int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx + \left(6c^2d^2\right) \int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx + \left(4cd^3\right) \int \frac{x}{(a+b}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \left(4c^3d\right) \operatorname{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) - \frac{\left(2ad^4\right) \int \frac{x}{(a+bx^3)}}{3b}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\right.}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\right.}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\right.}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} -$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{2ad^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}} + \frac{c^4x\left(1+\frac{bx^3}{a}\right.}{}$$

Mathematica [A] time = 0.144735, size = 166, normalized size = 0.54

$$\frac{d\left(x^2\left(6bc^3-ad^3\right) {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right) + d\left(\left(a+bx^3\right)\left(18c^2+d^2x^2\right) + 3bcdx^4\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right) + 3bc^4x}{3b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b*c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*((6*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)] + d*((18*c^2 + d^2*x^2)*(a + b*x^3) + 3*b*c*d*x^4*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])))/(3*b*(a + b*x^3)^(2
/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (dx+c)^4 \left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 4.32191, size = 204, normalized size = 0.67

$$6c^2d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(2/3),x)
```

```
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b
, True)) + c**4*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*a**(2/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((2/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + 4*c*d**3*x**4*g
amma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*g
amma(7/3)) + d**4*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^4}{(bx^3 + a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

3.37 $\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=187

$$-\frac{3c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{\sqrt{3}c^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2bc^3-ad^3\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{3cd^2\sqrt[3]{a+bx^3}}{b}$$

```
[Out] (3*c*d^2*(a + b*x^3)^(1/3))/b + (d^3*x*(a + b*x^3)^(1/3))/(2*b) - (Sqrt[3]*
c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/b^(2/3) + ((2*
b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3)) - (3*c^2*d*Log[b^(1/3)*x - (a + b*x^3)^
(1/3)])/(2*b^(2/3))
```

Rubi [A] time = 0.23701, antiderivative size = 239, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1888, 1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{c^2d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}+\frac{c^2d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{2b^{2/3}}-\frac{\sqrt{3}c^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2bc^3-ad^3\right){}_2F_1\left(\frac{1}{3},\ldots\right)}{2b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(1/3))/b + (d^3*x*(a + b*x^3)^(1/3))/(2*b) - (Sqrt[3]*
c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/b^(2/3) + ((2*
b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3)) - (c^2*d*Log[1 - (b^(1/3)*x)/(a + b*x^3
)^(1/3)])/b^(2/3) + (c^2*d*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/
3)*x)/(a + b*x^3)^(1/3)])/(2*b^(2/3))
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \frac{2bc^3-ad^3+6bc^2dx+6bcd^2x^2}{\left(a+bx^3\right)^{2/3}}\,dx}{2b}$$

$$= \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \frac{2bc^3-ad^3+6bc^2dx}{\left(a+bx^3\right)^{2/3}}\,dx}{2b} + \left(3cd^2\right)\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \left(\frac{2bc^3\left(1-\frac{ad^3}{2bc^3}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{6bc^2dx}{\left(a+bx^3\right)^{2/3}}\right)dx}{2b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \left(3c^2d\right)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx + \frac{\left(2bc^3-ad^3\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \left(3c^2d\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(\left(2bc^3-ad^3\right)\left(1+\frac{bx}{a}\right.\right.}{2b\left(a+\right.}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} + \frac{\left(c^2d\right)\text{Subst}\left(\int \frac{}{1-}\right.}{\sqrt[3]{}}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} - \frac{c^2d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx}}\right.}{b^{2/3}}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} - \frac{c^2d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx}}\right.}{b^{2/3}}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} - \frac{\sqrt{3}c^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\right.}{2b\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.0992056, size = 145, normalized size = 0.78

$$\frac{d\left(6bc^2x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)+d\left(12c\left(a+bx^3\right)+bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right)+4bc^3x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\right.}{4b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (4*b*c^3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)] +
 d*(12*c*(a + b*x^3) + b*d*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3,
 4/3, 7/3, -((b*x^3)/a)])))/(4*b*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (dx+c)^3 \left(bx^3+a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(bx^3+a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.2615, size = 153, normalized size = 0.82

$$3cd^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for}\ b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x**3+a)**(2/3),x)
```

```
[Out] 3*c*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, T
rue)) + c**3*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(2/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(a**(2/3)*gamma(5/3)) + d**3*x**4*gamma(4/3)*h
yper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

3.38 $\int \frac{(c+dx)^2}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=141

$$-\frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}-\frac{2cd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

```
[Out] (d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (c*d*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/b^(2/3)
```

Rubi [A] time = 0.147777, antiderivative size = 195, normalized size of antiderivative = 1.38, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{2cd\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{cd\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}}-\frac{2cd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c*d*Log[1 - (b^
(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) + (c*d*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3))
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{\left(a+bx^3\right)^{2/3}}\,dx = d^2\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx + \int \frac{c^2+2cdx}{\left(a+bx^3\right)^{2/3}}\,dx$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \int \left(\frac{c^2}{\left(a+bx^3\right)^{2/3}} + \frac{2cdx}{\left(a+bx^3\right)^{2/3}}\right)dx$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + c^2\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + (2cd)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + (2cd)\,\mathrm{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(c^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{(2cd)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{(2cd)}{}$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{(cd)\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{b}+2}{1+\sqrt[3]{b}x}\right.}{3b}$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{cd\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}\right.}{3b^{2/3}}$$
$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}$$

Mathematica [A] time = 0.0433338, size = 95, normalized size = 0.67

$$\frac{bc^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(bcx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)+d\left(a+bx^3\right)\right)}{b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (b*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3) + b*c*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)]))/(b*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (dx+c)^2\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.60459, size = 109, normalized size = 0.77

$$d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, True)
) + c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(2/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

3.39 $\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=121

$$-\frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

```
[Out] -((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3
))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)])/(a + b*x^3)^(2/3) - (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)
)
```

Rubi [A] time = 0.114944, antiderivative size = 172, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] -((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3
))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)])/(a + b*x^3)^(2/3) - (d*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(
2/3)) + (d*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3
)^(1/3)])/(6*b^(2/3))
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx = \int \left(\frac{c}{\left(a+bx^3\right)^{2/3}} + \frac{dx}{\left(a+bx^3\right)^{2/3}}\right) dx$$

$$= c\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + d\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= d\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(c\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{d\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{d\,\text{Subst}\left(\int \frac{1-\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,d\right.}{3\sqrt[3]{b}}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}} + \frac{d\,\text{Sub}}{}$$

$$= -\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\log\left(1+\frac{b^{2/3}}{(a+bx}\right.}{6b}$$

Mathematica [A] time = 0.031943, size = 78, normalized size = 0.64

$$\frac{x\left(2c\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)\right)}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(2*c*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
] + d*x*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(2*(a + b*x^3
)^(2/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (dx+c)\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\left(bx^3 + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\left(bx^3 + a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] time = 1.89121, size = 78, normalized size = 0.64

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3}, \frac{2}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3}, \frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\left(bx^3 + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

3.40 $\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=332

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}}-\frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{\ldots}\right)}{2\left(bc^3-ad^3\right)^{2/\ldots}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d^3*x^
3)/c^3)])/(c*(a + b*x^3)^(2/3)) + (d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x
)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(2/3)) - (d*Arc
Tan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
(b*c^3 - a*d^3)^(2/3)) - (d*Log[c^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(2/3)) +
 (d*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3
)^(2/3)) + (d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 -
 a*d^3)^(2/3))
```

Rubi [F] time = 0.0913542, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.041249, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\left(bx^3+a\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

3.41 $\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=760

$$-\frac{d^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\left(a+bx^3\right)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\left(a+bx^3\right)^{2/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)}+\frac{}{(c^3\cdots}$$

```
[Out] (c^2*d^2*(a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (d^4*x^2*(a
 + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/
3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^2*(a + b*
x^3)^(2/3)) - (d^3*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 2, 7/3, -((
b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*(a + b*x^3)^(2/3)) + (2*a*d^4*ArcTan[(
1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]
*c*(b*c^3 - a*d^3)^(5/3)) + (2*d*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 -
a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^
3)^(5/3)) - (2*b*c^2*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(5/3)) - (b*c^2*d*Log[c^3 + d^3*x
^3])/(3*(b*c^3 - a*d^3)^(5/3)) - (a*d^4*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a
*d^3)^(5/3)) - (d*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3
)^(5/3)) + (a*d^4*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*
c*(b*c^3 - a*d^3)^(5/3)) + (d*(3*b*c^3 - a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*
x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (b*c^2*d*Log[(b*c^
3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(b*c^3 - a*d^3)^(5/3)
```

Rubi [F] time = 0.0848816, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.323316, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]

[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)

[Out] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(2/3),x)

[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

3.42 $\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=1357

result too large to display

```
[Out] (3*c^4*d^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (3*c^
2*d^4*x^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (5*b*c
^4*d^2*(a + b*x^3)^(1/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(b
*c^3 - 6*a*d^3)*(a + b*x^3)^(1/3))/(6*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) +
(d^4*(9*b*c^3 - 4*a*d^3)*x^2*(a + b*x^3)^(1/3))/(6*c*(b*c^3 - a*d^3)^2*(c^3
 + d^3*x^3)) + (d^4*(3*b*c^3 + 2*a*d^3)*x^2*(a + b*x^3)^(1/3))/(3*c*(b*c^3
- a*d^3)^2*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3
, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a + b*x^3)^(2/3)) - (7*d^3*x^
4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 3, 7/3, -((b*x^3)/a), -((d^3*x^3
)/c^3)])/(4*c^6*(a + b*x^3)^(2/3)) + (d^6*x^7*(1 + (b*x^3)/a)^(2/3)*AppellF
1[7/3, 2/3, 3, 10/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(7*c^9*(a + b*x^3)^(2
/3)) + (2*a*d^4*(6*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(
c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)^(8/3)) + (d*
(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3
)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)^(8/3))
 - (10*b^2*c^4*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))
/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Arc
Tan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3
]*(b*c^3 - a*d^3)^(8/3)) - (5*b^2*c^4*d*Log[c^3 + d^3*x^3])/(9*(b*c^3 - a*d
^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Log[c^3 + d^3*x^3])/(18*(b*c^3 - a*d^
3)^(8/3)) - (a*d^4*(6*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c^2*(b*c^3 - a*
d^3)^(8/3)) - (d*(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*Log[c^3 + d^3*x^3]
)/(18*c^2*(b*c^3 - a*d^3)^(8/3)) + (a*d^4*(6*b*c^3 - a*d^3)*Log[((b*c^3 - a
*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c^2*(b*c^3 - a*d^3)^(8/3)) + (d*(
9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a
 + b*x^3)^(1/3)])/(6*c^2*(b*c^3 - a*d^3)^(8/3)) + (5*b^2*c^4*d*Log[(b*c^3 -
 a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(3*(b*c^3 - a*d^3)^(8/3)) - (b*c*d*(b
*c^3 - 6*a*d^3)*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(6*(b*c^3
 - a*d^3)^(8/3))
```

Rubi [F] time = 0.0831781, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.43289, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^3 \left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^3} \left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}} (dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}} (c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}}(dx + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

3.43 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=37

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rubi [A] time = 0.104899, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2137, 203}

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx &= \left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right)\\ &= \frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.430227, size = 326, normalized size = 8.81

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6\right.}{\sqrt{3}\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i\sqrt{x^3+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S
qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.034, size = 258, normalized size = 7.

$$-4\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+6\,\frac{2^{2}}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*2^(2/3)*(3/2-1/2*I*3^(1/2))*((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [B] time = 3.099, size = 198, normalized size = 5.35

$$\frac{1}{3}\,\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}2^{\frac{1}{6}}\left(2\,x^5+2\,x^2-2^{\frac{2}{3}}\left(7\,x^4+4\,x\right)-2^{\frac{1}{3}}\left(5\,x^3+2\right)\right)\sqrt{x^3+1}}{12\left(2\,x^6+3\,x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*(
7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int \frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.44 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Rubi [A] time = 0.122525, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2137, 203}

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx &= -\left(\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.365045, size = 327, normalized size = 8.18

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}\right.\right.\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I
)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)
+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I
 + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2
^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.037, size = 253, normalized size = 6.3

$$\frac{4i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/
3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [B] time = 2.50024, size = 200, normalized size = 5.

$$-\frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}2^{\frac{1}{6}}\left(2\,x^5-2\,x^2+2^{\frac{2}{3}}\left(7\,x^4-4\,x\right)-2^{\frac{1}{3}}\left(5\,x^3-2\right)\right)\sqrt{-x^3+1}}{12\left(2\,x^6-3\,x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int \frac{2x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integ
ral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.45 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Rubi [A] time = 0.112427, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 206}

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx &= -\left(\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)\right)\\
&= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.304679, size = 325, normalized size = 8.55

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\right.\right.\right.}{\sqrt{3}\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i\sqrt{x^3-1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I
)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)
+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I
 + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2
^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [C] time = 0.029, size = 262, normalized size = 6.9

$$-4\,{\frac {-3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}-1}}\sqrt {{\frac {x-1}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {x-1}{-3/2-i/2\sqrt {3}}}},\sqrt {{\frac {3/2+i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}} \right) }-6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-6*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2
^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x + 2^{\frac{2}{3}}}{\sqrt{x^{3} - 1}\left(x - 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [B] time = 2.42924, size = 717, normalized size = 18.87

$$\frac{1}{6}\,\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18} + 1440\,x^{15} + 17400\,x^{12} - 21056\,x^{9} - 10368\,x^{6} + 15360\,x^{3} + 2\,\sqrt{6}2^{\frac{1}{6}}\left(126\,x^{14} + 2664\,x^{11} - 4608\,x^{5} +\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368*
x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x) +
 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqrt
(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 6
4*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128*
x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 40
96))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int \frac{2x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integ
ral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.46 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=39

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Rubi [A] time = 0.113543, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 206}

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx &= \left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right)\\ &= \frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.283514, size = 328, normalized size = 8.41

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i\sqrt{-x^3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S
qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [C] time = 0.032, size = 249, normalized size = 6.4

$$\frac{4i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3)),(
I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x-2^{\frac{2}{3}}}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [B] time = 2.22002, size = 718, normalized size = 18.41

$$\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}-1440x^{15}+17400x^{12}+21056x^{9}-10368x^{6}-15360x^{3}-2\sqrt{6}2^{\frac{1}{6}}\left(126x^{14}-2664x^{11}+4608x^{5}+230\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368*
x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x) -
 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqrt
(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128
*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx-\int \frac{2x}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - In
tegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.47 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=63

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.178709, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2137, 203}

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a +
 b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$
$$= \frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 1.10104, size = 325, normalized size = 5.16

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{bx}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{2\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{bx}\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{bx}}{\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx}+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{bx}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{3\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{bx}}{\sqrt[3]{a}}+1}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\right.\right.}{\sqrt[3]{-1}+2^{2/3}}\right.}{\sqrt{3}\sqrt[3]{b}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((2*3^(1/4)*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))] - (3*(-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*
x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^
(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(1/3)*Sqrt[a + b*x
^3])
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}-2\sqrt[3]{bx}\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^
(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx-\int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)
*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(a
+ b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^
(2/3)*a^(1/3))), x)
```

3.48 $\int \frac{2^{2/3} \sqrt[3]{a}+2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx$

Optimal. Leaf size=65

$$-\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.199191, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 203}

$$-\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a -
 b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3} \sqrt[3]{a}+2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx &= -\frac{\left(2\ 2^{2/3} \sqrt[3]{a}\right) \text{Subst}\left(\int \frac{1}{1+3 a x^2} \, dx,x,\frac{1-\frac{\sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a}}}{\sqrt{a-b x^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 1.11903, size = 336, normalized size = 5.17

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{2\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}+\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{3b^{2/3}x^2}{a^{2/3}}+\frac{3\sqrt[3]{b}x}{\sqrt[3]{a}}+3}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\right.\right.}{\sqrt[3]{-1}+2^{2/3}}\right.}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3)*
a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3
))*a^(1/3)*Sqrt[3 + (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3)
)))/(b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}+2\sqrt[3]{b}x\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1
/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1
/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)
*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

3.49 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.200721, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 206}

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
+ b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx &= -\frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\
&= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.467082, size = 390, normalized size = 5.91

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(2\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\sqrt[3]{-1}2^{2/3}\sqrt{3}\left(1\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(2*((-1)^(1/3) +
 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - (-1)
^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2
/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqr
t[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)
]))/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}+2\sqrt[3]{bx}\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^
(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx-\int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^
(2/3)*a^(1/3))), x)
```

3.50 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.193079, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 206}

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
 - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx &= \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.653784, size = 375, normalized size = 5.68

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\sqrt[3]{-1}2^{2/3}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3)
 + 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x
)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4) + (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 +
 (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticP
i[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]))/(((-1)^(1/3) + 2^(2/3))
*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[-a - b*x^3])
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(2^{\frac{2}{3}}\sqrt[3]{a}-2\sqrt[3]{b}x\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1
/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1
/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx-\int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

3.51 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sq
rt[c]*d)
```

Rubi [A] time = 0.123039, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sq
rt[c]*d)
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx &= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{d}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}\end{aligned}$$

Mathematica [C] time = 1.06629, size = 373, normalized size = 7.61

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(2\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)-\sqrt[3]{-1}2^{2/3}\sqrt{3}\right.}{\left(2+\sqrt[3]{-2}\right)d\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(2*Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)
*Sqrt[3]*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(
I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*
d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]))/((2 + (-2)^(1/3))*d*Sqrt
[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3*x^3]
)
```

Maple [C] time = 0.036, size = 889, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/
2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(
1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+
c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(
1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2
^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/
2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)
-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)
*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(
((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^
(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,dx - c}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [B] time = 3.16113, size = 657, normalized size = 13.41

$$\left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\left(4\,cd^4x^4-10\,c^2d^3x^3-18\,c^3d^2x^2-8\,c^4dx-c^5\right)\sqrt{4\,d^3x^3+c^3}\sqrt{-\frac{1}{c}}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,d}, -\frac{\sqrt{3}\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 28
*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x^3
 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(d^6
*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c
^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*
d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 + 2*c^3*
d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx - \int \frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**
3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,dx - c}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")

[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.52 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=158

$$\frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.213155, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 218, 2137, 203}

$$\frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```


```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx + \frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(2-3\ 2^{2/3}\right.\right.$$

$$= \frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.470773, size = 336, normalized size = 2.13

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(3\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i\sqrt{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.028, size = 262, normalized size = 1.7

$$6\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+2\,\frac{(2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(2-3*2^(2/3))*(3/2-1/2*I*3^(1/2
))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,x^3+2\,x^2-2^{\frac{2}{3}}\left(3\,x^2+2\,x\right)+2\cdot 2^{\frac{1}{3}}(3\,x+2)\right)\sqrt{x^3+1}}{x^6+5\,x^3+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^3 + 2*x^2 - 2^(2/3)*(3*x^2 + 2*x) + 2*2^(1/3)*(3*x + 2))*sqrt
(x^3 + 1)/(x^6 + 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.53 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.242265, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2139, 218, 2137, 203}

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(2+3\ 2^{2/3}\right)\right.$$

$$= -\frac{2\left(2+3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{}}{1+\sqrt{}}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.47346, size = 335, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3i\sqrt{-2ix+\sqrt{3}-i}\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.028, size = 257, normalized size = 1.5

$$2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*I*(-2-3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2
)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^3+2x^2+2^{\frac{2}{3}}\left(3x^2+2x\right)+2\cdot 2^{\frac{1}{3}}(3x+2)\right)\sqrt{-x^3+1}}{x^6-5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^3 + 2*x^2 + 2^(2/3)*(3*x^2 + 2*x) + 2*2^(1/3)*(3*x + 2))*sqrt
(-x^3 + 1)/(x^6 - 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int \frac{2}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(2
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.54 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=176

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rubi [A] time = 0.224916, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```


```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}\left(2\left(2+3\,2^{2/3}\right.\right.$$

$$= -\frac{2\left(2+3\,2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.32222, size = 333, normalized size = 1.89

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3i\sqrt{-2ix+\sqrt{3}-i}\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i\sqrt{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] time = 0.02, size = 266, normalized size = 1.5

$$-6\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -6*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-2-3*2^(2/3))*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1
/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3\,x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int \frac{2}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(2
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3\,x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.55 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=169

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rubi [A] time = 0.229296, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx+\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{1}{3}\left(2\left(2-3\,2^{2/}\right.\right.$$

$$= \frac{2\left(2-3\,2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+}{1-}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.327312, size = 338, normalized size = 2.

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(3\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i\sqrt{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.021, size = 253, normalized size = 1.5

$$-2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*
I*3^(1/2)))^(1/2))-2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(
2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,x^3+2\,x^2-2^{\frac{2}{3}}\left(3\,x^2+2\,x\right)+2\cdot 2^{\frac{1}{3}}(3\,x+2)\right)\sqrt{-x^3-1}}{x^6+5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*x^3 + 2*x^2 - 2^(2/3)*(3*x^2 + 2*x) + 2*2^(1/3)*(3*x + 2))*sqr
t(-x^3 - 1)/(x^6 + 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.56 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=159

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.23216, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 218, 2137, 203}

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx + \frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(e-2^{2/3}f\right)\right.$$

$$= \frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+\right.\right.}{1+\sqrt{3}+}\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.470311, size = 340, normalized size = 2.14

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(f\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.023, size = 264, normalized size = 1.7

$$2\,\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*3^(1
/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(
1/2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.57 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.25963, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2139, 218, 2137, 203}

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```


```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\left(-\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{1}{3}\left(2\left(e+2^{2/3}f\right)\right.$$

$$= -\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.489026, size = 340, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt[3]{2}e+2f\right)\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-if\sqrt{-2ix+\sqrt{3}-}\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.024, size = 261, normalized size = 1.5

$$\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^
(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx^3+ex^2+2^{\frac{2}{3}}\left(fx^2+ex\right)+2\cdot 2^{\frac{1}{3}}\left(fx+e\right)\right)\sqrt{-x^3+1}}{x^6-5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + 2^(2/3)*(f*x^2 + e*x) + 2*2^(1/3)*(f*x + e))*sqrt
(-x^3 + 1)/(x^6 - 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int \frac{fx}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(f*x
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.58 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=178

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rubi [A] time = 0.224403, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\left(-\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}\left(2\left(e+2^{2/3}\right.\right.$$

$$= -\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+}{1-}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.350988, size = 338, normalized size = 1.9

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt[3]{2}e+2f\right)\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-if\sqrt{-2ix+\sqrt{3}-\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] time = 0.023, size = 270, normalized size = 1.5

$$-2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-e-2^(2/3)*f)*(-3/2-1/2*I*3^
(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^
(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(fx^3+ex^2+2^{\frac{2}{3}}\left(fx^2+ex\right)+2\cdot 2^{\frac{1}{3}}\left(fx+e\right)\right)\sqrt{x^3-1}}{x^6-5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^3 + e*x^2 + 2^(2/3)*(f*x^2 + e*x) + 2*2^(1/3)*(f*x + e))*sqr
t(x^3 - 1)/(x^6 - 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int \frac{fx}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(f*x
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.59 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=170

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rubi [A] time = 0.221745, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx+\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{1}{3}\left(2\left(e-2^{2/3}f\right.\right.$$

$$= \frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.443712, size = 342, normalized size = 2.01

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(f\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.023, size = 255, normalized size = 1.5

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))
*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)
+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(fx^3+ex^2-2^{\frac{2}{3}}\left(fx^2+ex\right)+2\cdot 2^{\frac{1}{3}}\left(fx+e\right)\right)\sqrt{-x^3-1}}{x^6+5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^3 + e*x^2 - 2^(2/3)*(f*x^2 + e*x) + 2*2^(1/3)*(f*x + e))*sqr
t(-x^3 - 1)/(x^6 + 5*x^3 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.60 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=316

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\cdots}{3\sqrt{3}\cdots}$$

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/
3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 +
Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rubi [A] time = 0.408653, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\cdots}{3\sqrt{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/
3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 +
Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx+\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx$$

$$=\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\cdots\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$=\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\cdots}{3\sqrt[4]{3}\sqrt[3]{b}\cdots}$$

Mathematica [C] time = 1.54848, size = 336, normalized size = 1.06

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2^{2/3}\sqrt[3]{a}f-\sqrt[3]{b}e\right)\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}-\frac{\sqrt[4]{3}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}F\left(\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\cdots}{\left(1+\sqrt[3]{-1}\right)\cdots}}}\right.}{\sqrt{3}b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(1/4)*f*((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Elli
pticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f
)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[
3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*
```

```
b^(2/3)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{a+bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.61 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=324

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3\sqrt{}}$$

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 +
 Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/
3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x
^3])
```

Rubi [A] time = 0.429667, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2139, 218, 2137, 203}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 +
 Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/
3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x
^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{a-bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\right|-}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right.}{3\sqrt[4]{3}\sqrt[3]{}}$$

Mathematica [C] time = 1.2711, size = 399, normalized size = 1.23

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{}}{}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((
-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1
```

```
/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*s
qrt(a - b*x**3)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.62 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3}$$

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(
1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2
 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rubi [A] time = 0.41287, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2139, 219, 2137, 206}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(
1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2
 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a+bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\right.}{3\sqrt[4]{3}}$$

Mathematica [C] time = 0.428471, size = 400, normalized size = 1.2

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{}}{}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((
-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1
```

```
/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*
x*sqrt(-a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.63 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=329

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\text{t}}{3\sqrt{}}$$

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1
/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rubi [A] time = 0.397515, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\text{t}}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```


```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1
/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx+\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= \frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3\sqrt[4]{3}\sqrt[3]{}}$$

Mathematica [C] time = 0.807576, size = 387, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2^{2/3}\sqrt[3]{a}f-\sqrt[3]{b}e\right)\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}}-\frac{\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\right.}{}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3) +
 2^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x
)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4)) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(
-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/
```

```
3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(
((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-a-bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.64 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=265

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\right.}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])
/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c + 2^
(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c
+ 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sq
rt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^
(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] time = 0.296453, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2139, 218, 2137, 203}

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\right.}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```


```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])
/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c + 2^
(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c
+ 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sq
rt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^
(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3cd}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$= \frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{}}$$

Mathematica [C] time = 1.40619, size = 380, normalized size = 1.43

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}2^{2/3}(1}{}\right)}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt[((-2)^(1/
3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))
*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1
)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(
2/3)*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((
1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*Ell
ipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-
1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 +
(-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*S
```

```
qrt[c^3 + 4*d^3*x^3])
```

Maple [B] time = 0.007, size = 900, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2*f/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x
+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d)
)^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x
^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
)^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2
)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2))+2*(-c*f+d*e)/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/
3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/
4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/
d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/
d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((
1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(
1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,d^3x^3+c^3}(fx+e)}{4\,d^4x^4+4\,cd^3x^3+c^3dx+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*d^3*x^3 + c^3)*(f*x + e)/(4*d^4*x^4 + 4*c*d^3*x^3 + c^3*d*x
 + c^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.65 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.20909, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{1}{3}\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x}\right.$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.316457, size = 207, normalized size = 1.43

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2-x+1}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 + x^3]
```

Maple [B] time = 0.022, size = 258, normalized size = 1.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-2\,\frac{2^{2/3}}{\sqrt{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
```

```
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))*((1
+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^
(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/
(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^3-2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{x^3+1}}{x^6+5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^3 - 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(x^3 + 1)/(x^6 + 5*x^3 + 4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.66 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.241281, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.313085, size = 209, normalized size = 1.31

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2+x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 - x^3]
```

Maple [A] time = 0.021, size = 253, normalized size = 1.6

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(
2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^3+2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{-x^3+1}}{x^6-5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(-x^3 + 1)/(x^6 - 5*x^3 + 4)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.67 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.224578, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}\left(2\ 2^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{1-3 \cdots}\right.$$

$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3} \cdots\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.150705, size = 207, normalized size = 1.27

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2+x+1}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 + x^3]
```

Maple [B] time = 0.02, size = 262, normalized size = 1.6

$$-2\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-2 \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
```

```
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2
^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(x^3+2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{x^3-1}}{x^6-5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^3 + 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(x^3 - 1)/(x^6 - 5*x^3 + 4)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.68 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.227826, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3\ldots}\right.$$

$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\ldots\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.160598, size = 209, normalized size = 1.34

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2-x+1}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 - x^3]
```

Maple [A] time = 0.019, size = 249, normalized size = 1.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(x^3-2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{-x^3-1}}{x^6+5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^3 - 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(-x^3 - 1)/(x^6 + 5*x^3 + 4
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.69 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=275

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rubi [A] time = 0.359081, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```


```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \ldots$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\ldots\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\ldots}}$$

Mathematica [C] time = 1.05643, size = 324, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}} - \frac{\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\ldots}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\ldots\right)}{\sqrt{3}b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(1/4)*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))]) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(
2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(2/3)*Sqrt[a + b*x^
```

```
3])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int x \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)^{-1} \frac{1}{\sqrt{b x^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b x^3 + a} \left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + b x^3} \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.70 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*S
qrt[a - b*x^3])
```

Rubi [A] time = 0.375863, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```


```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*S
qrt[a - b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-$$

$$=-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\mathrm{si}\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}}}$$

Mathematica [C] time = 0.896838, size = 388, normalized size = 1.37

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{\cdots}}{\cdots}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/
a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
```

```
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3
]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x \left(2^{\frac{2}{3}} \sqrt[3]{a} - \sqrt[3]{bx}\right)^{-1} \frac{1}{\sqrt{-bx^3 + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-bx^3 + a}\left(b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{a - bx^3} + \sqrt[3]{b}x\sqrt{a - bx^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.71 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=292

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.387256, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```



```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - $$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

Mathematica [C] time = 0.309039, size = 389, normalized size = 1.33

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{}}{}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/
a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
```

```
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3
]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int x\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^{3}}+\sqrt[3]{b}x\sqrt{-a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.72 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=288

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.370384, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-$$

$$=-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\cdot}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

Mathematica [C] time = 0.506538, size = 375, normalized size = 1.3

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}}-\frac{\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\cdot}}{\cdot}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3) +
 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/
a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4)) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1
/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqr
t[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
```

```
((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3)
)*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.73 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^
(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*
c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] time = 0.270559, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2139, 218, 2137, 203}

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```


```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^
(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*
c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}} - \frac{(2c \ldots}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+\ldots}{\left(1+\sqrt{3}\right)c+\ldots}\right.\right.}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Mathematica [C] time = 0.970301, size = 372, normalized size = 1.51

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}2^{2/3}\left(1+\ldots\right.}{\ldots}\right.}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(2/
3)*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1
/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/(
(1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 + (-2)^(1/3))*d^2
*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3
```

```
*x^3])
```

Maple [B] time = 0.009, size = 892, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^
(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^
(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*
2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1
/2))-2*c/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1
/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1
/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/
3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(
4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*Elliptic
Pi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)
*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)
+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)
*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,d^3x^3+c^3}x}{4\,d^4x^4+4\,cd^3x^3+c^3dx+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*d^3*x^3 + c^3)*x/(4*d^4*x^4 + 4*c*d^3*x^3 + c^3*d*x + c^4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.74 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx$

Optimal. Leaf size=23

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rubi [A] time = 0.0592311, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2138, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx &= 2\,\text{Subst}\left(\int \frac{1}{9-x^2} dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0093062, size = 46, normalized size = 2.

$$\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)-\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] -Log[3 - (1 + x)^2/Sqrt[1 + x^3]]/3 + Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

Maple [C] time = 0.02, size = 240, normalized size = 10.4

$$-2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+2\frac{3/}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi
(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [B] time = 1.99485, size = 117, normalized size = 5.09

$$\frac{1}{3}\log\left(\frac{x^3+12x^2+6\sqrt{x^3+1}(x+1)-6x+10}{x^3-6x^2+12x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int\frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt(
x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.75 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx$

Optimal. Leaf size=27

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3

Rubi [A] time = 0.0650662, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]

[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2} \, dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0098922, size = 54, normalized size = 2.

$$\frac{1}{3}\log\left(3-\frac{(1-x)^2}{\sqrt{1-x^3}}\right)-\frac{1}{3}\log\left(\frac{(1-x)^2}{\sqrt{1-x^3}}+3\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]

```
[Out] Log[3 - (1 - x)^2/Sqrt[1 - x^3]]/3 - Log[3 + (1 - x)^2/Sqrt[1 - x^3]]/3
```

Maple [C] time = 0.024, size = 240, normalized size = 8.9

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(2+x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I
*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [B] time = 1.90357, size = 120, normalized size = 4.44

$$\frac{1}{3}\log\left(-\frac{x^3-12x^2-6\sqrt{-x^3+1}(x-1)-6x-10}{x^3+6x^2+12x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int -\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqrt
(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.76 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx$

Optimal. Leaf size=25

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rubi [A] time = 0.0588041, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2138, 203}

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9+x^2} dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0087205, size = 25, normalized size = 1.

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Maple [C] time = 0.023, size = 240, normalized size = 9.6

$$-2\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(2+x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*
((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(
((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [B] time = 2.25901, size = 105, normalized size = 4.2

$$-\frac{1}{3}\arctan\left(\frac{\left(x^3-12\,x^2-6\,x-10\right)\sqrt{x^3-1}}{6\left(x^4-x^3-x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int -\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqrt
(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.77 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\, dx$

Optimal. Leaf size=25

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

```
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rubi [A] time = 0.0674888, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2138, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]

[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\, dx &= 2\,\text{Subst}\left(\int \frac{1}{9+x^2}\, dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0088987, size = 25, normalized size = 1.

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Maple [C] time = 0.023, size = 240, normalized size = 9.6

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1
+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x
^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3
^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [A] time = 2.09533, size = 107, normalized size = 4.28

$$-\frac{1}{3}\arctan\left(\frac{\left(x^3+12x^2-6x+10\right)\sqrt{-x^3-1}}{6\left(x^4+x^3+x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sqr
t(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.78 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=50

$$\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rubi [A] time = 0.132317, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2138, 206}

$$\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx &= \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.0247096, size = 51, normalized size = 1.02

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2}{3\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[a + b*x^3])])/(3*a
^(1/6)*b^(1/3))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\left(2\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx-\int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x
)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*s
qrt(a + b*x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.79 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=52

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rubi [A] time = 0.138134, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2138, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx &= -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.0234995, size = 53, normalized size = 1.02

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{3\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[a - b*x^3])])/(3*
a^(1/6)*b^(1/3))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),
x)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sq
rt(a - b*x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.80 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rubi [A] time = 0.14184, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2138, 203}

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx &= -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.0220353, size = 54, normalized size = 1.02

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{3\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[-a + b*x^3])])/(3*
a^(1/6)*b^(1/3))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)\left(2\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx-\int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x
)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x
*sqrt(-a + b*x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.81 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rubi [A] time = 0.141368, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2138, 203}

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx &= \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.0231941, size = 54, normalized size = 1.02

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2}{3\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[-a - b*x^3])])/(3*a
^(1/6)*b^(1/3))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx-\int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),
x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*
x*sqrt(-a - b*x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.82 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Rubi [A] time = 0.115931, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2138, 206}

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx &= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}\end{aligned}$$

Mathematica [A] time = 0.0244973, size = 46, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Maple [C] time = 0.143, size = 650, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/
2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2)
)*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((
x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2
*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*
I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)
)^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(
-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+6*c/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/
2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2
*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)
/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3
*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-1/2
-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))
*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1
/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1
/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,dx - c}{\sqrt{-8\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [B] time = 2.6736, size = 651, normalized size = 14.15

$$\left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\left(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4\right)\sqrt{-8\,d^3x^3+c^3}\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,\sqrt{c}d}, -\frac{\sqrt{-c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 15
6*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*
d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c
*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c
^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^
2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^
3 - 2*c^4*d*x + c^5))/(c*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx-\int \frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**
3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.83 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.149903, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 218, 2138, 206}

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{1+x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}(2(e+2f))\,\text{Subst}\left(\int \frac{}{9-}\right.$$

$$= \frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) + \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{\right.}}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.297003, size = 273, normalized size = 1.96

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}(e+2f)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\sqrt{3}+i}\left(\left(\sqrt{3}-\right.\right.\right.}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.005, size = 246, normalized size = 1.8

$$-2\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+\frac{(2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
```

```
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(1/2)
)*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2
)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3+1}\left(fx+e\right)}{x^4-2\,x^3+x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 + 1)*(f*x + e)/(x^4 - 2*x^3 + x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int \frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(f*x/(x*sqr
t(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.84 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=153

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.160596, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 218, 2138, 206}

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```


```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}(2(e-2f))\,\mathrm{Subst}\Bigg(\int$$

$$= -\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.271002, size = 271, normalized size = 1.77

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(3f\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+}\right.}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.007, size = 246, normalized size = 1.6

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
```

```
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\left(fx+e\right)}{x^4+2\,x^3-x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*(f*x + e)/(x^4 + 2*x^3 - x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.85 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=156

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.145891, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2139, 219, 2138, 203}

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}(2(e-2f))\,\mathrm{Subst}\left(\int\right.$$

$$= -\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.178481, size = 269, normalized size = 1.72

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(3f\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}(e\right.}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[-1 + x^3])
```

Maple [A] time = 0.004, size = 246, normalized size = 1.6

$$2\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2+x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
```

```
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1/2))
*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*
EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3-1}\left(fx+e\right)}{x^4+2\,x^3-x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*(f*x + e)/(x^4 + 2*x^3 - x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.86 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=150

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.162972, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 219, 2138, 203}

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{-1-x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}(2(e+2f))\,\mathrm{Subst}\Big(\int$$

$$= \frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.191926, size = 275, normalized size = 1.83

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}(e+2f)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\sqrt{3}+i}\left(\left(\sqrt{3}\right.\right.\right.}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.006, size = 246, normalized size = 1.6

$$\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
```

```
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3-1}\left(fx+e\right)}{x^4-2\,x^3+x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 - 1)*(f*x + e)/(x^4 - 2*x^3 + x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int \frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(f*x/(x*s
qrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.87 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=297

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan}{9\sqrt{a}b^2}$$

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.314986, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2139, 218, 2138, 206}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan}{9\sqrt{a}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```


```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx\right)-\frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

Mathematica [C] time = 1.31497, size = 419, normalized size = 1.41

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\frac{\sqrt[4]{3}f}{}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(3^(1/4)*f*((I
+ Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*b^
(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(
1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/(2*Sqrt[2]) + I*(
b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2
/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) +
```

```
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3
```

```
)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.88 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\text{ta}}{9\sqrt{ab}}$$

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sq
rt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.317509, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2139, 218, 2138, 206}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\text{ta}}{9\sqrt{ab}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sq
rt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

Mathematica [C] time = 1.21878, size = 447, normalized size = 1.47

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-i\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{\right.\right.\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/}}{\left(1+\sqrt[3]{-1}\right.}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-I/2)*f*Sqrt[((
-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))
]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[S
qrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3
))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3
) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3
]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqr
```

```
t[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(
1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{-bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=313

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\text{ta}}{9\sqrt{ab}}$$

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.320473, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2139, 219, 2138, 203}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\text{ta}}{9\sqrt{ab}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
```

```
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7+\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] time = 0.88537, size = 448, normalized size = 1.43

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-i\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{\cdots}\right.\right.\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/\cdots}}{\left(1+\sqrt[3]{-1}\cdots\right.}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-I/2)*f*Sqrt[((
-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))
]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[S
qrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3
))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3
) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3
]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqr
```

```
t[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(
1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.90 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=310

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan}{9\sqrt{a}b^2}$$

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt
[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.337208, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2139, 219, 2138, 203}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan}{9\sqrt{a}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt
[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
```

```
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx\right)-\frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7+4\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

Mathematica [C] time = 0.335488, size = 422, normalized size = 1.36

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\frac{\sqrt[4]{3}f\cdots}{\cdots}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(3^(1/4)*f*((I
+ Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*b^
(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(
1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/(2*Sqrt[2]) + I*(
b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2
/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) +
```

```
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}-\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
```

```
x**3)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")

[Out] Exception raised: TypeError
```

3.91 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=221

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(
9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 + 2
*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*
d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3]
)
```

Rubi [A] time = 0.284123, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2139, 218, 2138, 206}

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```


```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(
9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 + 2
*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*
d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3]
)
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3cd}$$

$$= -\frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left((1+\sqrt{3})c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})c-2dx}{(1+\sqrt{3})c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left((1+\sqrt{3})c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} - \frac{(2(de-\cdots}{\cdots}$$

$$= -\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} - \frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left((1+\sqrt{3})c-2dx\right)^2}}F\left(\sin^{-1}\left(\cdots\right.\right.}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left((1+\sqrt{3})c-2dx\right)^2}}\sqrt{c^3-8d\cdots}}$$

Mathematica [C] time = 1.1169, size = 384, normalized size = 1.74

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(4\sqrt{2}\sqrt{\frac{ic+\sqrt{3}dx+idx}{-\sqrt{3}c+3ic}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}}(de-cf)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{2}\sqrt{\frac{ic+\sqrt{3}dx+idx}{3ic-\sqrt{3}c}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+f\sqrt{\frac{(\sqrt{3}-i)c+2(\sqrt{\cdots}}{(\sqrt{3}-3i)\cdots}}\cdots}{2\left(\sqrt[3]{-1}-2\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] ((-I/2)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqrt[3])*c +
2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqrt[3])*c - 2*(3*I + S
qrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)/((3*
I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[(I*c +
 I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^
2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt[(I*c +
I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))/((-2 + (
-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 -
 8*d^3*x^3])
```

Maple [B] time = 0.008, size = 661, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2*f/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(
-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1
/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)
*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+
1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1
/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c
/d))^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+2*(-c*f+d*e)/d^2*(1/2*(-1/2+1/2
*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*
c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x
-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I
*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))
^(1/2)/(-8*d^3*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi
(((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-
1/2*I*3^(1/2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^
(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c
/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-8\,d^3x^3+c^3}\left(fx+e\right)}{8\,d^4x^4+8\,cd^3x^3-c^3dx-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-8*d^3*x^3 + c^3)*(f*x + e)/(8*d^4*x^4 + 8*c*d^3*x^3 - c^3*d
*x - c^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(-c+2dx)\left(c^2+2cdx+4d^2x^2\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c +
 d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.92 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=129

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.142337, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2139, 218, 2138, 206}

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{9-x^2}\,dx,x,\frac{(1+x}{\sqrt{1+}}\right.$$

$$= \frac{4}{9}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.251201, size = 193, normalized size = 1.5

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 + x^3]
```

Maple [B] time = 0.005, size = 240, normalized size = 1.9

$$-2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+\frac{6-2i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
```

```
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((1+x)/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elliptic
Pi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3+1}x}{x^4-2x^3+x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 + 1)*x/(x^4 - 2*x^3 + x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.93 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.149138, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2139, 218, 2138, 206}

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2]*Sqrt[1 - x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-}{\sqrt{1}}\right.$$

$$= \frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.230673, size = 195, normalized size = 1.34

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

Maple [A] time = 0.006, size = 240, normalized size = 1.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
```

```
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}x}{x^4+2x^3-x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*x/(x^4 + 2*x^3 - x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)\left(x^2+x+1\right)}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.94 $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=148

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
- x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.134564, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2139, 219, 2138, 203}

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
- x)^2)]*Sqrt[-1 + x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-}{\sqrt{-1}}\right.$$

$$= \frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.10447, size = 193, normalized size = 1.3

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

Maple [B] time = 0.004, size = 240, normalized size = 1.6

$$2\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right) - \frac{-6-}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
```

```
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3-1}\,x}{x^4+2\,x^3-x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*x/(x^4 + 2*x^3 - x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.95 $\int \frac{x}{(2-x)\sqrt{-1-x^3}} \, dx$

Optimal. Leaf size=140

$$\frac{4}{9}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.154992, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2139, 219, 2138, 203}

$$\frac{4}{9}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{4}{3}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, -\right.$$

$$= \frac{4}{9}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.183339, size = 195, normalized size = 1.39

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 - x^3]
```

Maple [B] time = 0.004, size = 240, normalized size = 1.7

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
```

```
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3-1}x}{x^4-2x^3+x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 - 1)*x/(x^4 - 2*x^3 + x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.96 $\int \frac{x}{\left(2 \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=260

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.280211, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2139, 218, 2138, 206}

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```


```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\left(4\sqrt[3]{\ldots}\right.}{}$$

$$= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ldots}}\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 1.34486, size = 407, normalized size = 1.57

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(8i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\sqrt{2}\sqrt[4]{3}\left(\left(\sqrt{3}+i\right.\right.\right.}{2\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2]*3^(1/4)*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/
3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqr
t[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*
x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2 + (-1)^(1/3))*
b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
```

```
qrt[a + b*x^3])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int x \left(2 \sqrt[3]{a} - \sqrt[3]{b} x\right)^{-1} \frac{1}{\sqrt{b x^3 + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{b x^3 + a} \left(b^{\frac{1}{3}} x - 2 a^{\frac{1}{3}}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2 \sqrt[3]{a} \sqrt{a + b x^3} + \sqrt[3]{b} x \sqrt{a + b x^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.97 $\int \frac{x}{\left(2 \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx$

Optimal. Leaf size=268

$$\frac{4 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a-b x^3}}\right)}{9 \sqrt[6]{a} b^{2/3}}-\frac{2 \sqrt{2+\sqrt{3}} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right)^2}} F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}\right)|-7-4 \sqrt{3}\right)}{3 \sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right)^2}} \sqrt{a-b x^3}}$$

```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.287474, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2139, 218, 2138, 206}

$$\frac{4 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a-b x^3}}\right)}{9 \sqrt[6]{a} b^{2/3}}-\frac{2 \sqrt{2+\sqrt{3}} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right)^2}} F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}\right)|-7-4 \sqrt{3}\right)}{3 \sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right)^2}} \sqrt{a-b x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```


```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} + \frac{(4\sqrt[3]{\cdots}}{\cdots}$$

$$= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}\cdots\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] time = 0.688964, size = 371, normalized size = 1.38

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2\cdots}}{\left(1+\sqrt[3]{-1}\cdots\right.}}\cdots}{\cdots}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
```

```
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.98 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=277

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.338323, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2139, 219, 2138, 203}

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```


```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```


Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} + \frac{(4 \cdots}{\cdots}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots}\right.\right.\cdots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.231237, size = 372, normalized size = 1.34

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\cdots}{\left(1+\sqrt[3]{-1}\cdots\right.}}\cdots}{\cdots}\right.\cdots}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
```

```
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x + 2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.99 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=273

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.321756, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2139, 219, 2138, 203}

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{(4}{}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.286387, size = 410, normalized size = 1.5

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(8i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\sqrt{2}\sqrt[4]{3}\left(\left(\sqrt{3}+i\right.\right.\right.}{2\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2]*3^(1/4)*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/
3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqr
t[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*
x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2 + (-1)^(1/3))*
b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
```

```
qrt[-a - b*x^3])
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2\sqrt[3]{a}\sqrt{-a-bx^{3}}+\sqrt[3]{b}x\sqrt{-a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.100 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=202

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^2
) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + S
qrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt
[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1
+ Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rubi [A] time = 0.255957, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2139, 218, 2138, 206}

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^2
) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + S
qrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt
[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1
+ Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3d}$$

$$= -\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} + \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{9-c \cdots}\right.}{3 \cdots}$$

$$= \frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2} - \frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4 \cdots\right.}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Mathematica [C] time = 0.680195, size = 295, normalized size = 1.46

$$\frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}F\left(\sin^{-1}\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right)|\sqrt[3]{-1}\right) + \frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\frac{c^2+2cdx+4d^{\cdots}}{c^2}}\cdots}{\sqrt{3}\cdots}\right.}{\left(\sqrt[3]{-1}-2\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^(1/3)*c +
2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (-1)^(1/3))*c)]*Ellip
ticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1/3)]
 + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*
I + Sqrt[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-
1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1
 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Maple [B] time = 0.007, size = 653, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1
/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2
))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*(
(x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/
2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2
*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d
))^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*
(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))-2*c/d^2*(1/2*(-1/2+1/2*I*3^(1/2))
*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(
-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(
1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c
/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*
d^3*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-
1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/
2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)
/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/
2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-8\,d^3x^3+c^3}x}{8\,d^4x^4+8\,cd^3x^3-c^3dx-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-8*d^3*x^3 + c^3)*x/(8*d^4*x^4 + 8*c*d^3*x^3 - c^3*d*x - c^4
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\left(-c+2dx\right)\left(c^2+2cdx+4d^2x^2\right)}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.101 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rubi [A] time = 0.113517, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 0.433573, size = 267, normalized size = 6.36

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-i)\right)\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.051, size = 245, normalized size = 5.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-4\,\frac{3/2-i/\ldots}{\sqrt{x^3\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [B] time = 1.50241, size = 532, normalized size = 12.67

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x^3-\sqrt{3}\left(x^6-12x\ldots\right.\right.}{x^8+8x^7+16\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*s
qrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x
 + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*
x^2 - 64*x + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.102 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rubi [A] time = 0.114319, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 0.455526, size = 269, normalized size = 5.85

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(\left((1+2i)-i\right.\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.049, size = 243, normalized size = 5.3

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)+3^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

Fricas [B] time = 1.50751, size = 533, normalized size = 11.59

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8x^3-\sqrt{3}\left(x^6+\right.\right.}{x^8-8x^7+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt(2*
sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*
x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64
*x^2 + 64*x + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

3.103 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rubi [A] time = 0.104045, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.368423, size = 267, normalized size = 6.07

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(\left((1+2i)-i\sqrt{3}\right)\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [C] time = 0.034, size = 245, normalized size = 5.6

$$2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-4\,\frac{-3/}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

Fricas [A] time = 1.60483, size = 155, normalized size = 3.52

$$\frac{1}{3}\,\sqrt{3}\sqrt{2\,\sqrt{3}+3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2+4\,x-2\right)-6\,x+6\right)\sqrt{2\,\sqrt{3}+3}}{6\,\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x +
 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

3.104 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rubi [A] time = 0.0936181, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.3728, size = 269, normalized size = 6.11

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [C] time = 0.046, size = 247, normalized size = 5.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

Fricas [A] time = 1.60785, size = 170, normalized size = 3.86

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\sqrt{-x^3-1}\left(\sqrt{3}\left(x^2-4x-2\right)+6x+6\right)\sqrt{2\sqrt{3}+3}}{6\left(x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 - 4
*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

3.105 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.204068, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.629755, size = 322, normalized size = 4.67

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\ldots}{\left(1+\sqrt[3]{-1}\right)\ldots}}\right)\ldots\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3))))/Sqrt[a +
 b*x^3]
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [A] time = 8.17214, size = 3237, normalized size = 46.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b
^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt(3)
*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a + bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sqr
t(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.106 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.192222, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 1.34442, size = 446, normalized size = 6.28

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\cdots}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[
ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3]
)*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3)
+ (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)
*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3*I + (1 + 2*I)*Sqrt[3])*b
^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[a - b*x^3])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [B] time = 6.66262, size = 3351, normalized size = 47.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
- sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x
))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*
x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3
 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 + a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3
+ a)*(sqrt(3)*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 - a
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.107 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.185351, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{-a+bx^3}}\,dx = \frac{(2\sqrt[3]{a})\,\text{Subst}\left(\int \frac{1}{1-(3-2\sqrt{3})ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.432119, size = 447, normalized size = 6.21

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\cdots}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[
ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3]
)*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3)
+ (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)
*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3*I + (1 + 2*I)*Sqrt[3])*b
^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[-a + b*x^3])
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [A] time = 6.32018, size = 3240, normalized size = 45.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/
(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt
(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{-a + bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.108 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.168736, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.530354, size = 325, normalized size = 4.51

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3))))/Sqrt[-a
- b*x^3]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [B] time = 6.39838, size = 3348, normalized size = 46.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
 - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*
x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
- 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7
*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^
3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(-(2
*sqrt(3) + 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 -
 a)*(sqrt(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 + a))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.109 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.198814, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1+(3-2\sqrt{3})ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.40683, size = 663, normalized size = 9.08

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)+bx^3\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2}\right.\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - s
qrt(3) + 1)), x)
```

Fricas [A] time = 4.16916, size = 3236, normalized size = 44.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a
^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
+ 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt((2*sqrt(3
) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x
^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*
b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a - a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}}{\sqrt{a + bx^3}\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.110 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.201201, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.25664, size = 648, normalized size = 8.64

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-bx^3\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\right.\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)+(\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

Fricas [B] time = 4.12895, size = 3345, normalized size = 44.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
+ 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x
^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
+ a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(
a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
- 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 - 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 - 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt
```

```
(3)*a - a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.111 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.192214, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.567806, size = 649, normalized size = 8.54

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-bx^3\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\right.\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)+(\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + s
qrt(3) - 1)), x)
```

Fricas [A] time = 4.13755, size = 3239, normalized size = 42.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*
a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt(-(2*sqrt
(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7
*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x - sqrt(3)*(5*
a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a -
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{-a + bx^3}\left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.112 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.180013, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.872654, size = 666, normalized size = 8.76

$$\frac{x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)+bx^3\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2}\right.\right.\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{}}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

Fricas [B] time = 4.10442, size = 3343, normalized size = 43.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 -
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*
x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 -
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)
*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 + 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 + 2*sqr
t(-b*x^3 - a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 - a)*(sqrt(
```

```
3)*a - a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.113 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Rubi [A] time = 0.0895657, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.426026, size = 269, normalized size = 6.4

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.019, size = 245, normalized size = 5.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-4\,\frac{3/2-i/}{\sqrt{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [A] time = 1.2032, size = 155, normalized size = 3.69

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2-4x-2\right)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.114 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]
```

Rubi [A] time = 0.101986, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/((((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.434934, size = 267, normalized size = 5.8

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\sqrt{3}-i}\sqrt{x}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.025, size = 247, normalized size = 5.4

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [A] time = 1.67513, size = 170, normalized size = 3.7

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}\left(x^2+4x-2\right)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4
*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1+\sqrt{3}}{\sqrt{-(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.115 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Rubi [A] time = 0.0898745, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}} \end{aligned}$$

Mathematica [C] time = 0.289503, size = 265, normalized size = 6.02

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+\cdots}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [C] time = 0.02, size = 245, normalized size = 5.6

$$2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-4\,\frac{-3/\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [B] time = 1.88947, size = 532, normalized size = 12.09

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2-4\left(2x^6+18x^5+42x^4+8x^3+\sqrt{3}\left(x^6+12\cdots\right.\right.}{x^8-8x^7+16\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*s
qrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x
 - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*
x^2 + 64*x + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.116 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

Rubi [A] time = 0.0866499, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.342778, size = 271, normalized size = 6.16

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right.\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[-1 - x^3])
```

Maple [C] time = 0.015, size = 243, normalized size = 5.5

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [B] time = 1.87355, size = 533, normalized size = 12.11

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4\left(2x^6-18x^5+42x^4-8x^3+\sqrt{3}\left(x^6-\right.\right.}{x^8+8x^7+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*
sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-(x+1)\left(x^2 - x + 1\right)}\left(x + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}\left(x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.117 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.175879, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\mathrm{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.573558, size = 320, normalized size = 4.64

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\ldots\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))/Sqrt[a + b*x^3
]
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [A] time = 8.39679, size = 3240, normalized size = 46.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/
(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt
(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a + bx^3}\left(\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a**
(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.118 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.184845, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.805781, size = 329, normalized size = 4.63

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(\ldots}{(1+\ldots}}\right.\right.}{3+(2+i)\sqrt{3}}\right.}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(
1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*S
qrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqrt[3])))/(b^(1/3
)*Sqrt[a - b*x^3])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [B] time = 8.17429, size = 3348, normalized size = 47.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
 + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*
x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
+ 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7
*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^
3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-(2
*sqrt(3) - 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 + a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3 +
 a)*(sqrt(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 - a))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-sq
rt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.119 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.188483, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.3542, size = 330, normalized size = 4.58

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}} - \frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(\ldots}{(1+\ldots}}\right.\right.}{3+(2+i)\sqrt{3}}\right)}{\sqrt[3]{b}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(
1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*S
qrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqrt[3])))/(b^(1/3
)*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [A] time = 8.47777, size = 3237, normalized size = 44.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b
^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt(3)
*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.120 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.167514, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.322102, size = 323, normalized size = 4.49

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\ldots\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))/Sqrt[-a - b*x^
3]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [B] time = 8.27117, size = 3351, normalized size = 46.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
+ sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x
))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 -
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*
x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^3
 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 - a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3
- a)*(sqrt(3)*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 + a
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{-a - bx^3}\left(\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.121 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.175801, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.24396, size = 667, normalized size = 9.14

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-bx^3\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(8(5+3\sqrt{\ldots}\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\ldots\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + s
qrt(3) + 1)), x)
```

Fricas [A] time = 5.32696, size = 3239, normalized size = 44.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*
a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt(-(2*sqrt
(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7
*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x + sqrt(3)*(5*
a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a +
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1}{\sqrt{a + bx^3}\left(x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) +
 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.122 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]
])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.18544, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]
])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.27369, size = 649, normalized size = 8.65

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+bx^3\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3}\right.\right.\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)+8\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

Fricas [B] time = 5.67356, size = 3343, normalized size = 44.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 +
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*
x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 +
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)
*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 - 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 + 2*sqr
t(-b*x^3 + a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(
```

```
3)*a + a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}}{\sqrt{a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.123 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.185271, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1-(3+2\sqrt{3})ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.721246, size = 650, normalized size = 8.55

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+bx^3\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3}\right.\right.\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)+8\left(\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - s
qrt(3) - 1)), x)
```

Fricas [A] time = 5.52566, size = 3236, normalized size = 42.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a
^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
- 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt((2*sqrt(3
) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x
^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*
b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a + a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}}{\sqrt{-a + bx^3}\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.124 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.174189, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.781634, size = 670, normalized size = 8.82

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-bx^3\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(8\left(5+3\sqrt{\right.\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{}{6\sqrt{}}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

Fricas [B] time = 6.06471, size = 3345, normalized size = 44.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
- 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x
^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(
a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
+ 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 + 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 - 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 - a)*(sqrt
```

```
(3)*a + a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1}{\sqrt{-a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.125 $\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
+ (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.239785, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 218, 2140, 203}

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
+ (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{12}\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.378037, size = 269, normalized size = 1.86

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.021, size = 245, normalized size = 1.7

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-2\,\frac{3/2-i/}{\sqrt{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}{x^4+x^3-3\,x^2+4\,x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(x^4 + x^3 - 3*x^2 + 4*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.126 $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3]
]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.227722, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2141, 218, 2140, 206}

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3]
]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{12}\int \frac{\left(1-\sqrt{3}\right)\left(-22+\left(1-\sqrt{3}\right)^3\right)+6x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$
$$= \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\right.$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.360116, size = 267, normalized size = 1.84

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-i)\right)\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (2*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.02, size = 245, normalized size = 1.7

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-2\,\frac{3/2-i/}{\sqrt{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}{x^4+x^3-3\,x^2+4\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(x^4 + x^3 - 3*x^2 + 4*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.127 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{\left(e-\sqrt{3}f-f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] ((e - f - Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/S
qrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.247163, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2141, 218, 2140, 203}

$$\frac{\left(e-\sqrt{3}f-f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```


```
[Out] ((e - f - Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/S
qrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])
```


Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\left(e-\left(1\right.\right.\right.}{}$$

$$= \frac{\left(e-f-\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\right.}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.550921, size = 291, normalized size = 1.68

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\left(3+\sqrt{3}\right)f-\sqrt{3}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+3f\sqrt{2ix+\sqrt{\cdot}}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [A] time = 0.026, size = 260, normalized size = 1.5

$$2\,\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+\frac{\left(2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-f-f*3^(1/2))*(3/2-1/2*I*
3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+
1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx^2+\left(e+f\right)x-\sqrt{3}\left(fx+e\right)+e\right)\sqrt{x^3+1}}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + (e + f)*x - sqrt(3)*(f*x + e) + e)*sqrt(x^3 + 1)/(x^5 + 2
*x^4 - 2*x^3 + x^2 + 2*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.128 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(e+\sqrt{3}f+f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])
/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
 x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.276521, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2141, 218, 2140, 203}

$$-\frac{\left(e+\sqrt{3}f+f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])
/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
 x)^2]*Sqrt[1 - x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(6e-\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)f\right)\int \frac{1}{\sqrt{1-}}}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(12\left(-e-\right.\right.}{}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.542383, size = 291, normalized size = 1.56

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3if\sqrt{-2ix+}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.026, size = 264, normalized size = 1.4

$$\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-f-f*3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx^2+\left(e-f\right)x+\sqrt{3}\left(fx+e\right)-e\right)\sqrt{-x^3+1}}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + (e - f)*x + sqrt(3)*(f*x + e) - e)*sqrt(-x^3 + 1)/(x^5 -
2*x^4 - 2*x^3 - x^2 + 2*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx-\int \frac{fx}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
) - Integral(f*x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.129 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(e+\sqrt{3}f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.245686, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2141, 219, 2140, 206}

$$-\frac{\left(e+\sqrt{3}f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(-6e-\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)f\right)}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right.}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\left(-e\right.\right.}{}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+\right.}}$$

Mathematica [C] time = 0.433259, size = 289, normalized size = 1.52

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] time = 0.023, size = 262, normalized size = 1.4

$$-2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-e-f-f*3^(1/2))*(-3/2-1/2*
I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1
/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(fx^2+\left(e-f\right)x+\sqrt{3}\left(fx+e\right)-e\right)\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^2 + (e - f)*x + sqrt(3)*(f*x + e) - e)*sqrt(x^3 - 1)/(x^5 -
2*x^4 - 2*x^3 - x^2 + 2*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx-\int \frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.130 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=183

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 +
x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.23645, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2141, 219, 2140, 206}

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```



```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 +
x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)]*Sqrt[-1 - x^3])
```


Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(e-\left(1+\right.\right.}{}$$

$$= \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\right.}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1}}$$

Mathematica [C] time = 0.457237, size = 293, normalized size = 1.6

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\left(3+\sqrt{3}\right)f-\sqrt{3}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+3f\sqrt{2ix+\sqrt{}}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.023, size = 258, normalized size = 1.4

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-f*3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/
2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(fx^2+\left(e+f\right)x-\sqrt{3}\left(fx+e\right)+e\right)\sqrt{-x^3-1}}{x^5+2x^4-2x^3+x^2+2x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^2 + (e + f)*x - sqrt(3)*(f*x + e) + e)*sqrt(-x^3 - 1)/(x^5 +
 2*x^4 - 2*x^3 + x^2 + 2*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.131 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{b}e-(1\cdots\right.}{\cdots}$$

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.540226, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2141, 218, 2140, 206}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{b}e-(1\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx = \frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx}{12\sqrt{3}a^{4/3}b^{4/3}} - \frac{\left(6ab^{4/3}e-\left(1-\sqrt{\right.}\right.}{\left(1-\sqrt{\right.}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{}}{(1+\sqrt{}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\right.\right.}{}$$

Mathematica [C] time = 1.76671, size = 438, normalized size = 1.32

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}-1\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/2)*3^(1/4)
*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[
I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1
/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2])/Sqrt[2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/
3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)
*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*
I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
+ Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*
Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a +
b*x^3])
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int (fx+e)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/
3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="giac")
```

```
[Out] sage0*x
```

3.132 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\ldots$$

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.567915, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2141, 218, 2140, 206}

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```


```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```


Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} + \frac{\left(6ab^{4/3}e-\left(1-\sqrt{\cdots}\right.\right.}{\left(1-\sqrt{\cdots}\right.}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} + \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{\cdots}\right.\right.}{\cdots}$$

Mathematica [C] time = 1.64916, size = 466, normalized size = 1.39

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right|\cdots\right.}{\left(3-(2-i)\sqrt{\cdots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (fx+e)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{2\left(2\,bfx^4+2\,bex^3-2\,afx-2\,ae-\sqrt{3}\left(bfx^4+bex^3+2\,afx+2\,ae\right)\right)\sqrt{-bx^3+a}a^{\frac{2}{3}}+\left(bfx^5+bex^4+8\,afx^2\right.}{b^3x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral((2*(2*b*f*x^4 + 2*b*e*x^3 - 2*a*f*x - 2*a*e - sqrt(3)*(b*f*x^4 + b
*e*x^3 + 2*a*f*x + 2*a*e))*sqrt(-b*x^3 + a)*a^(2/3) + (b*f*x^5 + b*e*x^4 +
8*a*f*x^2 + 8*a*e*x - sqrt(3)*(b*f*x^5 + b*e*x^4 - 4*a*f*x^2 - 4*a*e*x))*sq
rt(-b*x^3 + a)*a^(1/3)*b^(1/3) + (b*f*x^6 + b*e*x^5 - 10*a*f*x^3 - 10*a*e*x
```

```
^2 - 6*sqrt(3)*(a*f*x^3 + a*e*x^2))*sqrt(-b*x^3 + a)*b^(2/3))/(b^3*x^9 - 21
*a*b^2*x^6 + 12*a^2*b*x^3 + 8*a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{fx}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(a - b*x*
*3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] sage0*x
```

3.133 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}\right.}{\ldots}$$

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)
*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.491108, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2141, 219, 2140, 203}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```


```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)
*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```


Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} - -\frac{\left(-6ab^{4/3}e-\left(1\right.\right.}{\left(1\right.}$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{}}{(1-\sqrt{}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{}\right.\right.}{}$$

Mathematica [C] time = 1.10096, size = 467, normalized size = 1.35

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right|\right.\right.}{\left(3-(2-i)\sqrt{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (fx+e)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{bx^3-a}\left(2\left(2\,bfx^4+2\,bex^3-2\,afx-2\,ae-\sqrt{3}\left(bfx^4+bex^3+2\,afx+2\,ae\right)\right)a^{\frac{2}{3}}+\left(bfx^5+bex^4+8\,afx^2\right.\right.}{b^3x^9-21\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(b*x^3 - a)*(2*(2*b*f*x^4 + 2*b*e*x^3 - 2*a*f*x - 2*a*e - sqr
t(3)*(b*f*x^4 + b*e*x^3 + 2*a*f*x + 2*a*e))*a^(2/3) + (b*f*x^5 + b*e*x^4 +
8*a*f*x^2 + 8*a*e*x - sqrt(3)*(b*f*x^5 + b*e*x^4 - 4*a*f*x^2 - 4*a*e*x))*a^
(1/3)*b^(1/3) + (b*f*x^6 + b*e*x^5 - 10*a*f*x^3 - 10*a*e*x^2 - 6*sqrt(3)*(a
```

```
*f*x^3 + a*e*x^2))*b^(2/3))/(b^3*x^9 - 21*a*b^2*x^6 + 12*a^2*b*x^3 + 8*a^3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx-\int \frac{fx}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(-a +
b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

3.134 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\left(\sqrt[3]{b}e-\left(1\right.\right.}{}$$

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/
6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.474991, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2141, 219, 2140, 203}

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\left(\sqrt[3]{b}e-\left(1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/
6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} + -\frac{\left(-6ab^{4/3}e-\left(1\right.\right.}{\left(1\right.}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1+}{(1-}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\right.\right.}{}$$

Mathematica [C] time = 0.482795, size = 441, normalized size = 1.28

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}-1\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/2)*3^(1/4)
*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[
I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1
/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2])/Sqrt[2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/
3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)
*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*
I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
+ Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*
Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a -
 b*x^3])
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int (fx+e)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2\left(2bfx^4+2bex^3+2afx+2ae-\sqrt{3}\left(bfx^4+bex^3-2afx-2ae\right)\right)\sqrt{-bx^3-a}a^{\frac{2}{3}}-\left(bfx^5+bex^4-8afx^2\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(2*(2*b*f*x^4 + 2*b*e*x^3 + 2*a*f*x + 2*a*e - sqrt(3)*(b*f*x^4 +
b*e*x^3 - 2*a*f*x - 2*a*e))*sqrt(-b*x^3 - a)*a^(2/3) - (b*f*x^5 + b*e*x^4 -
 8*a*f*x^2 - 8*a*e*x - sqrt(3)*(b*f*x^5 + b*e*x^4 + 4*a*f*x^2 + 4*a*e*x))*s
qrt(-b*x^3 - a)*a^(1/3)*b^(1/3) + (b*f*x^6 + b*e*x^5 + 10*a*f*x^3 + 10*a*e*
```

```
x^2 + 6*sqrt(3)*(a*f*x^3 + a*e*x^2))*sqrt(-b*x^3 - a)*b^(2/3))/(b^3*x^9 + 2
1*a*b^2*x^6 + 12*a^2*b*x^3 - 8*a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt{-a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1
/3)*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

3.135 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4)) +
(Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.220413, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2141, 218, 2140, 203}

$$\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4)) +
(Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(-22+\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-28+\left(1+\sqrt{3}\right)^3}$$

$$= \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\left(-1-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right.}\right.}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{\right.\right.}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.48615, size = 209, normalized size = 1.54

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[1 + x^3]
```

Maple [B] time = 0.022, size = 255, normalized size = 1.9

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+\frac{\left(-2\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(-3^(1/2)-1)*(3/2-1/2*I*3^(1/
2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x^2-\sqrt{3}x+x\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x^2 - sqrt(3)*x + x)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2
*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.136 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3^(3/4)) +
(Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.220981, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2141, 218, 2140, 203}

$$\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3^(3/4)) +
(Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} - \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right.}{3-\sqrt{3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.626104, size = 232, normalized size = 1.53

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(2\left(1+\sqrt{3}\right)\sqrt{x^2+x+1}\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\frac{i\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(3+(2+i)\sqrt{3}\right)x+(1+2i)\sqrt{3}+3i\right)F\left(\sin^{-1}\left(\right.\right.}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [B] time = 0.02, size = 257, normalized size = 1.7

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*(-3^(1/2)-1)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2
)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x^2+\sqrt{3}x-x\right)}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x^2 + sqrt(3)*x - x)/(x^5 - 2*x^4 - 2*x^3 - x^2 +
2*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.137 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=164

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.218447, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 219, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} - \frac{\left(-22+\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{-28+\left(1+\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\left(-1-\sqrt{3}\right)\right)\text{Subst}\left(\int\right.}{\left(1+\sqrt{3}\right)\left(-28\right.}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.282499, size = 230, normalized size = 1.4

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(2\left(1+\sqrt{3}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\frac{i\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(3+(2+i)\sqrt{3}\right)x+(1+2i)\sqrt{3}+3i\right)F\left(\sin^{-1}\left(\right.\right.}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.017, size = 255, normalized size = 1.6

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-\frac{(-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-3^(1/2)-1)*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1
/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3
^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x^2+\sqrt{3}x-x\right)}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x^2 + sqrt(3)*x - x)/(x^5 - 2*x^4 - 2*x^3 - x^2 +
2*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.138 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.207727, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 219, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```


Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} + \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx\right.}{3-\sqrt{3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.204653, size = 211, normalized size = 1.35

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[-1 - x^3]
```

Maple [A] time = 0.018, size = 253, normalized size = 1.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*I*(-3^(1/2)-1)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)+3^
(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x^2-\sqrt{3}x+x\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x^2 - sqrt(3)*x + x)/(x^5 + 2*x^4 - 2*x^3 + x^2 +
 2*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.139 $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=147

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 + 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.224673, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 218, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 + 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\left(-22+\left(1-\sqrt{3}\right)^3\right)+6x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{6\left(3+\sqrt{3}\right)} + \frac{\left(-22+\left(1-\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-28+\left(1-\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx,\right.}{3+\sqrt{3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.557941, size = 225, normalized size = 1.53

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left((1+2i)\sqrt{3}-3i\right)x-(2+i)\sqrt{3}+3\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} - 2\left(\sqrt{3}-1\right)\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-}{}}\right.\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sqrt[3])*x)*EllipticF[
ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 + x^3]
)
```

Maple [B] time = 0.022, size = 255, normalized size = 1.7

$$2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+\frac{\left(2-2\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(1-3^(1/2))*(3/2-1/2*I*3^(1/2
))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x^2+\sqrt{3}x+x\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x^2 + sqrt(3)*x + x)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2
*x - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.140 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=278

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqr
t[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.407351, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2141, 218, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqr
t[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```


Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\dots\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\dots\right)}}}$$

Mathematica [C] time = 1.19775, size = 427, normalized size = 1.54

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\dots\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx\dots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/2)*3^(1/4)
*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[I
+ Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3
```

```
) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/
2])/Sqrt[2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(
2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSi
n[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)
)]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int x\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\left(2\left(2\,bx^4+2\,ax-\sqrt{3}\left(bx^4-2\,ax\right)\right)a^{\frac{2}{3}}-\left(bx^5-8\,ax^2-\sqrt{3}\left(bx^5+4\,ax^2\right)\right)a^{\frac{1}{3}}b^{\frac{1}{3}}+\left(bx^6+6\,\sqrt{3}ax^3+\right.\right.}{b^3x^9+21\,ab^2x^6+12\,a^2bx^3-8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(2*(2*b*x^4 + 2*a*x - sqrt(3)*(b*x^4 - 2*a*x))*a^(
2/3) - (b*x^5 - 8*a*x^2 - sqrt(3)*(b*x^5 + 4*a*x^2))*a^(1/3)*b^(1/3) + (b*x
^6 + 6*sqrt(3)*a*x^3 + 10*a*x^3)*b^(2/3))/(b^3*x^9 + 21*a*b^2*x^6 + 12*a^2*
b*x^3 - 8*a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

3.141 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=286

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.41277, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2141, 218, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\ldots\right)}}}$$

Mathematica [C] time = 1.23231, size = 454, normalized size = 1.59

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+\ldots\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
```

```
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\left(2\,bx^4-2\,ax-\sqrt{3}\left(bx^4+2\,ax\right)\right)\sqrt{-bx^3+a}a^{\frac{2}{3}}+\left(bx^5+8\,ax^2-\sqrt{3}\left(bx^5-4\,ax^2\right)\right)\sqrt{-bx^3+a}a^{\frac{1}{3}}b^{\frac{1}{3}}+\left(bx^6-\right.}{b^3x^9-21\,ab^2x^6+12\,a^2bx^3+8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral((2*(2*b*x^4 - 2*a*x - sqrt(3)*(b*x^4 + 2*a*x))*sqrt(-b*x^3 + a)*a^
(2/3) + (b*x^5 + 8*a*x^2 - sqrt(3)*(b*x^5 - 4*a*x^2))*sqrt(-b*x^3 + a)*a^(1
/3)*b^(1/3) + (b*x^6 - 6*sqrt(3)*a*x^3 - 10*a*x^3)*sqrt(-b*x^3 + a)*b^(2/3)
)/(b^3*x^9 - 21*a*b^2*x^6 + 12*a^2*b*x^3 + 8*a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

3.142 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt
[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

Rubi [A] time = 0.441004, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2141, 219, 2140, 203}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt
[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{\left(2\sqrt[3]{\right.}\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\cdots\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right.\cdots}}}$$

Mathematica [C] time = 0.405748, size = 455, normalized size = 1.61

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+\cdots\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
```

```
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int x\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
maxima")
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{bx^3-a}\left(2\left(2\,bx^4-2\,ax-\sqrt{3}\left(bx^4+2\,ax\right)\right)a^{\frac{2}{3}}+\left(bx^5+8\,ax^2-\sqrt{3}\left(bx^5-4\,ax^2\right)\right)a^{\frac{1}{3}}b^{\frac{1}{3}}+\left(bx^6-6\,\sqrt{3}ax^3-\right.\right.}{b^3x^9-21\,ab^2x^6+12\,a^2bx^3+8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(b*x^3 - a)*(2*(2*b*x^4 - 2*a*x - sqrt(3)*(b*x^4 + 2*a*x))*a^
(2/3) + (b*x^5 + 8*a*x^2 - sqrt(3)*(b*x^5 - 4*a*x^2))*a^(1/3)*b^(1/3) + (b*
x^6 - 6*sqrt(3)*a*x^3 - 10*a*x^3)*b^(2/3))/(b^3*x^9 - 21*a*b^2*x^6 + 12*a^2
*b*x^3 + 8*a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

3.143 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt
[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rubi [A] time = 0.41432, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2141, 219, 2140, 203}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt
[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\cdots\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\cdots}$$

Mathematica [C] time = 0.492128, size = 430, normalized size = 1.55

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\cdots}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-b\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/2)*3^(1/4)
*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[I
+ Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3
```

```
) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/
2])/Sqrt[2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(
2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSi
n[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)
)]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int x\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1-\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2\left(2\,bx^{4}+2\,ax-\sqrt{3}\left(bx^{4}-2\,ax\right)\right)\sqrt{-bx^{3}-a}\,a^{\frac{2}{3}}-\left(bx^{5}-8\,ax^{2}-\sqrt{3}\left(bx^{5}+4\,ax^{2}\right)\right)\sqrt{-bx^{3}-a}\,a^{\frac{1}{3}}b^{\frac{1}{3}}+\left(bx^{6}\right.}{b^{3}x^{9}+21\,ab^{2}x^{6}+12\,a^{2}bx^{3}-8\,a^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-(2*(2*b*x^4 + 2*a*x - sqrt(3)*(b*x^4 - 2*a*x))*sqrt(-b*x^3 - a)*a
^(2/3) - (b*x^5 - 8*a*x^2 - sqrt(3)*(b*x^5 + 4*a*x^2))*sqrt(-b*x^3 - a)*a^(
1/3)*b^(1/3) + (b*x^6 + 6*sqrt(3)*a*x^3 + 10*a*x^3)*sqrt(-b*x^3 - a)*b^(2/3
))/(b^3*x^9 + 21*a*b^2*x^6 + 12*a^2*b*x^3 - 8*a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a-bx^{3}}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{bx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

3.144 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=319

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\right.\right.}$$

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) - (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 1.23982, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2142, 2113, 537, 571, 93, 205}

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```


```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) - (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```


Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \ldots\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\left(1-\sqrt{3}\right)d\right)^2-\ldots\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2\ldots\right.}{\ldots}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2\ldots\right.}{\ldots}$$

$$= -\frac{\left(c-d-\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-\ldots}{(1+\ldots}}}{\left(c-(\ldots\right.}$$

Mathematica [C] time = 0.633153, size = 214, normalized size = 0.67

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c-\left(1+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-
1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] time = 0.036, size = 275, normalized size = 0.9

$$2\,\frac{3/2-i/2\sqrt{3}}{d\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+2\,\frac{(d\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(d*3^(1/2)-c+d)/d^2*(3/2-1/2*
I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.145 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=331

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+}}$$

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) + (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3]
- x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 1.29782, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2142, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) + (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3]
- x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}dx, x, \frac{-1+\sqrt{3}+}{1+\sqrt{3}-x}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\sqrt{3}\right)\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2}; -\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle| -7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\right.\right.}{}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2}; -\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle| -7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\right.\right.}{}$$

$$= -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+}{\left(1+\sqrt{3}-\right.}}}{\left(c+d-\right.}$$

Mathematica [C] time = 0.749066, size = 235, normalized size = 0.71

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}d}\right)}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3
])
```

Maple [A] time = 0.044, size = 264, normalized size = 0.8

$$\frac{\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(c+d+d*3^(1/2))/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx-\int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx-\int -\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x
/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3
) + d*x*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.146 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=327

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{t}}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{}}$$

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) + (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*E
llipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3]
 - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x
)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.783295, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2142, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{t}}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```


```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) + (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*E
llipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3]
 - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x
)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```


Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1}{1}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-(c+\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\left(\right.}}{(c+}$$

Mathematica [C] time = 0.243086, size = 233, normalized size = 0.71

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^
3])
```

Maple [A] time = 0.036, size = 273, normalized size = 0.8

$$-2\frac{-3/2-i/2\sqrt{3}}{d\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(c+d+d*3^(1/2))/d^2*(-3/2-1/2
*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}{dx^4+cx^3-dx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x - sqrt(3) - 1)/(d*x^4 + c*x^3 - d*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int -\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x
/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1
) + d*x*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.147 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=323

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{\right.\right.}}$$

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) - (4
*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.854269, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2142, 2113, 537, 571, 93, 205}

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```


```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) - (4
*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```


Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+}{1+}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\left(1-\sqrt{3}\right)d\right)^2-(-c\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{\left(c-d-\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+}{(1+\sqrt{3}}}}{\left(c-\left(1-\right.\right.}$$

Mathematica [C] time = 0.251142, size = 216, normalized size = 0.67

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c-\left(1+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-
1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A] time = 0.039, size = 266, normalized size = 0.8

$$\frac{-\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(d*3^(1/2)-c+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.148 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=360

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\Pi\left(\frac{(c-(1-\sqrt{3})d)^2}{(c-(1+\sqrt{3})d)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{t}}{\sqrt{d}\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{x^3+}}$$

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2)])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[1 + x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3
])*d)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[1 + x^3])
```

Rubi [A] time = 1.00413, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2143, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\Pi\left(\frac{(c-(1-\sqrt{3})d)^2}{(c-(1+\sqrt{3})d)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{t}}{\sqrt{d}\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{x^3+}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2)])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[1 + x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3
])*d)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[1 + x^3])
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
```

```
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx, x,\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(-c+\left(1+\sqrt{3}\right)d\right)^2-\right.}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(2\sqrt[4]{3}\sqrt{2+\right.}{}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2+\right.}{}$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1}{}$$

Mathematica [C] time = 0.531157, size = 213, normalized size = 0.59

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c+\left(\sqrt{3}-1\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (
-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] time = 0.02, size = 275, normalized size = 0.8

$$2\,\frac{3/2-i/2\sqrt{3}}{d\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-2\,\frac{(d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)+c-d)/d^2*(3/2-1/2*
I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.149 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=348

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\sin}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d\right.}$$

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) -
 (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.984906, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2143, 2113, 537, 571, 93, 205}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\sin}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) -
 (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx, x, \frac{1+\sqrt{3}}{-1+\sqrt{3}}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(c+\left(1-\sqrt{3}\right.\right.\right.}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\right.}{}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\right.}{}$$

$$= -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right.}}}{\left(c+d+\right.}$$

Mathematica [C] time = 0.670372, size = 235, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(\sqrt{3}-3\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}d}\right)}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^
3])
```

Maple [A] time = 0.019, size = 268, normalized size = 0.8

$$\frac{\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{3}}{c\sqrt{1 - x^3} + dx\sqrt{1 - x^3}}\,dx - \int \frac{x}{c\sqrt{1 - x^3} + dx\sqrt{1 - x^3}}\,dx - \int -\frac{1}{c\sqrt{1 - x^3} + dx\sqrt{1 - x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x/
(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3)
 + d*x*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.150 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=344

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\cdots\right.}$$

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]))
- (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sq
rt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.695947, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2143, 2113, 537, 571, 93, 205}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```


```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]))
- (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sq
rt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```


Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx, x,\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-(c+\right.}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(2\sqrt[4]{3}\sqrt{2\right.}{}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2\right.}{}$$

$$= -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1}{(1}}}{(c+}$$

Mathematica [C] time = 0.258475, size = 233, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(\sqrt{3}-3\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x
^3])
```

Maple [A] time = 0.018, size = 277, normalized size = 0.8

$$-2\,\frac{-3/2-i/2\sqrt{3}}{d\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)-c-d)/d^2*(-3/2-1/2
*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}{dx^4+cx^3-dx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x + sqrt(3) - 1)/(d*x^4 + c*x^3 - d*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int -\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x/
(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1)
 + d*x*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.151 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=364

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[
3])*d)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.792385, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2143, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[
3])*d)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
```

```
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}dx, x, \frac{1}{-}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(-c+\left(1+\sqrt{3}\right)d\right)^2-(-\right.}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2}; -\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle| -7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(2\sqrt[4]{3}\sqrt{2+\right.}{}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2}; -\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle| -7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2+\right.}{}$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+}{}$$

Mathematica [C] time = 0.200449, size = 215, normalized size = 0.59

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c+\left(\sqrt{3}-1\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (
-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(c + (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A] time = 0.019, size = 266, normalized size = 0.7

$$\frac{-\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)+c-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-(x+1)\left(x^2-x+1\right)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.152 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=125

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.0455044, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1832, 266, 63, 207, 218}

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x,\right.$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx\right.$$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.0255547, size = 39, normalized size = 0.31

$$x\,_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right) - \frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2,
 4/3, -x^3]
```

Maple [A] time = 0.026, size = 132, normalized size = 1.1

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right) - \frac{2+2\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*arctanh((x^3+1)^(1/2))*(1+3^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(x^4 + x), x)
```

Sympy [A] time = 3.12711, size = 56, normalized size = 0.45

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.153 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.0534554, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1832, 266, 63, 206, 218}

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x$$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.029213, size = 40, normalized size = 0.29

$$-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) - \frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2,
 4/3, x^3]
```

Maple [A] time = 0.032, size = 125, normalized size = 0.9

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)/(x^4 - x), x)
```

Sympy [A] time = 5.91333, size = 99, normalized size = 0.71

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/
2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.154 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=142

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.0497874, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1832, 266, 63, 203, 219}

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x$$

$$= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.0371313, size = 58, normalized size = 0.41

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right) - \frac{x\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Maple [A] time = 0.026, size = 140, normalized size = 1.

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+\frac{2\sqrt{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/x/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*arctan((x^3-1)^(1/2))*3^(1/2)
+2/3*arctan((x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x - sqrt(3) - 1)/(x^4 - x), x)
```

Sympy [A] time = 5.93225, size = 94, normalized size = 0.66

$$\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + Piecewise((
2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, True)) + s
qrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.155 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.0499955, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1832, 266, 63, 204, 219}

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx &= \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{1}{\sqrt{-1-x^3}}\,dx \\
&= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1-xx}}\,dx, x\right. \\
&= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right. \\
&= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

Mathematica [C] time = 0.0380568, size = 61, normalized size = 0.45

$$\frac{x\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}} + \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Maple [A] time = 0.028, size = 135, normalized size = 1.

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*arctan((-x^3-1)^(1/2))*3^(1/2)+2/3*arctan((-x^3-1)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x + sqrt(3) + 1)/(x^4 + x), x)
```

Sympy [A] time = 3.22138, size = 61, normalized size = 0.45

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3/2))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.156 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.0456327, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1832, 266, 63, 207, 218}

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx &= \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \right. \\
&= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, \right. \\
&= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.0222647, size = 41, normalized size = 0.32

$$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2,
 4/3, -x^3]
```

Maple [A] time = 0.017, size = 132, normalized size = 1.

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+\frac{2\sqrt{3}-2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)-1)*arctanh((x^3+1)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(x^4 + x), x)
```

Sympy [A] time = 3.96104, size = 56, normalized size = 0.44

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.157 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.0514084, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1832, 266, 63, 206, 218}

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx,x,x$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,$$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0243574, size = 42, normalized size = 0.3

$$-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) - \frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2,
 4/3, x^3]
```

Maple [A] time = 0.014, size = 125, normalized size = 0.9

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)-1)*arctanh((-x^3+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^4 - x), x)
```

Sympy [A] time = 8.3775, size = 99, normalized size = 0.7

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin
(x**(-3/2))/3, True)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.158 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.046942, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1832, 266, 63, 203, 219}

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, \right.$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.$$

$$= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.031346, size = 60, normalized size = 0.42

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right) - \frac{x\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Maple [A] time = 0.012, size = 140, normalized size = 1.

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-\frac{2\sqrt{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/x/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*arctan((x^3-1)^(1/2))*3^(1/2)
+2/3*arctan((x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x + sqrt(3) - 1)/(x^4 - x), x)
```

Sympy [A] time = 8.45124, size = 94, normalized size = 0.65

$$\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - sqrt(3)*Pie
cewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, Tr
ue)) + Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.159 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=138

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.0481781, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1832, 266, 63, 204, 219}

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx &= \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx+\int \frac{1}{\sqrt{-1-x^3}}\,dx\\
&= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}x}\,dx,x\right.\\
&= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}-\frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.\\
&= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] time = 0.0323671, size = 63, normalized size = 0.46

$$\frac{x\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}}+\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Maple [A] time = 0.013, size = 135, normalized size = 1.

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*arctan((-x^3-1)^(1/2))*3^(1/2)+2/3*arctan((-x^3-1)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}{x^4+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^4 + x), x)
```

Sympy [A] time = 4.08734, size = 61, normalized size = 0.44

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3/2))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.160 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=334

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*Sqrt[2*(97 +
56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.640057, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 204}

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[1 + x^3]),x]
```


```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*Sqrt[2*(97 +
56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```


Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x\right.}{}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x\right.}{}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)}{}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)}{}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.224484, size = 194, normalized size = 0.58

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2-x+1}\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[1 + x^3]
```

Maple [A] time = 0.007, size = 240, normalized size = 0.7

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-3\,\frac{3/2}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}x}{x^4+3x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.161 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=379

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)\ 1}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\cdots$$

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2*(37
+ 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqr
t[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sq
rt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/
(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.700103, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 206}

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)\ 1}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```


```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2*(37
+ 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqr
t[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sq
rt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/
(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```


Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\right.}{}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\right.}{}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{}{(}}}{}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{}{(}}}{}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-}{1+\sqrt{3}-}\right.\right.}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.225812, size = 195, normalized size = 0.51

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-3}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

Maple [A] time = 0.006, size = 240, normalized size = 0.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*
I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}x}{x^4+3x^3-x-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)\left(x^2+x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.162 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=375

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-12\sqrt[4]{3}\sqrt{2+}$$

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[2]*(1
 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqrt
[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(1
3*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.596848, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 206}

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-12\sqrt[4]{3}\sqrt{2+}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[2]*(1
 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqrt
[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(1
3*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right.}{\left(4+\right.}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right.}{}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}{13}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}{13}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.113545, size = 193, normalized size = 0.51

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-3}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

Maple [A] time = 0.007, size = 240, normalized size = 0.6

$$2\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-\frac{-\frac{9}{2}-}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3-1}x}{x^4+3x^3-x-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.163 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=343

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{14+8\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{12\sqrt[4]{3}(x\cdots}{\cdots}$$

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) - (2*Sqrt[14 + 8*
Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 +
x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.603785, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 204}

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{14+8\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{12\sqrt[4]{3}(x\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) - (2*Sqrt[14 + 8*
Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 +
x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```


Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-}{(1+}}\right.}{}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-}{(1+}}\right.}{}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-}{(1+\sqrt{}}}}{}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-}{(1+\sqrt{}}}}{}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.136634, size = 196, normalized size = 0.57

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2-x+1}\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[-1 - x^3]
```

Maple [A] time = 0.005, size = 240, normalized size = 0.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((
1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-
x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^3-1}x}{x^4+3x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x+1)\left(x^2-x+1\right)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.164 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=452

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(de-cf)\Pi\left(\frac{(c-(1+\sqrt{3})d)^2}{(c-(1-\sqrt{3})d)^2};-\right.}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}\left(c^2-2cd-2\right.}$$

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt
[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*(c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) +
(4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d
 - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 1.06011, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 205}

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(de-cf)\Pi\left(\frac{(c-(1+\sqrt{3})d)^2}{(c-(1-\sqrt{3})d)^2};-\right.}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}\left(c^2-2cd-2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt
[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*(c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) +
(4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d
 - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
```

```
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\right.}{}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\right.}{}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)}{\sqrt[4]{3}\left(c-d-\sqrt{3}\right.}$$

Mathematica [C] time = 0.544059, size = 211, normalized size = 0.47

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}(cf-de)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] time = 0.007, size = 274, normalized size = 0.6

$$2\frac{f\left(3/2-i/2\sqrt{3}\right)}{d\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2*f/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(3/2-1/2*I*3
^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1
)^(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.165 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=476

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\cdots\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c^2+2cd-\cdots\right.}$$

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 +
Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -
ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d
 - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 1.10559, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 208}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\cdots\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c^2+2cd-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 +
Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -
ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d
 - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
```

```
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}dx = \frac{\left(e+f+\sqrt{3}f\right)\int \frac{1}{\sqrt{1-x^3}}dx}{c+d+\sqrt{3}d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}dx}{c+d+\sqrt{3}d}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(\ldots\right.}{}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(\ldots\right.}{}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}(de-cf)(1\ldots}{\ldots}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}(de-cf)(1\ldots}{\ldots}$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\ldots}}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\ldots\right.}$$

Mathematica [C] time = 0.6949, size = 233, normalized size = 0.49

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}(cf-de)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}d-c}\right)}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

Maple [A] time = 0.007, size = 265, normalized size = 0.6

$$\frac{-\frac{2i}{3}f\sqrt{3}}{d}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(
1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x-1)\left(x^2+x+1\right)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.166 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=477

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{(c+\sqrt{3}d+d}{(c-\sqrt{3}d+d}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c^2+2c\right.}$$

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 -
 Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
 x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2
)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)
^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 +
2*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.924345, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 208}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{(c+\sqrt{3}d+d}{(c-\sqrt{3}d+d}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c^2+2c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 -
 Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
 x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2
)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)
^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 +
2*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
```

```
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{\left(e+f+\sqrt{3}f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}d}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}(de-c}{}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}(de-c}{}$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-}{\sqrt[4]{3}\left(c+d+\right.}$$

Mathematica [C] time = 0.297834, size = 231, normalized size = 0.48

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}(cf-de)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}d-c}\right)}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

Maple [A] time = 0.007, size = 274, normalized size = 0.6

$$2\frac{f\left(-3/2-i/2\sqrt{3}\right)}{d\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] 2*f/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(-3/2-1/2*I*3
^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)
^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(
1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.167 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=465

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c^2-2cd-2d^2\right)}$$

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(
1/4)*(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3
]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)
^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 -
2*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 1.03446, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 205}

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c^2-2cd-2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(
1/4)*(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3
]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)
^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 -
2*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
```

```
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(d\right.}{}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\right(}{}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(d}{}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(d}{}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{}}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)}$$

Mathematica [C] time = 0.172981, size = 213, normalized size = 0.46

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{i\sqrt{x^2-x+1}(cf-de)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A] time = 0.007, size = 265, normalized size = 0.6

$$\frac{-\frac{2i}{3}f\sqrt{3}}{d}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x+1)\left(x^2-x+1\right)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.168 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=120

$$\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rubi [A] time = 0.0492872, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1832, 266, 63, 207, 12, 218}

$$\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx &= e\int \frac{1}{x\sqrt{1+x^3}}\,dx+\int \frac{f}{\sqrt{1+x^3}}\,dx\\ &= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)+f\int \frac{1}{\sqrt{1+x^3}}\,dx\\ &= \frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+\right.\\ &= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1+x^3}\right)+\frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

Mathematica [C] time = 0.0169898, size = 34, normalized size = 0.28

$$fx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3
]
```

Maple [A] time = 0.004, size = 129, normalized size = 1.1

$$2\,\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-\frac{2e}{3}\text{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*e*arctanh((x^3+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(fx+e\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(f*x + e)/(x^4 + x), x)
```

Sympy [A] time = 2.41032, size = 42, normalized size = 0.35

$$-\frac{2e\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp
_polar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

3.169 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=134

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rubi [A] time = 0.0588458, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1832, 266, 63, 206, 12, 218}

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx &= e\int \frac{1}{x\sqrt{1-x^3}}\,dx+\int \frac{f}{\sqrt{1-x^3}}\,dx\\
&= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx,x,x^3\right)+f\int \frac{1}{\sqrt{1-x^3}}\,dx\\
&= -\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^{3}}\right)\\
&= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] time = 0.0186727, size = 34, normalized size = 0.25

$$fx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]
```

Maple [A] time = 0.004, size = 122, normalized size = 0.9

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*e*arctanh((-x^3+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\left(fx+e\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*(f*x + e)/(x^4 - x), x)
```

Sympy [A] time = 2.61708, size = 65, normalized size = 0.49

$$e\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right)+\frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

```
[Out] e*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/
3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

3.170 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 + x
 + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqr
t[-1 + x^3])
```

Rubi [A] time = 0.0512827, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1832, 266, 63, 203, 12, 219}

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 + x
 + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqr
t[-1 + x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx &= e\int \frac{1}{x\sqrt{-1+x^3}}\,dx+\int \frac{f}{\sqrt{-1+x^3}}\,dx\\ &= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx,x,x^3\right)+f\int \frac{1}{\sqrt{-1+x^3}}\,dx\\ &= -\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}+\frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1\right.\\ &= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

Mathematica [C] time = 0.0269654, size = 52, normalized size = 0.38

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{fx\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (f*x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3,
1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Maple [A] time = 0.006, size = 129, normalized size = 0.9

$$2\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{x-1}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(x^3-1)^(1/2),x)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan((x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^3-1}\left(fx+e\right)}{x^4-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*(f*x + e)/(x^4 - x), x)
```

Sympy [A] time = 2.41762, size = 60, normalized size = 0.44

$$e\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

```
[Out] e*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/
3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

3.171 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqr
t[-1 - x^3])
```

Rubi [A] time = 0.0535521, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1832, 266, 63, 204, 12, 219}

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqr
t[-1 - x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx &= e\int \frac{1}{x\sqrt{-1-x^3}}\,dx+\int \frac{f}{\sqrt{-1-x^3}}\,dx\\ &= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-xx}}\,dx,x,x^3\right)+f\int \frac{1}{\sqrt{-1-x^3}}\,dx\\ &= \frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}-\frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{-1\cdots}\right.\\ &= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1-x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] time = 0.0319325, size = 56, normalized size = 0.43

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{fx\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (f*x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3,
1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Maple [A] time = 0.004, size = 122, normalized size = 0.9

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan((-x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(fx+e\right)}{x^4+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(f*x + e)/(x^4 + x), x)
```

Sympy [A] time = 2.55529, size = 46, normalized size = 0.35

$$\frac{2ie\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*
exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

3.172 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=95

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d
) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*
d)
```

Rubi [A] time = 0.124666, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2151}

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```


```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d
) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*
d)
```


Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

Mathematica [F] time = 0.139853, size = 0, normalized size = 0.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{-dx+c}{dx+c}\frac{1}{\sqrt[3]{d^3x^3+2\,c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

```
[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx-\int \frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] -Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/
3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*
x**3)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.173 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=234

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd^2}+\frac{\sqrt{3}(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\tan^{-1}\left(\right.}{\sqrt{3}}$$

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (S
qrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2
^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e
- c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^
2)
```

Rubi [A] time = 0.220086, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2152, 239, 2148}

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd^2}+\frac{\sqrt{3}(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\tan^{-1}\left(\right.}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```


```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (S
qrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2
^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e
- c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^
2)
```


Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
```

```
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{f\int \frac{1}{\sqrt[3]{-c^3+d^3x^3}}\,dx}{d} + \frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx}{d}$$

$$= \frac{f\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt{3}(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2} + \frac{(de-cf)\log\left((c-dx)(c+d\right.}{4\sqrt[3]{2}cd^2}$$

Mathematica [F] time = 0.183859, size = 0, normalized size = 0.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]

[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{fx+e}{dx+c}\frac{1}{\sqrt[3]{d^3x^3-c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)

[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")

[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt[3]{(-c + dx)\left(c^2 + cdx + d^2x^2\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\left(d^3x^3 - c^3\right)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.174 $\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$

Optimal. Leaf size=160

$$\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad(a+b}{b^6(n+}$$

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3
*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n
))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rubi [A] time = 0.105788, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad(a+b}{b^6(n+}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```


```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3
*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n
))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx=\int\left(\frac{\left(a^2b^3c-a^5d\right)(a+bx)^n}{b^5}+\frac{a\left(-2b^3c+5a^3d\right)(a+bx)^{1+n}}{b^5}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{2+n}}{b^5}\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{3+n}}{b^6(3+n)}+$$

Mathematica [A] time = 0.125001, size = 133, normalized size = 0.83

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(b^3c-10a^3d\right)}{n+3}+\frac{a(a+bx)\left(5a^3d-2b^3c\right)}{n+2}+\frac{a^2b^3c-a^5d}{n+1}+\frac{10a^2d(a+bx)^3}{n+4}+\frac{d(a+bx)^5}{n+6}-\frac{5ad(a+bx)^4}{n+5}\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*(
a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*(a
 + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 + n))
)/b^6
```

Maple [B] time = 0.007, size = 451, normalized size = 2.8

$$-\frac{(bx+a)^{1+n}\left(-b^5dn^5x^5-15\,b^5dn^4x^5+5\,ab^4dn^4x^4-85\,b^5dn^3x^5+50\,ab^4dn^3x^4-b^5cn^5x^2-225\,b^5dn^2x^5-20\,a^2b^3dn^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^5*d*n^5*x^5-15*b^5*d*n^4*x^5+5*a*b^4*d*n^4*x^4-85*b^5*d*
n^3*x^5+50*a*b^4*d*n^3*x^4-b^5*c*n^5*x^2-225*b^5*d*n^2*x^5-20*a^2*b^3*d*n^3
*x^3+175*a*b^4*d*n^2*x^4-18*b^5*c*n^4*x^2-274*b^5*d*n*x^5-120*a^2*b^3*d*n^2
*x^3+2*a*b^4*c*n^4*x+250*a*b^4*d*n*x^4-121*b^5*c*n^3*x^2-120*b^5*d*x^5+60*a
^3*b^2*d*n^2*x^2-220*a^2*b^3*d*n*x^3+32*a*b^4*c*n^3*x+120*a*b^4*d*x^4-372*b
^5*c*n^2*x^2+180*a^3*b^2*d*n*x^2-2*a^2*b^3*c*n^3-120*a^2*b^3*d*x^3+178*a*b^
4*c*n^2*x-508*b^5*c*n*x^2-120*a^4*b*d*n*x+120*a^3*b^2*d*x^2-30*a^2*b^3*c*n^
2+388*a*b^4*c*n*x-240*b^5*c*x^2-120*a^4*b*d*x-148*a^2*b^3*c*n+240*a*b^4*c*x
+120*a^5*d-240*a^2*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+7
20)
```

Maxima [A] time = 1.1094, size = 342, normalized size = 2.14

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{\left(\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^6x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2
 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5
 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*
b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d
/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Fricas [B] time = 1.21846, size = 1073, normalized size = 6.71

$$\frac{\left(2\,a^3b^3cn^3+30\,a^3b^3cn^2+148\,a^3b^3cn+240\,a^3b^3c-120\,a^6d+\left(b^6dn^5+15\,b^6dn^4+85\,b^6dn^3+225\,b^6dn^2+274\,b^6dn+1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 120
*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b^6
```

```
*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 + 50
*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*
a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c
+ (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127
*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b^5*c*
n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d)*n)*x^2
 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*a^2*b^4*c
 - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6
*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [A] time = 6.54228, size = 6392, normalized size = 39.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 47*a**5*d/(60*a**5*b**6 + 300*a**4*b**7
*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*
x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a
**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 17
5*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**
2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*x**2*log(
a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 200*a**3*b**2*d*x**2/(60*a**5
*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b
**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 300*a**4*b**7*x
+ 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**
5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 60
0*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) -
 10*a*b**4*c*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a
**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*log(a
/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**
9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 150*a*b**4*d*x**4/(60*a**5*b**
6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10
*x**4 + 60*b**11*x**5) - 20*b**5*c*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
+ 60*b**5*d*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b*
*8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 90*b**5*
d*x**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d*log(a/b
+ x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 65*a**5*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x*
*2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d*x*log(a/b + x)/(12*a**4
*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4
) - 200*a**4*b*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*
a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d*x**2*log(a/b + x)/(12*a**4*b
**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4)
- 180*a**3*b**2*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 +
 48*a*b**9*x**3 + 12*b**10*x**4) - a**2*b**3*c/(12*a**4*b**6 + 48*a**3*b**7
*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*
x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a
*b**9*x**3 + 12*b**10*x**4) - 4*a*b**4*c*x/(12*a**4*b**6 + 48*a**3*b**7*x +
 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*d*x**4*log
```

(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x*
*3 + 12*b**10*x**4) + 60*a*b**4*d*x**4/(12*a**4*b**6 + 48*a**3*b**7*x + 72*
a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*b**5*c*x**2/(12*a**4*b
6 + 48*a3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4)
+ 12*b**5*d*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*
b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d*log(a/b + x)/(6*a**3*b**
6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*d/(6*a**3*b**
6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*d*x*log(a/b
 + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a
4*b*d*x/(6*a3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 1
80*a**3*b**2*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*
x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2/(6*a**3*b**6 + 18*a**2*b**7*x +
18*a*b**8*x**2 + 6*b**9*x**3) - 2*a**2*b**3*c/(6*a**3*b**6 + 18*a**2*b**7*x
 + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d*x**3*log(a/b + x)/(6*a**3
*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*a*b**4*c*x/(6*a*
*3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*d*x**4
/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*b**5*c*x
2/(6*a3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*
d*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n,
 -4)), (-60*a**5*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) -
 90*a**5*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d*x*log(a
/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d*x/(6*a**2*
b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/b + x)/(6*a**
2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*log(a/b + x)/(6*a**2*b*
*6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**3*c/(6*a**2*b**6 + 12*a*b**7*x
+ 6*b**8*x**2) + 20*a**2*b**3*d*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x*
*2) + 12*a*b**4*c*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2)
+ 12*a*b**4*c*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4*d*x**4
/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*b**5*c*x**2*log(a/b + x)/(6*
a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d*x**5/(6*a**2*b**6 + 12*a*
b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*log(a/b + x)/(12*a*b**6 + 12*
b**7*x) + 60*a**5*d/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*d*x*log(a/b + x)/(1
2*a*b**6 + 12*b**7*x) - 30*a**3*b**2*d*x**2/(12*a*b**6 + 12*b**7*x) - 24*a*
*2*b**3*c*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 24*a**2*b**3*c/(12*a*b**6
+ 12*b**7*x) + 10*a**2*b**3*d*x**3/(12*a*b**6 + 12*b**7*x) - 24*a*b**4*c*x*
log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*d*x**4/(12*a*b**6 + 12*b**7
*x) + 12*b**5*c*x**2/(12*a*b**6 + 12*b**7*x) + 3*b**5*d*x**5/(12*a*b**6 + 1
2*b**7*x), Eq(n, -2)), (-a**5*d*log(a/b + x)/b**6 + a**4*d*x/b**5 - a**3*d*
x**2/(2*b**4) + a**2*c*log(a/b + x)/b**3 + a**2*d*x**3/(3*b**3) - a*c*x/b**
2 - a*d*x**4/(4*b**2) + c*x**2/(2*b) + d*x**5/(5*b), Eq(n, -1)), (-120*a**6
d(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*d*n*x*(a + b*x)**n/(
b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n**2*x**2*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 60*a**4*b**2*d*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
5 + 175*b6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + 2*a**3*b**3*c*n**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*
n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 30*a**3*b
3*c*n2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**
6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 148*a**3*b**3*c*n*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b*
*6*n**2 + 1764*b**6*n + 720*b**6) + 240*a**3*b**3*c*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) + 20*a**3*b**3*d*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 60*a**3*b**3*d*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) +
40*a**3*b**3*d*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**

```
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 2*a**2*b**4*
c*n**4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*c*n**3*x*(a
+ b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*
b**6*n**2 + 1764*b**6*n + 720*b**6) - 148*a**2*b**4*c*n**2*x*(a + b*x)**n/(
b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
 1764*b**6*n + 720*b**6) - 240*a**2*b**4*c*n*x*(a + b*x)**n/(b**6*n**6 + 21
*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
 720*b**6) - 5*a**2*b**4*d*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
 - 30*a**2*b**4*d*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**
2*b**4*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*d*
n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*c*n**5*x**2*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 16*a*b**5*c*n**4*x**2*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 89*a*b**5*c*n**3*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 194*a*b**5*c*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 12
0*a*b**5*c*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*d*n**5*x*
*5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*d*n**4*x**5*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*d*n**3*x**5*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 50*a*b**5*d*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 24*a*b**5*d*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**
6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*c*
n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*b**6*c*n**4*x**3*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 121*b**6*c*n**3*x**3*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 372*b**6*c*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 508*b**6*c*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 240*b
**6*c*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**
6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d*n**5*x**6*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d*n**4*x**6*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 85*b**6*d*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) + 225*b**6*d*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b*
*6*d*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b*
*6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*d*x**6*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6), True))
```

Giac [B] time = 1.15569, size = 1127, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*b
^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*x^
6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*x +
 a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b^5*c
*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3*x^4
+ 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(b*x +
a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^n*a^3*b
^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*b^5*d*n*
x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x + 89*(b*x +
 a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*c*n
^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n^2*x^2 +
 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3 + 2*(b*x + a)
^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x + a)^n*a*b^5*
c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b^6*c*x^3 + 30*(
b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x + 120*(b*x + a)^n*
a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^n*a^3*b^3*c - 120
*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 162
4*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

3.175 $\int x(a+bx)^n \left(c+dx^3\right) dx$

Optimal. Leaf size=126

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)
*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n
)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(
5 + n))
```

Rubi [A] time = 0.0682909, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1620}

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)
*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n
)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(
5 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^3\right)dx &= \int\left(\frac{a\left(-b^3c+a^3d\right)(a+bx)^n}{b^4}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{1+n}}{b^4}+\frac{6a^2d(a+bx)^{2+n}}{b^4}-\frac{4ad(a+b\right.}{b^4}\\ &= -\frac{a\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^5(1+n)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+bx)^{\cdot}}{b^5(4+n)}\end{aligned}$$

Mathematica [A] time = 0.078184, size = 104, normalized size = 0.83

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)\left(b^3c-4a^3d\right)}{n+2}+\frac{a\left(a^3d-b^3c\right)}{n+1}+\frac{6a^2d(a+bx)^2}{n+3}+\frac{d(a+bx)^4}{n+5}-\frac{4ad(a+bx)^3}{n+4}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
+ b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4 +
n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Maple [B] time = 0.004, size = 283, normalized size = 2.3

$$\frac{(bx+a)^{1+n}\left(b^4dn^4x^4+10\,b^4dn^3x^4-4\,ab^3dn^3x^3+35\,b^4dn^2x^4-24\,ab^3dn^2x^3+b^4cn^4x+50\,b^4dnx^4+12\,a^2b^2dn^2x^2-44\,\dots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^
2*x^4-24*a*b^3*d*n^2*x^3+b^4*c*n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44
*a*b^3*d*n*x^3+13*b^4*c*n^3*x+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-2
4*a*b^3*d*x^3+59*b^4*c*n^2*x-24*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2
+107*b^4*c*n*x-24*a^3*b*d*x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)/b^
5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A] time = 1.03665, size = 248, normalized size = 1.97

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^nc}{\left(n^2+3\,n+2\right)b^2}+\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4x^4-4\left(n^3+\dots\right.\right.}{\left(n^5+15\,n^4+85\,n^3+225\,n^2\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*
b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24
*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*
n + 120)*b^5)
```

Fricas [B] time = 1.15695, size = 744, normalized size = 5.9

$$-\frac{\left(a^2b^3cn^3+12\,a^2b^3cn^2+47\,a^2b^3cn+60\,a^2b^3c-24\,a^5d-\left(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b^5d\right)x^5-\left(ab^4dn\dots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^5
*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5
- (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(a^2
*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5*c*n
^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^2*d)*
n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4*c - 2
*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^
```

```
2 + 274*b^5*n + 120*b^5)
```

Sympy [A] time = 4.14797, size = 3703, normalized size = 29.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 13*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 40*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3
*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 36*a**2*b**2
*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x
**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a*
*4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**
4) - 4*b**4*c*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b
**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a
**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 12*b**4*d
*x**4/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 +
 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6*a**3*b**5 + 18*a**2
*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6*a**3*b**5 + 18*a**2*
b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a**3
*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 108*a**3*b*d*x/(6*
a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b**2*d
*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*
x**3) - 72*a**2*b**2*d*x**2/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2
+ 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 +
6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x
+ 18*a*b**7*x**2 + 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6*x
+ 18*a*b**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/(2*
a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**6*x
 + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*
b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a*
*2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a*b*
*3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**
5 + 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b*
*7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3))
, (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 + 3*
b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d*
x**2/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x)
+ 3*a*b**3*c/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x)
+ 3*b**4*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3
*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*d*x*
*2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**2) + c*x/b + d*x**4/(4
*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**
5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*
```

```
n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a + b*x)**n/(b**5*n**5 + 15*
b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2
*b**3*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b*
*5*n**2 + 274*b**5*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 6
0*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b
**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85
*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x**3*
(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274
*b**5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a*b**4*c*n**
3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
 274*b**5*n + 120*b**5) + 47*a*b**4*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*b**
4*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*
b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**n/(
b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*
b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**2*(a + b*x)*
*n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
120*b**5) + 13*b**5*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85
*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 59*b**5*c*n**2*x**2*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) + 107*b**5*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*b**5*c*x**2
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*n**
3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*b
**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b*
*5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5*n**5 +
 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), True
))
```

Giac [B] time = 1.21134, size = 779, normalized size = 6.18

$(bx+a)^n b^5 d n^4 x^5 + (bx+a)^n a b^4 d n^4 x^4 + 10\,(bx+a)^n b^5 d n^3 x^5 + 6\,(bx+a)^n a b^4 d n^3 x^4 + 35\,(bx+a)^n b^5 d n^2 x^5 + (bx+a)^n b^5$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x^5
 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x +
a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c*n^4
*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b
*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a*b^4*c
*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 -
```

```
8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x + a)^n*
a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^
2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 24*(b*x + a
)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*b^3*c*n - 6
0*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*
b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.176 $\int (a+bx)^n \left(c+dx^3\right) dx$

Optimal. Leaf size=94

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Rubi [A] time = 0.0460304, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1850}

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n\left(c+dx^3\right)dx &= \int \left(\frac{\left(b^3c-a^3d\right)(a+bx)^n}{b^3}+\frac{3a^2d(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+bx)^{3+n}}{b^3}\right)dx\\ &= \frac{\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.0698985, size = 94, normalized size = 1.

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Maple [A] time = 0.004, size = 167, normalized size = 1.8

$$-\frac{(bx+a)^{1+n}\left(-b^3dn^3x^3-6\,b^3dn^2x^3+3\,ab^2dn^2x^2-11\,b^3dnx^3+9\,ab^2dnx^2-b^3cn^3-6\,dx^3b^3-6\,a^2bdnx+6\,adx^2b^2-\right.}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n
*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^
3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24*b^3*c)/b^4/(n^4+10*n^3+35*n^2+50*
n+24)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.20149, size = 473, normalized size = 5.03

$$\frac{\left(ab^3cn^3+9\,ab^3cn^2+26\,ab^3cn+24\,ab^3c-6\,a^4d+\left(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d\right)x^4+\left(ab^3dn^3+3\,ab^3dn^2+2\,a\right.}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*d
*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d*n
^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3 +
9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^
4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [A] time = 3.0571, size = 1906, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**3*
```

```
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*
a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**
2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*
b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b*
*5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**
4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4
+ 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a
/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*log(a/b + x)/b + d*x**
3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35
*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2*x
**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a + b*x
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*a
*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
 + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(
a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**4*c*n**2*x*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*
x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b*
*4) + 24*b**4*c*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d
*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4), True))
```

Giac [B] time = 1.18777, size = 487, normalized size = 5.18

$$\frac{(bx+a)^n b^4 d n^3 x^4 + (bx+a)^n a b^3 d n^3 x^3 + 6\,(bx+a)^n b^4 d n^2 x^4 + 3\,(bx+a)^n a b^3 d n^2 x^3 + 11\,(bx+a)^n b^4 d n x^4 + (bx+a)^n b^4 c n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 +
(b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a
*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b*x +
 a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^3*c*n
```

```
^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x + a)^n*
a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x + a)^n
*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.177 $\int \frac{(a+bx)^n(c+dx^3)}{x}\,dx$

Optimal. Leaf size=99

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(2
 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rubi [A] time = 0.0578469, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1620, 65}

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(2
 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx &= \int \left(\frac{a^2d(a+bx)^n}{b^2}+\frac{c(a+bx)^n}{x}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right)dx\\
&= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}+c\int \frac{(a+bx)^n}{x}\,dx\\
&= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}-\frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.056102, size = 94, normalized size = 0.95

$$\frac{(a+bx)^{n+1}\left(ad\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)-b^3c\left(n^2+5n+6\right){}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)\right)}{ab^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] ((a + b*x)^(1 + n)*(a*d*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2)
 - b^3*c*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/
(a*b^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\left(dx^3+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^3+c\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x + a)^n/x, x)
```

Sympy [B] time = 5.25384, size = 741, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(n +
 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) + d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*
b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**
5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/
(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)
/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**
3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b
**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3
*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) - b*b**n*c*n*x*(a/b + x)**
n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*x*
(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)(bx + a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

3.178 $\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=294

$$\frac{\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}+$$

```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a
^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3
*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c
 - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(
a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n)
)/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(
a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rubi [A] time = 0.201699, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1620}

$$\frac{\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```


```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a
^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3
*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c
 - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(
a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n)
)/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(
a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = \int \left(\frac{\left(ab^3c-a^4d\right)^2(a+bx)^n}{b^8}-\frac{2\left(ab^6c^2-5a^4b^3cd+4a^7d^2\right)(a+bx)^{1+n}}{b^8}+\frac{\left(b^6c^2-20a^3\right.}{}\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^6c^2-20a^3b^3\right.}{}$$

Mathematica [A] time = 0.278485, size = 252, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)}{n+3}+\frac{2d(a+bx)^5\left(b^3c-28a^3d\right)}{n+6}+\frac{10ad(a+bx)^4\left(7a^3d-b^3c\right)}{n+5}+\frac{4a^2d(a+bx)^3\left(5b^3c-14a^3d\right)}{n+4}-\frac{2a(a+bx)\left(b^3c-\right.}{}\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(b
^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)
*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 + n)
+ (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*a^3*
d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*(a +
b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

Maple [B] time = 0.014, size = 1565, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(1+n)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a*b^7*d^2*n^7*x^7+546*b
^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^5+4536*b^8*d^2*n^5*x^8
+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78*b^8*c*d*n^7*x^5+22449*b^8
*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c*d*n^7*x^4-15680*a*b^7*d^2*
n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*x^8-336*a^3*b^5*d^2*n^5*x^5+
9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*a*b^7*d^2*n^3*x^7+b^8*
c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*x^8-5040*a^3*b^5*d^2*n
^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x^6-4660*a*b^7*c*d*n^5*
x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58938*b^8*c*d*n^4*x^5+10958
4*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-28560*a^3*b^5*d^2*n^3*x^5+1200*a^2
*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*
d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5
+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*
a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*
b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5
*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*
x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+7680
0*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b
^7*c*d*n^2*x^4+41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n*x^5-40320*a^5*b^3*d^2
*n^2*x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^4-31800*a^3*b^5*c*d*n^
3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760*a^2*b^6*c*d*n^2*x^3-1
1780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468*b^8*c^2*n^3*x^2+120960*b
^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-73920*a^5*b^3*d^2*n*x^3+6000*a^4*b^4*c
*d*n^3*x+40320*a^4*b^4*d^2*x^4-135000*a^3*b^5*c*d*n^2*x^2+1250*a^2*b^6*c^2*
n^4+267600*a^2*b^6*c*d*n*x^3-59678*a*b^7*c^2*n^3*x-120960*a*b^7*c*d*x^4+290
276*b^8*c^2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b^3*c*d*n^3-40320*a^5*b
^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d*n*x^2+10530*a^2*b^6*c
^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+301872*b^8*c^2*n*x^2-4
0320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760*a^5*b^3*c*d*n^2+166800*a^4*b^
4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49148*a^2*b^6*c^2*n^2-241392*a*b^7*c^2*n*x
+120960*b^8*c^2*x^2-40320*a^7*b*d^2*x-45840*a^5*b^3*c*d*n+120960*a^4*b^4*c*
d*x+120432*a^2*b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8*d^2-120960*a^5*b^3*c*
d+120960*a^2*b^6*c^2)/b^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4
+723680*n^3+1172700*n^2+1026576*n+362880)
```

Maxima [B] time = 1.19546, size = 811, normalized size = 2.76

$$\frac{\left(\left(n^2+3n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^nc^2}{\left(n^3+6n^2+11n+6\right)b^3}+\frac{2\left(\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) +
 ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 +
 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4
+ 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 7
35*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n
^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 +
50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4
- 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 4032
0*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870*n^7 + 9450*n^
6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)
*b^9)
```

Fricas [B] time = 1.05735, size = 3567, normalized size = 12.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^3
*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2*n
^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*d^2
*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*
d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*
a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*
n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5 + 73
5*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 720*a^2*b
^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d + 4*(159*b^
9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5 + (29469*
b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 +
 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*
d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*
(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*b^5*d^2)*n
^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a*b^8*c*d - 350*
a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5 - 10*(a^2*b^7*c*
d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a
^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b
^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 +
 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*n^7 +
 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(401*b^9*c^2 + 60
*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6*c*d)*n^4 + 12*(12039*b^9
```

```
*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^2)*n^3 + 4*(72569*b^9*c^2 + 18940*a
^3*b^6*c*d - 5040*a^6*b^3*d^2)*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 2
80*a^6*b^3*d^2)*n)*x^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*
b^8*c^2*n^8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12
*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10*(8479*a*b
^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3475*a^4*b^5*c*d + 84
0*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a^4*b^5*c*d + a^7*b^2*d^2)*n)*x
^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^
2*b^7*c^2*n^6 + 625*a^2*b^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*
n^4 + 2*(12287*a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 -
 955*a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d^2)*n
)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273
*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*
n + 362880*b^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25503, size = 3591, normalized size = 12.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b^9
*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2*n^
7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^5*x^
9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 - 168*
(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8 + 2244
9*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1272*(b*
x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 1400*(b*x +
 a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 67284*(b*x
 + a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x + a)^n*a^2*
b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x + a)^n*b^9
*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x + a)^n*a^2*b
^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*(b*x + a)^n*b
^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)^n*b^9*c^2*n^7
*x^3 - 300*(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^n*a*b^8*c*d*n^5
*x^5 - 336*(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a)^n*b^9*c*d*n^4*
x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x + a)^n*a^2*b^7*d^2*
n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b*x + a)^n*b^9*d^2*
n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)^n*b^9*c^2*n^6*x^3
+ 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a)^n*a^2*b^7*c*d*n^5*x^4
 + 25898*(b*x + a)^n*a*b^8*c*d*n^4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^2*n^4*x
```

```
^5 + 185022*(b*x + a)^n*b^9*c*d*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^2*n^3
*x^6 - 14112*(b*x + a)^n*a^2*b^7*d^2*n^2*x^7 + 5040*(b*x + a)^n*a*b^8*d^2*n
*x^8 + 40320*(b*x + a)^n*b^9*d^2*x^9 - 2*(b*x + a)^n*a^2*b^7*c^2*n^7*x + 66
4*(b*x + a)^n*a*b^8*c^2*n^6*x^2 + 7218*(b*x + a)^n*b^9*c^2*n^5*x^3 + 1080*(
b*x + a)^n*a^3*b^6*c*d*n^5*x^3 - 19200*(b*x + a)^n*a^2*b^7*c*d*n^4*x^4 + 16
80*(b*x + a)^n*a^5*b^4*d^2*n^4*x^4 + 55532*(b*x + a)^n*a*b^8*c*d*n^3*x^5 -
11760*(b*x + a)^n*a^4*b^5*d^2*n^3*x^5 + 337228*(b*x + a)^n*b^9*c*d*n^2*x^6
+ 15344*(b*x + a)^n*a^3*b^6*d^2*n^2*x^6 - 5760*(b*x + a)^n*a^2*b^7*d^2*n*x^
7 - 78*(b*x + a)^n*a^2*b^7*c^2*n^6*x + 5890*(b*x + a)^n*a*b^8*c^2*n^5*x^2 -
 120*(b*x + a)^n*a^4*b^5*c*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^2*n^4*x^3 +
10600*(b*x + a)^n*a^3*b^6*c*d*n^4*x^3 - 52690*(b*x + a)^n*a^2*b^7*c*d*n^3*x
^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n^3*x^4 + 59568*(b*x + a)^n*a*b^8*c*d*n^
2*x^5 - 16800*(b*x + a)^n*a^4*b^5*d^2*n^2*x^5 + 322032*(b*x + a)^n*b^9*c*d*
n*x^6 + 6720*(b*x + a)^n*a^3*b^6*d^2*n*x^6 + 2*(b*x + a)^n*a^3*b^6*c^2*n^6
- 1250*(b*x + a)^n*a^2*b^7*c^2*n^5*x + 29839*(b*x + a)^n*a*b^8*c^2*n^4*x^2
- 3000*(b*x + a)^n*a^4*b^5*c*d*n^4*x^2 + 144468*(b*x + a)^n*b^9*c^2*n^3*x^3
 + 45000*(b*x + a)^n*a^3*b^6*c*d*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^2*n^3
*x^3 - 66900*(b*x + a)^n*a^2*b^7*c*d*n^2*x^4 + 18480*(b*x + a)^n*a^5*b^4*d^
2*n^2*x^4 + 24192*(b*x + a)^n*a*b^8*c*d*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^
2*n*x^5 + 120960*(b*x + a)^n*b^9*c*d*x^6 + 78*(b*x + a)^n*a^3*b^6*c^2*n^5 -
 10530*(b*x + a)^n*a^2*b^7*c^2*n^4*x + 240*(b*x + a)^n*a^5*b^4*c*d*n^4*x +
84790*(b*x + a)^n*a*b^8*c^2*n^3*x^2 - 25800*(b*x + a)^n*a^4*b^5*c*d*n^3*x^2
 + 290276*(b*x + a)^n*b^9*c^2*n^2*x^3 + 75760*(b*x + a)^n*a^3*b^6*c*d*n^2*x
^3 - 20160*(b*x + a)^n*a^6*b^3*d^2*n^2*x^3 - 30240*(b*x + a)^n*a^2*b^7*c*d*
n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n*x^4 + 1250*(b*x + a)^n*a^3*b^6*c^2*
n^4 - 49148*(b*x + a)^n*a^2*b^7*c^2*n^3*x + 5760*(b*x + a)^n*a^5*b^4*c*d*n^
3*x + 120696*(b*x + a)^n*a*b^8*c^2*n^2*x^2 - 83400*(b*x + a)^n*a^4*b^5*c*d*
n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n^2*x^2 + 301872*(b*x + a)^n*b^9*c^
2*n*x^3 + 40320*(b*x + a)^n*a^3*b^6*c*d*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d
^2*n*x^3 + 10530*(b*x + a)^n*a^3*b^6*c^2*n^3 - 240*(b*x + a)^n*a^6*b^3*c*d*
n^3 - 120432*(b*x + a)^n*a^2*b^7*c^2*n^2*x + 45840*(b*x + a)^n*a^5*b^4*c*d*
n^2*x + 60480*(b*x + a)^n*a*b^8*c^2*n*x^2 - 60480*(b*x + a)^n*a^4*b^5*c*d*n
*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n*x^2 + 120960*(b*x + a)^n*b^9*c^2*x^3
 + 49148*(b*x + a)^n*a^3*b^6*c^2*n^2 - 5760*(b*x + a)^n*a^6*b^3*c*d*n^2 - 1
20960*(b*x + a)^n*a^2*b^7*c^2*n*x + 120960*(b*x + a)^n*a^5*b^4*c*d*n*x - 40
320*(b*x + a)^n*a^8*b*d^2*n*x + 120432*(b*x + a)^n*a^3*b^6*c^2*n - 45840*(b
*x + a)^n*a^6*b^3*c*d*n + 120960*(b*x + a)^n*a^3*b^6*c^2 - 120960*(b*x + a)
^n*a^6*b^3*c*d + 40320*(b*x + a)^n*a^9*d^2)/(b^9*n^9 + 45*b^9*n^8 + 870*b^9
*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 117
2700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

3.179 $\int x(a+bx)^n\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=248

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{ad\left(8b^3c-35a^3d\right)}{b^8(n+}$$

```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*
d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7
*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b
*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^
8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b
*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rubi [A] time = 0.14838, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{ad\left(8b^3c-35a^3d\right)}{b^8(n+}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```


```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*
d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7
*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b
*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^
8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b
*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=\int\left(-\frac{a\left(-b^3c+a^3d\right)^2(a+bx)^n}{b^7}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^7}-\frac{3a^2d\left(-4b^3c+7a\right.}{b^7}\right.$$

$$=-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^8(2+n)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a}{b^8(3+n)}$$

Mathematica [A] time = 0.202781, size = 211, normalized size = 0.85

$$\frac{(a+bx)^{n+1}\left(\frac{d(a+bx)^4\left(2b^3c-35a^3d\right)}{n+5}+\frac{ad(a+bx)^3\left(35a^3d-8b^3c\right)}{n+4}+\frac{3a^2d(a+bx)^2\left(4b^3c-7a^3d\right)}{n+3}+\frac{(a+bx)\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)}{n+2}-\frac{a\left(b^3c-a^3d\right)^2}{n+1}+\frac{21a^2d^2(a}{n+6}\right.}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^
2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3*c
- 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (7*a*
d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

Maple [B] time = 0.013, size = 1142, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^7*d^2*n^7*x^7-28*b^7*d^2*n^6*x^7+7*a*b^6*d^2*n^6*x^6-322
*b^7*d^2*n^5*x^7+147*a*b^6*d^2*n^5*x^6-2*b^7*c*d*n^7*x^4-1960*b^7*d^2*n^4*x
^7-42*a^2*b^5*d^2*n^5*x^5+1225*a*b^6*d^2*n^4*x^6-62*b^7*c*d*n^6*x^4-6769*b^
7*d^2*n^3*x^7-630*a^2*b^5*d^2*n^4*x^5+8*a*b^6*c*d*n^6*x^3+5145*a*b^6*d^2*n^
3*x^6-782*b^7*c*d*n^5*x^4-13132*b^7*d^2*n^2*x^7+210*a^3*b^4*d^2*n^4*x^4-357
0*a^2*b^5*d^2*n^3*x^5+216*a*b^6*c*d*n^5*x^3+11368*a*b^6*d^2*n^2*x^6-b^7*c^2
*n^7*x-5162*b^7*c*d*n^4*x^4-13068*b^7*d^2*n*x^7+2100*a^3*b^4*d^2*n^3*x^4-24
*a^2*b^5*c*d*n^5*x^2-9450*a^2*b^5*d^2*n^2*x^5+2264*a*b^6*c*d*n^4*x^3+12348*
a*b^6*d^2*n*x^6-34*b^7*c^2*n^6*x-19088*b^7*c*d*n^3*x^4-5040*b^7*d^2*x^7-840
*a^4*b^3*d^2*n^3*x^3+7350*a^3*b^4*d^2*n^2*x^4-576*a^2*b^5*c*d*n^4*x^2-11508
*a^2*b^5*d^2*n*x^5+a*b^6*c^2*n^6+11592*a*b^6*c*d*n^3*x^3+5040*a*b^6*d^2*x^6
-478*b^7*c^2*n^5*x-39128*b^7*c*d*n^2*x^4-5040*a^4*b^3*d^2*n^2*x^3+48*a^3*b^
4*c*d*n^4*x+10500*a^3*b^4*d^2*n*x^4-5064*a^2*b^5*c*d*n^3*x^2-5040*a^2*b^5*d
^2*x^5+33*a*b^6*c^2*n^5+29984*a*b^6*c*d*n^2*x^3-3580*b^7*c^2*n^4*x-40608*b^
7*c*d*n*x^4+2520*a^5*b^2*d^2*n^2*x^2-9240*a^4*b^3*d^2*n*x^3+1056*a^3*b^4*c*
d*n^3*x+5040*a^3*b^4*d^2*x^4-19584*a^2*b^5*c*d*n^2*x^2+445*a*b^6*c^2*n^4+36
576*a*b^6*c*d*n*x^3-15289*b^7*c^2*n^3*x-16128*b^7*c*d*x^4+7560*a^5*b^2*d^2*
n*x^2-48*a^4*b^3*c*d*n^3-5040*a^4*b^3*d^2*x^3+8016*a^3*b^4*c*d*n^2*x-31200*
a^2*b^5*c*d*n*x^2+3135*a*b^6*c^2*n^3+16128*a*b^6*c*d*x^3-36706*b^7*c^2*n^2*
x-5040*a^6*b*d^2*n*x+5040*a^5*b^2*d^2*x^2-1008*a^4*b^3*c*d*n^2+23136*a^3*b^
4*c*d*n*x-16128*a^2*b^5*c*d*x^2+12154*a*b^6*c^2*n^2-44712*b^7*c^2*n*x-5040*
a^6*b*d^2*x-7008*a^4*b^3*c*d*n+16128*a^3*b^4*c*d*x+24552*a*b^6*c^2*n-20160*
b^7*c^2*x+5040*a^7*d^2-16128*a^4*b^3*c*d+20160*a*b^6*c^2)/b^8/(n^8+36*n^7+5
46*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)
```

Maxima [A] time = 1.15948, size = 640, normalized size = 2.58

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^n c^2}{\left(n^2+3\,n+2\right)b^2} + \frac{2\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5 + \left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4x^4 - 4\right.}{\left(n^5+15\,n^4+85\,n^3+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
```

```
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2
+ 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 1313
2*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*
n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 27
4*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3
*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2
+ 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8
)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n
^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

Fricas [B] time = 0.971381, size = 2719, normalized size = 10.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b^
6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6 +
322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^2 +
 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 +
175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2
*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*
a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*b^6*d
^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b^8*c*d +
 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (9544*b^8*c*d
 + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2 + 72*(282*
b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283
*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d
- 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16
*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5*b^3*c*d)*
n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*c*d*n^4 + 3*(27
2*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d - 63*a^5*b^3*d^2)*
n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 50
4*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239
*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (1
5289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5*c*d
- 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d - 35*a^6*b^2*d^
2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d)*n - (a*b^7*c^2*n^7 + 33
*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^
4 + 2*(6077*a*b^7*c^2 - 504*a^4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4
*b^4*c*d)*n^2 + 1008*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x)*(b
*x + a)^n/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^
4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17035, size = 2746, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b^8
*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2*n^
6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^4*x^
8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 - 105*
(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 + 6769*
(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782*(b*x +
 a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b*x + a)^
n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132*(b*x + a
)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n*a^2*b^6*c
*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^n*b^8*c*d*n
^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n*a^2*b^6*d^2
*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^2*n
*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n^6*x^2 - 192*(
b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*n^4*x^4 - 210*(
b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*n^3*x^5 + 1470*(
b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*d^2*n^2*x^6 + 720
*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x^8 + 33*(b*x + a)^
n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24*(b*x + a)^n*a^3*b^
5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3 + 7496*(b*x + a)^n*a*b
^7*c*d*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^2*n^3*x^4 + 39128*(b*x + a)^n*b
^8*c*d*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^2*n^2*x^5 - 840*(b*x + a)^n*a^2
*b^6*d^2*n*x^6 - (b*x + a)^n*a^2*b^6*c^2*n^6 + 445*(b*x + a)^n*a*b^7*c^2*n^
5*x + 3580*(b*x + a)^n*b^8*c^2*n^4*x^2 + 528*(b*x + a)^n*a^3*b^5*c*d*n^4*x^
2 - 6528*(b*x + a)^n*a^2*b^6*c*d*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^2*n^3*
x^3 + 9144*(b*x + a)^n*a*b^7*c*d*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^2*n^2
*x^4 + 40608*(b*x + a)^n*b^8*c*d*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^2*n*x^5
 - 33*(b*x + a)^n*a^2*b^6*c^2*n^5 + 3135*(b*x + a)^n*a*b^7*c^2*n^4*x - 48*(
b*x + a)^n*a^4*b^4*c*d*n^4*x + 15289*(b*x + a)^n*b^8*c^2*n^3*x^2 + 4008*(b*
x + a)^n*a^3*b^5*c*d*n^3*x^2 - 10400*(b*x + a)^n*a^2*b^6*c*d*n^2*x^3 + 2520
*(b*x + a)^n*a^5*b^3*d^2*n^2*x^3 + 4032*(b*x + a)^n*a*b^7*c*d*n*x^4 - 1260*
(b*x + a)^n*a^4*b^4*d^2*n*x^4 + 16128*(b*x + a)^n*b^8*c*d*x^5 - 445*(b*x +
a)^n*a^2*b^6*c^2*n^4 + 12154*(b*x + a)^n*a*b^7*c^2*n^3*x - 1008*(b*x + a)^n
*a^4*b^4*c*d*n^3*x + 36706*(b*x + a)^n*b^8*c^2*n^2*x^2 + 11568*(b*x + a)^n*
a^3*b^5*c*d*n^2*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2*n^2*x^2 - 5376*(b*x + a)
^n*a^2*b^6*c*d*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^2*n*x^3 - 3135*(b*x + a)^
n*a^2*b^6*c^2*n^3 + 48*(b*x + a)^n*a^5*b^3*c*d*n^3 + 24552*(b*x + a)^n*a*b^
7*c^2*n^2*x - 7008*(b*x + a)^n*a^4*b^4*c*d*n^2*x + 44712*(b*x + a)^n*b^8*c^
2*n*x^2 + 8064*(b*x + a)^n*a^3*b^5*c*d*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2
*n*x^2 - 12154*(b*x + a)^n*a^2*b^6*c^2*n^2 + 1008*(b*x + a)^n*a^5*b^3*c*d*n
^2 + 20160*(b*x + a)^n*a*b^7*c^2*n*x - 16128*(b*x + a)^n*a^4*b^4*c*d*n*x +
5040*(b*x + a)^n*a^7*b*d^2*n*x + 20160*(b*x + a)^n*b^8*c^2*x^2 - 24552*(b*x
 + a)^n*a^2*b^6*c^2*n + 7008*(b*x + a)^n*a^5*b^3*c*d*n - 20160*(b*x + a)^n*
a^2*b^6*c^2 + 16128*(b*x + a)^n*a^5*b^3*c*d - 5040*(b*x + a)^n*a^8*d^2)/(b^
8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8
*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

3.180 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=203

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}$$

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3
*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)
^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(
4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)
^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rubi [A] time = 0.114845, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1850}

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3
*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)
^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(
4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)
^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \int \left(\frac{\left(b^3c-a^3d\right)^2 (a+bx)^n}{b^6}-\frac{6a^2d\left(-b^3c+a^3d\right)(a+bx)^{1+n}}{b^6}+\frac{3ad\left(-2b^3c+5a^3d\right)(a+bx)^{2+n}}{b^6}\right.$$

$$=\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^7(2+n)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{3+n}}{b^7(3+n)}+$$

Mathematica [A] time = 0.167198, size = 172, normalized size = 0.85

$$\frac{(a+bx)^{n+1}\left(\frac{2d(a+bx)^3\left(b^3c-10a^3d\right)}{n+4}+\frac{3ad(a+bx)^2\left(5a^3d-2b^3c\right)}{n+3}+\frac{6a^2d(a+bx)\left(b^3c-a^3d\right)}{n+2}+\frac{\left(b^3c-a^3d\right)^2}{n+1}+\frac{15a^2d^2(a+bx)^4}{n+5}+\frac{d^2(a+bx)^6}{n+7}-\frac{6ad^2(a+bx)^5}{n+6}\right.}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(a
 + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*d*
(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 + n)
- (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

Maple [B] time = 0.01, size = 793, normalized size = 3.9

$$\frac{(bx+a)^{1+n}\left(b^6d^2n^6x^6+21\,b^6d^2n^5x^6-6\,ab^5d^2n^5x^5+175\,b^6d^2n^4x^6-90\,ab^5d^2n^4x^5+2\,b^6cdn^6x^3+735\,b^6d^2n^3x^6+30\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(1+n)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+175*b
^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+2*b^6*c*d*n^6*x^3+735*b^6*d^2*n^3*x^6+3
0*a^2*b^4*d^2*n^4*x^4-510*a*b^5*d^2*n^3*x^5+48*b^6*c*d*n^5*x^3+1624*b^6*d^2
*n^2*x^6+300*a^2*b^4*d^2*n^3*x^4-6*a*b^5*c*d*n^5*x^2-1350*a*b^5*d^2*n^2*x^5
+452*b^6*c*d*n^4*x^3+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+1050*a^2*b^
4*d^2*n^2*x^4-126*a*b^5*c*d*n^4*x^2-1644*a*b^5*d^2*n*x^5+b^6*c^2*n^6+2112*b
^6*c*d*n^3*x^3+720*b^6*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+12*a^2*b^4*c*d*n^4*x
+1500*a^2*b^4*d^2*n*x^4-978*a*b^5*c*d*n^3*x^2-720*a*b^5*d^2*x^5+27*b^6*c^2*
n^5+5090*b^6*c*d*n^2*x^3+360*a^4*b^2*d^2*n^2*x^2-1320*a^3*b^3*d^2*n*x^3+228
*a^2*b^4*c*d*n^3*x+720*a^2*b^4*d^2*x^4-3402*a*b^5*c*d*n^2*x^2+295*b^6*c^2*n
^4+5904*b^6*c*d*n*x^3+1080*a^4*b^2*d^2*n*x^2-12*a^3*b^3*c*d*n^3-720*a^3*b^3
*d^2*x^3+1500*a^2*b^4*c*d*n^2*x-5064*a*b^5*c*d*n*x^2+1665*b^6*c^2*n^3+2520*
b^6*c*d*x^3-720*a^5*b*d^2*n*x+720*a^4*b^2*d^2*x^2-216*a^3*b^3*c*d*n^2+3804*
a^2*b^4*c*d*n*x-2520*a*b^5*c*d*x^2+5104*b^6*c^2*n^2-720*a^5*b*d^2*x-1284*a^
3*b^3*c*d*n+2520*a^2*b^4*c*d*x+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+
5040*b^6*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5
040)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.862773, size = 1963, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 25
20*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*
n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^
7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^
3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*
b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x
^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 15*a^3
*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c*d + 33*
a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(555*a*b^6*c
^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c
*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4
*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2
- 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b^5*c*d - 2
*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*n + (b^7*c^2*n^
6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b^4*c*d)*n^4 + 9*
(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2
 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*
n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^
2 + 13068*b^7*n + 5040*b^7)
```

Sympy [A] time = 13.9771, size = 11664, normalized size = 57.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**9*d*
*2*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200
*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**1
3*x**6) + 37*a**9*d**2/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2
 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a*
*3*b**13*x**6) + 360*a**8*b*d**2*x*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b*
*8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 36
0*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 162*a**8*b*d**2*x/(60*a**9*b**7 +
 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**
11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 900*a**7*b**2*d**2*x*
*2*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200
*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**1
3*x**6) + 225*a**7*b**2*d**2*x**2/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**
7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x
**5 + 60*a**3*b**13*x**6) + 1200*a**6*b**3*d**2*x**3*log(a/b + x)/(60*a**9*
b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a*
*5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 900*a**5*b**4*d
**2*x**4*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2
+ 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**
3*b**13*x**6) - 300*a**5*b**4*d**2*x**4/(60*a**9*b**7 + 360*a**8*b**8*x + 9
00*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b
**12*x**5 + 60*a**3*b**13*x**6) + 360*a**4*b**5*d**2*x**5*log(a/b + x)/(60*
a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 9
00*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) - 300*a**4*b
**5*d**2*x**5/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a
**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*
x**6) - 10*a**3*b**6*c**2/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x
**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60
```

```
*a**3*b**13*x**6) + 60*a**3*b**6*d**2*x**6*log(a/b + x)/(60*a**9*b**7 + 360
*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x
**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) - 110*a**3*b**6*d**2*x**6/(
60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3
+ 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 30*a**2
*b**7*c*d*x**4/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*
a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13
*x**6) + 12*a*b**8*c*d*x**5/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9
*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 +
60*a**3*b**13*x**6) + 2*b**9*c*d*x**6/(60*a**9*b**7 + 360*a**8*b**8*x + 900
*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**
12*x**5 + 60*a**3*b**13*x**6), Eq(n, -7)), (-60*a**8*d**2*log(a/b + x)/(10*
a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*
a**3*b**11*x**4 + 10*a**2*b**12*x**5) - 47*a**8*d**2/(10*a**7*b**7 + 50*a**
6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 +
10*a**2*b**12*x**5) - 300*a**7*b*d**2*x*log(a/b + x)/(10*a**7*b**7 + 50*a**
6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 +
10*a**2*b**12*x**5) - 175*a**7*b*d**2*x/(10*a**7*b**7 + 50*a**6*b**8*x + 10
0*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12
*x**5) - 600*a**6*b**2*d**2*x**2*log(a/b + x)/(10*a**7*b**7 + 50*a**6*b**8*
x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2
*b**12*x**5) - 200*a**6*b**2*d**2*x**2/(10*a**7*b**7 + 50*a**6*b**8*x + 100
*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*
x**5) - 600*a**5*b**3*d**2*x**3*log(a/b + x)/(10*a**7*b**7 + 50*a**6*b**8*x
 + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*
b**12*x**5) - 300*a**4*b**4*d**2*x**4*log(a/b + x)/(10*a**7*b**7 + 50*a**6*
b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10
*a**2*b**12*x**5) + 150*a**4*b**4*d**2*x**4/(10*a**7*b**7 + 50*a**6*b**8*x
+ 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b
**12*x**5) - 60*a**3*b**5*d**2*x**5*log(a/b + x)/(10*a**7*b**7 + 50*a**6*b*
*8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a
**2*b**12*x**5) + 90*a**3*b**5*d**2*x**5/(10*a**7*b**7 + 50*a**6*b**8*x + 1
00*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**1
2*x**5) - 2*a**2*b**6*c**2/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x
**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) + 10*a
**2*b**6*d**2*x**6/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 10
0*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) + 5*a*b**7*c*d
*x**4/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*
x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) + b**8*c*d*x**5/(10*a**7*b*
*7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b*
*11*x**4 + 10*a**2*b**12*x**5), Eq(n, -6)), (60*a**7*d**2*log(a/b + x)/(4*a
**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b*
*11*x**4) + 65*a**7*d**2/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2
+ 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 240*a**6*b*d**2*x*log(a/b + x)/(4*
a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b
**11*x**4) + 200*a**6*b*d**2*x/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9
*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 360*a**5*b**2*d**2*x**2*log(
a/b + x)/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*
x**3 + 4*a*b**11*x**4) + 180*a**5*b**2*d**2*x**2/(4*a**5*b**7 + 16*a**4*b**
8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 240*a**4*b
**3*d**2*x**3*log(a/b + x)/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**
2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 60*a**3*b**4*d**2*x**4*log(a/b +
 x)/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3
+ 4*a*b**11*x**4) - 60*a**3*b**4*d**2*x**4/(4*a**5*b**7 + 16*a**4*b**8*x +
24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) - 12*a**2*b**5*d**
2*x**5/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x*
*3 + 4*a*b**11*x**4) - a*b**6*c**2/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*
b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 2*a*b**6*d**2*x**6/(4*a*
*5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**
```

11*x**4) + 2*b**7*c*d*x**4/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**
2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/
b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a
6*d2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180
*a**5*b*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 +
3*b**10*x**3) - 270*a**5*b*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x
2 + 3*b10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9
*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a
3*b7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*a**3*b**3*c*d*
log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) +
11*a**3*b**3*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x*
*3) - 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*
a*b**9*x**2 + 3*b**10*x**3) + 18*a**2*b**4*c*d*x*log(a/b + x)/(3*a**3*b**7
+ 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 27*a**2*b**4*c*d*x/(3*a**
3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**2*b**4*d**2*
x**4/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a*b*
*5*c*d*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b
10*x3) + 18*a*b**5*c*d*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b
9*x2 + 3*b**10*x**3) - b**6*c**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**
9*x**2 + 3*b**10*x**3) + 6*b**6*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2
*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 + 9*a
2*b8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*log(a
/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*b**
7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b**7
+ 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**
8*x + 4*b**9*x**2) - 24*a**3*b**3*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*
x + 4*b**9*x**2) - 36*a**3*b**3*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2
) - 20*a**3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a*
*2*b**4*c*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a*
*2*b**4*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x
4/(4*a2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a*b**5*c*d*x**2*log(a/b +
x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2*a*b**5*d**2*x**5/(4*a**2*b
7 + 8*a*b8*x + 4*b**9*x**2) - 2*b**6*c**2/(4*a**2*b**7 + 8*a*b**8*x + 4
*b**9*x**2) + 8*b**6*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b*
*6*d**2*x**6/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-60*a**
6*d**2*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 60*a**6*d**2/(10*a*b**7 + 10*
b**8*x) - 60*a**5*b*d**2*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) + 30*a**4*b
2*d2*x**2/(10*a*b**7 + 10*b**8*x) + 60*a**3*b**3*c*d*log(a/b + x)/(10*a
*b**7 + 10*b**8*x) + 60*a**3*b**3*c*d/(10*a*b**7 + 10*b**8*x) - 10*a**3*b**
3*d**2*x**3/(10*a*b**7 + 10*b**8*x) + 60*a**2*b**4*c*d*x*log(a/b + x)/(10*a
*b**7 + 10*b**8*x) + 5*a**2*b**4*d**2*x**4/(10*a*b**7 + 10*b**8*x) - 30*a*b
5*c*d*x2/(10*a*b**7 + 10*b**8*x) - 3*a*b**5*d**2*x**5/(10*a*b**7 + 10*b
8*x) - 10*b6*c**2/(10*a*b**7 + 10*b**8*x) + 10*b**6*c*d*x**3/(10*a*b**7
+ 10*b**8*x) + 2*b**6*d**2*x**6/(10*a*b**7 + 10*b**8*x), Eq(n, -2)), (a**6
*d**2*log(a/b + x)/b**7 - a**5*d**2*x/b**6 + a**4*d**2*x**2/(2*b**5) - 2*a*
*3*c*d*log(a/b + x)/b**4 - a**3*d**2*x**3/(3*b**4) + 2*a**2*c*d*x/b**3 + a*
*2*d**2*x**4/(4*b**3) - a*c*d*x**2/b**2 - a*d**2*x**5/(5*b**2) + c**2*log(a
/b + x)/b + 2*c*d*x**3/(3*b) + d**2*x**6/(6*b), Eq(n, -1)), (720*a**7*d**2*
(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 720*a**6*b*d*
*2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
4 + 6769*b7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a*
*5*b**2*d**2*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n*
*5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 504
0*b**7) + 360*a**5*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) - 12*a**4*b**3*c*d*n**3*(a + b*x)**n/(b**7*n**7 + 28*b

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**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) - 216*a**4*b**3*c*d*n**2*(a + b*x)**n/(b**7*n
**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1313
2*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1284*a**4*b**3*c*d*n*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2520*a**4*b**3*c*d*(a + b
*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b*
*7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*
n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360
*a**4*b**3*d**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) - 240*a**4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
*6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13
068*b**7*n + 5040*b**7) + 12*a**3*b**4*c*d*n**4*x*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 216*a**3*b**4*c*d*n**3*x*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1284*a**3*b**4*c*d*n**2*x*
(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*a**3*b**
4*c*d*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*
a**3*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) + 330*a**3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13
132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b
**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*c*d*n*
*5*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 114*a
**2*b**5*c*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n*
*5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 504
0*b**7) - 750*a**2*b**5*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) - 1902*a**2*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1260*a**2*b**5*c*d*n*x**2*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5
*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 60*a**2
*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d**2*n*x**5*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*c**2*n**6*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b
**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27*a*b**6*c**2*n**
5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 295*a*b**6*
c**2*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
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7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 166
5*a*b**6*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) + 5104*a*b**6*c**2*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b*
*7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
+ 5040*b**7) + 8028*a*b**6*c**2*n*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) + 5040*a*b**6*c**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + 2*a*b**6*c*d*n**6*x**3*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 42*a*b**6*c*d*n**5*x**3*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 326*a*b**6*c*d*n**4*x**3*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1134*a*b**6*c*d*n
**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1688
*a*b**6*c*d*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 840*a*b**6*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + a*b**6*d**2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7
 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b
**7*n**2 + 13068*b**7*n + 5040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**
6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*
d**2*n**2*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
+ 120*a*b**6*d**2*n*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + b**7*c**2*n**6*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) + 27*b**7*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) + 295*b**7*c**2*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b*
*7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 1665*b**7*c**2*n**3*x*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5104*b**7*c**2*n**2*x*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 8028*b**7*c**2*n*x*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*b**7*c**2*x*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769
*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*
x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 48*b**7*
c*d*n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960
*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +
 452*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 2112*b**7*c*d*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 5090*b**7*c*d*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*
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b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) + 5904*b**7*c*d*n*x**4*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*b**7*c*d*x**4*(a + b*x)**n/(b
**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**
7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d*
*2*n**4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*
b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +
735*b**7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7), True))
```

Giac [B] time = 1.24295, size = 1994, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7
*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2*n^
5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7
+ 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 60*(b*
x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 1624*(b*
x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b*x + a)
^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x + a)^n*a
^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b
^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*c*d*n^5*x^
2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d*n^3*x^4 +
180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 +
 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7 + (b*x + a)^
n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a)^n*a^2*b^5*c*
d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^
2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*
n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)^n*a*b^6*c^2*n^5
+ 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*c*d*n^4*x - 750*(b
*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c*d*n^2*x^3 - 360*(b
*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7*c*d*n*x^4 + 180*(b*x +
 a)^n*a^3*b^4*d^2*n*x^4 + 295*(b*x + a)^n*a*b^6*c^2*n^4 + 1665*(b*x + a)^n*
b^7*c^2*n^3*x + 216*(b*x + a)^n*a^3*b^4*c*d*n^3*x - 1902*(b*x + a)^n*a^2*b^
5*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n^2*x^2 + 840*(b*x + a)^n*a*b^6
*c*d*n*x^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 2520*(b*x + a)^n*b^7*c*d*x
^4 + 1665*(b*x + a)^n*a*b^6*c^2*n^3 - 12*(b*x + a)^n*a^4*b^3*c*d*n^3 + 5104
*(b*x + a)^n*b^7*c^2*n^2*x + 1284*(b*x + a)^n*a^3*b^4*c*d*n^2*x - 1260*(b*x
 + a)^n*a^2*b^5*c*d*n*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n*x^2 + 5104*(b*x +
```

```
 a)^n*a*b^6*c^2*n^2 - 216*(b*x + a)^n*a^4*b^3*c*d*n^2 + 8028*(b*x + a)^n*b^
7*c^2*n*x + 2520*(b*x + a)^n*a^3*b^4*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 8028*(b*x + a)^n*a*b^6*c^2*n - 1284*(b*x + a)^n*a^4*b^3*c*d*n + 5040*(b
*x + a)^n*b^7*c^2*x + 5040*(b*x + a)^n*a*b^6*c^2 - 2520*(b*x + a)^n*a^4*b^3
*c*d + 720*(b*x + a)^n*a^7*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*
b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

3.181 $\int \frac{(a+bx)^n\left(c+dx^3\right)^2}{x}\,dx$

Optimal. Leaf size=209

$$\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad}{}$$

```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c -
 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n)) -
(5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6
 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x
)/a])/(a*(1 + n))
```

Rubi [A] time = 0.126826, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1620, 65}

$$\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```


```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c -
 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n)) -
(5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6
 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x
)/a])/(a*(1 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^2}{x} dx = \int \left(-\frac{a^2d\left(-2b^3c+a^3d\right)(a+bx)^n}{b^5} + \frac{c^2(a+bx)^n}{x} + \frac{ad\left(-4b^3c+5a^3d\right)(a+bx)^{1+n}}{b^5} + \frac{2d\left(b^3c\right.}{}\right.$$
$$= \frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d\left(b^3c-5a^3d\right)(a+bx)^{3+n}}{b^6(3+n)} -$$
$$= \frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d\left(b^3c-5a^3d\right)(a+bx)^{3+n}}{b^6(3+n)} -$$

Mathematica [A] time = 0.163573, size = 188, normalized size = 0.9

$$(a+bx)^{n+1}\left(\frac{2d(a+bx)^2\left(b^3c-5a^3d\right)}{b^6(n+3)} + \frac{ad(a+bx)\left(5a^3d-4b^3c\right)}{b^6(n+2)} + \frac{a^2d\left(2b^3c-a^3d\right)}{b^6(n+1)} + \frac{10a^2d^2(a+bx)^3}{b^6(n+4)} + \frac{d^2(a+bx)^5}{b^6(n+6)} -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*((a^2*d*(2*b^3*c - a^3*d))/(b^6*(1 + n)) + (a*d*(-4*b^3*c
 + 5*a^3*d)*(a + b*x))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*x)^2)/
(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)
^4)/(b^6*(5 + n)) + (d^2*(a + b*x)^5)/(b^6*(6 + n)) - (c^2*Hypergeometric2F
1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n \left(dx^3+c\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^2 (bx+a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^3+c^2\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] time = 10.4853, size = 4755, normalized size = 22.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 2*c*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x
)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4
*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5
*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log
(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*lo
g(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b +
 x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(
a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**
n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + d**2*Piecewise
((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) + 47*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 60
0*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*log(a/b
 + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*
x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 175*a**4*b*x/(60*a**5*b**6 + 300
*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 +
 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) + 200*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b
**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**
2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x*
*4*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*
a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 150*a*b**4*x**4/(60*a*
*5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a
*b**10*x**4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) - 90*b**5*x**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*
b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -
```

```
6)), (-60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x
**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 65*a**5/(12*a**4*b**6 + 48*a**3*b**
7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*x*lo
g(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x
**3 + 12*b**10*x**4) - 200*a**4*b*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*x**2*log(a/b
 + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 180*a**3*b**2*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3*log(a/b
 + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 60*a*b**4*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 60*a*b**4*x**4/(12
*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10
*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 +
48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*log(a/b + x)/(6*a**3*
b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5/(6*a**3*b*
*6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*x*log(a/b
 + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a*
*4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*
a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2
 + 6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b*
*8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a
**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*x**4/(6*a**3*b**6 +
18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*x**5/(6*a**3*b**6 +
 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5*log(
a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5/(6*a**2*b**6 +
 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(a/b + x)/(6*a**2*b**6 + 12*a
*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x
**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x
**2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b*
*4*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*x**5/(6*a**2*b**
6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*log(a/b + x)/(12*a*b**
6 + 12*b**7*x) + 60*a**5/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*x*log(a/b + x)
/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2/(12*a*b**6 + 12*b**7*x) + 10*a
**2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*x**4/(12*a*b**6 + 12*b**7*
x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*log(a/b + x)/b
**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b**3) - a*x**4/(4*b**
2) + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + 120*a**5*b*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*
n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n*x**2*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*n**3*x**3*(a + b*x)**n/(b
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*x**3*(a + b*x)**n/(b**6*n**6 +
21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
 + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
 5*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**
4*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b*
*6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b**4*n**2*x**4
*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1
624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n*x**4*(a + b*x)**n/
(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
+ 1764*b**6*n + 720*b**6) + a*b**5*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
```

```
20*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*
a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*n**2*x*
*5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b**5*n*x**5*(a + b*x)**n/(
b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
 1764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6
*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*
n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*n**2*x**6*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b*
*6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*n*x**6*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), Tru
e)) - b*b**n*c**2*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n +
1)/(a*gamma(n + 2)) - b*b**n*c**2*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n +
 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

3.182 $\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx$

Optimal. Leaf size=459

$$\frac{\left(b^3c-a^3d\right)\left(-29a^3b^3cd+55a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}+\frac{3a^2d\left(-56a^3b^3cd+55a^6d^2+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{15ad\left(-14a^3b\right.}{}$$

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11
*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3
*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n
)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))
/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)
^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a
 + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x
)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n)
)/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 +
n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)
^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rubi [A] time = 0.316699, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1620}

$$\frac{\left(b^3c-a^3d\right)\left(-29a^3b^3cd+55a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}+\frac{3a^2d\left(-56a^3b^3cd+55a^6d^2+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{15ad\left(-14a^3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```


```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11
*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3
*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n
)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))
/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)
^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a
 + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x
)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n)
)/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 +
n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)
^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 dx = \int \left(-\frac{a^2\left(-b^3c+a^3d\right)^3 (a+bx)^n}{b^{11}} + \frac{a\left(-b^3c+a^3d\right)^2 \left(-2b^3c+11a^3d\right)(a+bx)^{1+n}}{b^{11}} + \frac{\left(b^3c\right.}{}\right.$$

$$= \frac{a^2\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{12}(1+n)} - \frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{12}(2+n)} + \frac{\left(b^3c-a^3d\right)\left(\right.}{}$$

Mathematica [A] time = 0.469966, size = 402, normalized size = 0.88

$$(a+bx)^{n+1}\left(\frac{3d(a+bx)^5\left(-56a^3b^3cd+154a^6d^2+b^6c^2\right)}{n+6} - \frac{15ad(a+bx)^4\left(-14a^3b^3cd+22a^6d^2+b^6c^2\right)}{n+5} + \frac{3a^2d(a+bx)^3\left(-56a^3b^3cd+55a^6d^2+10b^6c^2\right)}{n+4} + \frac{(a+bx}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2*
(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 56*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a^3*b
^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d
+ 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a +
 b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3
*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10
 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n)))/b^
12
```

Maple [B] time = 0.027, size = 3780, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*b^10*d^3*n^1
0*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c*d^2*n^11*x^8
-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10*d^3*n^8*x^10-
207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^9*d^3*n^8*x^9+2
4*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^11*c*d^2*n^9*x^8-
2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^3*n^7*x^9
+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*b^11*c^2*d*n^11*x^
5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+35640*a^3*b^8*d^3*n^
7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6*x^9+38592*a*b^10*c*
d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d*n^10*x^5-1251927*b^1
1*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b^7*d^3*n^7*x^7+540540*
a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3*n^5*x^9+
15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^7+37586230*a*b^10*d^3*n^
4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*n^6*x^8-105258076*b^11*d^
3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^2*n^8*x^5+4490640*a^
3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-29625750*a^2*b^9*d^3*n^4*x^9
+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2*n^6*x^7+92504500*a*b^10*d^3
*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2*d*n^8*x^5-49357662*b^11*c*d^2*n
```

^5*x^8-150917976*b^11*d^3*n^2*x^11+55440*a^5*b^6*d^3*n^6*x^6-2550240*a^4*b^
7*d^3*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4*x^8-60*a
^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*a^2*b^9*d^3*n^3
*x^9+29250*a*b^10*c^2*d*n^8*x^4+32905656*a*b^10*c*d^2*n^5*x^7+140289336*a*b
^10*d^3*n^2*x^10-75*b^11*c^3*n^10*x^2-1491309*b^11*c^2*d*n^7*x^5-173991492*
b^11*c*d^2*n^4*x^8-120543840*b^11*d^3*n*x^11+1164240*a^5*b^6*d^3*n^5*x^6-50
40*a^4*b^7*c*d^2*n^7*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b^8*c*d^2*
n^6*x^5+66611160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19647432*a^
2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^10*c^3*n^10*x+48465
0*a*b^10*c^2*d*n^7*x^4+131616048*a*b^10*c*d^2*n^4*x^7+116915040*a*b^10*d^3*
n*x^10-2492*b^11*c^3*n^9*x^2-11832048*b^11*c^2*d*n^6*x^5-405697080*b^11*c*d
^2*n^3*x^8-39916800*b^11*d^3*x^11-332640*a^6*b^5*d^3*n^5*x^5+9702000*a^5*b^
6*d^3*n^4*x^6-216720*a^4*b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^3*x^7+180
*a^3*b^8*c^2*d*n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3*b^8*d^3*
n^2*x^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^6-1129233
60*a^2*b^9*d^3*n*x^9+146*a*b^10*c^3*n^9*x+5033295*a*b^10*c^2*d*n^6*x^4+3390
03552*a*b^10*c*d^2*n^3*x^7+39916800*a*b^10*d^3*x^10-48294*b^11*c^3*n^8*x^2-
63978405*b^11*c^2*d*n^5*x^5-590770944*b^11*c*d^2*n^2*x^8-4989600*a^6*b^5*d^
3*n^4*x^5+20160*a^5*b^6*c*d^2*n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6-3664080*
a^4*b^7*c*d^2*n^5*x^4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*c^2*d*n^7
*x^2+58735152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*a^2*b^9*c
^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*x^6-399168
00*a^2*b^9*d^3*x^9+4692*a*b^10*c^3*n^8*x+33993765*a*b^10*c^2*d*n^5*x^4+5335
48224*a*b^10*c*d^2*n^2*x^7-604581*b^11*c^3*n^7*x^2-234340020*b^11*c^2*d*n^4
*x^5-477740160*b^11*c*d^2*n*x^8+1663200*a^7*b^4*d^3*n^4*x^4-28274400*a^6*b^
5*d^3*n^3*x^5+786240*a^5*b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^2*x^6-360
*a^4*b^7*c^2*d*n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*b^7*d^3*n
*x^7+273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+39916800*
a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n^5*x^3-47140934
4*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^10*c^3*n^7*x+149923200*a*b^10*c^2*d*n^4*x
^4+457781760*a*b^10*c*d^2*n*x^7-5112891*b^11*c^3*n^6*x^2-565580388*b^11*c^2
*d*n^3*x^5-159667200*b^11*c*d^2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-60480*a^6*
b^5*c*d^2*n^5*x^2-74844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*d^2*n^4*x
^3+97796160*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*a^4*b^7*c
*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x^2+4033491
84*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*a^2*b^9*c^2*d*n^4*x^
3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^10*c^3*n^6*x+422084100*a*b^10*c
^2*d*n^3*x^4+159667200*a*b^10*c*d^2*x^7-29651558*b^11*c^3*n^5*x^2-848562336
*b^11*c^2*d*n^2*x^5-6652800*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d^3*n^2*x^
4-2177280*a^6*b^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5*b^6*c^2*
d*n^6+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-504720*a^4*b^
7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b^8*c^2*d*n^4*x^
2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*n^6-279934320*a^2*b^9*c^2
*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8156274*a*b^10*c^3*n^5*x+717481440*a
*b^10*c^2*d*n^2*x^4-117115476*b^11*c^3*n^4*x^2-703304640*b^11*c^2*d*n*x^5-3
9916800*a^8*b^3*d^3*n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^7*b^4*d^3
*n*x^4-28002240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+20520*a^5*b^
6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d*n^4*x-376
427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^2*d*n^3*x^2+159667200*a^3*b^
8*c*d^2*x^5-952098*a^2*b^9*c^3*n^5-568599120*a^2*b^9*c^2*d*n^2*x^3+42990568
*a*b^10*c^3*n^4*x+655404480*a*b^10*c^2*d*n*x^4-305860408*b^11*c^3*n^3*x^2-2
39500800*b^11*c^2*d*x^5+19958400*a^9*b^2*d^3*n^2*x^2-73180800*a^8*b^3*d^3*n
*x^3+4112640*a^7*b^4*c*d^2*n^3*x+39916800*a^7*b^4*d^3*x^4-149506560*a^6*b^5
*c*d^2*n^2*x^2+484200*a^5*b^6*c^2*d*n^4+336510720*a^5*b^6*c*d^2*n*x^3-48336
840*a^4*b^7*c^2*d*n^3*x-159667200*a^4*b^7*c*d^2*x^4+396700560*a^3*b^8*c^2*d
*n^2*x^2-7204176*a^2*b^9*c^3*n^4-595529280*a^2*b^9*c^2*d*n*x^3+148249816*a*
b^10*c^3*n^3*x+239500800*a*b^10*c^2*d*x^4-496433664*b^11*c^3*n^2*x^2+598752
00*a^9*b^2*d^3*n*x^2-120960*a^8*b^3*c*d^2*n^3-39916800*a^8*b^3*d^3*x^3+4777
9200*a^7*b^4*c*d^2*n^2*x-283288320*a^6*b^5*c*d^2*n*x^2+6053400*a^5*b^6*c^2*

```
d*n^3+159667200*a^5*b^6*c*d^2*x^3-198727920*a^4*b^7*c^2*d*n^2*x+515695680*a
^3*b^8*c^2*d*n*x^2-35786392*a^2*b^9*c^3*n^3-239500800*a^2*b^9*c^2*d*x^3+315
221184*a*b^10*c^3*n^2*x-442258560*b^11*c^3*n*x^2-39916800*a^10*b*d^3*n*x+39
916800*a^9*b^2*d^3*x^2-3991680*a^8*b^3*c*d^2*n^2+203454720*a^7*b^4*c*d^2*n*
x-159667200*a^6*b^5*c*d^2*x^2+42283440*a^5*b^6*c^2*d*n^2-395945280*a^4*b^7*
c^2*d*n*x+239500800*a^3*b^8*c^2*d*x^2-112463424*a^2*b^9*c^3*n^2+362424960*a
*b^10*c^3*n*x-159667200*b^11*c^3*x^2-39916800*a^10*b*d^3*x-43787520*a^8*b^3
*c*d^2*n+159667200*a^7*b^4*c*d^2*x+156444480*a^5*b^6*c^2*d*n-239500800*a^4*
b^7*c^2*d*x-202757760*a^2*b^9*c^3*n+159667200*a*b^10*c^3*x+39916800*a^11*d^
3-159667200*a^8*b^3*c*d^2+239500800*a^5*b^6*c^2*d-159667200*a^2*b^9*c^3)/b^
12/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634*n^7+44990231*n^6+20
6070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^2+1486442880*n+479001
600)
```

Maxima [B] time = 1.15203, size = 1557, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
 + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n
^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*
n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5
 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 +
85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^
3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*
x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 -
40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 870*n^7 + 9
450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 3
62880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 263755
8*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 12054
3840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9 + 18150*n^8 + 157
773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640*
n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 870*n^8 + 9450*n^7 + 632
73*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362880*n)*a^
2*b^10*x^10 + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*n^
4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 32
2*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 +
 7920*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5*
b^7*x^7 - 55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6
*x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5 - 1663200
*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^3 + 3*n^2 + 2*n)*a^9
*b^3*x^3 - 19958400*(n^2 + n)*a^10*b^2*x^2 + 39916800*a^11*b*n*x - 39916800
*a^12)*(b*x + a)^n*d^3/((n^12 + 78*n^11 + 2717*n^10 + 55770*n^9 + 749463*n^
8 + 6926634*n^7 + 44990231*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888
*n^3 + 1931559552*n^2 + 1486442880*n + 479001600)*b^12)
```

Fricas [B] time = 1.04659, size = 8852, normalized size = 19.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 159667200
*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 39916800
*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 32670*b
^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^12*d^
3*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3
*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 +
55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 157773*a*
b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 8409500*a*b
^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 + 3628800*a
*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b
^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b
^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*
a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a^3*b^9*d^3
)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(417309*b^12*c*d^2 +
 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b^9*d^3)*n^6 + 6*
(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 20
5590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 +
 288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2
+ 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^10 + 1
608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3)*n^8 + 21*(107
13*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289*a*b^11*c*d^2 - 5060*a^4*
b^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 - 323400*a^4*b^8*d^3)*n^5 + 2*(70625
74*a*b^11*c*d^2 - 1116885*a^4*b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 1641
5*a^4*b^8*d^3)*n^3 + 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 166
3200*(4*a*b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*d^2 - 55*a
^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7*d^3)*n^6 + 42*(1315
3*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10*(161702*a^2*b^10*c*d^2 - 2425
5*a^5*b^7*d^3)*n^4 + 24*(116917*a^2*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 3
60*(7192*a^2*b^10*c*d^2 - 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2
- a^5*b^7*d^3)*n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^
12*c^2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*c^2*d
 + 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9*c*d^2)*n^8 + 3
*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 48*(82167*b^12*c^2*d + 114
10*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6 + (21326135*b^12*c^2*d + 3263064*a^
3*b^9*c*d^2 - 277200*a^6*b^6*d^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3
*b^9*c*d^2 - 130900*a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3
*b^9*c*d^2 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 3080*a^3*b^9
*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^3 - 3420*a^6*b^6*c^2
*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^2*d*n^10 + 1950*a*b^11*c^2*d*n
^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d
- 4816*a^4*b^8*c*d^2)*n^7 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n
^6 + 560*(17848*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 +
4*(7034735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*n^4 +
 96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*b^5*d^3)*n^3 +
576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 9625*a^7*b^5*d^3)*n^2 + 266
1120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 + a^7*b^5*d^3)*n)*x^5 + 72*(100058*a
^3*b^9*c^3 - 6725*a^6*b^6*c^2*d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^1
0*c^2*d*n^9 + 1698*a^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7
```

```
*c*d^2)*n^7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*c^2*d - 32
4324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(789721*a^2*b^10*c^2*d - 3
38800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3 + 144*(68927*a^2*b^10*c^2*d -
38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*
a^5*b^7*c*d^2 + a^8*b^4*d^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b
^6*c^2*d + 15120*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 1
59667200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683*b^12
*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a^3*b^9*c^2*d)*n
^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2240*a^6*b^6*c*d^2)*n^6 +
2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c^2*d - 362880*a^6*b^6*c*d^2)*n^5 +
12*(9759623*b^12*c^3 + 4102800*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 +
8*(38232551*b^12*c^3 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831
600*a^9*b^3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 + 13860*a^3*
b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d^3)*n)*x^3 + 144*(780996*a^3
*b^9*c^3 - 293635*a^6*b^6*c^2*d + 27720*a^9*b^3*c*d^2)*n^2 + (a*b^11*c^3*n^
11 + 73*a*b^11*c^3*n^10 + 2346*a*b^11*c^3*n^9 + 6*(7267*a*b^11*c^3 - 30*a^4
*b^8*c^2*d)*n^8 + 3*(172459*a*b^11*c^3 - 3480*a^4*b^8*c^2*d)*n^7 + 3*(13593
79*a*b^11*c^3 - 84120*a^4*b^8*c^2*d)*n^6 + 4*(5373821*a*b^11*c^3 - 817200*a
^4*b^8*c^2*d + 15120*a^7*b^5*c*d^2)*n^5 + 4*(18531227*a*b^11*c^3 - 6042105*
a^4*b^8*c^2*d + 514080*a^7*b^5*c*d^2)*n^4 + 72*(2189036*a*b^11*c^3 - 138005
5*a^4*b^8*c^2*d + 331800*a^7*b^5*c*d^2)*n^3 + 1440*(125842*a*b^11*c^3 - 137
481*a^4*b^8*c^2*d + 70644*a^7*b^5*c*d^2 - 13860*a^10*b^2*d^3)*n^2 + 1995840
0*(4*a*b^11*c^3 - 6*a^4*b^8*c^2*d + 4*a^7*b^5*c*d^2 - a^10*b^2*d^3)*n)*x^2
+ 2880*(70402*a^3*b^9*c^3 - 54321*a^6*b^6*c^2*d + 15204*a^9*b^3*c*d^2)*n -
2*(a^2*b^10*c^3*n^10 + 72*a^2*b^10*c^3*n^9 + 2274*a^2*b^10*c^3*n^8 + 36*(11
48*a^2*b^10*c^3 - 5*a^5*b^7*c^2*d)*n^7 + 3*(158683*a^2*b^10*c^3 - 3420*a^5*
b^7*c^2*d)*n^6 + 36*(100058*a^2*b^10*c^3 - 6725*a^5*b^7*c^2*d)*n^5 + 4*(447
3299*a^2*b^10*c^3 - 756675*a^5*b^7*c^2*d + 15120*a^8*b^4*c*d^2)*n^4 + 72*(7
80996*a^2*b^10*c^3 - 293635*a^5*b^7*c^2*d + 27720*a^8*b^4*c*d^2)*n^3 + 1440
*(70402*a^2*b^10*c^3 - 54321*a^5*b^7*c^2*d + 15204*a^8*b^4*c*d^2)*n^2 + 199
58400*(4*a^2*b^10*c^3 - 6*a^5*b^7*c^2*d + 4*a^8*b^4*c*d^2 - a^11*b*d^3)*n)*
x)*(b*x + a)^n/(b^12*n^12 + 78*b^12*n^11 + 2717*b^12*n^10 + 55770*b^12*n^9
+ 749463*b^12*n^8 + 6926634*b^12*n^7 + 44990231*b^12*n^6 + 206070150*b^12*n
^5 + 657206836*b^12*n^4 + 1414014888*b^12*n^3 + 1931559552*b^12*n^2 + 14864
42880*b^12*n + 479001600*b^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.183 $\int x(a+bx)^n \left(c+dx^3\right)^3 dx$

Optimal. Leaf size=396

$$-\frac{3ad\left(-35a^3b^3cd+40a^6d^2+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(-35a^3b^3cd+70a^6d^2+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right.}{b^{11}(n+}$$

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^
3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*
c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*
b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) +
(3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 +
n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21
*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c
 - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 +
 n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*
(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rubi [A] time = 0.264233, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$-\frac{3ad\left(-35a^3b^3cd+40a^6d^2+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(-35a^3b^3cd+70a^6d^2+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right.}{b^{11}(n+}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```


```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^
3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*
c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*
b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) +
(3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 +
n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21
*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c
 - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 +
 n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*
(a + b*x)^(11 + n))/(b^11*(11 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^3\right)^3 dx=\int\left(\frac{a\left(-b^3c+a^3d\right)^3(a+bx)^n}{b^{10}}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^{10}}+\frac{9a^2d\left(2b^3c-\right.}{}\right.$$

$$=-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{11}(1+n)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{11}(2+n)}+\frac{9a^2d\left(2b^3c-5a\right.}{}$$

Mathematica [A] time = 0.380743, size = 345, normalized size = 0.87

$$(a+bx)^{n+1}\left(\frac{3d(a+bx)^4\left(-35a^3b^3cd+70a^6d^2+b^6c^2\right)}{n+5}-\frac{3ad(a+bx)^3\left(-35a^3b^3cd+40a^6d^2+4b^6c^2\right)}{n+4}+\frac{3d^2(a+bx)^7\left(b^3c-40a^3d\right)}{n+8}+\frac{21ad^2(a+bx)^6\left(10a^3d-b^3c\right)}{n+7}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)*
(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^3*
c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40
*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^
2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(6 + n
) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^3*c -
40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) - (10*a*d
^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

Maple [B] time = 0.021, size = 2972, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] (b*x+a)^(1+n)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9
+1320*b^10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b
^10*d^3*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d
^2*n^9*x^7+157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2
*n^9*x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^
5*x^10-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n
^8*x^6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*
x^7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n
^8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d
^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d^
3*n^3*x^10+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8
*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*
b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+3363066
*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*x^6-6
30*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c*d^2*n
^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a*b^9*c*
d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7
*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3
*n^5*x^5+882000*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3
*b^7*d^3*n^3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^5+106
31160*a^2*b^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*a*b^9*c*d^2*n^4
*x^6-10265760*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+703719*b^10*c^2*d*n^6*x^4+3
3074574*b^10*c*d^2*n^3*x^7+3628800*b^10*d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5
+2520*a^4*b^6*c*d^2*n^6*x^3+3704400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^
2*n^5*x^4-9455040*a^3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^
2*b^8*c*d^2*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^
2*d*n^6*x^3-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+1797*b^10*c^
3*n^8*x+4394079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*n^2*x^7+151200*a^6*b
^4*d^3*n^4*x^4-2570400*a^5*b^5*d^3*n^3*x^5+90720*a^4*b^6*c*d^2*n^5*x^3+8184
```

```
960*a^4*b^6*d^3*n^2*x^6-72*a^3*b^7*c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^
4-9408960*a^3*b^7*d^3*n*x^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c*
d^2*n^3*x^5+3628800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^
5*x^3-45292716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*c^2*d
*n^4*x^4+43332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x^4-7560*a^5*b^5
*c*d^2*n^5*x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800*a^4*b^6*c*d^2*n^4*x^3+88
90560*a^4*b^6*d^3*n*x^6-3744*a^3*b^7*c^2*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3
*x^4-3628800*a^3*b^7*d^3*x^7+551232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8*
c*d^2*n^2*x^5-1734*a*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9
*c*d^2*n*x^6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^1
0*c*d^2*x^7-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^4-249480*a
^5*b^5*c*d^2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b^6*c^2*d*n^6+7862400
*a^4*b^6*c*d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^6-81072*a^3*b^7*c^2*d*n^5*x-31
701600*a^3*b^7*c*d^2*n^2*x^4+4054644*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8
*c*d^2*n*x^5-27342*a*b^9*c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^
9*c*d^2*x^6+2039016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7*
b^3*d^3*n^2*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-2955960
*a^5*b^5*c*d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*c^2*d*n^5+23710
680*a^4*b^6*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4*x-35705880*a^3*b^7*c*d^2
*n*x^4+17731656*a^2*b^8*c^2*d*n^3*x^2+14968800*a^2*b^8*c*d^2*x^5-271929*a*b
^9*c^3*n^5-61656336*a*b^9*c^2*d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^1
0*c^2*d*n*x^4+1814400*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720*
a^6*b^4*c*d^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+
77400*a^4*b^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*b^7*c^2*d*
n^3*x-14968800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d*n^2*x^2-1767087*a*b
^9*c^3*n^4-61548768*a*b^9*c^2*d*n*x^3+27472724*b^10*c^3*n^3*x+23950080*b^10
*c^2*d*x^4+5443200*a^8*b^2*d^3*n*x^2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^
3*d^3*x^3+4974480*a^6*b^4*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a
^4*b^6*c^2*d*n^3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+53
565408*a^2*b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*x^3+5
0312628*b^10*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2*d^3*x^2-453600
*a^7*b^3*c*d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-14968800*a^5*b^5*c*d^2*x^2+53
65728*a^4*b^6*c^2*d*n^2-41590368*a^3*b^7*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x
^2-19978308*a*b^9*c^3*n^2+50292720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880
*a^7*b^3*c*d^2*n+14968800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080
*a^3*b^7*c^2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3-
14968800*a^7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)/b^11/(n^1
1+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n^6+13339535*n^5+45995730*n
^4+105258076*n^3+150917976*n^2+120543840*n+39916800)
```

Maxima [B] time = 1.09508, size = 1287, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^
2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)
*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*
```

```
n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040
*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 6
7284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8
 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n^8 + 9450*
n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362
880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67
284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7
 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*
x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*
a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^5*
b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*b^5*x^5 + 15120
0*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 604800*(n^3 + 3*n^2 + 2*n)*a^8
*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2 - 3628800*a^10*b*n*x + 3628800*a^1
1)*(b*x + a)^n*d^3/((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2
637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 +
120543840*n + 39916800)*b^11)
```

Fricas [B] time = 1.01276, size = 7065, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a^
2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^11*
d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*d^3
*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4 + 8
409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 3628800
*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d^3*n^8
 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^5 + 723
680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^2 + 3628
80*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 + 546*a^2*
b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*
d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d
^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^11*c*d^2 + 3
*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 + 140*a^3*b^8*d
^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*b^11*c*d^2
 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b^8*d^3)*n^4 + 2
*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867*b^11*c*d^2 + 10
890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*
(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*d^2*n^8 + 6*(2209*
a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d^2 - 240*a^4*b^7*d^
3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*b^10*
c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^10*c*d^2 - 32480*a^4*b^7*d^
3)*n^3 + 360*(5449*a*b^10*c*d^2 - 1176*a^4*b^7*d^3)*n^2 + 21600*(33*a*b^10*
c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6
 + 21*(a^2*b^9*c*d^2*n^9 + 45*a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 3
0*(275*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a
^5*b^6*d^3)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(7706
9*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2 - 1096*a^
5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n)*x^6 + 3*(90643*
a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^11*c^2*d*n^10 + 61*b^11*c^2*d*
n^9 + 7983360*b^11*c^2*d + 6*(270*b^11*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(
117*b^11*c^2*d + 8*a^3*b^8*c*d^2)*n^7 + 3*(78191*b^11*c^2*d + 8876*a^3*b^8*
```

```
c*d^2)*n^6 + 3*(488231*b^11*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)
*n^5 + 2*(3008035*b^11*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^3)*n^
4 + 20*(795769*b^11*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6*b^5*d^3)*n^3 +
 72*(356683*b^11*c^2*d + 33061*a^3*b^8*c*d^2 - 7000*a^6*b^5*d^3)*n^2 + 288*
(78167*b^11*c^2*d + 3465*a^3*b^8*c*d^2 - 840*a^6*b^5*d^3)*n)*x^5 + 9*(19634
3*a^2*b^9*c^3 - 8600*a^5*b^6*c^2*d)*n^4 - 3*(a*b^10*c^2*d*n^10 + 57*a*b^10*
c^2*d*n^9 + 1392*a*b^10*c^2*d*n^8 + 6*(3167*a*b^10*c^2*d - 35*a^4*b^7*c*d^2
)*n^7 + 15*(10571*a*b^10*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^10*
c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^10*c^2*d - 327600*a^4*b^7
*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2*d - 47045*a^4*b^7*c
*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^10*c^2*d - 36995*a^4*b^7*c*d^2
 + 7700*a^7*b^4*d^3)*n^2 + 7560*(264*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*
a^7*b^4*d^3)*n)*x^4 + 8*(936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a
^8*b^3*c*d^2)*n^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2
*b^9*c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3413*a
^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2*d - 13685*a^5*
b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*a^5*b^6*c*d^2 + 5040*a^8*
b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d - 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3
*d^3)*n^2 + 2520*(264*a^2*b^9*c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n
)*x^3 + 36*(554953*a^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2
)*n^2 - (b^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2*d)*n^7 +
3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(84959*b^11*c^3 + 19590*a
^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261503*b^11*c^3 + 3114324*a^3*b^8
*c^2*d - 234360*a^6*b^5*c*d^2)*n^4 + 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*
c^2*d - 621810*a^6*b^5*c*d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c
^2*d - 270690*a^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3
+ 16632*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2 + 14
4*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3*c*d^2)*n - (a*
b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^3*n^8 + 18*(1519*a*b^10*c
^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 +
 9*(196343*a*b^10*c^3 - 8600*a^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 10
7865*a^4*b^7*c^2*d + 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 1490
48*a^4*b^7*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c^3 - 264*a
^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^3)*n)*x)*(b*x + a)^n/(b^11*n
^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^11*n^8 + 357423*b^11*n^7 + 263
7558*b^11*n^6 + 13339535*b^11*n^5 + 45995730*b^11*n^4 + 105258076*b^11*n^3
+ 150917976*b^11*n^2 + 120543840*b^11*n + 39916800*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)

[Out] Timed out
```

Giac [B] time = 1.34142, size = 6661, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b*
x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b*x
 + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x + a
)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(b*x
+ a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(b*x +
 a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450*(b*x
+ a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 + 153*(b*
x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8 + 90*(b
*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x^9 + 6327
3*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n^5*x^11 +
3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2*n^9*x^6 +
3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*c*d^2*n^7*x^
8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a^2*b^9*d^3*n^
6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x + a)^n*b^11*d
^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x + a)^n*b^11*c^
2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762*(b*x + a)^n*a*b
^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 + 568701*(b*x + a)^
n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*x^8 - 224490*(b*x
+ a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d^3*n^4*x^10 + 84095
00*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a*b^10*c^2*d*n^9*x^4 + 4
860*(b*x + a)^n*b^11*c^2*d*n^8*x^5 + 126*(b*x + a)^n*a^3*b^8*c*d^2*n^8*x^5
 - 17514*(b*x + a)^n*a^2*b^9*c*d^2*n^7*x^6 + 290367*(b*x + a)^n*a*b^10*c*d^2
*n^6*x^7 - 15120*(b*x + a)^n*a^4*b^7*d^3*n^6*x^7 + 3363066*(b*x + a)^n*b^11
*c*d^2*n^5*x^8 + 176400*(b*x + a)^n*a^3*b^8*d^3*n^5*x^8 - 672840*(b*x + a)^
n*a^2*b^9*d^3*n^4*x^9 + 1172700*(b*x + a)^n*a*b^10*d^3*n^3*x^10 + 12753576*
(b*x + a)^n*b^11*d^3*n^2*x^11 + (b*x + a)^n*b^11*c^3*n^10*x^2 - 12*(b*x + a
)^n*a^2*b^9*c^2*d*n^9*x^3 + 4176*(b*x + a)^n*a*b^10*c^2*d*n^8*x^4 + 73710*(
b*x + a)^n*b^11*c^2*d*n^7*x^5 + 5040*(b*x + a)^n*a^3*b^8*c*d^2*n^7*x^5 - 17
3250*(b*x + a)^n*a^2*b^9*c*d^2*n^6*x^6 + 5040*(b*x + a)^n*a^5*b^6*d^3*n^6*x
^6 + 1330497*(b*x + a)^n*a*b^10*c*d^2*n^5*x^7 - 126000*(b*x + a)^n*a^4*b^7*
d^3*n^5*x^7 + 13114077*(b*x + a)^n*b^11*c*d^2*n^4*x^8 + 609210*(b*x + a)^n*
a^3*b^8*d^3*n^4*x^8 - 1181240*(b*x + a)^n*a^2*b^9*d^3*n^3*x^9 + 1026576*(b*
x + a)^n*a*b^10*d^3*n^2*x^10 + 10628640*(b*x + a)^n*b^11*d^3*n*x^11 + (b*x
+ a)^n*a*b^10*c^3*n^10*x + 64*(b*x + a)^n*b^11*c^3*n^9*x^2 - 648*(b*x + a)^
n*a^2*b^9*c^2*d*n^8*x^3 + 57006*(b*x + a)^n*a*b^10*c^2*d*n^7*x^4 - 630*(b*x
 + a)^n*a^4*b^7*c*d^2*n^7*x^4 + 703719*(b*x + a)^n*b^11*c^2*d*n^6*x^5 + 798
84*(b*x + a)^n*a^3*b^8*c*d^2*n^6*x^5 - 993069*(b*x + a)^n*a^2*b^9*c*d^2*n^5
*x^6 + 75600*(b*x + a)^n*a^5*b^6*d^3*n^5*x^6 + 3800598*(b*x + a)^n*a*b^10*c
*d^2*n^4*x^7 - 529200*(b*x + a)^n*a^4*b^7*d^3*n^4*x^7 + 33074574*(b*x + a)^
n*b^11*c*d^2*n^3*x^8 + 1181880*(b*x + a)^n*a^3*b^8*d^3*n^3*x^8 - 1095840*(b
*x + a)^n*a^2*b^9*d^3*n^2*x^9 + 362880*(b*x + a)^n*a*b^10*d^3*n*x^10 + 3628
800*(b*x + a)^n*b^11*d^3*x^11 + 63*(b*x + a)^n*a*b^10*c^3*n^9*x + 1797*(b*x
 + a)^n*b^11*c^3*n^8*x^2 + 36*(b*x + a)^n*a^3*b^8*c^2*d*n^8*x^2 - 14760*(b*
x + a)^n*a^2*b^9*c^2*d*n^7*x^3 + 475695*(b*x + a)^n*a*b^10*c^2*d*n^6*x^4 -
22680*(b*x + a)^n*a^4*b^7*c*d^2*n^6*x^4 + 4394079*(b*x + a)^n*b^11*c^2*d*n^
5*x^5 + 640080*(b*x + a)^n*a^3*b^8*c*d^2*n^5*x^5 - 30240*(b*x + a)^n*a^6*b^
5*d^3*n^5*x^5 - 3355065*(b*x + a)^n*a^2*b^9*c*d^2*n^4*x^6 + 428400*(b*x + a
)^n*a^5*b^6*d^3*n^4*x^6 + 6470388*(b*x + a)^n*a*b^10*c*d^2*n^3*x^7 - 116928
0*(b*x + a)^n*a^4*b^7*d^3*n^3*x^7 + 51177636*(b*x + a)^n*b^11*c*d^2*n^2*x^8
 + 1176120*(b*x + a)^n*a^3*b^8*d^3*n^2*x^8 - 403200*(b*x + a)^n*a^2*b^9*d^3
*n*x^9 - (b*x + a)^n*a^2*b^9*c^3*n^9 + 1734*(b*x + a)^n*a*b^10*c^3*n^8*x +
29076*(b*x + a)^n*b^11*c^3*n^7*x^2 + 1872*(b*x + a)^n*a^3*b^8*c^2*d*n^7*x^2
 - 183744*(b*x + a)^n*a^2*b^9*c^2*d*n^6*x^3 + 2520*(b*x + a)^n*a^5*b^6*c*d^
2*n^6*x^3 + 2491299*(b*x + a)^n*a*b^10*c^2*d*n^5*x^4 - 308700*(b*x + a)^n*a
^4*b^7*c*d^2*n^5*x^4 + 18048210*(b*x + a)^n*b^11*c^2*d*n^4*x^5 + 2758014*(b
*x + a)^n*a^3*b^8*c*d^2*n^4*x^5 - 302400*(b*x + a)^n*a^6*b^5*d^3*n^4*x^5 -
6473796*(b*x + a)^n*a^2*b^9*c*d^2*n^3*x^6 + 1134000*(b*x + a)^n*a^5*b^6*d^3
```

```
*n^3*x^6 + 5884920*(b*x + a)^n*a*b^10*c*d^2*n^2*x^7 - 1270080*(b*x + a)^n*a
^4*b^7*d^3*n^2*x^7 + 43332840*(b*x + a)^n*b^11*c*d^2*n*x^8 + 453600*(b*x +
a)^n*a^3*b^8*d^3*n*x^8 - 63*(b*x + a)^n*a^2*b^9*c^3*n^8 + 27342*(b*x + a)^n
*a*b^10*c^3*n^7*x - 72*(b*x + a)^n*a^4*b^7*c^2*d*n^7*x + 299271*(b*x + a)^n
*b^11*c^3*n^6*x^2 + 40536*(b*x + a)^n*a^3*b^8*c^2*d*n^6*x^2 - 1351548*(b*x
+ a)^n*a^2*b^9*c^2*d*n^5*x^3 + 83160*(b*x + a)^n*a^5*b^6*c*d^2*n^5*x^3 + 80
83014*(b*x + a)^n*a*b^10*c^2*d*n^4*x^4 - 1965600*(b*x + a)^n*a^4*b^7*c*d^2*
n^4*x^4 + 151200*(b*x + a)^n*a^7*b^4*d^3*n^4*x^4 + 47746140*(b*x + a)^n*b^1
1*c^2*d*n^3*x^5 + 6340320*(b*x + a)^n*a^3*b^8*c*d^2*n^3*x^5 - 1058400*(b*x
+ a)^n*a^6*b^5*d^3*n^3*x^5 - 6449940*(b*x + a)^n*a^2*b^9*c*d^2*n^2*x^6 + 13
80960*(b*x + a)^n*a^5*b^6*d^3*n^2*x^6 + 2138400*(b*x + a)^n*a*b^10*c*d^2*n*
x^7 - 518400*(b*x + a)^n*a^4*b^7*d^3*n*x^7 + 14968800*(b*x + a)^n*b^11*c*d^
2*x^8 - 1734*(b*x + a)^n*a^2*b^9*c^3*n^7 + 271929*(b*x + a)^n*a*b^10*c^3*n^
6*x - 3672*(b*x + a)^n*a^4*b^7*c^2*d*n^6*x + 2039016*(b*x + a)^n*b^11*c^3*n
^5*x^2 + 470160*(b*x + a)^n*a^3*b^8*c^2*d*n^5*x^2 - 7560*(b*x + a)^n*a^6*b^
5*c*d^2*n^5*x^2 - 5910552*(b*x + a)^n*a^2*b^9*c^2*d*n^4*x^3 + 985320*(b*x +
 a)^n*a^5*b^6*c*d^2*n^4*x^3 + 15414084*(b*x + a)^n*a*b^10*c^2*d*n^3*x^4 - 5
927670*(b*x + a)^n*a^4*b^7*c*d^2*n^3*x^4 + 907200*(b*x + a)^n*a^7*b^4*d^3*n
^3*x^4 + 77043528*(b*x + a)^n*b^11*c^2*d*n^2*x^5 + 7141176*(b*x + a)^n*a^3*
b^8*c*d^2*n^2*x^5 - 1512000*(b*x + a)^n*a^6*b^5*d^3*n^2*x^5 - 2494800*(b*x
+ a)^n*a^2*b^9*c*d^2*n*x^6 + 604800*(b*x + a)^n*a^5*b^6*d^3*n*x^6 - 27342*(
b*x + a)^n*a^2*b^9*c^3*n^6 + 72*(b*x + a)^n*a^5*b^6*c^2*d*n^6 + 1767087*(b*
x + a)^n*a*b^10*c^3*n^5*x - 77400*(b*x + a)^n*a^4*b^7*c^2*d*n^5*x + 9261503
*(b*x + a)^n*b^11*c^3*n^4*x^2 + 3114324*(b*x + a)^n*a^3*b^8*c^2*d*n^4*x^2 -
 234360*(b*x + a)^n*a^6*b^5*c*d^2*n^4*x^2 - 14600400*(b*x + a)^n*a^2*b^9*c^
2*d*n^3*x^3 + 4906440*(b*x + a)^n*a^5*b^6*c*d^2*n^3*x^3 - 604800*(b*x + a)^
n*a^8*b^3*d^3*n^3*x^3 + 15387192*(b*x + a)^n*a*b^10*c^2*d*n^2*x^4 - 7990920
*(b*x + a)^n*a^4*b^7*c*d^2*n^2*x^4 + 1663200*(b*x + a)^n*a^7*b^4*d^3*n^2*x^
4 + 67536288*(b*x + a)^n*b^11*c^2*d*n*x^5 + 2993760*(b*x + a)^n*a^3*b^8*c*d
^2*n*x^5 - 725760*(b*x + a)^n*a^6*b^5*d^3*n*x^5 - 271929*(b*x + a)^n*a^2*b^
9*c^3*n^5 + 3672*(b*x + a)^n*a^5*b^6*c^2*d*n^5 + 7494416*(b*x + a)^n*a*b^10
*c^3*n^4*x - 862920*(b*x + a)^n*a^4*b^7*c^2*d*n^4*x + 15120*(b*x + a)^n*a^7
*b^4*c*d^2*n^4*x + 27472724*(b*x + a)^n*b^11*c^3*n^3*x^2 + 11503008*(b*x +
a)^n*a^3*b^8*c^2*d*n^3*x^2 - 2487240*(b*x + a)^n*a^6*b^5*c*d^2*n^3*x^2 - 17
855136*(b*x + a)^n*a^2*b^9*c^2*d*n^2*x^3 + 8991360*(b*x + a)^n*a^5*b^6*c*d^
2*n^2*x^3 - 1814400*(b*x + a)^n*a^8*b^3*d^3*n^2*x^3 + 5987520*(b*x + a)^n*a
*b^10*c^2*d*n*x^4 - 3742200*(b*x + a)^n*a^4*b^7*c*d^2*n*x^4 + 907200*(b*x +
 a)^n*a^7*b^4*d^3*n*x^4 + 23950080*(b*x + a)^n*b^11*c^2*d*x^5 - 1767087*(b*
x + a)^n*a^2*b^9*c^3*n^4 + 77400*(b*x + a)^n*a^5*b^6*c^2*d*n^4 + 19978308*(
b*x + a)^n*a*b^10*c^3*n^3*x - 5365728*(b*x + a)^n*a^4*b^7*c^2*d*n^3*x + 453
600*(b*x + a)^n*a^7*b^4*c*d^2*n^3*x + 50312628*(b*x + a)^n*b^11*c^3*n^2*x^2
 + 20795184*(b*x + a)^n*a^3*b^8*c^2*d*n^2*x^2 - 9744840*(b*x + a)^n*a^6*b^5
*c*d^2*n^2*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n^2*x^2 - 7983360*(b*x + a
)^n*a^2*b^9*c^2*d*n*x^3 + 4989600*(b*x + a)^n*a^5*b^6*c*d^2*n*x^3 - 1209600
*(b*x + a)^n*a^8*b^3*d^3*n*x^3 - 7494416*(b*x + a)^n*a^2*b^9*c^3*n^3 + 8629
20*(b*x + a)^n*a^5*b^6*c^2*d*n^3 - 15120*(b*x + a)^n*a^8*b^3*c*d^2*n^3 + 30
334320*(b*x + a)^n*a*b^10*c^3*n^2*x - 17640288*(b*x + a)^n*a^4*b^7*c^2*d*n^
2*x + 4520880*(b*x + a)^n*a^7*b^4*c*d^2*n^2*x + 50292720*(b*x + a)^n*b^11*c
^3*n*x^2 + 11975040*(b*x + a)^n*a^3*b^8*c^2*d*n*x^2 - 7484400*(b*x + a)^n*a
^6*b^5*c*d^2*n*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n*x^2 - 19978308*(b*x
+ a)^n*a^2*b^9*c^3*n^2 + 5365728*(b*x + a)^n*a^5*b^6*c^2*d*n^2 - 453600*(b*
x + a)^n*a^8*b^3*c*d^2*n^2 + 19958400*(b*x + a)^n*a*b^10*c^3*n*x - 23950080
*(b*x + a)^n*a^4*b^7*c^2*d*n*x + 14968800*(b*x + a)^n*a^7*b^4*c*d^2*n*x - 3
628800*(b*x + a)^n*a^10*b*d^3*n*x + 19958400*(b*x + a)^n*b^11*c^3*x^2 - 303
34320*(b*x + a)^n*a^2*b^9*c^3*n + 17640288*(b*x + a)^n*a^5*b^6*c^2*d*n - 45
20880*(b*x + a)^n*a^8*b^3*c*d^2*n - 19958400*(b*x + a)^n*a^2*b^9*c^3 + 2395
0080*(b*x + a)^n*a^5*b^6*c^2*d - 14968800*(b*x + a)^n*a^8*b^3*c*d^2 + 36288
00*(b*x + a)^n*a^11*d^3)/(b^11*n^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*
b^11*n^8 + 357423*b^11*n^7 + 2637558*b^11*n^6 + 13339535*b^11*n^5 + 4599573
```

```
0*b^11*n^4 + 105258076*b^11*n^3 + 150917976*b^11*n^2 + 120543840*b^11*n + 3
9916800*b^11)
```

3.184 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=337

$$\frac{3d\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\cdots$$

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^
3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c
- a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d
 + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14
*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a
+ b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n
))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^
3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 +
n))
```

Rubi [A] time = 0.208595, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1850}

$$\frac{3d\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^
3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c
- a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d
 + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14
*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a
+ b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n
))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^
3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 +
n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \int \left(\frac{\left(b^3c-a^3d\right)^3 (a+bx)^n}{b^9}+\frac{9d\left(ab^3c-a^4d\right)^2 (a+bx)^{1+n}}{b^9}+\frac{9ad\left(b^3c-4a^3d\right)\left(-b^3c+a^3d\right)\cdots}{b^9}\cdots\right.$$

$$=\frac{\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{10}(1+n)}+\frac{9a^2d\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{10}(2+n)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)\cdots}{b^{10}(3+n)}$$

Mathematica [A] time = 0.358814, size = 290, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{3d(a+bx)^3\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)}{n+4}+\frac{3d^2(a+bx)^6\left(b^3c-28a^3d\right)}{n+7}+\frac{18ad^2(a+bx)^5\left(7a^3d-b^3c\right)}{n+6}+\frac{9a^2d^2(a+bx)^4\left(5b^3c-14a^3d\right)}{n+5}-\frac{9ad(a+bx)^2(\cdots}{}\right.}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(a
 + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/(3
 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n) +
 (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^3*c)
 + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^6)/(
7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9 + n) +
 (d^3*(a + b*x)^9)/(10 + n)))/b^10
```

Maple [B] time = 0.018, size = 2280, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^9*d^3*n^9*x^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870
*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6
*x^9-72*a^2*b^7*d^3*n^7*x^7+4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63
273*b^9*d^3*n^5*x^9-2016*a^2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a
*b^8*d^3*n^5*x^8-2952*b^9*c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*
d^3*n^6*x^6-23184*a^2*b^7*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*
d^3*n^4*x^8-3*b^9*c^2*d*n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*
x^9+10584*a^3*b^6*d^3*n^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n
^4*x^7+13176*a*b^8*c*d^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8
*x^3-236817*b^9*c*d^2*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*
x^5+88200*a^3*b^6*d^3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3
*n^3*x^7+9*a*b^8*c^2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3
*n^2*x^8-3348*b^9*c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n
*x^9-45360*a^4*b^5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3
*n^3*x^6-49230*a^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c
^2*d*n^7*x^2+678942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-
41058*b^9*c^2*d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*
a^5*b^4*d^3*n^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+
818496*a^3*b^6*d^3*n^2*x^6-18*a^2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*
x^4-940896*a^2*b^7*d^3*n*x^7+8748*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*d^2*n
^3*x^5+362880*a*b^8*d^3*x^8-54*b^9*c^3*n^8-309087*b^9*c^2*d*n^5*x^3-4846824
*b^9*c*d^2*n^2*x^6+151200*a^5*b^4*d^3*n^3*x^4-1080*a^4*b^5*c*d^2*n^5*x^2-68
0400*a^4*b^5*d^3*n^2*x^5+149400*a^3*b^6*c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*
x^6-828*a^2*b^7*c^2*d*n^6*x-1533960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^
3*x^7+96930*a*b^8*c^2*d*n^5*x^2+4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^
7-1469817*b^9*c^2*d*n^4*x^3-4332960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x
^3+529200*a^5*b^4*d^3*n^2*x^4-32400*a^4*b^5*c*d^2*n^4*x^2-828576*a^4*b^5*d^
3*n*x^5+18*a^3*b^6*c^2*d*n^6+891000*a^3*b^6*c*d^2*n^3*x^3+362880*a^3*b^6*d^
3*x^6-15840*a^2*b^7*c^2*d*n^5*x-3415320*a^2*b^7*c*d^2*n^2*x^4+636471*a*b^8*
c^2*d*n^4*x^2+4073760*a*b^8*c*d^2*n*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d
```

```
*n^3*x^3-1555200*b^9*c*d^2*x^6-362880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^
2*n^4*x+756000*a^5*b^4*d^3*n*x^4-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^
5*d^3*x^5+810*a^3*b^6*c^2*d*n^5+2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a^2*b^
7*c^2*d*n^4*x-3762720*a^2*b^7*c*d^2*n*x^4+2500038*a*b^8*c^2*d*n^3*x^2+15552
00*a*b^8*c*d^2*x^5-140889*b^9*c^3*n^5-7742412*b^9*c^2*d*n^2*x^3+181440*a^7*
b^2*d^3*n^2*x^2-665280*a^6*b^3*d^3*n*x^3+60480*a^5*b^4*c*d^2*n^3*x+362880*a
^5*b^4*d^3*x^4-1620000*a^4*b^5*c*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+337392
0*a^3*b^6*c*d^2*n*x^3-948582*a^2*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+
5614452*a*b^8*c^2*d*n^2*x^2-761166*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+5443
20*a^7*b^2*d^3*n*x^2-2160*a^6*b^3*c*d^2*n^3-362880*a^6*b^3*d^3*x^3+581040*a
^5*b^4*c*d^2*n^2*x-2855520*a^4*b^5*c*d^2*n*x^2+147150*a^3*b^6*c^2*d*n^3+155
5200*a^3*b^6*c*d^2*x^3-3102912*a^2*b^7*c^2*d*n^2*x+6383880*a*b^8*c^2*d*n*x^
2-2655764*b^9*c^3*n^3-2721600*b^9*c^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7
*b^2*d^3*x^2-58320*a^6*b^3*c*d^2*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*
b^5*c*d^2*x^2+801432*a^3*b^6*c^2*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*
b^8*c^2*d*x^2-5753736*b^9*c^3*n^2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*d^2*n
+1555200*a^5*b^4*c*d^2*x+2301480*a^3*b^6*c^2*d*n-2721600*a^2*b^7*c^2*d*x-69
99840*b^9*c^3*n+362880*a^9*d^3-1555200*a^6*b^3*c*d^2+2721600*a^3*b^6*c^2*d-
3628800*b^9*c^3)/b^10/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^5
+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.920244, size = 5404, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3
- 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10*d
^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*b^1
0*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^
2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*
d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67
284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40320*a*b
^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b^8*d^3*n
^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d^3*n^3 +
13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^
10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6
*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7
*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d
^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d^3)*n^2 +
1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b^9*c^3 - a^4*b^
6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 732*a*b^9*c*d^2*n^
```

```
7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a
^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*
b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)
*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c^3 -
270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*c*d^2*n^7 + 547
*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5*b^5*d^3)*n^5 + 4*(4261*a
^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*
d^3)*n^3 + 48*(871*a^2*b^8*c*d^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c
*d^2 - 7*a^5*b^5*d^3)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4
 + 3*(b^10*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*
c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c*d^2)*n^6
+ 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(163313*b^10*c^2*d + 24
750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2*(728587*b^10*c^2*d + 107160*a
^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7
*c*d^2 - 1540*a^6*b^4*d^3)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 -
 84*a^6*b^4*d^3)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080
*a^7*b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*b^9*c
^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(23573*a*b^9*c^2*
d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d - 6500*a^4*b^6*c*d^2)*n^
4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 1
20*(5911*a*b^9*c^2*d - 2644*a^4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(10
5*a*b^9*c^2*d - 60*a^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9
*c^3 - 11131*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8
+ 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*c^2*d - 12
*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b^5*c*d^2)*n^4 + 8*(2
1548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3 + 60*(4651*a^2*b^8*c^2*d - 192
4*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b
^5*c*d^2 + 14*a^8*b^2*d^3)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2
*d + 1452*a^7*b^3*c*d^2)*n + (b^10*c^3*n^9 + 54*b^10*c^3*n^8 + 3628800*b^10
*c^3 + 6*(211*b^10*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^10*c^3 + 45*a^3*b
^7*c^2*d)*n^6 + 3*(46963*b^10*c^3 + 5010*a^3*b^7*c^2*d)*n^5 + 18*(42287*b^1
0*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^4 + 4*(663941*b^10*c^3 +
200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d^2)*n^3 + 72*(79913*b^10*c^3 + 3196
5*a^3*b^7*c^2*d - 7260*a^6*b^4*c*d^2)*n^2 + 1440*(4861*b^10*c^3 + 1890*a^3*
b^7*c^2*d - 1080*a^6*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x)*(b*x + a)^n/(b^10*n^1
0 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055
*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628
640*b^10*n + 3628800*b^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37085, size = 5230, normalized size = 15.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)^n
*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^2*b
^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b^10*
d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^10*c*d
^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a*b^9*d
^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a*b^9*c*
d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*a^3*b^7*
d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x + a)^n*a*b^
9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a)^n*b^10*c
^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x + a)^n*a*b^
9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(b*x + a)^n*a
^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 67284*(b*x + a
)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10 + 3*(b*x + a)^
n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 - 666*(b*x + a)^
n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6*x^6 - 504*(b*x
+ a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2*n^5*x^7 + 12600*
(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b^8*d^3*n^4*x^8 + 1
18124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^n*b^10*d^3*n^2*x^10
 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x + a)^n*b^10*c^2*d*n^7*x^
4 + 90*(b*x + a)^n*a^3*b^7*c*d^2*n^7*x^4 - 9846*(b*x + a)^n*a^2*b^8*c*d^2*n
^6*x^5 + 113157*(b*x + a)^n*a*b^9*c*d^2*n^5*x^6 - 7560*(b*x + a)^n*a^4*b^6*
d^3*n^5*x^6 + 1048446*(b*x + a)^n*b^10*c*d^2*n^4*x^7 + 52920*(b*x + a)^n*a^
3*b^7*d^3*n^4*x^7 - 118188*(b*x + a)^n*a^2*b^8*d^3*n^3*x^8 + 109584*(b*x +
a)^n*a*b^9*d^3*n^2*x^9 + 1026576*(b*x + a)^n*b^10*d^3*n*x^10 + (b*x + a)^n*
b^10*c^3*n^9*x - 9*(b*x + a)^n*a^2*b^8*c^2*d*n^8*x^2 + 2916*(b*x + a)^n*a*b
^9*c^2*d*n^7*x^3 + 41058*(b*x + a)^n*b^10*c^2*d*n^6*x^4 + 2970*(b*x + a)^n*
a^3*b^7*c*d^2*n^6*x^4 - 74430*(b*x + a)^n*a^2*b^8*c*d^2*n^5*x^5 + 3024*(b*x
 + a)^n*a^5*b^5*d^3*n^5*x^5 + 369504*(b*x + a)^n*a*b^9*c*d^2*n^4*x^6 - 4284
0*(b*x + a)^n*a^4*b^6*d^3*n^4*x^6 + 2911668*(b*x + a)^n*b^10*c*d^2*n^3*x^7
+ 116928*(b*x + a)^n*a^3*b^7*d^3*n^3*x^7 - 117612*(b*x + a)^n*a^2*b^8*d^3*n
^2*x^8 + 40320*(b*x + a)^n*a*b^9*d^3*n*x^9 + 362880*(b*x + a)^n*b^10*d^3*x^
10 + (b*x + a)^n*a*b^9*c^3*n^9 + 54*(b*x + a)^n*b^10*c^3*n^8*x - 414*(b*x +
 a)^n*a^2*b^8*c^2*d*n^7*x^2 + 32310*(b*x + a)^n*a*b^9*c^2*d*n^6*x^3 - 360*(
b*x + a)^n*a^4*b^6*c*d^2*n^6*x^3 + 309087*(b*x + a)^n*b^10*c^2*d*n^5*x^4 +
37350*(b*x + a)^n*a^3*b^7*c*d^2*n^5*x^4 - 306792*(b*x + a)^n*a^2*b^8*c*d^2*
n^4*x^5 + 30240*(b*x + a)^n*a^5*b^5*d^3*n^4*x^5 + 694644*(b*x + a)^n*a*b^9*
c*d^2*n^3*x^6 - 113400*(b*x + a)^n*a^4*b^6*d^3*n^3*x^6 + 4846824*(b*x + a)^
n*b^10*c*d^2*n^2*x^7 + 127008*(b*x + a)^n*a^3*b^7*d^3*n^2*x^7 - 45360*(b*x
+ a)^n*a^2*b^8*d^3*n*x^8 + 54*(b*x + a)^n*a*b^9*c^3*n^8 + 1266*(b*x + a)^n*
b^10*c^3*n^7*x + 18*(b*x + a)^n*a^3*b^7*c^2*d*n^7*x - 7920*(b*x + a)^n*a^2*
b^8*c^2*d*n^6*x^2 + 212157*(b*x + a)^n*a*b^9*c^2*d*n^5*x^3 - 10800*(b*x + a
)^n*a^4*b^6*c*d^2*n^5*x^3 + 1469817*(b*x + a)^n*b^10*c^2*d*n^4*x^4 + 222750
*(b*x + a)^n*a^3*b^7*c*d^2*n^4*x^4 - 15120*(b*x + a)^n*a^6*b^4*d^3*n^4*x^4
- 683064*(b*x + a)^n*a^2*b^8*c*d^2*n^3*x^5 + 105840*(b*x + a)^n*a^5*b^5*d^3
*n^3*x^5 + 678960*(b*x + a)^n*a*b^9*c*d^2*n^2*x^6 - 138096*(b*x + a)^n*a^4*
b^6*d^3*n^2*x^6 + 4332960*(b*x + a)^n*b^10*c*d^2*n*x^7 + 51840*(b*x + a)^n*
a^3*b^7*d^3*n*x^7 + 1266*(b*x + a)^n*a*b^9*c^3*n^7 + 16884*(b*x + a)^n*b^10
*c^3*n^6*x + 810*(b*x + a)^n*a^3*b^7*c^2*d*n^6*x - 81090*(b*x + a)^n*a^2*b^
8*c^2*d*n^5*x^2 + 1080*(b*x + a)^n*a^5*b^5*c*d^2*n^5*x^2 + 833346*(b*x + a)
^n*a*b^9*c^2*d*n^4*x^3 - 117000*(b*x + a)^n*a^4*b^6*c*d^2*n^4*x^3 + 4371522
*(b*x + a)^n*b^10*c^2*d*n^3*x^4 + 642960*(b*x + a)^n*a^3*b^7*c*d^2*n^3*x^4
- 90720*(b*x + a)^n*a^6*b^4*d^3*n^3*x^4 - 752544*(b*x + a)^n*a^2*b^8*c*d^2*
n^2*x^5 + 151200*(b*x + a)^n*a^5*b^5*d^3*n^2*x^5 + 259200*(b*x + a)^n*a*b^9
*c*d^2*n*x^6 - 60480*(b*x + a)^n*a^4*b^6*d^3*n*x^6 + 1555200*(b*x + a)^n*b^
10*c*d^2*x^7 + 16884*(b*x + a)^n*a*b^9*c^3*n^6 - 18*(b*x + a)^n*a^4*b^6*c^2
*d*n^6 + 140889*(b*x + a)^n*b^10*c^3*n^5*x + 15030*(b*x + a)^n*a^3*b^7*c^2*
```

d*n^5*x - 474291*(b*x + a)^n*a^2*b^8*c^2*d*n^4*x^2 + 30240*(b*x + a)^n*a^5*b^5*c*d^2*n^4*x^2 + 1871484*(b*x + a)^n*a*b^9*c^2*d*n^3*x^3 - 540000*(b*x + a)^n*a^4*b^6*c*d^2*n^3*x^3 + 60480*(b*x + a)^n*a^7*b^3*d^3*n^3*x^3 + 7742412*(b*x + a)^n*b^10*c^2*d*n^2*x^4 + 843480*(b*x + a)^n*a^3*b^7*c*d^2*n^2*x^4 - 166320*(b*x + a)^n*a^6*b^4*d^3*n^2*x^4 - 311040*(b*x + a)^n*a^2*b^8*c*d^2*n*x^5 + 72576*(b*x + a)^n*a^5*b^5*d^3*n*x^5 + 140889*(b*x + a)^n*a*b^9*c^3*n^5 - 810*(b*x + a)^n*a^4*b^6*c^2*d*n^5 + 761166*(b*x + a)^n*b^10*c^3*n^4*x + 147150*(b*x + a)^n*a^3*b^7*c^2*d*n^4*x - 2160*(b*x + a)^n*a^6*b^4*c*d^2*n^4*x - 1551456*(b*x + a)^n*a^2*b^8*c^2*d*n^3*x^2 + 290520*(b*x + a)^n*a^5*b^5*c*d^2*n^3*x^2 + 2127960*(b*x + a)^n*a*b^9*c^2*d*n^2*x^3 - 951840*(b*x + a)^n*a^4*b^6*c*d^2*n^2*x^3 + 181440*(b*x + a)^n*a^7*b^3*d^3*n^2*x^3 + 7291080*(b*x + a)^n*b^10*c^2*d*n*x^4 + 388800*(b*x + a)^n*a^3*b^7*c*d^2*n*x^4 - 90720*(b*x + a)^n*a^6*b^4*d^3*n*x^4 + 761166*(b*x + a)^n*a*b^9*c^3*n^4 - 15030*(b*x + a)^n*a^4*b^6*c^2*d*n^4 + 2655764*(b*x + a)^n*b^10*c^3*n^3*x + 801432*(b*x + a)^n*a^3*b^7*c^2*d*n^3*x - 58320*(b*x + a)^n*a^6*b^4*c*d^2*n^3*x - 2511540*(b*x + a)^n*a^2*b^8*c^2*d*n^2*x^2 + 1038960*(b*x + a)^n*a^5*b^5*c*d^2*n^2*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n^2*x^2 + 907200*(b*x + a)^n*a*b^9*c^2*d*n*x^3 - 518400*(b*x + a)^n*a^4*b^6*c*d^2*n*x^3 + 120960*(b*x + a)^n*a^7*b^3*d^3*n*x^3 + 2721600*(b*x + a)^n*b^10*c^2*d*x^4 + 2655764*(b*x + a)^n*a*b^9*c^3*n^3 - 147150*(b*x + a)^n*a^4*b^6*c^2*d*n^3 + 2160*(b*x + a)^n*a^7*b^3*c*d^2*n^3 + 5753736*(b*x + a)^n*b^10*c^3*n^2*x + 2301480*(b*x + a)^n*a^3*b^7*c^2*d*n^2*x - 522720*(b*x + a)^n*a^6*b^4*c*d^2*n^2*x - 1360800*(b*x + a)^n*a^2*b^8*c^2*d*n*x^2 + 777600*(b*x + a)^n*a^5*b^5*c*d^2*n*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n*x^2 + 5753736*(b*x + a)^n*a*b^9*c^3*n^2 - 801432*(b*x + a)^n*a^4*b^6*c^2*d*n^2 + 58320*(b*x + a)^n*a^7*b^3*c*d^2*n^2 + 6999840*(b*x + a)^n*b^10*c^3*n*x + 2721600*(b*x + a)^n*a^3*b^7*c^2*d*n*x - 1555200*(b*x + a)^n*a^6*b^4*c*d^2*n*x + 362880*(b*x + a)^n*a^9*b*d^3*n*x + 6999840*(b*x + a)^n*a*b^9*c^3*n - 2301480*(b*x + a)^n*a^4*b^6*c^2*d*n + 522720*(b*x + a)^n*a^7*b^3*c*d^2*n + 3628800*(b*x + a)^n*b^10*c^3*x + 3628800*(b*x + a)^n*a*b^9*c^3 - 2721600*(b*x + a)^n*a^4*b^6*c^2*d + 1555200*(b*x + a)^n*a^7*b^3*c*d^2 - 362880*(b*x + a)^n*a^10*d^3)/(b^10*n^10 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628640*b^10*n + 3628800*b^10)

3.185 $\int \frac{(a+bx)^n\left(c+dx^3\right)^3}{x}\,dx$

Optimal. Leaf size=358

$$\frac{a^2d\left(-3a^3b^3cd+a^6d^2+3b^6c^2\right)(a+bx)^{n+1}}{b^9(n+1)}-\frac{ad\left(-15a^3b^3cd+8a^6d^2+6b^6c^2\right)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d\left(-30a^3b^3cd+28a^6d^2+\right.}{b^9(n+}$$

```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 + n
)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^9*
(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/
(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4
 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (
d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a
+ b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 + n)) +
 (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rubi [A] time = 0.224214, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1620, 65}

$$\frac{a^2d\left(-3a^3b^3cd+a^6d^2+3b^6c^2\right)(a+bx)^{n+1}}{b^9(n+1)}-\frac{ad\left(-15a^3b^3cd+8a^6d^2+6b^6c^2\right)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d\left(-30a^3b^3cd+28a^6d^2+\right.}{b^9(n+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```


```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 + n
)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^9*
(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/
(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4
 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (
d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a
+ b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 + n)) +
 (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```


Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^3}{x} dx = \int \left(\frac{a^2d\left(3b^6c^2-3a^3b^3cd+a^6d^2\right)(a+bx)^n}{b^8} + \frac{c^3(a+bx)^n}{x} - \frac{ad\left(6b^6c^2-15a^3b^3cd+8a^6d^2\right)(}{b^8}\right.$$
$$= \frac{a^2d\left(3b^6c^2-3a^3b^3cd+a^6d^2\right)(a+bx)^{1+n}}{b^9(1+n)} - \frac{ad\left(6b^6c^2-15a^3b^3cd+8a^6d^2\right)(a+bx)^{2+n}}{b^9(2+n)} + \frac{d(}{}$$
$$= \frac{a^2d\left(3b^6c^2-3a^3b^3cd+a^6d^2\right)(a+bx)^{1+n}}{b^9(1+n)} - \frac{ad\left(6b^6c^2-15a^3b^3cd+8a^6d^2\right)(a+bx)^{2+n}}{b^9(2+n)} + \frac{d}{}$$

Mathematica [A] time = 0.345472, size = 332, normalized size = 0.93

$$(a+bx)^{n+1}\left(\frac{d(a+bx)^2\left(-30a^3b^3cd+28a^6d^2+3b^6c^2\right)}{b^9(n+3)} - \frac{ad(a+bx)\left(-15a^3b^3cd+8a^6d^2+6b^6c^2\right)}{b^9(n+2)} + \frac{a^2d\left(-3a^3b^3cd+a^6d\right.}{b^9(n+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*((a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2))/(b^9*(1 +
n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x))/(b^9*(2 + n)
) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(b^9*(3 + n))
 + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^3)/(b^9*(4 + n)) + (5*a*d^2*(
-3*b^3*c + 14*a^3*d)*(a + b*x)^4)/(b^9*(5 + n)) + (d^2*(3*b^3*c - 56*a^3*d)
*(a + b*x)^5)/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x)^6)/(b^9*(7 + n)) - (8*a
*d^3*(a + b*x)^7)/(b^9*(8 + n)) + (d^3*(a + b*x)^8)/(b^9*(9 + n)) - (c^3*Hy
pergeometric2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n \left(dx^3+c\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^3 (bx+a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^{3}x^{9}+3\,cd^{2}x^{6}+3\,c^{2}dx^{3}+c^{3}\right)(bx+a)^{n}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dx^{3}+c\right)^{3}(bx+a)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

3.186 $\int \frac{x^5(e+fx)^n}{a+bx^3} \, dx$

Optimal. Leaf size=324

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;\right.}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e\right.}$$

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(2
 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*
b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1
+ n))
```

Rubi [A] time = 0.864549, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6725, 68}

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;\right.}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```


```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(2
 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*
b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1
+ n))
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{e^2(e+fx)^n}{bf^2} - \frac{2e(e+fx)^{1+n}}{bf^2} + \frac{(e+fx)^{2+n}}{bf^2} - \frac{ax^2(e+fx)^n}{b\left(a+bx^3\right)}\right) dx$$
$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{b}$$
$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e\cdots}{3b^{2/3}\left((-1)\cdots\right.}\right.}{b}$$
$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{(e\cdots}{(-1)^{2/3}\cdots}}{3b\cdots}$$
$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} + \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{a(e\cdots}{\cdots}$$

Mathematica [A] time = 0.599307, size = 284, normalized size = 0.88

$$\frac{(e+fx)^{n+1}\left(\frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)} + \frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)} + \frac{3b^{2/3}e^2}{f^3(n+1)} - \frac{6b^{2/3}e(e+fx)}{f^3(n+2)} + \cdots\right)}{3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((3*b^(2/3)*e^2)/(f^3*(1 + n)) - (6*b^(2/3)*e*(e + f*x))
/(f^3*(2 + n)) + (3*b^(2/3)*(e + f*x)^2)/(f^3*(3 + n)) + (a*Hypergeometric2
F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)
*e - a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/((b^(1/3)*e + (-1)^(1/3)*a^(
1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/((b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*
(1 + n))))/(3*b^(5/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{x^5\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^5}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^5}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^5}{bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

3.187 $\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=332

$$-\frac{a^{2/3}(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}a^{2/3}}{}$$

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n)
) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(
3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)
*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/3)*b^(1/
3)*e + a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.862376, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6725, 68}

$$-\frac{a^{2/3}(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}a^{2/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n)
) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(
3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)
*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/3)*b^(1/
3)*e + a^(1/3)*f)*(1 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{e(e+fx)^n}{bf} + \frac{(e+fx)^{1+n}}{bf} - \frac{ax(e+fx)^n}{b\left(a+bx^3\right)}\right) dx$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) d}{b}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} + \frac{a^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\right)\int \frac{}{3}}{3b^{4/3}}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{1+n}\,{}_2}{3b^{4/3}\left((-1\right.}$$

Mathematica [A] time = 0.68365, size = 292, normalized size = 0.88

$$\frac{(e+fx)^{n+1}\left(-\frac{a^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{\sqrt[3]{-1}a^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{(-1)^{2/3}a^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)} + \frac{3\sqrt[3]{b}(e+fx)}{f^2(n+2)} - \right.}{3b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((-3*b^(1/3)*e)/(f^2*(1 + n)) + (3*b^(1/3)*(e + f*x))/(f
^2*(2 + n)) - (a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3
)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))
/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(
1 + n)) + ((-1)^(2/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3
)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(((-1)^(1/3)*b^(1
/3)*e + a^(1/3)*f)*(1 + n))))/(3*b^(4/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^4\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^4}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x^{4}}{bx^{3}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x^{4}}{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

3.188 $\int \frac{x^3(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=293

$$\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1\right.}{3b(n+1)\left(\right.}$$

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a
^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e + a^(
1/3)*f)*(1 + n))
```

Rubi [A] time = 0.476178, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6725, 68}

$$\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1\right.}{3b(n+1)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a
^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e + a^(
1/3)*f)*(1 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{(e+fx)^n}{b} - \frac{a(e+fx)^n}{b\left(a+bx^3\right)}\right)dx$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \frac{(e+fx)^n}{a+bx^3}\,dx}{b}$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b}$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{3b\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1}$$

Mathematica [A] time = 0.235932, size = 239, normalized size = 0.82

$$\frac{(e+fx)^{n+1}\left(\frac{\sqrt[3]{a}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\sqrt[3]{a}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\sqrt[3]{a}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e} + \frac{3}{f}\right)}{3b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(3/f + (a^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1
/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (a^(1/3)
*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2
/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - (a^(1/3)*
Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*b*(1 +
n))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^3\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^3}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

3.189 $\int \frac{x^2(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=253

$$-\frac{(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}{}_2F_1\left(1,n+1\right.}{3b^{2/3}(n+1)\left(\sqrt[3]{be}\right.}$$

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(
1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e -
(-1)^(2/3)*a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.278007, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6725, 68}

$$-\frac{(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}{}_2F_1\left(1,n+1\right.}{3b^{2/3}(n+1)\left(\sqrt[3]{be}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```


```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(
1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e -
(-1)^(2/3)*a^(1/3)*f)*(1 + n))
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)} \right) dx$$
$$= \frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$
$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} - \frac{(e+f\dots}{}$$

Mathematica [A] time = 0.149467, size = 213, normalized size = 0.84

$$\frac{(e+fx)^{n+1}\left(-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)]/(b^(1
/3)*e + (-1)^(1/3)*a^(1/3)*f) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)]/(b^(1/3)*e - (-1)^(2/3)*a^(
1/3)*f)))/(3*b^(2/3)*(1 + n))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^2\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x^{2}}{bx^{3}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x^{2}}{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

3.190 $\int \frac{x(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=288

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\,{}_2F_1(\ldots}{3\sqrt[3]{a}\sqrt[3]{b}(n+1\ldots}$$

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)
) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^
(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^
(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1/3)*e
 + a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.28337, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6725, 68}

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\,{}_2F_1(\ldots}{3\sqrt[3]{a}\sqrt[3]{b}(n+1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)
) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^
(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^
(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1/3)*e
 + a^(1/3)*f)*(1 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$
$$= -\frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$
$$= \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{-1}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} -$$

Mathematica [A] time = 0.157851, size = 237, normalized size = 0.82

$$\frac{(e+fx)^{n+1}\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\sqrt[3]{-1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{(-1)^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f) - ((-1)^(1/3)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e
 - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - ((-1)^(2/3)*Hypergeome
tric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)
*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(1/3)*b^(1/3)*(
1 + n))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x}{bx^{3}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x}{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

3.191 $\int \frac{(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=263

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+\right.}{3a^{2/3}(n+1)\left(\sqrt[3]{}\right.}$$

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e +
 f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3)*
e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3
*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.158192, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6725, 68}

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+\right.}{3a^{2/3}(n+1)\left(\sqrt[3]{}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e +
 f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3)*
e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3
*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$
$$= -\frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$
$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(e+fx}{}$$

Mathematica [A] time = 0.116736, size = 222, normalized size = 0.84

$$\frac{(e+fx)^{n+1}\left(-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3a^{2/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3
)*f)]/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)]/((-1)
^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(2/3)*(1 + n))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}}{bx^{3}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

3.192 $\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,\right.}{3a(n+1)\left(\sqrt[3]{\right.}}$$

```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e +
 f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (b
^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)
*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rubi [A] time = 0.559143, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6725, 65, 68}

$$\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,\right.}{3a(n+1)\left(\sqrt[3]{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e +
 f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (b
^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)
*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))]])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x\left(a+bx^3\right)}dx = \int \left(\frac{(e+fx)^n}{ax} - \frac{bx^2(e+fx)^n}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{(e+fx)^n}{x}dx}{a} - \frac{b\int \frac{x^2(e+fx)^n}{a+bx^3}dx}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{b\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}\right.}\right.}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a} - \frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a} - \frac{\sqrt[3]{b}\int \frac{(\cdots}{(-1)^{2}\cdots}}{3\cdots}$$

$$= \frac{\sqrt[3]{b}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{b}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} + \cdots$$

Mathematica [A] time = 0.246914, size = 244, normalized size = 0.81

$$\frac{(e+fx)^{n+1}\left(\frac{\sqrt[3]{b}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\sqrt[3]{b}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e} + \frac{\sqrt[3]{b}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f} - \frac{3\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{e}\right.}{3a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((b^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
 + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (b^(1/3)*Hyper
geometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a
^(1/3)*f)])/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) + (b^(1/3)*Hypergeometric2F1
[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/
(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f) - (3*Hypergeometric2F1[1, 1 + n, 2 + n,
1 + (f*x)/e])/e))/(3*a*(1 + n))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{x\left(bx^{3}+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(fx+e)^n}{bx^4+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^4 + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

3.193 $\int \frac{(e+fx)^n}{x^2\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=326

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}b^{2/3}}{}$$

```
[Out] -(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n
)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a
^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
 f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(1/3)*
e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[2, 1 + n,
2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rubi [A] time = 0.611083, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6725, 65, 68}

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}b^{2/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```


```
[Out] -(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n
)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a
^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
 f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(1/3)*
e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[2, 1 + n,
2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \left(\frac{(e+fx)^n}{ax^2} - \frac{bx(e+fx)^n}{a(a+bx^3)}\right)dx$$

$$= \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a} - \frac{b\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{a}$$

$$= \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} - \frac{b\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\right.}\right.}{a}$$

$$= \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} + \frac{b^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{((-}{}$$

$$= -\frac{b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+}{(-1)^{2/3}\sqrt[3]{b}e-}\right.}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] time = 0.264991, size = 273, normalized size = 0.84

$$\frac{(e+fx)^{n+1}\left(-\frac{b^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\sqrt[3]{-1}b^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{(-1)^{2/3}b^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e} + \frac{3\sqrt[3]{a}f\,{}_2F_1(2,}{}\right.}{3a^{4/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f)) + ((-1)^(1/3)
*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/
((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + (
(-1)^(2/3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1
/3)*f) + (3*a^(1/3)*f*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/e^2)
)/(3*a^(4/3)*(1 + n))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{x^2(bx^3+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{bx^5+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

3.194 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3} dx$

Optimal. Leaf size=253

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n\right.}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\right.}$$

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/
(b^(1/3)*c - a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) - ((c
 + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(
1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/3)
*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/
3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c -
(-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

Rubi [A] time = 0.585119, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6725, 68}

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n\right.}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/
(b^(1/3)*c - a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) - ((c
 + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(
1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/3)
*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/
3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c -
(-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \left(\frac{(c+dx)^{1+n}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)} + \frac{(c+dx)^{1+n}}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx}\right)} + \frac{(c+dx)^{1+n}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{bx}\right)} \right) dx$$
$$= \frac{\int \frac{(c+dx)^{1+n}}{\sqrt[3]{a}+\sqrt[3]{bx}}\,dx}{3b^{2/3}} + \frac{\int \frac{(c+dx)^{1+n}}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx}}\,dx}{3b^{2/3}} + \frac{\int \frac{(c+dx)^{1+n}}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{bx}}\,dx}{3b^{2/3}}$$
$$= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d\right)(2+n)} - \ldots$$

Mathematica [A] time = 0.37598, size = 213, normalized size = 0.84

$$\frac{(c+dx)^{n+2}\left(-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d} - \frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c} - \frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x)
)/(b^(1/3)*c - a^(1/3)*d)]/(b^(1/3)*c - a^(1/3)*d)) - Hypergeometric2F1[1,
2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(b^(1
/3)*c + (-1)^(1/3)*a^(1/3)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)
*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(b^(1/3)*c - (-1)^(2/3)*a^(
1/3)*d)))/(3*b^(2/3)*(2 + n))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^2\left(dx+c\right)^{1+n}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

```
[Out] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx+c\right)^{n+1}x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{n+1}x^2}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

3.195 $\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=211

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e}\right.}{3a(m+1)}$$

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3)
*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*Appel
lF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3*a*
(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
+ m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x
)/e)^n)
```

Rubi [A] time = 0.462599, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6725, 135, 133}

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e}\right.}{3a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3)
*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*Appel
lF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3*a*
(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
+ m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x
)/e)^n)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)} \right) dx$$

$$= -\frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= -\frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\left((e+fx)^n\left(1\right.\right.}{}$$

$$= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)} + \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m\right.}{3a(1+m}$$

Mathematica [F] time = 0.168223, size = 0, normalized size = 0.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^m\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^m}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

3.196 $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

Optimal. Leaf size=1482

result too large to display

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*d^(1/3)]*Sqrt[b^2*c^
(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2
/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3
)*d^(1/3) + a^2*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c^(1/3
) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*Sqrt[(c^(1/3)*(c^(1/3) +
d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2 + Sqrt[
3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*
d^(1/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2
/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticPi[((1 +
Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2,
-ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d
^(2/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 2.81458, antiderivative size = 1482, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {2147, 261, 1878, 218, 1877, 2136, 2142, 2113, 537, 571, 93, 208}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x),x]
```


```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*d^(1/3)]*Sqrt[b^2*c^
(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2
/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3
)*d^(1/3) + a^2*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1
```

```
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c^(1/3
) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*Sqrt[(c^(1/3)*(c^(1/3) +
d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2 + Sqrt[
3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*
d^(1/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2
/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticPi[((1 +
Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2,
-ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d
^(2/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 2147

```
Int[Sqrt[(a_) + (b_.)*(x_)^3]/((c_) + (d_.)*(x_)), x_Symbol] :> Dist[b/d, I
nt[x^2/Sqrt[a + b*x^3], x], x] + (Dist[(b*c)/d^3, Int[(c - d*x)/Sqrt[a + b*
x^3], x], x] - Dist[(b*c^3 - a*d^3)/d^3, Int[1/((c + d*x)*Sqrt[a + b*x^3]),
 x], x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \frac{(ad)\int \frac{a-bx}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{d\int \frac{x^2}{\sqrt{c+dx^3}}\,dx}{b} - \left(-c+\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{c+dx^3}}\,dx$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{\left(ad^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{b^2} + \frac{\left(a\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)d\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{\left(b\left(c-\frac{a^3 d}{b^3}\right)\right)\int \frac{1+\sqrt{\cdots}}{(a+bx\cdots}}{b+\sqrt{3}b-\frac{a}{\cdots}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin\cdots\right.}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx\cdots}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin\cdots\right.}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx\cdots}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin\cdots\right.}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx\cdots}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin\cdots\right.}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx\cdots}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{b^2c^{2/3}+ab\sqrt[3]{c}\sqrt[3]{d}+a^2d^{2/3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}{b^{5/2}\sqrt{\frac{\cdots}{\left(\left(1\cdots\right.\right.}}}$$

Mathematica [C] time = 2.0305, size = 820, normalized size = 0.55

$$2\left(\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}d\sqrt{\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\sqrt{\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1}\Pi\left(\frac{i\sqrt{3}b\sqrt[3]{c}}{\sqrt[3]{d}a+\sqrt[3]{-1}b\sqrt[3]{c}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\right)|\sqrt[3]{-1}\right)a^3}{b^2\left(\sqrt[3]{d}a+\sqrt[3]{-1}b\sqrt[3]{c}\right)}-\frac{3^{3/4}d^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\sqrt{\sqrt[6]{-1}-\cdots}}{b^2\sqrt{\frac{(-1)^{2/3}\sqrt[3]{c}\cdots}{\left(1+\sqrt[3]{-1}\cdots\right.}}}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*(c + d*x^3 - (3^(3/4)*a^2*d^(2/3)*((-1)^(1/3)*c^(1/3) - d^(1/3)*x)*Sqrt[
(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[(-1)^(1/6) - (I*d^(1
/3)*x)/c^(1/3)]*EllipticF[ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1
+ (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1
/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) + (3^(3/4)*a*c^(1/3)*d^(1/3)*((-1)^(1/3
)*c^(1/3) - d^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*d^(1/3)*x)/c^(1/3)]*Sqrt[(
I*(1 + (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[
ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
(-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^
(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/
3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/
3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x
^2)/c^(2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(
1/3)), ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/
3))]], (-1)^(1/3)])/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)) + ((-1)^(1/3)*Sqrt[3
]*(1 + (-1)^(1/3))*a^3*c^(1/3)*d*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*Elliptic
Pi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[(c
^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(
b^2*((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)))))/(3*b*Sqrt[c + d*x^3])
```

Maple [A] time = 0.151, size = 1126, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*(
(x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
```

```
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
)+2/3*I*(a^3*d-b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*
(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)+a/b)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)+a/b),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{bx + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

3.197 $\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$

Optimal. Leaf size=135

$$\frac{(d^3+e^3x^3)^p\left(1+\frac{2(d+ex)}{(-3+i\sqrt{3})d}\right)^{-p}\left(1-\frac{2(d+ex)}{(3+i\sqrt{3})d}\right)^{-p}F_1\left(p;-p,-p;p+1;-\frac{2(d+ex)}{(-3+i\sqrt{3})d},\frac{2(d+ex)}{(3+i\sqrt{3})d}\right)}{ep}$$

```
[Out] ((d^3 + e^3*x^3)^p*AppellF1[p, -p, -p, 1 + p, (-2*(d + e*x))/((-3 + I*Sqrt[
3])*d), (2*(d + e*x))/((3 + I*Sqrt[3])*d)])/(e*p*(1 + (2*(d + e*x))/((-3 +
I*Sqrt[3])*d))^p*(1 - (2*(d + e*x))/((3 + I*Sqrt[3])*d))^p)
```

Rubi [F] time = 0.0856207, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

```
[Out] Defer[Int][(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Rubi steps

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx=\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

Mathematica [F] time = 0.0492502, size = 0, normalized size = 0.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

```
[Out] Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

```
[Out] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e^3x^3 + d^3)^p}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + d^3)^p}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Sympy [B] time = 56.4874, size = 636, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

```
[Out] 0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)/
(4*pi**2*e) + 0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(-1/3)
*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*exp(2*I*
pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**
2*e*gamma(4/3)) - 0**p*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamma(1/3
)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*exp_po
lar(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) + 0**p*exp
(-2*I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/
(12*pi**2*e*gamma(4/3)) + 0**p*exp(-I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)
/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3))
- d**2*e**(3*p)*p*x**(3*p)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)*ga
mma(p)*gamma(2/3 - p)*hyper((1 - p, 2/3 - p), (5/3 - p,), d**3*exp_polar(I*
pi)/(e**3*x**3))/(4*pi**2*e**3*x**2*gamma(5/3 - p)*gamma(p + 1)) - d*e**(3*
p)*p*x**(3*p)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4/3)*gamma(p)*gamma(1
/3 - p)*hyper((1 - p, 1/3 - p), (4/3 - p,), d**3*exp_polar(I*pi)/(e**3*x**3
))/(4*pi**2*e**2*x*gamma(4/3 - p)*gamma(p + 1)) - d**(3*p)*e**2*x**3*gamma(
1/3)**2*gamma(2/3)**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e*
*3*x**3*exp_polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamma(p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e^3x^3 + d^3\right)^p}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

3.198 $\int \frac{2-2x-x^2}{\left(2+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=16

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rubi [A] time = 0.0767319, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2146, 203}

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+x^2\right)\sqrt{1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\\ &= 2\tan^{-1}\left(\frac{1+x}{\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.828515, size = 296, normalized size = 18.5

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}\right.\cdots}{\cdots}\right.}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(
2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*
Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqrt[2]
 + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[1 + x^3])
```

Maple [C] time = 0.036, size = 1640, normalized size = 102.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2
))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I
*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+
1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))-2^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)
/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+
1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1
/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/
2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1
/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)/(I*2^(1/2)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((1+x)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2
*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I
```

```
/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/
2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(
1/2)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(
I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1
/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/
(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2
*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}\left(x^2 + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Fricas [A] time = 1.48543, size = 54, normalized size = 3.38

$$-\arctan\left(\frac{x^2 - 2x}{2\sqrt{x^3 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x}{x^2\sqrt{x^3 + 1} + 2\sqrt{x^3 + 1}}\, dx - \int \frac{x^2}{x^2\sqrt{x^3 + 1} + 2\sqrt{x^3 + 1}}\, dx - \int -\frac{2}{x^2\sqrt{x^3 + 1} + 2\sqrt{x^3 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/
(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**3
+ 1) + 2*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}\left(x^2 + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

3.199 $\int \frac{2+2x-x^2}{\left(2+x^2\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=20

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rubi [A] time = 0.0850639, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2146, 203}

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]

[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2+x^2\right)\sqrt{1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\right)\\ &= -2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.639948, size = 280, normalized size = 14.

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+\frac{3\left(1-i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{}}{-i+2\sqrt{}}\right.}{}\right.}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*
Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*El
lipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt[1 -
x^3])
```

Maple [C] time = 0.039, size = 732, normalized size = 36.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(
1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*
3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1
/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2
)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1
/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1
/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+I*2^(1/2))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1
/2*I*3^(1/2)+I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*2^(1/2)
*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/
(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)
^(1/2)/(-1/2+1/2*I*3^(1/2)+I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+I*2^(1/2)),(I*3^(1/
2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 2x - 2}{\sqrt{-x^3 + 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Fricas [A] time = 1.3752, size = 69, normalized size = 3.45

$$-\arctan\left(\frac{\sqrt{-x^3+1}\left(x^2+2\,x\right)}{2\,\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int \frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int -\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**2
/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1 -
x**3) + 2*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

3.200 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=18

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rubi [A] time = 0.0792075, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2146, 206}

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\right)\\ &= -2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.227572, size = 278, normalized size = 15.44

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+\frac{3\left(1-i\sqrt{2}\right)\Pi\left(\cdots\right.}{\cdots}\right.}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*
Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*El
lipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt[-1 +
 x^3])
```

Maple [C] time = 0.032, size = 1656, normalized size = 92.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3
/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1
/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1
/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^
(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/
(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-
1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2
/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I
*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))
+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1
/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2
))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3
/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I
*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/
2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*El
lipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+
1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*I*2^(1/2)*(1/
(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))
*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2
```

```
+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))+2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-2x-2}{\sqrt{x^3-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Fricas [A] time = 1.37915, size = 62, normalized size = 3.44

$$\log\left(\frac{x^2+2x+2\sqrt{x^3-1}}{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int \frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int -\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x**3
 - 1) + 2*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-2x-2}{\sqrt{x^3-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

3.201 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=18

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rubi [A] time = 0.0829336, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2146, 206}

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\\ &= 2\tanh^{-1}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.525577, size = 298, normalized size = 16.56

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}-\right.}{}\right.}{3\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(
2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*
Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqrt[2]
 + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[-1 - x^3])
```

Maple [C] time = 0.035, size = 724, normalized size = 40.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticP
i(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*
3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/
2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+I*2^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*3^(1/
2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)
))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3
^(1/2)+I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Fricas [A] time = 1.37878, size = 65, normalized size = 3.61

$$\log\left(-\frac{x^2 - 2\,x - 2\,\sqrt{-x^3 - 1}}{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int -\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt(-
x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 + 2\,x - 2}{\sqrt{-x^3 - 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

3.202 $\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

```
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rubi [A] time = 0.0930462, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2145, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{1+x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-(2+2d)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [C] time = 1.20414, size = 424, normalized size = 14.13

$$\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-1}\sqrt{d^2-4d-8}+4\sqrt{d^2-4d}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (
-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]
 + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3]
)/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))
*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^
2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(
1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d
+ d^2])))/(3*Sqrt[1 + x^3])
```

Maple [C] time = 0.04, size = 4397, normalized size = 146.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2+2*I/(d^2-4*d-8)^(1/
2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi
(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-
4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+
3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^
(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticP
i(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2
-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-2*I/(d^
2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3
/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+
1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)
)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2
*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)
)*d*3^(1/2)+6/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1
/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*
```

```
(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2))*d+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2
)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*
I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)
))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(
1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(
-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^
(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2
)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/
2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(
d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*
3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I
*3^(1/2)))^(1/2))*d^2*3^(1/2)+12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x
+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(
1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1
/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(
1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2))
)^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I
*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/
2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3
^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)+3/2/(d^2-4*d-8
)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*
I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*Ellip
ticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*
(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2+I
*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/
2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1
/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))
*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi((
(1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*
d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3/2*
(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2
))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/
(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d
-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(3
/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-
1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2
+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/
(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(
1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-6/(d^2-4
*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-
1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2
```

```
))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*E
llipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+
1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d
+4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2
)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*
I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-
8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)
))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(
1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(
-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^
(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2))
)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2)
)+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^
2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^
(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))*3^(1/2)-12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/
2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-
1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3
/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(
-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2
),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I
*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(
-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*
I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*
d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2
+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4872, size = 458, normalized size = 15.27

$$\left[-\frac{\sqrt{-d-1}\log\left(-\frac{2\,(3\,d+4)x^3-x^4-\left(d^2+2\,d+4\right)x^2-d^2+4\,\sqrt{x^3+1}\left((d+2)x-x^2+d\right)\sqrt{-d-1}-2\left(d^2+2\,d\right)x+4\,d+4}{2\,dx^3+x^4+\left(d^2+2\,d+4\right)x^2+d^2+2\left(d^2+2\,d\right)x+4\,d+4}\right)}{2\,(d+1)},-\frac{\arctan\left(-\frac{\sqrt{x^3+1}\left((d+2)x-x^2+d\right)\sqrt{d}}{2\left((d+1)x^3+d+1\right)}\right.}{\sqrt{d+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x + 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d +
 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d + 1)/
((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int -\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
+ 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 +
 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqrt(x
**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+2x-2}{\sqrt{x^3+1}\left(dx+x^2+d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

3.203 $\int \frac{2+2x-x^2}{\left(2-d+dx+x^2\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rubi [A] time = 0.107712, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2145, 204}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2-d+dx+x^2\right)\sqrt{1-x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{-2-(2-2d)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}\end{aligned}$$

Mathematica [C] time = 1.34781, size = 427, normalized size = 11.24

$$\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}-4\sqrt{d\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 +
(-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
] + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[
3])/(2*(-1)^(1/3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1/3))*
d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1
)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((-2*I)*Sqrt[3])/(-2*(-1)
^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)]))/((-2 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4
*d + d^2])))/(3*Sqrt[1 - x^3])
```

Maple [C] time = 0.048, size = 1908, normalized size = 50.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^
(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-
I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*
3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*
(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1
/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2+4*d-8)
^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2)
)*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(
1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-1
/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/
2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2
```

```
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^
(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1
/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/
2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)))^(1/2))*d-4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d
-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50649, size = 459, normalized size = 12.08

$$\left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-\left(d^2-2\,d+4\right)x^2-4\,\sqrt{-x^3+1}\left((d-2)x-x^2-d\right)\sqrt{d-1}-d^2+2\left(d^2-2\,d\right)x-4\,d+4}{2\,dx^3+x^4+\left(d^2-2\,d+4\right)x^2+d^2-2\left(d^2-2\,d\right)x-4\,d+4}\right)}{2\,\sqrt{d-1}},\,-\frac{\sqrt{-d+1}\arctan\left(-\frac{\sqrt{-x^3+1}\left((d-2)x-x^2-d\right)\sqrt{-d}}{2\left((d-1)x^3-d+1\right)}\right.}{d-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*(
(d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d -
1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt(-d
+ 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int \frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3)
 + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x
**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sqrt(
1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-2x-2}{\sqrt{-x^3+1}\left(dx+x^2-d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

3.204 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

```
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rubi [A] time = 0.0930514, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2145, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2-d+dx+x^2\right)\sqrt{-1+x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{-2-(-2+2d)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}\end{aligned}$$

Mathematica [C] time = 0.452205, size = 425, normalized size = 11.81

$$\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}-4\sqrt{d}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 +
(-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
] + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[
3])/(2*(-1)^(1/3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1/3))*
d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1
)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((-2*I)*Sqrt[3])/(-2*(-1)
^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)]))/((-2 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4
*d + d^2])))/(3*Sqrt[-1 + x^3])
```

Maple [C] time = 0.028, size = 4437, normalized size = 123.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(
3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3
^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I
*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+1/2*I/(d^2+4*d-8)^(1/2)*(1/(
-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*
x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+
1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))
,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)-3/2*(1/(-3/2-
1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2
/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I
*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2
*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-I*(1/(-3/2-1/2*I*3^(1/2))*x-
1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/
2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(
3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(
1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1
/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*
I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*
x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^
(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)
```

))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d+2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1
/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2
))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1
/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x
-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1
/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/
(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/
(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/
2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))
/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))
+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4
*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/
2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))*3^(1/2)-12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/
2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/
(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*
3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/
2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2
*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2))+4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1
/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I
/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I
*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1
/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/
2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2))*3^(1/2)-3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-
1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/
2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(
3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(
1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))*d^2-4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2
))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*
3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+
1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1
/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1
/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/
2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3
/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2))*d-2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1
/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2
))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3
/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1
+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)
,(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2
*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)

```
^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2
)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3
/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1
/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1
/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2
*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/
2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1
/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d
^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)
*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)
)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*
d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d
*3^(1/2)+12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+
1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*
d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2
))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))-1/2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^
(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2
))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2
+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(
1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2))*d^2*3^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63272, size = 458, normalized size = 12.72

$$\left[-\frac{\sqrt{-d+1}\log\left(-\frac{2\,(3\,d-4)x^3-x^4-\left(d^2-2\,d+4\right)x^2-d^2+4\,\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{-d+1}+2\left(d^2-2\,d\right)x-4\,d+4}{2\,dx^3+x^4+\left(d^2-2\,d+4\right)x^2+d^2-2\left(d^2-2\,d\right)x-4\,d+4}\right)}{2\,(d-1)},-\frac{\arctan\left(-\frac{\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{d-1}}{2\left((d-1)x^3-d+1\right)}\right)}{\sqrt{d-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x - 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d +
 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d - 1)/
((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int \frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1)
 + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3
- 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sqrt(
x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}\left(dx+x^2-d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

3.205 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

```
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rubi [A] time = 0.0926577, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2145, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-(-2-2d)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [C] time = 0.462222, size = 426, normalized size = 13.31

$$\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-1}\sqrt{d^2-4d-8}+4\sqrt{d^2-4d\ldots}\right.\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (
-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]
 + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3]
)/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))
*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^
2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(
1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d
+ d^2])))/(3*Sqrt[-1 - x^3])
```

Maple [C] time = 0.035, size = 1888, normalized size = 59.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1
/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/
2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)
+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-
1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2
)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I
*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*
3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1
/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2
)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*
3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-1/3*I/(d^2-4*d-8)^(1/2)*3^(1
/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1
/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/
(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*
(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/
2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^
2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2-4*d-8)^
(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*
x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I
*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(
1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2-4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58844, size = 459, normalized size = 14.34

$$\left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-\left(d^2+2\,d+4\right)x^2-4\,\sqrt{-x^3-1}\left((d+2)x-x^2+d\right)\sqrt{d+1}-d^2-2\,\left(d^2+2\,d\right)x+4\,d+4}{2\,dx^3+x^4+\left(d^2+2\,d+4\right)x^2+d^2+2\,\left(d^2+2\,d\right)x+4\,d+4}\right)}{2\,\sqrt{d+1}}, -\frac{\sqrt{-d-1}\arctan\left(-\frac{\sqrt{-x^3-1}\left((d+2)x-x^2+d\right)\sqrt{-d-1}}{2\,\left((d+1)x^3+d+1\right)}\right)}{d+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*(
(d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d +
1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt(-d
- 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int\frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-
x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(d*
x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3
 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}\left(dx + x^2 + d + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

3.206 $\int (d+ex)^3\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=355

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 + (
e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*
x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.232614, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1885, 1177, 1198, 220, 1196, 1248, 641, 195, 217, 206}

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```


```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 + (
e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*
x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4])
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \left(\left(d^3+3de^2x^2\right)\sqrt{a+cx^4}+x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\right)dx$$
$$= \int \left(d^3+3de^2x^2\right)\sqrt{a+cx^4}\,dx + \int x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\,dx$$
$$= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{15}\int \frac{10ad^3+18ade^2x^2}{\sqrt{a+cx^4}}dx + \frac{1}{2}\text{Subst}\left(\int \left(3d^2e+e^3x\right)\sqrt{a+c\right.$$
$$= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+cx^4\right)^{3/2}}{6c} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right) - \frac{(6a}{}$$
$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+cx^4\right)^{3/2}}{6c} - \frac{6a^{5/4}}{}$$
$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+cx^4\right)^{3/2}}{6c} - \frac{6a^{5/4}}{}$$
$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+cx^4\right)^{3/2}}{6c} + \frac{3ad^2}{}$$

Mathematica [C] time = 0.13794, size = 186, normalized size = 0.52

$$\frac{\sqrt{a+cx^4}\left(9cd^2ex^2\sqrt{\frac{cx^4}{a}+1}+9\sqrt{a}\sqrt{c}d^2e\sinh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+12cd^3x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+12cde^2x^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)+2c\right.}{12c\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(2*a*e^3*Sqrt[1 + (c*x^4)/a] + 9*c*d^2*e*x^2*Sqrt[1 + (c*x
^4)/a] + 2*c*e^3*x^4*Sqrt[1 + (c*x^4)/a] + 9*Sqrt[a]*Sqrt[c]*d^2*e*ArcSinh[
(Sqrt[c]*x^2)/Sqrt[a]] + 12*c*d^3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)] + 12*c*d*e^2*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)]))
/(12*c*Sqrt[1 + (c*x^4)/a])
```

Maple [C] time = 0.048, size = 334, normalized size = 0.9

$$\frac{e^3}{6c}\left(cx^4+a\right)^{\frac{3}{2}} + \frac{3de^2x^3}{5}\sqrt{cx^4+a} + \frac{6i}{5}de^2a^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/6*e^3*(c*x^4+a)^(3/2)/c+3/5*d*e^2*x^3*(c*x^4+a)^(1/2)+6/5*I*d*e^2*a^(3/2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/
2),I)-6/5*I*d*e^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
```

```
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticE(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+3/4*e*d^2*a/c^
(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))+1/3*d^3*x*(c*x^4+a)^(1/2)+2/3*d^3*a/(
I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+a}(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^4 + a), x)
```

Sympy [A] time = 3.84379, size = 175, normalized size = 0.49

$$\frac{\sqrt{a}d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3\sqrt{a}d^2ex^2\sqrt{1+\frac{cx^4}{a}}}{4}+\frac{3\sqrt{a}de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{3ad^2e\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}+e^3\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*sqrt(a)
*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(7/4)) + 3*a*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) + e**
3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+a}(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

3.207 $\int (d+ex)^2\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=326

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\cdots\right)\right)}{5c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*S
qrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(
3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.189014, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1885, 275, 195, 217, 206, 1177, 1198, 220, 1196}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\cdots\right)\right)}{5c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*S
qrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(
3/4)*Sqrt[a + c*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \left(2dex\sqrt{a+cx^4}+\left(d^2+e^2x^2\right)\sqrt{a+cx^4}\right)dx$$
$$= (2de)\int x\sqrt{a+cx^4}\,dx + \int \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\,dx$$
$$= \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{1}{15}\int \frac{10ad^2+6ae^2x^2}{\sqrt{a+cx^4}}\,dx+(de)\,\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2$$
$$= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{1}{2}(ade)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)-\frac{(2}{}$$
$$= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)}{5}$$
$$= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{ade\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [C] time = 0.135089, size = 146, normalized size = 0.45

$$\frac{\sqrt{a+cx^4}\left(6\sqrt{c}d^2x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+e\left(3d\left(\sqrt{c}x^2\sqrt{\frac{cx^4}{a}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\right)+2\sqrt{c}ex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)\right)}{6\sqrt{c}\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(6*Sqrt[c]*d^2*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^
4)/a)] + e*(3*d*(Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]
*x^2)/Sqrt[a]]) + 2*Sqrt[c]*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^
4)/a)])))/(6*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Maple [C] time = 0.006, size = 310, normalized size = 1.

$$\frac{e^2x^3}{5}\sqrt{cx^4+a}+\frac{2\,i}{5}e^2a^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}-\frac{2\,i}{5}e^2a^{\frac{3}{2}}\sqrt{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/5*e^2*x^3*(c*x^4+a)^(1/2)+2/5*I*e^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2
)/c^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-2/5*I*e^2*a^(3/2)/(I/a^(
1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^
2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1
/2*d*e*x^2*(c*x^4+a)^(1/2)+1/2*d*e*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2)
)+1/3*d^2*x*(c*x^4+a)^(1/2)+2/3*d^2*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
```

```
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+a}(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+a}\left(e^2x^2+2\,dex+d^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [C] time = 3.36225, size = 138, normalized size = 0.42

$$\frac{\sqrt{a}d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}dex^2\sqrt{1+\frac{cx^4}{a}}}{2}+\frac{\sqrt{a}e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ade\,\text{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + sqrt(a)*e**2*x
**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamm
a(7/4)) + a*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+a}(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

3.208 $\int (d+ex)\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=158

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0890388, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1885, 195, 220, 275, 217, 206}

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a + c*x^4],x]
```


```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\sqrt{a+cx^4}\,dx &= \int \left(d\sqrt{a+cx^4}+ex\sqrt{a+cx^4}\right)dx\\
&= d\int \sqrt{a+cx^4}\,dx+e\int x\sqrt{a+cx^4}\,dx\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{3}(2ad)\int \frac{1}{\sqrt{a+cx^4}}\,dx+\frac{1}{2}e\,\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{4}(ae)\text{S}\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{4}(ae)\text{S}\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan\right.}{3\sqrt[4]{c}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.0628603, size = 109, normalized size = 0.69

$$\frac{\sqrt{a+cx^4}\left(4\sqrt{c}dx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+\sqrt{c}ex^2\sqrt{\frac{cx^4}{a}+1}+\sqrt{a}e\sinh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\right)}{4\sqrt{c}\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*e*ArcSinh[(Sq
rt[c]*x^2)/Sqrt[a]] + 4*Sqrt[c]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)]))/(4*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Maple [C] time = 0.004, size = 127, normalized size = 0.8

$$\frac{ex^2}{4}\sqrt{cx^4+a}+\frac{ae}{4}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}+\frac{dx}{3}\sqrt{cx^4+a}+\frac{2\,ad}{3}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e*x^2*(c*x^4+a)^(1/2)+1/4*e*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))+1
/3*d*x*(c*x^4+a)^(1/2)+2/3*d*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}(ex + d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + a}(ex + d), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e*x + d), x)
```

Sympy [C] time = 2.88789, size = 88, normalized size = 0.56

$$\frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}ex^2\sqrt{1 + \frac{cx^4}{a}}}{4} + \frac{ae\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 + a*e*asinh(sqrt(c)*x
**2/sqrt(a))/(4*sqrt(c))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}(ex + d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

3.209 $\int \sqrt{a+cx^4}\,dx$

Optimal. Leaf size=105

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3
*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0194917, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 220}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3
*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+cx^4}\,dx &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{1}{3}(2a)\int \frac{1}{\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.105744, size = 89, normalized size = 0.85

$$\frac{x\left(a+cx^4\right)-\frac{2ia\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4],x]
```

```
[Out] (x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

Maple [C] time = 0.001, size = 85, normalized size = 0.8

$$\frac{x}{3}\sqrt{cx^4+a}+\frac{2a}{3}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a), x)
```

Sympy [C] time = 0.657534, size = 37, normalized size = 0.35

$$\frac{\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

3.210 $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

Optimal. Leaf size=730

$$-\frac{\sqrt{-ae^4-cd^4}\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2e^3}+\frac{\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt{ae^4+cd^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)}{}$$

```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[
c]*x^2)) - (Sqrt[-(c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*S
qrt[a + c*x^4])])/(2*e^3) + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x
^4]])/(2*e^3) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^
4 + a*e^4]*Sqrt[a + c*x^4])])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*d*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a
 + c*x^4]) + (c^(1/4)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c
]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^
(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.725251, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1729, 1209, 1198, 220, 1196, 1217, 1707, 1248, 735, 844, 217, 206, 725}

$$-\frac{\sqrt{-ae^4-cd^4}\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2e^3}+\frac{\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt{ae^4+cd^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x),x]
```


```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[
c]*x^2)) - (Sqrt[-(c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*S
qrt[a + c*x^4])])/(2*e^3) + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x
^4]])/(2*e^3) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^
4 + a*e^4]*Sqrt[a + c*x^4])])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*d*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a
 + c*x^4]) + (c^(1/4)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c
]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^
(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx &= d\int \frac{\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx - e\int \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx\\
&= \left(d\left(a+\frac{cd^4}{e^4}\right)\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx - \frac{d\int \frac{cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{e^4} - \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{\sqrt{a+cx^2}}{d^2-e^2x}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+cx^4}}{2e} + \frac{\left(\sqrt{a}\sqrt{c}d\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{e^2} + \frac{\mathrm{Subst}\left(\int \frac{-ae^2-cd^2x}{\left(d^2-e^2x\right)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{2e} + \frac{\left(\sqrt{c}d\left(a+\frac{cd^4}{e^4}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2}\\
&= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\Big(}{e^2\sqrt{a+cx^4}}\\
&= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\Big(}{e^2\sqrt{a+cx^4}}\\
&= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3} - \frac{\sqrt{cd^4+ae^4}}{}
\end{aligned}$$

Mathematica [C] time = 0.818278, size = 405, normalized size = 0.55

$$\frac{2c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{a}e^2+i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-2\sqrt{a}c^{3/4}d^2e^2\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] (-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sq
rt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + S
qrt[(I*Sqrt[c])/Sqrt[a]]*(c^(1/4)*d*e*(e^2*(a + c*x^4) + Sqrt[c]*d^2*Sqrt[a
 + c*x^4]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]] - Sqrt[c*d^4 + a*e^4]*Sqrt
[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4
])]) - 2*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1 + (c*x^4)/a]*EllipticPi[
(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1])
)/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt[a + c*x^4])
```

Maple [C] time = 0.015, size = 565, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/e-c*d^3/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*d^2/e^3*ln(2*x^2*c^(1/2)+2*(c*x^4+
a)^(1/2))-I*c^(1/2)*d/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-1
/2/e/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1
/2)/(c*x^4+a)^(1/2))*a-1/2/e^5/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2
/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))*c*d^4+1/(I/a^(1/2)*c^(1/2))^
(1/2)/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*
x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*
e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a+1/e^4/(I/a^(1/2
)*c^(1/2))^(1/2)*d^3*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)
/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^4}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + a}}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

3.211 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2} \, dx$

Optimal. Leaf size=1221

result too large to display

```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*Sqrt[a + c
*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 - e^2*x^2) + (Sqrt[-(
c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(
2*d*e^3) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a +
 c*x^4])])/(2*d*e^3*Sqrt[-(c*d^4) - a*e^4]) - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4
+ a*e^4]*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*
c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(
Sqrt[c]*d^2 + Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/
4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt
[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 1.79971, antiderivative size = 1221, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.79, Rules used = {2153, 1227, 1198, 220, 1196, 1217, 1707, 1248, 733, 844, 217, 206, 725, 1336, 1209}

$$\frac{c \tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right) d^3}{e^3\sqrt{cd^4+ae^4}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{cx^4+a}}\right) d}{e^3} - \frac{\sqrt{cx^4+a}\, d}{e\left(d^2-e^2x^2\right)} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{e^2\sqrt{cx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```


```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*Sqrt[a + c
*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 - e^2*x^2) + (Sqrt[-(
c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(
2*d*e^3) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a +
 c*x^4])])/(2*d*e^3*Sqrt[-(c*d^4) - a*e^4]) - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4
+ a*e^4]*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
```

```
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*
c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(
Sqrt[c]*d^2 + Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/
4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt
[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*Sqrt[a + c*x^4])
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rule 1227

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[(x
*Sqrt[a + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2
)/Sqrt[a + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^
2, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
```

```
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
```

```
| IntegersQ[m, q])
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \left(\frac{d^2\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2} - \frac{2dex\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2} + \frac{e^2x^2\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\right)dx$$

$$= d^2\int \frac{\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}\,dx - (2de)\int \frac{x\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}\,dx + e^2\int \frac{x^2\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx$$

$$= \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)} + \frac{1}{2}\left(a-\frac{cd^4}{e^4}\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \frac{c\int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2e^4} - (de)\,\mathrm{Subst}\left(\int \frac{\sqrt{a+cx^2}}{\left(d^2-e^2x\right)^2}\right.$$

$$= -\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)} + d^2\int \frac{\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx + \frac{1}{2}\left(\sqrt{a}\left(\sqrt{a}-\frac{\sqrt{c}d^2}{e^2}\right)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+c}}$$

$$= \frac{\sqrt{c}x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{4de^3\sqrt{-cd^4-ae^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}{}$$

$$= \frac{\sqrt{c}x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{4de^3\sqrt{-cd^4-ae^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}{}$$

$$= \frac{2\sqrt{c}x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} + \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\right.}{2de^3\sqrt{-cd^4-}}$$

Mathematica [C] time = 1.89189, size = 382, normalized size = 0.31

$$\frac{2\sqrt[4]{-1}\sqrt[4]{a}c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right) - \frac{cd^3e\sqrt{a+cx^4}\tanh^{-1}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}} - \frac{2\sqrt{c}\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{a}e^2+i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{e^4\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (-((e^3*(a + c*x^4))/(d + e*x)) - Sqrt[c]*d*e*Sqrt[a + c*x^4]*ArcTanh[(Sqrt
[c]*x^2)/Sqrt[a + c*x^4]] - (c*d^3*e*Sqrt[a + c*x^4]*ArcTanh[(-(a*e^2) - c*
d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]])/Sqrt[c*d^4 + a*e^4] - (2*I
)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*Sqrt[c]*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)
*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])
/Sqrt[(I*Sqrt[c])/Sqrt[a]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[1 + (c*x
```

```
^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)
*x)/a^(1/4)], -1])/(e^4*Sqrt[a + c*x^4])
```

Maple [C] time = 0.016, size = 402, normalized size = 0.3

$$-\frac{1}{e\left(ex+d\right)}\sqrt{cx^4+a}+2\,\frac{cd^2}{e^4\sqrt{cx^4+a}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^2}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}-\frac{d}{e^3}\sqrt{c}\ln\left(2\,x^2\sqrt{c}+2\,\sqrt{cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] -1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(
1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-c^(1/2)*d/e^3*ln(2*x^2*c^(1/2)+2*(c*x
^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
)+c*d^3/e^5/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^
4+a)^(1/2)/(c*x^4+a)^(1/2))-2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elli
pticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c
^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+a}}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^4}}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.212 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.160057, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \left(\frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}+\frac{x\left(3d^2e+e^3x^2\right)}{\sqrt{a+cx^4}}\right)dx$$

$$= \int \frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx + \int \frac{x\left(3d^2e+e^3x^2\right)}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{3d^2e+e^3x}{\sqrt{a+cx^2}}\,dx, x, x^2\right) - \frac{\left(3\sqrt{a}de^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d\left(d^2+\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+\right.}{}$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+\right.}{}$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-}\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.132394, size = 157, normalized size = 0.53

$$\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{d^3x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}+\frac{de^2x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}+\frac{e^3\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4
]])/(2*Sqrt[c]) + (d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Maple [C] time = 0.014, size = 218, normalized size = 0.7

$$\frac{e^3}{2c}\sqrt{cx^4+a}+3\,ide^2\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^3*(c*x^4+a)^(1/2)/c+3*I*d*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/
c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^
(1/2))^(1/2),I))+3/2*e*d^2*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+d^3/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 3.19374, size = 141, normalized size = 0.48

$$e^{3}\left(\begin{cases}\frac{x^{4}}{4\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^{4}}}{2c} & \text{otherwise}\end{cases}\right)+\frac{3d^{2}e\operatorname{asinh}\left(\frac{\sqrt{c}x^{2}}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{d^{3}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{3de^{2}x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2*c), True)
) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d**3*x*gamma(1/4)*hy
per((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
3*d*e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{3}}{\sqrt{cx^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

3.213 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[
a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4
])
```

Rubi [A] time = 0.123506, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```


```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[
a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4
])
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \left(\frac{2dex}{\sqrt{a+cx^4}} + \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\right)dx$$

$$= (2de)\int \frac{x}{\sqrt{a+cx^4}}\,dx + \int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx$$

$$= (de)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}e^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d^2+\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2 \ldots\right.}{2\sqrt[4]{a}c \ldots}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{de\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d \ldots\right.}{\ldots}$$

Mathematica [C] time = 0.123195, size = 133, normalized size = 0.51

$$\frac{d^2x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{de\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{e^2x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] + (d^2*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (e
^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/
(3*Sqrt[a + c*x^4])
```

Maple [C] time = 0.006, size = 197, normalized size = 0.8

$$ie^2\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2
)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d*e*ln(x^2*c^
(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 2.66394, size = 105, normalized size = 0.4

$$\frac{de\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}+\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e**2*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

3.214 $\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.062415, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1885, 220, 275, 217, 206}

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx &= \int \left(\frac{d}{\sqrt{a+cx^4}}+\frac{ex}{\sqrt{a+cx^4}}\right)dx\\
&= d\int \frac{1}{\sqrt{a+cx^4}}\,dx+e\int \frac{x}{\sqrt{a+cx^4}}\,dx\\
&= \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)\\
&= \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)\\
&= \frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0415367, size = 79, normalized size = 0.65

$$\frac{dx\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Maple [C] time = 0.004, size = 96, normalized size = 0.8

$$\frac{e}{2}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}+d\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+d/(I/a^(1/2)*c^(1/2))^(1/2)*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex+d}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 1.91794, size = 61, normalized size = 0.5

$$\frac{e\,\text{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

3.215 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rubi [A] time = 0.0095494, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0326845, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.001, size = 70, normalized size = 0.8

$$\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 0.618706, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

3.216 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=405

$$\frac{e\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}+\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2}{\cdots}$$

```
[Out] (e*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*Sqrt[-(c*d^
4) - a*e^4]) - (e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a +
 c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[
c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqr
t[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.272437, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1725, 1217, 220, 1707, 1248, 725, 206}

$$\frac{e\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}+\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```


```
[Out] (e*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*Sqrt[-(c*d^
4) - a*e^4]) - (e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a +
 c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[
c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqr
t[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```


Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = d\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx - e\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\sqrt{a+cx^2}}\,dx, x, x^2\right)\right) + \frac{\left(\sqrt{c}d\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} + \frac{\left(\sqrt{a}de^2\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+}}}{\sqrt{c}d^2+\sqrt{a}e^2}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{}\right.}{}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.255431, size = 200, normalized size = 0.49

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(\sqrt[4]{c}d\log\left(\frac{e^2x^2-d^2}{ae^2\left(\sqrt{\frac{cx^4}{a}+1}\sqrt{\frac{cd^4}{ae^4}+1}+1\right)+cd^2x^2}\right)-2\sqrt[4]{-1}\sqrt[4]{a}e\sqrt{\frac{cd^4}{ae^4}+1}\Pi\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{2\sqrt[4]{c}de\sqrt{a+cx^4}\sqrt{\frac{cd^4}{ae^4}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e^4)]*e*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d^2*x^2 + a*e^2*(1 + Sqrt[1 + (
c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/(a*e
^4)]*e*Sqrt[a + c*x^4])
```

Maple [C] time = 0.008, size = 169, normalized size = 0.4

$$\frac{1}{e}\left(-\frac{1}{2}\mathrm{Artanh}\left(\frac{1}{2}\left(2\frac{cd^2x^2}{e^2}+2a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\frac{1}{\sqrt{cx^4+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}+\frac{e}{d}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticPi}\left(x\sqrt{i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+
a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x
*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^
(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.217 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=610

$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(ae^4+cd^4\right)}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}+\frac{\sqrt{c}e}{\left(\sqrt{a}+\sqrt{\right.}}$$

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e^2*x*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*e*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(-(c*d^4) - a*e^4)^(3/2) -
(c*d^3*e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(c*d^4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4
]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a
^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.760191, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1727, 1742, 12, 1248, 725, 206, 1715, 1196, 1709, 220, 1707}

$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(ae^4+cd^4\right)}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}+\frac{\sqrt{c}e}{\left(\sqrt{a}+\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e^2*x*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*e*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(-(c*d^4) - a*e^4)^(3/2) -
(c*d^3*e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(c*d^4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4
]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a
^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 1727

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[(
e^3*(d + e*x)^(q + 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)*Simp[d^3*(q + 1) - d^2
*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x])/Sqrt[a + c*x^4], x]
```

```
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
```

```
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} - \frac{c\int \frac{-d^3+d^2ex-de^2x^2-e^3x^3}{(d+ex)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} - \frac{c\int \frac{2d^3ex}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4} - \frac{c\int \frac{-d^4-2d^2e^2x^2+e^4x^4}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\int \frac{cd^4e^2+\sqrt{a}\sqrt{c}d^2e^4+\left(2cd^2e^4-e^4\left(cd^2+\sqrt{a}\sqrt{c}e^2\right)\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{e^2\left(cd^4+ae^4\right)} - \frac{\left(2cd^3e\right)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+\cdots}}}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(\cdots\right.}{\left(cd^4+ae^4\right)\sqrt{a+cx^4}}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{\left(-cd^4-ae^4\right)^{3/2}} - \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{\left(-cd^4-ae^4\right)^{3/2}} - \frac{cd^3e\tanh^{-1}\cdots}{\left(cd^4\cdots\right.}$$

Mathematica [C] time = 1.10013, size = 425, normalized size = 0.7

$$-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(2\sqrt[4]{-1}\sqrt[4]{a}c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}(d+ex)\sqrt{ae^4+cd^4}\,\Pi\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)+e^3\left(a+cx^4\right)\sqrt{ae^4+cd^4}+cd^3e\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*e^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*Sqrt[c]*(Sqrt[c]*d^2
+ I*Sqrt[a]*e^2)*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[(I*Sqrt[c])/Sqrt[a]]*(
e^3*Sqrt[c*d^4 + a*e^4]*(a + c*x^4) + c*d^3*e*(d + e*x)*Sqrt[a + c*x^4]*Arc
Tanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])] + 2*(-1)^(1
/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*E
llipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1
/4)], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^4 + a*e^4)^(3/2)*(d + e*x)*Sqrt
[a + c*x^4])
```

Maple [C] time = 0.014, size = 421, normalized size = 0.7

$$-\frac{e^3}{\left(ae^4+cd^4\right)(ex+d)}\sqrt{cx^4+a}-\frac{cd^2}{ae^4+cd^4}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] -e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-c*d^2/(a*e^4+c*d^4)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*c^(1/2)*e^2/(
a*e^4+c*d^4)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*c*d^3/(a*e^4
+c*d^4)/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^
4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(
1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

3.218 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=659

$$\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^4+cd^4\right)^2}+\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(ae^4+cd^4\right)}-\frac{3\sqrt[4]{a}c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{\sqrt{a+cx^4}\left(ae^4+cd\right.}$$

```
[Out] -(e^3*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (3*c*d^3*e^3*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)^2*(d + e*x)) + (3*c^(3/2)*d^3*e^2*x*Sqrt[a + c
*x^4])/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (3*c*d^2*e*(c*d^4 - a*
e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4)
 - a*e^4)^(5/2)) - (3*c*d^2*e*(c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)/(
Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*(c*d^4 + a*e^4)^(5/2)) - (3*a^(1/
4)*c^(5/4)*d^3*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)
^2*Sqrt[a + c*x^4]) + (c^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2
*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(3/4)*d*(Sqrt[c]*d^2 - Sqr
t[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2),
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a +
c*x^4])
```

Rubi [A] time = 1.15702, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1727, 1739, 1742, 12, 1248, 725, 206, 1715, 1196, 1709, 220, 1707}

$$\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^4+cd^4\right)^2}+\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(ae^4+cd^4\right)}-\frac{3\sqrt[4]{a}c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{\sqrt{a+cx^4}\left(ae^4+cd\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] -(e^3*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (3*c*d^3*e^3*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)^2*(d + e*x)) + (3*c^(3/2)*d^3*e^2*x*Sqrt[a + c
*x^4])/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (3*c*d^2*e*(c*d^4 - a*
e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4)
 - a*e^4)^(5/2)) - (3*c*d^2*e*(c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)/(
Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*(c*d^4 + a*e^4)^(5/2)) - (3*a^(1/
4)*c^(5/4)*d^3*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)
^2*Sqrt[a + c*x^4]) + (c^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2
*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(3/4)*d*(Sqrt[c]*d^2 - Sqr
t[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2),
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a +
c*x^4])
```

Rule 1727

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[(
e^3*(d + e*x)^(q + 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)*Simp[d^3*(q + 1) - d^2
*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x])/Sqrt[a + c*x^4], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1739

```
Int[((Px_)*((d_) + (e_.)*(x_))^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :
> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D =
Coeff[Px, x, 3]}, -Simp[((d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*(d + e*x)^(q +
 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist[1/((q + 1)*(c*d^4
 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[(q + 1)*(a*e*(d^2*
D - C*d*e + B*e^2) + A*d*(c*d^2)) - (e*(q + 1)*(A*c*d^2 + a*e*(d*D - C*e))
- B*d*(c*d^2*(q + 1)))*x + (q + 1)*(D*e*(a*e^2) + c*d*(C*d^2 - e*(B*d - A*e
)))*x^2 + c*(q + 3)*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x^3, x], x], x]] /;
 FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^
4 + a*e^4, 0] && LtQ[q, -1]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
```

```
&& NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{c\int \frac{-2d^3+2d^2ex-2de^2x^2}{(d+ex)^2\sqrt{a+cx^4}}\,dx}{2\left(cd^4+ae^4\right)}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\int \frac{2d^2\left(cd^4-2ae^4\right)-2de\left(2cd^4-ae^4\right)x+6cd^4e^2x^2+6cd}{(d+ex)\sqrt{a+cx^4}}}{2\left(cd^4+ae^4\right)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\int \frac{\left(-2d^2e\left(cd^4-2ae^4\right)-2d^2e\left(2cd^4-ae^4\right)\right)x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{2\left(cd^4+ae^4\right)^2} + \cdots$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} - \frac{\int \frac{-6\sqrt{a}c^{3/2}d^5e^4-2cd^3e^2\left(cd^4-2ae^4\right)+\left(6cd^3e^4\left(cd^2+\sqrt{\cdots}\right.\right.}{\left(d^2-e^2x^2\right)\sqrt{a\cdots}}}{2e^2\left(cd^4+\cdots\right.}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}c^{5/4}d^3\cdots}{\cdots}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3cd^2e\left(cd^4\cdots\right.}{2\cdots}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3cd^2e\left(cd^4\cdots\right.}{2\cdots}$$

Mathematica [C] time = 2.33085, size = 513, normalized size = 0.78

$$\frac{-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(e^3\sqrt{ae^4+cd^4}\left(a^2e^4+ac\left(6d^3ex+7d^4+e^4x^4\right)+c^2d^3x^4(7d+6ex)\right)+6\sqrt[4]{-1}\sqrt[4]{a}c^{3/4}d\sqrt{\frac{cx^4}{a}+1}(d+ex)^2\left(cd^4-a\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (6*Sqrt[a]*c^(3/2)*d^3*e^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)
/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (2*I)*c*d*(2*c*
d^4 + (3*I)*Sqrt[a]*Sqrt[c]*d^2*e^2 - a*e^4)*Sqrt[c*d^4 + a*e^4]*(d + e*x)^
2*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]
 - Sqrt[(I*Sqrt[c])/Sqrt[a]]*(e^3*Sqrt[c*d^4 + a*e^4]*(a^2*e^4 + c^2*d^3*x^
4*(7*d + 6*e*x) + a*c*(7*d^4 + 6*d^3*e*x + e^4*x^4)) + 3*c*d^2*e*(c*d^4 - a
*e^4)*(d + e*x)^2*Sqrt[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 +
 a*e^4]*Sqrt[a + c*x^4])] + 6*(-1)^(1/4)*a^(1/4)*c^(3/4)*d*(c*d^4 - a*e^4)*
Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e
^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(2*Sqrt[(I
*Sqrt[c])/Sqrt[a]]*(c*d^4 + a*e^4)^(5/2)*(d + e*x)^2*Sqrt[a + c*x^4])
```

Maple [C] time = 0.016, size = 483, normalized size = 0.7

$$-\frac{e^3}{\left(2\,ae^4+2\,cd^4\right)(ex+d)^2}\sqrt{cx^4+a}-3\,\frac{cd^3e^3\sqrt{cx^4+a}}{\left(ae^4+cd^4\right)^2(ex+d)}+\frac{cd\left(ae^4-2\,cd^4\right)}{\left(ae^4+cd^4\right)^2}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2
)/(a*e^4+c*d^4)^2/(e*x+d)+c*d*(a*e^4-2*c*d^4)/(a*e^4+c*d^4)^2/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3*I*c^(3/2)*e^2*
d^3/(a*e^4+c*d^4)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/
a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-3*c*d^2
*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c
*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2)
)^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/
d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

3.219 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} -$$

```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a*e^3
 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a + c*x^4]) + (3*d*e^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
 + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])
/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.135761, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1854, 1198, 220, 1196}

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} -$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a*e^3
 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a + c*x^4]) + (3*d*e^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
 + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])
/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{3/2}}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}-\frac{\int \frac{-d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}+\frac{\left(3de^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}}+\frac{\left(d\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan\right.}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0641065, size = 126, normalized size = 0.42

$$\frac{cd^3x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+2cde^2x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)-ae^3+3cd^2ex^2+cd^3x}{2ac\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-(a*e^3) + c*d^3*x + 3*c*d^2*e*x^2 + c*d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*H
ypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(2*a*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.025, size = 261, normalized size = 0.9

$$-\frac{e^3}{2\,c}\frac{1}{\sqrt{cx^4+a}}+3\,de^2\left(1/2\,\frac{x^3}{a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i/2}{\sqrt{a}\sqrt{cx^4+a}\sqrt{c}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{Ellipt}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^(3/2),x)
```

```
[Out] -1/2*e^3/c/(c*x^4+a)^(1/2)+3*d*e^2*(1/2*x^3/a/((x^4+a/c)*c)^(1/2)-1/2*I/a^(
1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
```

```
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+3/2*e*d^2/(c*x^4+a)^(
1/2)/a*x^2+d^3*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^
(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^4+a}}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^4 + a)/(c^2*x^8
 + 2*a*c*x^4 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

3.220 $\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=270

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{x(d}{2a}$$

```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.117386, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1855, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{x(d}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```


```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```


Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{\int \frac{-d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} + \frac{e^2\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}} + \frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{\left(d^2-\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.0617646, size = 108, normalized size = 0.4

$$\frac{x\left(3d^2\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+2e^2x^2\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)+3d(d+2ex)\right)}{6a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(3*d*(d + 2*e*x) + 3*d^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((c*x^4)/a)] + 2*e^2*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4,
3/2, 7/4, -((c*x^4)/a)]))/(6*a*Sqrt[a + c*x^4])
```

Maple [C] time = 0.006, size = 239, normalized size = 0.9

$$e^2\left(\frac{x^3}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i}{2}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^(3/2),x)
```

```
[Out] e^2*(1/2*x^3/a/((x^4+a/c)*c)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*
(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)))+d*e/(c*x^4+a)^(1/2)/a*x^2+d^2*(1/2*x/a/((x^4+a/c)*c)
^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))
```

```
^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+a}\left(e^2x^2+2\,dex+d^2\right)}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e^2*x^2 + 2*d*e*x + d^2)/(c^2*x^8 + 2*a*c*x^4 + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

3.221 $\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0474485, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1855, 12, 220}

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{\int \frac{d}{\sqrt{a+cx^4}}\,dx}{2a}$$
$$= \frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{d\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$
$$= \frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0299536, size = 59, normalized size = 0.52

$$\frac{x\left(d\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+d+ex\right)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x + d*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^4)/a)]))/(2*a*Sqrt[a + c*x^4])
```

Maple [C] time = 0.004, size = 115, normalized size = 1.

$$\frac{ex^2}{2a}\frac{1}{\sqrt{cx^4+a}}+d\left(\frac{x}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{1}{2a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] 1/2*e/(c*x^4+a)^(1/2)/a*x^2+d*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)
*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+a}(ex+d)}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e*x + d)/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [C] time = 7.74928, size = 61, normalized size = 0.54

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex+d}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

3.222 $\int \frac{1}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/
4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0190018, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {199, 220}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3/2),x]
```

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/
4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^{3/2}}\,dx=\frac{x}{2a\sqrt{a+cx^4}}+\frac{\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$=\frac{x}{2a\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0106048, size = 55, normalized size = 0.51

$$\frac{x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+x}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3/2),x]
```

```
[Out] (x + x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/
(2*a*Sqrt[a + c*x^4])
```

Maple [C] time = 0.002, size = 94, normalized size = 0.9

$$\frac{x}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{1}{2\,a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(3/2),x)
```

```
[Out] 1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+a}}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [C] time = 0.680081, size = 36, normalized size = 0.33

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

3.223 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=818

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{cx^4+a}}\right)e^5}{2\left(-cd^4-ae^4\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}-\frac{\left(\sqrt{c}d^2\right.}{}$$

```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c*d*x*(d^2
 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (Sqrt[c]*d*e^2*x*Sqrt[
a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^5*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4) - a*e^4)^(3/2)
) - (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d
^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a +
c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^
2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.600813, antiderivative size = 818, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1729, 1222, 1179, 1198, 220, 1196, 1217, 1707, 1248, 741, 12, 725, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{cx^4+a}}\right)e^5}{2\left(-cd^4-ae^4\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}-\frac{\left(\sqrt{c}d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```


```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c*d*x*(d^2
 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (Sqrt[c]*d*e^2*x*Sqrt[
a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^5*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4) - a*e^4)^(3/2)
) - (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d
^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a +
c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
```

```
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^
2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 1222

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(
c*d^2 + a*e^2), Int[(c*d - c*e*x^2)*(a + c*x^4)^p, x], x] + Dist[e^2/(c*d^2
 + a*e^2), Int[(a + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
```

```
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = d\int \frac{1}{\left(d^2-e^2x^2\right)\left(a+cx^4\right)^{3/2}}\,dx - e\int \frac{x}{\left(d^2-e^2x^2\right)\left(a+cx^4\right)^{3/2}}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\left(a+cx^2\right)^{3/2}}\,dx, x, x^2\right)\right) + \frac{d\int \frac{cd^2+ce^2x^2}{\left(a+cx^4\right)^{3/2}}\,dx}{cd^4+ae^4} + \frac{\left(de^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+c}}}{cd^4+ae^4}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{d\int \frac{-cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{2a\left(cd^4+ae^4\right)} - \frac{e\,\text{Subst}\left(\int \frac{ae}{\left(d^2-e^2x\right)}\right.}{2a\left(cd^4\right.}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\left(-cd^4-ae^4\right)^{3/2}} + \frac{\sqrt[4]{c}de^4\left(\sqrt{a}+\right.}{2\sqrt[4]{a}\left(\sqrt{\right.}}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e^5\tan^{-}}{2\left(-c\right.}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e^5\tan^{-}}{2\left(-c\right.}$$

Mathematica [C] time = 0.882911, size = 434, normalized size = 0.53

$$\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(\sqrt[4]{c}d\left(\sqrt{ae^4+cd^4}\left(ae^3+cdx\left(d^2-dex+e^2x^2\right)\right)-ae^5\sqrt{a+cx^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)\right)-2\sqrt[4]{-1}a^{5/4}e^4\sqrt{\frac{cx^4}{a}+1}\sqrt{ae}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*Elliptic
E[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4)*d^2*((-I)*Sqrt[c]*
d^2 + Sqrt[a]*e^2)*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcS
inh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c^(1/4)*
d*(Sqrt[c*d^4 + a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)) - a*e^5*Sqrt
[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4
])]) - 2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1]))/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*(c*d^4 + a*e^4)^(3/2)*Sq
rt[a + c*x^4])
```

Maple [C] time = 0.013, size = 496, normalized size = 0.6

$$-2\,c\left(-1/4\,\frac{de^2x^3}{a\left(ae^4+cd^4\right)}+1/4\,\frac{ed^2x^2}{a\left(ae^4+cd^4\right)}-1/4\,\frac{d^3x}{a\left(ae^4+cd^4\right)}-1/4\,\frac{e^3}{\left(ae^4+cd^4\right)c}\right)\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{cd^3}{2\,a\left(ae^4+cd^4\right)}\sqrt{1-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] -2*c*(-1/4/a*d*e^2/(a*e^4+c*d^4)*x^3+1/4/a*e*d^2/(a*e^4+c*d^4)*x^2-1/4/a*d^
3/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/((x^4+a/c)*c)^(1/2)+1/2/a*c*d^3/
(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-1/2*I/a^(1/2)*c^(1/2)*d*e^2/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I))+e^3/(a*e^4+c*d^4)*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(
2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1
/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/
2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^4\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

3.224 $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n\right.}{4b^{3/4}(n+1)\left(\right.}$$

```
[Out] -((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d
)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c
- (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c + (-a)^(1/4)*d)*(1 + n))
```

Rubi [A] time = 0.729258, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6725, 831, 68}

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n\right.}{4b^{3/4}(n+1)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] -((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d
)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c
- (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c + (-a)^(1/4)*d)*(1 + n))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx &= \int \left(\frac{x(c+dx)^n}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^n}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx\\
&= \frac{1}{2}\int \frac{x(c+dx)^n}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^n}{\sqrt{-a}\sqrt{b}+bx^2}\,dx\\
&= \frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx+\frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}+\frac{(}{2b^{3/4}(}\right.\\
&= -\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4b^{3/4}}\\
&= -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(1+n)}-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(1+n)}-\frac{(c+}{}
\end{aligned}$$

Mathematica [C] time = 0.393453, size = 274, normalized size = 0.79

$$\frac{(c+dx)^{n+1}\left(-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{3/4}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(1 + n))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^3\left(dx+c\right)^n}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^n x^3}{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

3.225 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2\right.}{4b^{3/4}(n+2)\left(\sqrt[4]{b}\right.}$$

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d
)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 +
 n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c
- (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c + (-a)^(1/4)*d)*(2 + n))
```

Rubi [A] time = 0.570632, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6725, 831, 68}

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2\right.}{4b^{3/4}(n+2)\left(\sqrt[4]{b}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```


```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d
)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 +
 n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c
- (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c + (-a)^(1/4)*d)*(2 + n))
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx &= \int\left(\frac{x(c+dx)^{1+n}}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^{1+n}}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx\\
&= \frac{1}{2}\int\frac{x(c+dx)^{1+n}}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int\frac{x(c+dx)^{1+n}}{\sqrt{-a}\sqrt{b}+bx^2}\,dx\\
&= \frac{1}{2}\int\left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx+\frac{1}{2}\int\left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}+\right.\\
&= -\frac{\int\frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}-\frac{\int\frac{(c+dx)^{1+n}}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int\frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int\frac{(c+dx)^{1+n}}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4b^{3/4}}\\
&= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(2+n)}-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(2+n)}
\end{aligned}
$$

Mathematica [C] time = 0.224354, size = 274, normalized size = 0.79

$$
\frac{(c+dx)^{n+2}\left(-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{3/4}(n+2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(2 + n))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$
\int \frac{x^3\left(dx+c\right)^{1+n}}{bx^4+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{n+1}x^3}{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

3.226 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}} dx$

Optimal. Leaf size=1605

result too large to display

```
[Out] -((e^2*ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4
- b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a
 + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e
+ 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])])) + (e^2*
ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4 + b*d*S
qrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^
4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e + 2*c^2
*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*ArcTanh[
(4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
+ b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e
^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (e^2*ArcTanh[(4*a*e^
2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e +
2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4
])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*
a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt[d^2 -
4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e
]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*
x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
+ d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*x^4]) + (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2
 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
- d*Sqrt[d^2 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*e])^2)
, 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]
*(d - Sqrt[d^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2
 - 4*c*e]))*Sqrt[a + b*x^4]) - (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2 - 2*c*e + d
*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2
 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d
^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e]))*
Sqrt[a + b*x^4])
```

Rubi [A] time = 9.67962, antiderivative size = 1605, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 1725, 1217, 220, 1707, 1248, 725, 206}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -((e^2*ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4
- b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a
 + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e
+ 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])])) + (e^2*
```

```
ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4 + b*d*S
qrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^
4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e + 2*c^2
*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*ArcTanh[
(4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
+ b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e
^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (e^2*ArcTanh[(4*a*e^
2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e +
2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4
])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*
a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt[d^2 -
4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e
]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*
x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
+ d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*x^4]) + (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2
 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
- d*Sqrt[d^2 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*e])^2)
, 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]
*(d - Sqrt[d^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2
 - 4*c*e]))*Sqrt[a + b*x^4]) - (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2 - 2*c*e + d
*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2
 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d
^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e]))*
Sqrt[a + b*x^4])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+dx+ex^2\right)\sqrt{a+bx^4}}\,dx = \int \left(\frac{2e}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}} - \frac{2e}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)\sqrt{}}\right.$$

$$= \frac{(2e)\int \frac{1}{\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{1}{\left(d+\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}}$$

$$= -\frac{\left(4e^2\right)\int \frac{x}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} + \frac{\left(4e^2\right)\int \frac{x}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} - \Big(2e\Big(1$$

$$= -\frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x\right)\sqrt{a+bx^2}}\,dx, x, x^2\right)}{\sqrt{d^2-4ce}} + \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e\right.}\right.}{\sqrt{d^2-4ce}}$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\left(d^2-2ce\right)}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-4ce}\right)}} + -$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\left(d^2-2ce\right)}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-4ce}\right)}} + -$$

Mathematica [C] time = 7.37276, size = 1416, normalized size = 0.88

$$-\frac{i\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1}\Pi\left(-\frac{2i\sqrt{a}e^2}{\sqrt{b}\left(-d^2+2ce-\sqrt{d^4-4cd^2e}\right)};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)d^2}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}e\left(-d^2+2ce-\sqrt{d^4-4cd^2e}\right)\left(\frac{d^2-2ce+\sqrt{d^4-4cd^2e}}{2e^2}-\frac{d^2-2ce-\sqrt{d^4-4cd^2e}}{2e^2}\right)\sqrt{bx^4+a}}-\frac{i\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1}\Pi\left(-\frac{}{\sqrt{b}\left(-d^2\right.}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}e\left(-d^2+2ce+\sqrt{d^4-4cd^2e}\right)\left(\frac{d}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -((Sqrt[2]*e^2*(ArcTanh[(2*a*e^2 + b*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e])*x^
2)/(Sqrt[4*a*e^4 + b*(2*d^4 - 8*c*d^2*e + 4*c^2*e^2 - 2*d^3*Sqrt[d^2 - 4*c*
e] + 4*c*d*e*Sqrt[d^2 - 4*c*e])]*Sqrt[a + b*x^4])]/(2*Sqrt[2*a*e^4 + b*(d^4
 - 4*c*d^2*e + 2*c^2*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e
])]) - ArcTanh[(2*a*e^2 + b*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e])*x^2)/(Sqrt[
4*a*e^4 + 2*b*(d^4 - 4*c*d^2*e + 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*
e*Sqrt[d^2 - 4*c*e])]*Sqrt[a + b*x^4])]/(2*Sqrt[2*a*e^4 + b*(d^4 - 4*c*d^2*
e + 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])])))/Sqrt
[d^2 - 4*c*e]) - (I*d^2*Sqrt[1 - (I*Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[
b]*x^2)/Sqrt[a]]*EllipticPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e - S
qrt[d^4 - 4*c*d^2*e])), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[
(I*Sqrt[b])/Sqrt[a]]*e*(-d^2 + 2*c*e - Sqrt[d^4 - 4*c*d^2*e])*(-(d^2 - 2*c*
e - Sqrt[d^4 - 4*c*d^2*e])/(2*e^2) + (d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])/
(2*e^2))*Sqrt[a + b*x^4]) - (I*Sqrt[d^4 - 4*c*d^2*e]*Sqrt[1 - (I*Sqrt[b]*x^
2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*EllipticPi[((-2*I)*Sqrt[a]*e^
2)/(Sqrt[b]*(-d^2 + 2*c*e - Sqrt[d^4 - 4*c*d^2*e])), I*ArcSinh[Sqrt[(I*Sqrt
[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*e*(-d^2 + 2*c*e - Sqrt[d^
4 - 4*c*d^2*e])*(-(d^2 - 2*c*e - Sqrt[d^4 - 4*c*d^2*e])/(2*e^2) + (d^2 - 2*
c*e + Sqrt[d^4 - 4*c*d^2*e])/(2*e^2))*Sqrt[a + b*x^4]) - (I*d^2*Sqrt[1 - (I
*Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*EllipticPi[((-2*I)
*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e])), I*ArcSinh[S
qrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*e*(-d^2 + 2*c*
e + Sqrt[d^4 - 4*c*d^2*e])*((d^2 - 2*c*e - Sqrt[d^4 - 4*c*d^2*e])/(2*e^2) -
 (d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])/(2*e^2))*Sqrt[a + b*x^4]) + (I*Sqrt[
d^4 - 4*c*d^2*e]*Sqrt[1 - (I*Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)
/Sqrt[a]]*EllipticPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4
 - 4*c*d^2*e])), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt
[b])/Sqrt[a]]*e*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e])*((d^2 - 2*c*e - Sqrt
[d^4 - 4*c*d^2*e])/(2*e^2) - (d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])/(2*e^2))
*Sqrt[a + b*x^4])
```

Maple [C] time = 0.069, size = 1153, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2+b/e^3*(-4*c*e+d^2)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2
*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*(-4*c*e+d^2
```

```
)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b/e^4
*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*(-4*c*e+d^2)^(1/2
)*c*d+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(
1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*(-4*c*e+
d^2)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^
4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*(-4*c*e+d^2)^(1/2)*c
*d+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*e/(-d+(-4*c*e+d^2)^(1/2))*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2)
)^(1/2),-4*I*a^(1/2)/b^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4
+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*(-4*c*e+d^2)^(1/2)*c*
d+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^
(1/2)-2*b/e^3*c*d^2-b/e^3*(-4*c*e+d^2)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)/(b*x^4+
a)^(1/2)*b*x^2/e^2*d^2+1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-
2*b/e^3*c*d^2-b/e^3*(-4*c*e+d^2)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/
2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^
2)^(1/2)-2*b/e^3*c*d^2-b/e^3*(-4*c*e+d^2)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)/(b*x
^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2-b/e^3*(-4*c*e+d^2)^(1/2)*c*d+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*
a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)/(d+(-4*c*e+d^2)^(1/2))*e*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-4*I*a^(1/2)/b^(1/2)/(d+(-4*c*e
+d^2)^(1/2))^2*e^2,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + a}\left(ex^2 + dx + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^4}\left(c + dx + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+a}\left(ex^2+dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

3.227 $\int x^m \left(c \left(a + bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a^3cx^{m+1}\sqrt{c\left(a+bx^2\right)^2}}{(m+1)\left(a+bx^2\right)}+\frac{3a^2bcx^{m+3}\sqrt{c\left(a+bx^2\right)^2}}{(m+3)\left(a+bx^2\right)}+\frac{3ab^2cx^{m+5}\sqrt{c\left(a+bx^2\right)^2}}{(m+5)\left(a+bx^2\right)}+\frac{b^3cx^{m+7}\sqrt{c\left(a+bx^2\right)^2}}{(m+7)\left(a+bx^2\right)}$$

```
[Out] (a^3*c*x^(1 + m)*Sqrt[c*(a + b*x^2)^2])/((1 + m)*(a + b*x^2)) + (3*a^2*b*c*
x^(3 + m)*Sqrt[c*(a + b*x^2)^2])/((3 + m)*(a + b*x^2)) + (3*a*b^2*c*x^(5 +
m)*Sqrt[c*(a + b*x^2)^2])/((5 + m)*(a + b*x^2)) + (b^3*c*x^(7 + m)*Sqrt[c*(
a + b*x^2)^2])/((7 + m)*(a + b*x^2))
```

Rubi [A] time = 0.127775, antiderivative size = 205, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{a^3cx^{m+1}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(m+1)\left(a+bx^2\right)}+\frac{3a^2bcx^{m+3}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(m+3)\left(a+bx^2\right)}+\frac{3ab^2cx^{m+5}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(m+5)\left(a+bx^2\right)}+\frac{b^3cx^{m+7}\sqrt{\cdots}}{(m\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^(1 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((1 + m)*(a + b*x^2
)) + (3*a^2*b*c*x^(3 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((3 + m)*(
a + b*x^2)) + (3*a*b^2*c*x^(5 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(
(5 + m)*(a + b*x^2)) + (b^3*c*x^(7 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^
4])/((7 + m)*(a + b*x^2))
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^m \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int x^m \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^m \left(abc+b^2cx^2\right)^3 dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \left(a^3b^3c^3x^m+3a^2b^4c^3x^{2+m}+3ab^5c^3x^{4+m}+b^6c^3x^{6+m}\right) dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{a^3cx^{1+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(1+m)\left(a+bx^2\right)} + \frac{3a^2bcx^{3+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(3+m)\left(a+bx^2\right)} + \frac{3ab^2cx^{5+m}\sqrt{a^2c+2a}}{(5+m)\left(a+\right.}$$

Mathematica [A] time = 0.0933486, size = 132, normalized size = 0.82

$$\frac{x^{m+1}\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(3a^2b\left(m^3+13m^2+47m+35\right)x^2+a^3\left(m^3+15m^2+71m+105\right)+3ab^2\left(m^3+11m^2+31m+21\right)\right.}{(m+1)(m+3)(m+5)(m+7)\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2
*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4
+ b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a +
 b*x^2)^3)
```

Maple [A] time = 0.007, size = 200, normalized size = 1.2

$$\frac{x^{1+m}\left(b^3m^3x^6+9\,b^3m^2x^6+3\,ab^2m^3x^4+23\,b^3mx^6+33\,ab^2m^2x^4+15\,b^3x^6+3\,a^2bm^3x^2+93\,ab^2mx^4+39\,a^2bm^2x^2+63\right.}{(7+m)\,(5+m)\,(3+m)\,(1+m)\left(bx^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] x^(1+m)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(c*(b
*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Maxima [A] time = 1.08889, size = 161, normalized size = 1.

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3c^{\frac{3}{2}}x^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2c^{\frac{3}{2}}x^5+3\left(m^3+13\,m^2+47\,m+35\right)a^2bc^{\frac{3}{2}}x^3+\left(m^3+1\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*c^(3/2)*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a
*b^2*c^(3/2)*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*c^(3/2)*x^3 + (m^3 +
15*m^2 + 71*m + 105)*a^3*c^(3/2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)
```

Fricas [A] time = 1.54038, size = 528, normalized size = 3.28

$$\frac{\left(\left(b^3cm^3 + 9\,b^3cm^2 + 23\,b^3cm + 15\,b^3c\right)x^7 + 3\left(ab^2cm^3 + 11\,ab^2cm^2 + 31\,ab^2cm + 21\,ab^2c\right)x^5 + 3\left(a^2bcm^3 + 13\,a^2bcm^2\right.\right.}{am^4 + 16\,am^3 + 86\,am^2 + \left(bm^4 + 16\,bm^3 + 86\,bm^2 + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 + 1
1*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*b*
c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*a^3
*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 + 16*
a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2 + 17
6*a*m + 105*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23075, size = 479, normalized size = 2.98

$$\frac{\left(b^3m^3x^7x^m\mathrm{sgn}\left(bx^2 + a\right) + 9\,b^3m^2x^7x^m\mathrm{sgn}\left(bx^2 + a\right) + 3\,ab^2m^3x^5x^m\mathrm{sgn}\left(bx^2 + a\right) + 23\,b^3mx^7x^m\mathrm{sgn}\left(bx^2 + a\right) + 33\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b^
2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2*m
^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*x^3
*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*x^3*
x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*sgn(b*
x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(b*x^2
+ a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 + a) + 1
05*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.228 $\int x^5 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$

Optimal. Leaf size=143

$$\frac{3a^2bcx^8\sqrt{c\left(a+bx^2\right)^2}}{8\left(a+bx^2\right)}+\frac{a^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{b^3cx^{12}\sqrt{c\left(a+bx^2\right)^2}}{12\left(a+bx^2\right)}+\frac{3ab^2cx^{10}\sqrt{c\left(a+bx^2\right)^2}}{10\left(a+bx^2\right)}$$

```
[Out] (a^3*c*x^6*Sqrt[c*(a + b*x^2)^2])/(6*(a + b*x^2)) + (3*a^2*b*c*x^8*Sqrt[c*(
a + b*x^2)^2])/(8*(a + b*x^2)) + (3*a*b^2*c*x^10*Sqrt[c*(a + b*x^2)^2])/(10
*(a + b*x^2)) + (b^3*c*x^12*Sqrt[c*(a + b*x^2)^2])/(12*(a + b*x^2))
```

Rubi [A] time = 0.161853, antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1111, 645}

$$\frac{c\left(a+bx^2\right)^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{12b^3}-\frac{ac\left(a+bx^2\right)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3}+\frac{a^2c\left(a+bx^2\right)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^2*c*(a + b*x^2)^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(8*b^3) - (a*c*
(a + b*x^2)^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*b^3) + (c*(a + b*x^
2)^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(12*b^3)
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
x]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\int x^5 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx = \int x^5 \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x^2 \left(a^2c+2abcx+b^2cx^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \left(\frac{a^2(abc+b^2cx)^3}{b^2} - \frac{2a(abc+b^2cx)^4}{b^3c} + \frac{(abc+b^2cx)^5}{b^4c^2}\right) dx, x, x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{a^2c\left(a+bx^2\right)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3} - \frac{ac\left(a+bx^2\right)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3} + \frac{c\left(a+bx\right.}{}$$

Mathematica [A] time = 0.0215343, size = 63, normalized size = 0.44

$$\frac{x^6\left(45a^2bx^2+20a^3+36ab^2x^4+10b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{120\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3
*x^6))/(120*(a + b*x^2)^3)
```

Maple [A] time = 0.004, size = 60, normalized size = 0.4

$$\frac{x^6\left(10\,b^3x^6+36\,ab^2x^4+45\,a^2bx^2+20\,a^3\right)}{120\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*(c*(b*x^2+a)^2)^(3/
2)/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41231, size = 166, normalized size = 1.16

$$\frac{\left(10\, b^{3} c x^{12} + 36\, a b^{2} c x^{10} + 45\, a^{2} b c x^{8} + 20\, a^{3} c x^{6}\right) \sqrt{b^{2} c x^{4} + 2\, a b c x^{2} + a^{2} c}}{120 \left(b x^{2} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sqr
t(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21836, size = 97, normalized size = 0.68

$$\frac{1}{120} \left(10\, b^{3} x^{12} \mathrm{sgn}\left(b x^{2} + a\right) + 36\, a b^{2} x^{10} \mathrm{sgn}\left(b x^{2} + a\right) + 45\, a^{2} b x^{8} \mathrm{sgn}\left(b x^{2} + a\right) + 20\, a^{3} x^{6} \mathrm{sgn}\left(b x^{2} + a\right)\right) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*b
*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)
```

3.229 $\int x^4 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=143

$$\frac{3a^2bcx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{a^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{b^3cx^{11}\sqrt{c\left(a+bx^2\right)^2}}{11\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}$$

```
[Out] (a^3*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2)) + (3*a^2*b*c*x^7*Sqrt[c*(
a + b*x^2)^2])/(7*(a + b*x^2)) + (a*b^2*c*x^9*Sqrt[c*(a + b*x^2)^2])/(3*(a
+ b*x^2)) + (b^3*c*x^11*Sqrt[c*(a + b*x^2)^2])/(11*(a + b*x^2))
```

Rubi [A] time = 0.10539, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^{11}\sqrt{a^2c+2abcx^2+b^2cx^4}}{11\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{a^3cx^5\sqrt{a^2c+2abc}}{5\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a^2*
b*c*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (a*b^2*c*x
^9*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (b^3*c*x^11*Sqr
t[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(11*(a + b*x^2))
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^4\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^4\left(abc+b^2cx^2\right)^3dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int\left(a^3b^3c^3x^4+3a^2b^4c^3x^6+3ab^5c^3x^8+b^6c^3x^{10}\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{a^2c+2abcx^2+b^2}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0213691, size = 63, normalized size = 0.44

$$\frac{x^5\left(495a^2bx^2+231a^3+385ab^2x^4+105b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{1155\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105
*b^3*x^6))/(1155*(a + b*x^2)^3)
```

Maple [A] time = 0.003, size = 60, normalized size = 0.4

$$\frac{x^5\left(105\,b^3x^6+385\,ab^2x^4+495\,a^2bx^2+231\,a^3\right)}{1155\,\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*(c*(b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.996651, size = 63, normalized size = 0.44

$$\frac{1}{11}\,b^3c^{\frac{3}{2}}x^{11}+\frac{1}{3}\,ab^2c^{\frac{3}{2}}x^9+\frac{3}{7}\,a^2bc^{\frac{3}{2}}x^7+\frac{1}{5}\,a^3c^{\frac{3}{2}}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/5
*a^3*c^(3/2)*x^5
```

Fricas [A] time = 1.40636, size = 171, normalized size = 1.2

$$\frac{\left(105\,b^3cx^{11}+385\,ab^2cx^9+495\,a^2bcx^7+231\,a^3cx^5\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{1155\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5)
*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27508, size = 97, normalized size = 0.68

$$\frac{1}{1155}\left(105\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+385\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+495\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+231\,a^3x^5\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a^
2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)
```

3.230 $\int x^3 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=66

$$\frac{c\left(a+bx^2\right)^4 \sqrt{c\left(a+bx^2\right)^2}}{10b^2}-\frac{ac\left(a+bx^2\right)^3 \sqrt{c\left(a+bx^2\right)^2}}{8b^2}$$

```
[Out] -(a*c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b^2) + (c*(a + b*x^2)^4*Sqrt[
c*(a + b*x^2)^2])/(10*b^2)
```

Rubi [A] time = 0.113314, antiderivative size = 78, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1989, 1111, 640, 609}

$$\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}-\frac{a\left(a+bx^2\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(8*b^2) + (a^2*c +
 2*a*b*c*x^2 + b^2*c*x^4)^(5/2)/(10*b^2*c)
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int x\left(a^2c+2abcx+b^2cx^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}-\frac{a\,\text{Subst}\left(\int\left(a^2c+2abcx+b^2cx^2\right)^{3/2}dx,x,x^2\right)}{2b}$$
$$=-\frac{a\left(a+bx^2\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8b^2}+\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}$$

Mathematica [A] time = 0.0213701, size = 63, normalized size = 0.95

$$\frac{x^4\left(20a^2bx^2+10a^3+15ab^2x^4+4b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{40\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*
x^6))/(40*(a + b*x^2)^3)
```

Maple [A] time = 0.005, size = 60, normalized size = 0.9

$$\frac{x^4\left(4\,b^3x^6+15\,ab^2x^4+20\,a^2bx^2+10\,a^3\right)}{40\,\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*(c*(b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42047, size = 162, normalized size = 2.45

$$\frac{\left(4\,b^3cx^{10}+15\,ab^2cx^8+20\,a^2bcx^6+10\,a^3cx^4\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{40\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(b
^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21654, size = 65, normalized size = 0.98

$$\frac{1}{40}\left(4\,b^{3}x^{10} + 15\,ab^{2}x^{8} + 20\,a^{2}bx^{6} + 10\,a^{3}x^{4}\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^{2} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*c^(3/2)*sgn(b*
x^2 + a)
```

3.231 $\int x^2 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=143

$$\frac{3a^2bcx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{a^3cx^3\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}+\frac{b^3cx^9\sqrt{c\left(a+bx^2\right)^2}}{9\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}$$

```
[Out] (a^3*c*x^3*Sqrt[c*(a + b*x^2)^2])/(3*(a + b*x^2)) + (3*a^2*b*c*x^5*Sqrt[c*(
a + b*x^2)^2])/(5*(a + b*x^2)) + (3*a*b^2*c*x^7*Sqrt[c*(a + b*x^2)^2])/(7*(
a + b*x^2)) + (b^3*c*x^9*Sqrt[c*(a + b*x^2)^2])/(9*(a + b*x^2))
```

Rubi [A] time = 0.10218, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^9\sqrt{a^2c+2abcx^2+b^2cx^4}}{9\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{a^3cx^3\sqrt{a^2c+2abc}}{3\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (3*a^2*
b*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a*b^2*c
*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (b^3*c*x^9*Sq
rt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(9*(a + b*x^2))
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx = \int x^2 \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int x^2 \left(abc+b^2cx^2\right)^3 dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int \left(a^3b^3c^3x^2+3a^2b^4c^3x^4+3ab^5c^3x^6+b^6c^3x^8\right) dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{a^3cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\left(a+bx^2\right)} + \frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)} + \frac{3ab^2cx^7\sqrt{a^2c+2abcx^2+b}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0171205, size = 63, normalized size = 0.44

$$\frac{\left(189a^2bx^5+105a^3x^3+135ab^2x^7+35b^3x^9\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{315\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*
b^3*x^9))/(315*(a + b*x^2)^3)
```

Maple [A] time = 0.006, size = 60, normalized size = 0.4

$$\frac{x^3\left(35\,b^3x^6+135\,ab^2x^4+189\,a^2bx^2+105\,a^3\right)}{315\,\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*(c*(b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 1.02553, size = 63, normalized size = 0.44

$$\frac{1}{9}\,b^3c^{\frac{3}{2}}x^9+\frac{3}{7}\,ab^2c^{\frac{3}{2}}x^7+\frac{3}{5}\,a^2bc^{\frac{3}{2}}x^5+\frac{1}{3}\,a^3c^{\frac{3}{2}}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*a
^3*c^(3/2)*x^3
```

Fricas [A] time = 1.38646, size = 167, normalized size = 1.17

$$\frac{\left(35\,b^3cx^9+135\,ab^2cx^7+189\,a^2bcx^5+105\,a^3cx^3\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{315\,\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*sq
rt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2272, size = 97, normalized size = 0.68

$$\frac{1}{315}\left(35\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+135\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+189\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+105\,a^3x^3\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*b
*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)
```

3.232 $\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

```
[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)
```

Rubi [A] time = 0.0223109, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^2\right)^{3/2}\,dx, x, a+bx^2\right)}{2b}\\
&= \frac{\left(c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int x^3\,dx, x, a+bx^2\right)}{2b\left(a+bx^2\right)}\\
&= \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}
\end{aligned}$$

Mathematica [A] time = 0.0123346, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

Maple [B] time = 0.003, size = 59, normalized size = 1.8

$$\frac{x^2\left(b^3x^6+4\,ab^2x^4+6\,a^2bx^2+4\,a^3\right)}{8\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*(c*(b*x^2+a)^2)^(3/2)/(b*x^
2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39286, size = 153, normalized size = 4.78

$$\frac{\left(b^3cx^8+4\,ab^2cx^6+6\,a^2bcx^4+4\,a^3cx^2\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{8\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x^
4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23664, size = 34, normalized size = 1.06

$$\frac{\left(bx^2+a\right)^4 c^{\frac{3}{2}} \operatorname{sgn}\left(bx^2+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b
```

3.233 $\int \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=135

$$\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

```
[Out] (a^3*c*x*Sqrt[c*(a + b*x^2)^2])/(a + b*x^2) + (a^2*b*c*x^3*Sqrt[c*(a + b*x^
2)^2])/(a + b*x^2) + (3*a*b^2*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2))
+ (b^3*c*x^7*Sqrt[c*(a + b*x^2)^2])/(7*(a + b*x^2))
```

Rubi [A] time = 0.0530723, antiderivative size = 175, normalized size of antiderivative = 1.3, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1988, 1088, 194}

$$\frac{b^3x^7\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2c+2a\right.}{\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*x*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*
(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2*
c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2*c + 2
*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(7*(a + b*x^2)^3)
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] &&  !TrinomialMatchQ[u, x]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} \int \left(2abc+2b^2cx^2\right)^3 dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} \int \left(8a^3b^3c^3+24a^2b^4c^3x^2+24ab^5c^3x^4+8b^6c^3x^6\right) dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{a^3x\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2c+2abcx^2+b^2c\right.}{5\left(a+bx^2\right)^3}$$

Mathematica [A] time = 0.014702, size = 61, normalized size = 0.45

$$\frac{\left(35a^2bx^3+35a^3x+21ab^2x^5+5b^3x^7\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{35\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^
7))/(35*(a + b*x^2)^3)
```

Maple [A] time = 0.003, size = 58, normalized size = 0.4

$$\frac{x\left(5\,b^3x^6+21\,ab^2x^4+35\,a^2bx^2+35\,a^3\right)}{35\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*(c*(b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

Maxima [A] time = 1.03473, size = 58, normalized size = 0.43

$$\frac{1}{7}b^3c^{\frac{3}{2}}x^7+\frac{3}{5}ab^2c^{\frac{3}{2}}x^5+a^2bc^{\frac{3}{2}}x^3+a^3c^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3/
2)*x
```

Fricas [A] time = 1.43896, size = 158, normalized size = 1.17

$$\frac{\left(5\,b^{3}cx^{7}+21\,ab^{2}cx^{5}+35\,a^{2}bcx^{3}+35\,a^{3}cx\right)\sqrt{b^{2}cx^{4}+2\,abcx^{2}+a^{2}c}}{35\,\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.18606, size = 62, normalized size = 0.46

$$\frac{1}{35}\left(5\,b^{3}x^{7}+21\,ab^{2}x^{5}+35\,a^{2}bx^{3}+35\,a^{3}x\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^2
 + a)
```

3.234 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=139

$$\frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{a^3c\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}$$

```
[Out] (3*a^2*b*c*x^2*Sqrt[c*(a + b*x^2)^2])/(2*(a + b*x^2)) + (3*a*b^2*c*x^4*Sqrt
[c*(a + b*x^2)^2])/(4*(a + b*x^2)) + (b^3*c*x^6*Sqrt[c*(a + b*x^2)^2])/(6*(
a + b*x^2)) + (a^3*c*Sqrt[c*(a + b*x^2)^2]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.103352, antiderivative size = 183, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^6\sqrt{a^2c+2abcx^2+b^2cx^4}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{a^3c\log(x)\sqrt{a^2c+2al}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*c*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (3*
a*b^2*c*x^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (b^3*c
*x^6*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(6*(a + b*x^2)) + (a^3*c*Sqrt[a
^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx = \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x}\,dx$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x}\,dx}{b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \frac{\left(abc+b^2cx\right)^3}{x}\,dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x}+3ab^5c^3x+b^6c^3x^2\right)dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{a^2c+2abcx^2+}}{6\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0226401, size = 62, normalized size = 0.45

$$\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(bx^2\left(18a^2+9abx^2+2b^2x^4\right)+12a^3\log(x)\right)}{12\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*L
og[x]))/(12*(a + b*x^2)^3)
```

Maple [A] time = 0.009, size = 59, normalized size = 0.4

$$\frac{2\,b^3x^6+9\,ab^2x^4+18\,a^2bx^2+12\,a^3\ln(x)}{12\,\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x,x)
```

```
[Out] 1/12*(c*(b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x)
)/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44902, size = 163, normalized size = 1.17

$$\frac{\left(2 b^3 c x^6 + 9 a b^2 c x^4 + 18 a^2 b c x^2 + 12 a^3 c \log\left(x\right)\right) \sqrt{b^2 c x^4 + 2 a b c x^2 + a^2 c}}{12 \left(b x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt(
b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c \left(a + b x^2\right)^2\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x, x)
```

Giac [A] time = 1.22856, size = 99, normalized size = 0.71

$$\frac{1}{12} \left(2 b^3 x^6 \mathrm{sgn}\left(b x^2 + a\right) + 9 a b^2 x^4 \mathrm{sgn}\left(b x^2 + a\right) + 18 a^2 b x^2 \mathrm{sgn}\left(b x^2 + a\right) + 6 a^3 \log\left(x^2\right) \mathrm{sgn}\left(b x^2 + a\right)\right) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2*
sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)
```

3.235 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=134

$$\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

```
[Out] -((a^3*c*Sqrt[c*(a + b*x^2)^2])/(x*(a + b*x^2))) + (3*a^2*b*c*x*Sqrt[c*(a +
 b*x^2)^2])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt[c*(a + b*x^2)^2])/(a + b*x^2) +
 (b^3*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2))
```

Rubi [A] time = 0.0933017, antiderivative size = 178, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}+\frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}-\frac{a^3c\sqrt{a^2c+2abcx^2+}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] -((a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(x*(a + b*x^2))) + (3*a^2*b
*c*x*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt
[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (b^3*c*x^5*Sqrt[a^2*c + 2*
a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2))
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^2}\,dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x^2}dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x^2}+3ab^5c^3x^2+b^6c^3x^4\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}+\frac{ab^2cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}$$

Mathematica [A] time = 0.0237818, size = 62, normalized size = 0.46

$$\frac{\left(15a^2bx^2-5a^3+5ab^2x^4+b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{5x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(
5*x*(a + b*x^2)^3)
```

Maple [A] time = 0.005, size = 60, normalized size = 0.5

$$-\frac{-b^3x^6-5\,ab^2x^4-15\,a^2bx^2+5\,a^3}{5\,x\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*(c*(b*x^2+a)^2)^(3/2)/x/(b*x
^2+a)^3
```

Maxima [A] time = 1.02658, size = 65, normalized size = 0.49

$$\frac{b^3c^{\frac{3}{2}}x^6+5\,ab^2c^{\frac{3}{2}}x^4+15\,a^2bc^{\frac{3}{2}}x^2-5\,a^3c^{\frac{3}{2}}}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*c^(3/2)*x^6 + 5*a*b^2*c^(3/2)*x^4 + 15*a^2*b*c^(3/2)*x^2 - 5*a^3*c
^(3/2))/x
```

Fricas [A] time = 1.41895, size = 151, normalized size = 1.13

$$\frac{\left(b^3cx^6+5\,ab^2cx^4+15\,a^2bcx^2-5\,a^3c\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{5\left(bx^3+ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)
```

Giac [A] time = 1.25013, size = 93, normalized size = 0.69

$$\frac{1}{5}\left(b^3x^5\mathrm{sgn}\left(bx^2+a\right)+5\,ab^2x^3\mathrm{sgn}\left(bx^2+a\right)+15\,a^2bx\mathrm{sgn}\left(bx^2+a\right)-\frac{5\,a^3\mathrm{sgn}\left(bx^2+a\right)}{x}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(b
*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)
```

3.236 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=140

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}$$

```
[Out] -(a^3*c*Sqrt[c*(a + b*x^2)^2])/(2*x^2*(a + b*x^2)) + (3*a*b^2*c*x^2*Sqrt[c*
(a + b*x^2)^2])/(2*(a + b*x^2)) + (b^3*c*x^4*Sqrt[c*(a + b*x^2)^2])/(4*(a +
 b*x^2)) + (3*a^2*b*c*Sqrt[c*(a + b*x^2)^2]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.104984, antiderivative size = 184, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}-\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -(a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b
^2*c*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (b^3*c*x^
4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (3*a^2*b*c*Sqrt[
a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^3}\,dx$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x^3}\,dx}{b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \frac{\left(abc+b^2cx\right)^3}{x^2}\,dx, x, x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \left(3ab^5c^3+\frac{a^3b^3c^3}{x^2}+\frac{3a^2b^4c^3}{x}+b^6c^3x\right)dx, x, x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$

$$= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\left(a+bx^2\right)} + \frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)} + \frac{b^3cx^4\sqrt{a^2c+2abcx^2+b^2c}}{4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0227166, size = 65, normalized size = 0.46

$$-\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(-12a^2bx^2\log(x)+2a^3-6ab^2x^4-b^3x^6\right)}{4x^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -((c*(a + b*x^2)^2)^(3/2)*(2*a^3 - 6*a*b^2*x^4 - b^3*x^6 - 12*a^2*b*x^2*Log
[x]))/(4*x^2*(a + b*x^2)^3)
```

Maple [A] time = 0.009, size = 61, normalized size = 0.4

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2b\ln(x)\,x^2-2\,a^3}{4\,\left(bx^2+a\right)^3x^2}\left(c\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x^3,x)
```

```
[Out] 1/4*(c*(b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*a^2*b*ln(x)*x^2-2*a^3)/(b
*x^2+a)^3/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45133, size = 163, normalized size = 1.16

$$\frac{\left(b^3cx^6 + 6\,ab^2cx^4 + 12\,a^2bcx^2\log\left(x\right) - 2\,a^3c\right)\sqrt{b^2cx^4 + 2\,abcx^2 + a^2c}}{4\left(bx^4 + ax^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)
```

Giac [A] time = 1.27034, size = 123, normalized size = 0.88

$$\frac{1}{4}\left(b^3x^4\mathrm{sgn}\left(bx^2 + a\right) + 6\,ab^2x^2\mathrm{sgn}\left(bx^2 + a\right) + 6\,a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2 + a\right) - \frac{2\left(3\,a^2bx^2\mathrm{sgn}\left(bx^2 + a\right) + a^3\mathrm{sgn}\left(bx^2 + a\right)\right)}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2)
*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2)*
c^(3/2)
```

3.237 $\int x^2 \left(c \left(a+bx^2\right)^3\right)^{3/2} dx$

Optimal. Leaf size=253

$$-\frac{21a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{2}{32}$$

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3]
)/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x
^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a
 + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*
x^2)^3])/12 - (21*a^(9/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(1024*b^(3/2)*(1 + (b*x^2)/a)^(3/2))
```

Rubi [A] time = 0.246433, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 279, 321, 217, 206}

$$-\frac{21a^6c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3]
)/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x
^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a
 + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*
x^2)^3])/12 - (21*a^6*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(1024*b^(3/2)*(a + b*x^2)^(3/2))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(a+bx^2\right)^{9/2}dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(3ac\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(a+bx^2\right)^{7/2}dx}{4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(21a^2c\sqrt{c\left(a+bx^2\right)^3}\right)}{40\left(a+b\right.}$$

$$= \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+b\right.}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+b\right.$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+b\right.$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+b\right.$$

Mathematica [A] time = 0.150343, size = 143, normalized size = 0.57

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\left(12144a^2b^3x^6+11432a^3b^2x^4+4910a^4bx^2+315a^5+6272ab^4x^8+1280b^5x^{10}\right)-315a^{11/2}\right.}{15360b^{3/2}\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(315*a^5 + 4910*a^4
*b*x^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*
x^10) - 315*a^(11/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(15360*b^(3/2)*(a + b*x
^2)^4*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.03, size = 236, normalized size = 0.9

$$\frac{1}{15360\, b\left(bx^2+a\right)^3 c}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(1280\, x^7\left(bcx^2+ac\right)^{5/2} b^3\sqrt{bc}+3712\sqrt{bc}\left(bcx^2+ac\right)^{5/2} x^5 ab^2+3440\sqrt{bc}\left(bcx^2 \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] 1/15360*(c*(b*x^2+a)^3)^(3/2)/b*(1280*x^7*(b*c*x^2+a*c)^(5/2)*b^3*(b*c)^(1/
2)+3712*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^5*a*b^2+3440*(b*c)^(1/2)*(b*c*x^2
+a*c)^(5/2)*x^3*a^2*b+840*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x*a^3-210*(b*c)^(
1/2)*(b*c*x^2+a*c)^(3/2)*x*a^4*c-315*(b*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*x*a^5*
c^2-315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a^6*c^3)/(b
*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(bx^2+a\right)^3 c\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)
```

Fricas [A] time = 1.98179, size = 969, normalized size = 3.83

$$\left[\frac{315\left(a^6bcx^2+a^7c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(1280\,b^5cx^{11}+6272\,ab^4cx^9+1214\right.}{30720\left(b^2x^2+ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*
sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2
```

```
*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3
*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15360*(
315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c)*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c))
+ (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2
*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3
*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30981, size = 239, normalized size = 0.94

$$\frac{1}{15360}\left(\frac{315\,a^6c\log\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\operatorname{sgn}\left(bx^2+a\right)}{\sqrt{bc}b}+\left(\frac{315\,a^5\operatorname{sgn}\left(bx^2+a\right)}{b}+2\left(2455\,a^4\operatorname{sgn}\left(bx^2+a\right)+4\left(1429\,a\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a)
+ 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b^4*
x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(b*c*
x^2 + a*c)*x)*c
```

3.238 $\int x \left(c \left(a+b x^2\right)^3\right)^{3/2} \, dx$

Optimal. Leaf size=32

$$\frac{c \left(a+b x^2\right)^4 \sqrt{c \left(a+b x^2\right)^3}}{11 b}$$

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rubi [A] time = 0.0265127, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$\frac{c \left(a+b x^2\right)^4 \sqrt{c \left(a+b x^2\right)^3}}{11 b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \left(c \left(a+b x^2\right)^3\right)^{3/2} \, dx &= \frac{\text{Subst}\left(\int \left(c x^3\right)^{3/2} \, dx,x,a+b x^2\right)}{2 b}\\
&= \frac{\left(c \sqrt{c \left(a+b x^2\right)^3}\right) \text{Subst}\left(\int x^{9/2} \, dx,x,a+b x^2\right)}{2 b \left(a+b x^2\right)^{3/2}}\\
&= \frac{c \left(a+b x^2\right)^4 \sqrt{c \left(a+b x^2\right)^3}}{11 b}
\end{aligned}$$

Mathematica [A] time = 0.0176661, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

Maple [A] time = 0.003, size = 26, normalized size = 0.8

$$\frac{bx^2+a}{11\,b}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] 1/11*(b*x^2+a)/b*(c*(b*x^2+a)^3)^(3/2)
```

Maxima [B] time = 1.0748, size = 95, normalized size = 2.97

$$\frac{\left(b^4c^{\frac{3}{2}}x^8+4\,ab^3c^{\frac{3}{2}}x^6+6\,a^2b^2c^{\frac{3}{2}}x^4+4\,a^3bc^{\frac{3}{2}}x^2+a^4c^{\frac{3}{2}}\right)\left(bx^2+a\right)^{\frac{3}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x^4 + 4*a^3
*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

Fricas [B] time = 1.62584, size = 181, normalized size = 5.66

$$\frac{\left(b^4cx^8+4\,ab^3cx^6+6\,a^2b^2cx^4+4\,a^3bcx^2+a^4c\right)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)*
sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral(x*(c*(a + b*x**2)**3)**(3/2), x)
```

Giac [B] time = 1.19807, size = 450, normalized size = 14.06

$$1155\left(bcx^2+ac\right)^{\frac{3}{2}}a^4\mathrm{sgn}\left(bx^2+a\right)-\frac{924\left(5\left(bcx^2+ac\right)^{\frac{3}{2}}ac-3\left(bcx^2+ac\right)^{\frac{5}{2}}\right)a^3\mathrm{sgn}\left(bx^2+a\right)}{c}+\frac{198\left(35\left(bcx^2+ac\right)^{\frac{3}{2}}a^2c^2-42\left(bcx^2+ac\right)^{\frac{5}{2}}ac+15\left(bcx^2\right.\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*(1155*(b*c*x^2 + a*c)^(3/2)*a^4*sgn(b*x^2 + a) - 924*(5*(b*c*x^2 + a
*c)^(3/2)*a*c - 3*(b*c*x^2 + a*c)^(5/2))*a^3*sgn(b*x^2 + a)/c + 198*(35*(b*
c*x^2 + a*c)^(3/2)*a^2*c^2 - 42*(b*c*x^2 + a*c)^(5/2)*a*c + 15*(b*c*x^2 + a
*c)^(7/2))*a^2*sgn(b*x^2 + a)/c^2 - 44*(105*(b*c*x^2 + a*c)^(3/2)*a^3*c^3 -
 189*(b*c*x^2 + a*c)^(5/2)*a^2*c^2 + 135*(b*c*x^2 + a*c)^(7/2)*a*c - 35*(b*
c*x^2 + a*c)^(9/2))*a*sgn(b*x^2 + a)/c^3 + (1155*(b*c*x^2 + a*c)^(3/2)*a^4*
c^4 - 2772*(b*c*x^2 + a*c)^(5/2)*a^3*c^3 + 2970*(b*c*x^2 + a*c)^(7/2)*a^2*c
^2 - 1540*(b*c*x^2 + a*c)^(9/2)*a*c + 315*(b*c*x^2 + a*c)^(11/2))*sgn(b*x^2
 + a)/c^4)/b
```

3.239 $\int \left(c \left(a + bx^2\right)^3\right)^{3/2} dx$

Optimal. Leaf size=207

$$\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^{7/2}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])
/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (
9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c
*(a + b*x^2)^3])/10 + (63*a^(7/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[(Sqrt[b]*
x)/Sqrt[a]])/(256*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rubi [A] time = 0.0733558, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6720, 195, 217, 206}

$$\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^5c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])
/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (
9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c
*(a + b*x^2)^3])/10 + (63*a^5*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(256*Sqrt[b]*(a + b*x^2)^(3/2))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{9/2}dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(9ac\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}dx}{10\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(63a^2c\sqrt{c\left(a+bx^2\right)^3}\right)\int}{80\left(a+bx^2\right.}$$

$$= \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c(}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right.}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx$$

Mathematica [A] time = 0.1129, size = 132, normalized size = 0.64

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\left(1368a^2b^2x^4+1490a^3bx^2+965a^4+656ab^3x^6+128b^4x^8\right)+315a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{1280\sqrt{b}\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2),x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(965*a^4 + 1490*a^3
*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) + 315*a^(9/2)*ArcS
inh[(Sqrt[b]*x)/Sqrt[a]]))/(1280*Sqrt[b]*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a])
```

Maple [A] time = 0.006, size = 205, normalized size = 1.

$$\frac{1}{1280\,c\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(128\,x^5\left(bcx^2+ac\right)^{5/2}b^2\sqrt{bc}+400\left(bcx^2+ac\right)^{5/2}\sqrt{bc}x^3ab+440\left(bcx^2+ac\right)^{5/2}\sqrt{bc}xa^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] 1/1280*(c*(b*x^2+a)^3)^(3/2)*(128*x^5*(b*c*x^2+a*c)^(5/2)*b^2*(b*c)^(1/2)+4
00*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^3*a*b+440*(b*c*x^2+a*c)^(5/2)*(b*c)^(1
/2)*x*a^2+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)*x*a^3*c+315*(b*c*x^2+a*c)^(1/
2)*(b*c)^(1/2)*x*a^4*c^2+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*
c)^(1/2))*a^5*c^3)/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2), x)
```

Fricas [A] time = 1.7172, size = 887, normalized size = 4.29

$$\left[\frac{315\left(a^5bcx^2+a^6c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(128\,b^4cx^9+656\,ab^3cx^7+1368\,a^2b^2cx\right.}{2560\left(bx^2+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^
2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*s
qrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*
c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(
-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*sq
rt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a*b^3*c*
x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 +
 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2), x)
```

Giac [A] time = 1.23069, size = 207, normalized size = 1.

$$-\frac{1}{1280}\left(\frac{315\,a^5c\log\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}}-\left(965\,a^4\mathrm{sgn}\left(bx^2+a\right)+2\left(745\,a^3b\mathrm{sgn}\left(bx^2+a\right)+4\left(171\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4*(
171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(b*x
^2 + a))*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

3.240 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=192

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{1}{7}ac\left(\right.$$

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2)
 + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*
(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^3*c*Sqrt
[c*(a + b*x^2)^3]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(1 + (b*x^2)/a)^(3/2)
```

Rubi [A] time = 0.217076, antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 50, 63, 208}

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\left(a+bx^2\right)^{3/2}}+\frac{1}{7}ac\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2)
 + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*
(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^(9/2)*c*
Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a + b*x^2)^(3/2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}\,dx, x, x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(ac\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\right.}{2\left(a+bx^2\right)^{3}}$$

$$= \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a\right.$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2$$

Mathematica [A] time = 0.0749732, size = 111, normalized size = 0.58

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{a+bx^2}\left(408a^2b^2x^4+506a^3bx^2+563a^4+185ab^3x^6+35b^4x^8\right)-315a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{315\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^
2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))
```

Maple [A] time = 0.013, size = 221, normalized size = 1.2

$$\frac{1}{315\,c\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(35\sqrt{ac}\left(bcx^2+ac\right)^{5/2}x^4b^2+115\sqrt{ac}\left(bcx^2+ac\right)^{5/2}x^2ab-315\ln\left(2\frac{\sqrt{ac}\sqrt{bcx^2+ac}+ac}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x,x)
```

```
[Out] 1/315*(c*(b*x^2+a)^3)^(3/2)*(35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^4*b^2+115
*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^2*a*b-315*ln(2*((a*c)^(1/2)*(b*c*x^2+a*c
)^(1/2)+a*c)/x)*a^5*c^3-46*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2+105*(a*c)^(1
/2)*(b*c*x^2+a*c)^(3/2)*a^3*c+315*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4*c^2+1
89*a^2*(c*(b*x^2+a))^(5/2)*(a*c)^(1/2))/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(
a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)
```

Fricas [A] time = 1.72856, size = 864, normalized size = 4.5

$$\left[\frac{315\left(a^4bcx^2+a^5c\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+2\left(35\,b^4cx^8+185\,ab^3cx^6+408\,a^2b^2cx^4+\right.}{630\left(bx^2+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 +
 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a
*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x
^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*
c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*ar
ctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^
```

```
2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2
*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 +
3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

Giac [A] time = 1.23287, size = 194, normalized size = 1.01

$$\frac{1}{315}\left(\frac{315\,a^5c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}}+\frac{315\sqrt{bcx^2+ac}a^4c^{36}+105\left(bcx^2+ac\right)^{\frac{3}{2}}a^3c^{35}+63\left(bcx^2+ac\right)^{\frac{5}{2}}a^2c^{34}+45\left(bcx^2+\right.}{c^{36}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^5*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/sqrt(-a*c) + (315*s
qrt(b*c*x^2 + a*c)*a^4*c^36 + 105*(b*c*x^2 + a*c)^(3/2)*a^3*c^35 + 63*(b*c*
x^2 + a*c)^(5/2)*a^2*c^34 + 45*(b*c*x^2 + a*c)^(7/2)*a*c^33 + 35*(b*c*x^2 +
 a*c)^(9/2)*c^32)/c^36)*c*sgn(b*x^2 + a)
```

3.241 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=208

$$\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{315a^{5/2}\sqrt{b}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+\right.}$$

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2
)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16
 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[
c*(a + b*x^2)^3])/x + (315*a^(5/2)*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[
(Sqrt[b]*x)/Sqrt[a]])/(128*(1 + (b*x^2)/a)^(3/2))
```

Rubi [A] time = 0.18896, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 277, 195, 217, 206}

$$\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{315a^4\sqrt{b}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128\left(a+bx^2\right)^{3/2}}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2
)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16
 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[
c*(a + b*x^2)^3])/x + (315*a^4*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*(a + b*x^2)^(3/2))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}dx}{\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(63abc\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+\right.}{8\left(a+bx^2\right)^{3/2}}$$

$$= \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx\right.}}{x}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bc$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bc$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bc$$

Mathematica [C] time = 0.0159161, size = 65, normalized size = 0.31

$$-\frac{a^4\left(c\left(a+bx^2\right)^3\right)^{3/2}\,{}_2F_1\left(-\frac{9}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] -((a^4*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[-9/2, -1/2, 1/2, -((b*x^2)
/a)])/(x*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a]))
```

Maple [A] time = 0.008, size = 215, normalized size = 1.

$$\frac{1}{128\,c\left(bx^2+a\right)^3 x}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(16\left(bcx^2+ac\right)^{5/2}\sqrt{bc}x^4b^2+56\left(bcx^2+ac\right)^{5/2}\sqrt{bc}x^2ab+210\left(bcx^2+ac\right)^{3/2}\sqrt{bc}x^2a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x^2,x)
```

```
[Out] 1/128*(c*(b*x^2+a)^3)^(3/2)*(16*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^4*b^2+56*
(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^2*a*b+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)
*x^2*a^2*b*c+315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*x^2*a^3*b*c^2+315*ln((b*c*
x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*x*a^4*b*c^3-128*(b*c*x^2+a*
c)^(5/2)*(b*c)^(1/2)*a^2)/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(bx^2+a\right)^3 c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)
```

Fricas [A] time = 1.74705, size = 873, normalized size = 4.2

$$\left[\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{bc}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{bc}x}{bx^2+a}\right)+2\left(16\,b^4cx^8+88\,ab^3cx^6+210\,a^2b^2cx^4\cdots\right.}{256\left(bx^3+ax\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^
4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c
*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(-b*c
)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-b*c)
*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 21
0*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x
^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**2, x)
```

Giac [A] time = 1.33512, size = 250, normalized size = 1.2

$$\frac{1}{256}\left(\frac{512\,\sqrt{bc}a^5c\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac}-315\,\sqrt{bc}a^4\log\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2\right)\mathrm{sgn}\left(bx^2+a\right)+2\left(325\,a^3b\mathrm{sgn}\left(bx^2+a\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/256*(512*sqrt(b*c)*a^5*c*sgn(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b*c*x^2 + a*
c))^2 - a*c) - 315*sqrt(b*c)*a^4*log((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2)
*sgn(b*x^2 + a) + 2*(325*a^3*b*sgn(b*x^2 + a) + 2*(105*a^2*b^2*sgn(b*x^2 +
a) + 4*(2*b^4*x^2*sgn(b*x^2 + a) + 11*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(
b*c*x^2 + a*c)*x)*c
```

3.242 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=202

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-$$

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*
(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a +
 b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3
])/(2*x^2) - (9*a^2*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[1 + (b*x^2)/a]])
/(2*(1 + (b*x^2)/a)^(3/2))
```

Rubi [A] time = 0.215951, antiderivative size = 204, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6720, 266, 47, 50, 63, 208}

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^{7/2}bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\left(a+bx^2\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*
(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a +
 b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3
])/(2*x^2) - (9*a^(7/2)*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*(a + b*x^2)^(3/2))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx, x, x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)}{4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} + \frac{\left(9abc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \right.}{4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} +$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a\right.}{}$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2$$

Mathematica [C] time = 0.0187556, size = 48, normalized size = 0.24

$$\frac{b\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 +
 (b*x^2)/a])/(11*a^2)
```

Maple [A] time = 0.01, size = 238, normalized size = 1.2

$$\frac{1}{70\,c\left(bx^2+a\right)^3x^2}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(10\sqrt{ac}\left(bcx^2+ac\right)^{5/2}x^4b^2 - 315\ln\left(2\,\frac{\sqrt{ac}\sqrt{bcx^2+ac}+ac}{x}\right)x^2a^4bc^3 - 4\sqrt{ac}\left(bcx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x^3,x)
```

```
[Out] 1/70*(c*(b*x^2+a)^3)^(3/2)*(10*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^4*b^2-315*
ln(2*((a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)+a*c)/x)*x^2*a^4*b*c^3-4*(a*c)^(1/2)*(
b*c*x^2+a*c)^(5/2)*x^2*a*b+105*(b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*b*c+
315*(b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a^3*b*c^2+42*a*b*(c*(b*x^2+a))^(5/2
)*(a*c)^(1/2)*x^2-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2)/(b*x^2+a)^3/(c*(b
*x^2+a))^(3/2)/c/(a*c)^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(bx^2+a\right)^3 c\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)
```

Fricas [A] time = 1.67576, size = 890, normalized size = 4.41

$$\left[\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+2\left(10\,b^4cx^8+58\,ab^3cx^6+156\,a^2b\right.}{140\left(bx^4+ax^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b
*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c
)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*
b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x
^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*
c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 58*a*b^3
*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3
*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**3, x)
```

Giac [A] time = 1.25614, size = 204, normalized size = 1.01

$$\frac{1}{70}\left(\frac{315\,a^4c^2\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}}-\frac{35\,\sqrt{bcx^2+ac}a^4c}{bx^2}+\frac{2\left(140\,\sqrt{bcx^2+ac}a^3c^{15}+35\left(bcx^2+ac\right)^{\frac{3}{2}}a^2c^{14}+14\left(bcx^2+ac\right)^{\frac{5}{2}}\right.}{c^{14}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*c^2*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/sqrt(-a*c) - 35*sq
rt(b*c*x^2 + a*c)*a^4*c/(b*x^2) + 2*(140*sqrt(b*c*x^2 + a*c)*a^3*c^15 + 35*
(b*c*x^2 + a*c)^(3/2)*a^2*c^14 + 14*(b*c*x^2 + a*c)^(5/2)*a*c^13 + 5*(b*c*x
^2 + a*c)^(7/2)*c^12)/c^14)*b*sgn(b*x^2 + a)
```

3.243 $\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a}c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (Sqrt[a]*c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b
*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] time = 0.14265, antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6720, 288, 217, 206}

$$\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2} dx = \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{x^2}{\left(a+bx^2\right)^{3/2}} dx$$
$$= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}} dx}{b}$$
$$= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b} + \frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.0660019, size = 89, normalized size = 1.16

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\left(\frac{c}{a+bx^2}\right)^{3/2}\left(\left(a+bx^2\right)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*(c/(a + b*x^2))^(3/2)*Sqrt[1 + (b*x^2)/a]*(-(Sqrt[a]*Sqrt[b]*x*Sqr
t[1 + (b*x^2)/a]) + (a + b*x^2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/b^(3/2)
```

Maple [A] time = 0.01, size = 59, normalized size = 0.8

$$\left(bx^2+a\right)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\left(-xb^{\frac{3}{2}}+\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)b\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(b*x^2+a))^(3/2),x)
```

```
[Out] (c/(b*x^2+a))^(3/2)*(b*x^2+a)*(-x*b^(3/2)+ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b*(
b*x^2+a)^(1/2))/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*(c/(b*x^2 + a))^(3/2), x)
```

Fricas [A] time = 1.55354, size = 294, normalized size = 3.82

$$\left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}} - c\sqrt{\frac{c}{b}}\log\left(-2\,bcx^2 - ac - 2\left(b^2x^3 + abx\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2\,b}, -\frac{cx\sqrt{\frac{c}{bx^2+a}} + c\sqrt{-\frac{c}{b}}\arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{b}}}{c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b^
2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a))
 + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

Giac [A] time = 1.20456, size = 96, normalized size = 1.25

$$-\left(\frac{cx\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,b} + \frac{c\log\left(\left|-\sqrt{bc}x + \sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}\,b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] -(c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*log(abs(-sqrt(b*c)*x + sqr
t(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b))*c
```

3.244 $\int x \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=21

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

Rubi [A] time = 0.0175141, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

[In] Int[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x}\right)^{3/2} dx, x, a+bx^2\right)}{2b}\\
&= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, a+bx^2\right)}{2b}\\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}
\end{aligned}$$

Mathematica [A] time = 0.0048062, size = 21, normalized size = 1.

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Maple [A] time = 0.004, size = 26, normalized size = 1.2

$$-\frac{bx^2+a}{b}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c/(b*x^2+a))^(3/2),x)
```

```
[Out] -(b*x^2+a)/b*(c/(b*x^2+a))^(3/2)
```

Maxima [A] time = 0.967719, size = 26, normalized size = 1.24

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Fricas [A] time = 1.49485, size = 35, normalized size = 1.67

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Sympy [A] time = 1.4509, size = 53, normalized size = 2.52

$$\begin{cases} -\frac{ac^{\frac{3}{2}}\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{b} - c^{\frac{3}{2}}x^2\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((-a*c**(3/2)*(1/(a + b*x**2))**(3/2)/b - c**(3/2)*x**2*(1/(a + b*
x**2))**(3/2), Ne(b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

Giac [A] time = 1.19169, size = 38, normalized size = 1.81

$$-\frac{c^2\operatorname{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] -c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

3.245 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=21

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Rubi [A] time = 0.017496, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6720, 191}

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx\\ &= \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}\end{aligned}$$

Mathematica [A] time = 0.0047915, size = 21, normalized size = 1.

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Maple [A] time = 0.003, size = 26, normalized size = 1.2

$$\frac{x\left(bx^2+a\right)}{a}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2),x)
```

```
[Out] (b*x^2+a)*x/a*(c/(b*x^2+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c/(b*x^2 + a))^(3/2), x)
```

Fricas [A] time = 1.48451, size = 36, normalized size = 1.71

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] c*x*sqrt(c/(b*x^2 + a))/a
```

Sympy [A] time = 1.46707, size = 66, normalized size = 3.14

$$\begin{cases} c^{\frac{3}{2}}x\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}+\frac{bc^{\frac{3}{2}}x^3\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for } a\neq 0\\ -\frac{c^{\frac{3}{2}}x\left(\frac{1}{b}\right)^{\frac{3}{2}}\left(\frac{1}{x^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x*(1/(a + b*x**2))**(3/2) + b*c**(3/2)*x**3*(1/(a + b*x
**2))**(3/2)/a, Ne(a, 0)), (-c**(3/2)*x*(1/b)**(3/2)*(x**(-2))**(3/2)/2, Tr
```

```
ue))
```

Giac [A] time = 1.2531, size = 38, normalized size = 1.81

$$\frac{c^2x\text{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+aca}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

3.246 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=71

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{a}$$

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*ArcT
anh[Sqrt[1 + (b*x^2)/a]])/a
```

Rubi [A] time = 0.135793, antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 51, 63, 208}

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx\\
&= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0089605, size = 38, normalized size = 0.54

$$
\frac{c\sqrt{\frac{c}{a+bx^2}}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a])/a
```

Maple [A] time = 0.005, size = 64, normalized size = 0.9

$$
-(bx^2+a)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\left(\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)a\sqrt{bx^2+a}-a^{\frac{3}{2}}\right)a^{-\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2)/x,x)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*x^
2+a)^(1/2)-a^(3/2))/a^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53063, size = 289, normalized size = 4.07

$$\left[\frac{c\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac-2\left(abx^2+a^2\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a},\ \frac{c\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+c\sqrt{\frac{c}{bx^2+a}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2 +
 a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a*s
qrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)
```

Giac [A] time = 1.17665, size = 88, normalized size = 1.24

$$c^3\left(\frac{\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}ac}+\frac{1}{\sqrt{bcx^2+ac}ac}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] c^3*(arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a*c) + 1/(sqrt(b*c*
x^2 + a*c)*a*c))*sgn(b*x^2 + a)
```

3.247 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/a^2
```

Rubi [A] time = 0.114111, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6720, 271, 191}

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/a^2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx\\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{\left(2bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0078871, size = 32, normalized size = 0.67

$$-\frac{c\left(a+2bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

Maple [A] time = 0.003, size = 37, normalized size = 0.8

$$-\frac{\left(bx^2+a\right)\left(2\,bx^2+a\right)}{xa^2}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2)/x^2,x)
```

```
[Out] -(b*x^2+a)*(2*b*x^2+a)*(c/(b*x^2+a))^(3/2)/x/a^2
```

Maxima [A] time = 1.00312, size = 62, normalized size = 1.29

$$-\frac{2\,b^2c^{\frac{3}{2}}x^4+3\,abc^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 + a)^(3/2)*a
^2*x)
```

Fricas [A] time = 1.4804, size = 65, normalized size = 1.35

$$-\frac{\left(2\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

Giac [A] time = 1.18548, size = 109, normalized size = 2.27

$$-\left(\frac{bcx\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}a^2}-\frac{2\sqrt{bc}c\mathrm{sgn}\left(bx^2+a\right)}{\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2
+ a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a))*c
```

3.248 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}+\frac{3bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}$$

```
[Out] (-3*b*c*Sqrt[c/(a + b*x^2)])/(2*a^2) - (c*Sqrt[c/(a + b*x^2)])/(2*a*x^2) +
(3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]]
)/(2*a^2)
```

Rubi [A] time = 0.152599, antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 51, 63, 208}

$$-\frac{3c\left(a+bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}+\frac{3bc\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/(a*x^2) - (3*c*Sqrt[c/(a + b*x^2)]*(a + b*x^2))/(2*
a^2*x^2) + (3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx\\
&= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}+\frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}-\frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^2}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}-\frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^2}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}+\frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0091202, size = 40, normalized size = 0.38

$$-\frac{bc\sqrt{\frac{c}{a+bx^2}}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/
a^2)
```

Maple [A] time = 0.007, size = 79, normalized size = 0.8

$$-\frac{bx^2+a}{2\,x^2}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\left(3\,a^{3/2}x^2b-3\,\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)\sqrt{bx^2+a}x^2ab+a^{\frac{5}{2}}\right)a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2)/x^3,x)
```

```
[Out] -1/2*(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(3*a^(3/2)*x^2*b-3*ln(2*(a^(1/2)*(b*x^2+
a)^(1/2)+a)/x)*(b*x^2+a)^(1/2)*x^2*a*b+a^(5/2))/a^(7/2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59462, size = 382, normalized size = 3.67

$$\left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac+2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)-2\,(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2}, -\frac{3\,bcx^2\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+(3\,bcx^2+ac}{2\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*sqrt(c/
(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^
2*x^2), -1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/
c) + (3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

Giac [A] time = 1.19783, size = 135, normalized size = 1.3

$$-\frac{1}{2}\,bc^4\left(\frac{3\,\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c^2}-\frac{3\,bcx^2+ac}{\left(\sqrt{bcx^2+ac}ac-(bcx^2+ac)^{\frac{3}{2}}\right)a^2c^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*c^4*(3*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2*c^2) -
 (3*b*c*x^2 + a*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2 + a*c)^(3/2))*a^2*c
^2))*sgn(b*x^2 + a)
```

3.249 $\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=138

$$\frac{6a^2\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^4}-\frac{2a^3\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^4}+\frac{2\left(a+bx^2\right)^4\left(c\sqrt{a+bx^2}\right)^{3/2}}{19b^4}-\frac{2a\left(a+bx^2\right)^3\left(c\sqrt{a+bx^2}\right)}{5b^4}$$

```
[Out] (-2*a^3*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^4) + (6*a^2*(c*Sqrt[a +
 b*x^2])^(3/2)*(a + b*x^2)^2)/(11*b^4) - (2*a*(c*Sqrt[a + b*x^2])^(3/2)*(a
+ b*x^2)^3)/(5*b^4) + (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)^4)/(19*b^4)
```

Rubi [A] time = 0.1879, antiderivative size = 152, normalized size of antiderivative = 1.1, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{6a^2c\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^4}-\frac{2a^3c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^4}+\frac{2c\left(a+bx^2\right)^{9/2}\sqrt{c\sqrt{a+bx^2}}}{19b^4}-\frac{2ac\left(a+bx^2\right)^{7/2}\sqrt{c\sqrt{a+}}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a^3*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^4) + (6*a^2*c*Sqr
t[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^4) - (2*a*c*Sqrt[c*Sqrt[a + b
*x^2]]*(a + b*x^2)^(7/2))/(5*b^4) + (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2
)^(9/2))/(19*b^4)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^7 \left(a+bx^2\right)^{3/4} dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^3(a+bx)^{3/4} dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{3/4}}{b^3}+\frac{3a^2(a+bx)^{7/4}}{b^3}-\frac{3a(a+bx)^{11/4}}{b^3}+\frac{(a+bx)^{15/4}}{b^3}\right) dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2a^3c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^4}+\frac{6a^2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^4}-\frac{2ac\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right.}{5b^4}$$

Mathematica [A] time = 0.0372595, size = 63, normalized size = 0.46

$$\frac{2\left(a+bx^2\right)\left(224a^2bx^2-128a^3-308ab^2x^4+385b^3x^6\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a*
b^2*x^4 + 385*b^3*x^6))/(7315*b^4)
```

Maple [A] time = 0.006, size = 58, normalized size = 0.4

$$-\frac{\left(2\,bx^2+2\,a\right)\left(-385\,b^3x^6+308\,ab^2x^4-224\,a^2bx^2+128\,a^3\right)}{7315\,b^4}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] -2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b*
x^2+a)^(1/2))^(3/2)/b^4
```

Maxima [A] time = 1.0125, size = 115, normalized size = 0.83

$$-\frac{2\left(1045\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}a^3c^6-1995\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}a^2c^4+1463\left(\sqrt{bx^2+a}c\right)^{\frac{15}{2}}ac^2-385\left(\sqrt{bx^2+a}c\right)^{\frac{19}{2}}\right)}{7315\,b^4c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1045*(sqrt(b*x^2 + a)*c)^(7/2)*a^3*c^6 - 1995*(sqrt(b*x^2 + a)*c)^
(11/2)*a^2*c^4 + 1463*(sqrt(b*x^2 + a)*c)^(15/2)*a*c^2 - 385*(sqrt(b*x^2 +
```

```
a)*c)^(19/2))/(b^4*c^8)
```

Fricas [A] time = 1.70495, size = 180, normalized size = 1.3

$$\frac{2\left(385\,b^{4}cx^{8}+77\,ab^{3}cx^{6}-84\,a^{2}b^{2}cx^{4}+96\,a^{3}bcx^{2}-128\,a^{4}c\right)\sqrt{bx^{2}+a}\sqrt{\sqrt{bx^{2}+a}c}}{7315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2
- 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16246, size = 81, normalized size = 0.59

$$\frac{2\left(385\left(bx^{2}+a\right)^{\frac{19}{4}}-1463\left(bx^{2}+a\right)^{\frac{15}{4}}a+1995\left(bx^{2}+a\right)^{\frac{11}{4}}a^{2}-1045\left(bx^{2}+a\right)^{\frac{7}{4}}a^{3}\right)c^{\frac{3}{2}}}{7315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/7315*(385*(b*x^2 + a)^(19/4) - 1463*(b*x^2 + a)^(15/4)*a + 1995*(b*x^2 +
a)^(11/4)*a^2 - 1045*(b*x^2 + a)^(7/4)*a^3)*c^(3/2)/b^4
```

3.250 $\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=102

$$\frac{2a^2\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^3}+\frac{2\left(a+bx^2\right)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b^3}-\frac{4a\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^3}$$

```
[Out] (2*a^2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^3) - (4*a*(c*Sqrt[a + b*
x^2])^(3/2)*(a + b*x^2)^2)/(11*b^3) + (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x
^2)^3)/(15*b^3)
```

Rubi [A] time = 0.157332, antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{2a^2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^3}+\frac{2c\left(a+bx^2\right)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{15b^3}-\frac{4ac\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a^2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^3) - (4*a*c*Sqrt[c
*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^3) + (2*c*Sqrt[c*Sqrt[a + b*x^2]
]*(a + b*x^2)^(7/2))/(15*b^3)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^5\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^2(a+bx)^{3/4}dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/4}}{b^2}-\frac{2a(a+bx)^{7/4}}{b^2}+\frac{(a+bx)^{11/4}}{b^2}\right)dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=\frac{2a^2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^3}-\frac{4ac\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^3}+\frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{7/2}}{15b^3}$$

Mathematica [A] time = 0.0234396, size = 52, normalized size = 0.51

$$\frac{2\left(a+bx^2\right)\left(32a^2-56abx^2+77b^2x^4\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{1155b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4)
)/(1155*b^3)
```

Maple [A] time = 0.006, size = 47, normalized size = 0.5

$$\frac{\left(2\,bx^2+2\,a\right)\left(77\,b^2x^4-56\,abx^2+32\,a^2\right)}{1155\,b^3}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] 2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*(c*(b*x^2+a)^(1/2))^(3/2)/b
^3
```

Maxima [A] time = 0.999604, size = 86, normalized size = 0.84

$$\frac{2\left(165\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}a^2c^4-210\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}ac^2+77\left(\sqrt{bx^2+a}c\right)^{\frac{15}{2}}\right)}{1155\,b^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(165*(sqrt(b*x^2 + a)*c)^(7/2)*a^2*c^4 - 210*(sqrt(b*x^2 + a)*c)^(11
/2)*a*c^2 + 77*(sqrt(b*x^2 + a)*c)^(15/2))/(b^3*c^6)
```

Fricas [A] time = 1.72647, size = 151, normalized size = 1.48

$$\frac{2\left(77\,b^{3}cx^{6}+21\,ab^{2}cx^{4}-24\,a^{2}bcx^{2}+32\,a^{3}c\right)\sqrt{bx^{2}+a}\sqrt{\sqrt{bx^{2}+a}c}}{1155\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*x
^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3
```

Sympy [A] time = 117.062, size = 116, normalized size = 1.14

$$\begin{cases} \frac{64a^{3}c^{\frac{3}{2}}\left(a+bx^{2}\right)^{\frac{3}{4}}}{1155b^{3}}-\frac{16a^{2}c^{\frac{3}{2}}x^{2}\left(a+bx^{2}\right)^{\frac{3}{4}}}{385b^{2}}+\frac{2ac^{\frac{3}{2}}x^{4}\left(a+bx^{2}\right)^{\frac{3}{4}}}{55b}+\frac{2c^{\frac{3}{2}}x^{6}\left(a+bx^{2}\right)^{\frac{3}{4}}}{15} & \text{for}\ b\neq 0\\ \frac{x^{6}\left(\sqrt{a}c\right)^{\frac{3}{2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((64*a**3*c**(3/2)*(a + b*x**2)**(3/4)/(1155*b**3) - 16*a**2*c**(3
/2)*x**2*(a + b*x**2)**(3/4)/(385*b**2) + 2*a*c**(3/2)*x**4*(a + b*x**2)**(
3/4)/(55*b) + 2*c**(3/2)*x**6*(a + b*x**2)**(3/4)/15, Ne(b, 0)), (x**6*(sqr
t(a)*c)**(3/2)/6, True))
```

Giac [A] time = 1.19837, size = 62, normalized size = 0.61

$$\frac{2\left(77\left(bx^{2}+a\right)^{\frac{15}{4}}-210\left(bx^{2}+a\right)^{\frac{11}{4}}a+165\left(bx^{2}+a\right)^{\frac{7}{4}}a^{2}\right)c^{\frac{3}{2}}}{1155\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*(77*(b*x^2 + a)^(15/4) - 210*(b*x^2 + a)^(11/4)*a + 165*(b*x^2 + a)^
(7/4)*a^2)*c^(3/2)/b^3
```

3.251 $\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=66

$$\frac{2\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^2}-\frac{2a\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^2}$$

```
[Out] (-2*a*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^2) + (2*(c*Sqrt[a + b*x^2
])^(3/2)*(a + b*x^2)^2)/(11*b^2)
```

Rubi [A] time = 0.136821, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{2c\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^2}-\frac{2ac\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^2) + (2*c*Sqrt[c*Sq
rt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^3\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x(a+bx)^{3/4}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(-\frac{a(a+bx)^{3/4}}{b}+\frac{(a+bx)^{7/4}}{b}\right)dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2ac\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^2}+\frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^2}$$

Mathematica [A] time = 0.0202312, size = 41, normalized size = 0.62

$$\frac{2\left(a+bx^2\right)\left(7bx^2-4a\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{77b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)
```

Maple [A] time = 0.006, size = 36, normalized size = 0.6

$$-\frac{\left(2\,bx^2+2\,a\right)\left(-7\,bx^2+4\,a\right)}{77\,b^2}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] -2/77*(b*x^2+a)*(-7*b*x^2+4*a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2
```

Maxima [A] time = 0.997918, size = 58, normalized size = 0.88

$$-\frac{2\left(11\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}ac^2-7\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}\right)}{77\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/(
b^2*c^4)
```

Fricas [A] time = 1.68915, size = 119, normalized size = 1.8

$$\frac{2\left(7\,b^{2}cx^{4}+3\,abcx^{2}-4\,a^{2}c\right)\sqrt{bx^{2}+a}\sqrt{\sqrt{bx^{2}+a}c}}{77\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2
+ a)*c)/b^2
```

Sympy [A] time = 52.6148, size = 87, normalized size = 1.32

$$\begin{cases} -\frac{8a^{2}c^{\frac{3}{2}}\left(a+bx^{2}\right)^{\frac{3}{4}}}{77b^{2}}+\frac{6ac^{\frac{3}{2}}x^{2}\left(a+bx^{2}\right)^{\frac{3}{4}}}{77b}+\frac{2c^{\frac{3}{2}}x^{4}\left(a+bx^{2}\right)^{\frac{3}{4}}}{11} & \text{for}\ b\neq 0\\ \frac{x^{4}\left(\sqrt{a}c\right)^{\frac{3}{2}}}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((-8*a**2*c**(3/2)*(a + b*x**2)**(3/4)/(77*b**2) + 6*a*c**(3/2)*x*
*2*(a + b*x**2)**(3/4)/(77*b) + 2*c**(3/2)*x**4*(a + b*x**2)**(3/4)/11, Ne(
b, 0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))
```

Giac [A] time = 1.23085, size = 43, normalized size = 0.65

$$\frac{2\left(7\left(bx^{2}+a\right)^{\frac{11}{4}}-11\left(bx^{2}+a\right)^{\frac{7}{4}}a\right)c^{\frac{3}{2}}}{77\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/77*(7*(b*x^2 + a)^(11/4) - 11*(b*x^2 + a)^(7/4)*a)*c^(3/2)/b^2
```

3.252 $\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=36

$$\frac{2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)
```

Rubi [A] time = 0.0195566, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1591, 15, 30}

$$\frac{2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int\left(c\sqrt{x}\right)^{3/2}\,dx,x,a+bx^2\right)}{2b}\\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^{3/4}\,dx,x,a+bx^2\right)}{2b\sqrt[4]{a+bx^2}}\\
&= \frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b}
\end{aligned}$$

Mathematica [A] time = 0.0067952, size = 31, normalized size = 0.86

$$\frac{2\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$\frac{2\,bx^2+2\,a}{7\,b}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] 2/7*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b
```

Maxima [A] time = 0.993363, size = 34, normalized size = 0.94

$$\frac{2\left(bx^2+a\right)\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b
```

Fricas [A] time = 1.6987, size = 85, normalized size = 2.36

$$\frac{2\left(bcx^2+ac\right)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b
```

Sympy [A] time = 20.1388, size = 58, normalized size = 1.61

$$\begin{cases} \frac{2ac^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}{7b}+\frac{2c^{\frac{3}{2}}x^2\left(a+bx^2\right)^{\frac{3}{4}}}{7} & \text{for}\ b\neq 0 \\ \frac{x^2\left(\sqrt{a}c\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((2*a*c**(3/2)*(a + b*x**2)**(3/4)/(7*b) + 2*c**(3/2)*x**2*(a + b*
x**2)**(3/4)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))
```

Giac [A] time = 1.13623, size = 23, normalized size = 0.64

$$\frac{2\left(bx^2+a\right)^{\frac{7}{4}}c^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b
```

3.253 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2))/3 + ((c*Sqrt[a + b*x^2])^(3/2)*ArcTan[(1 + (b
*x^2)/a)^(1/4)])/(1 + (b*x^2)/a)^(3/4) - ((c*Sqrt[a + b*x^2])^(3/2)*ArcTanh
[(1 + (b*x^2)/a)^(1/4)])/(1 + (b*x^2)/a)^(3/4)
```

Rubi [A] time = 0.157942, antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 266, 50, 63, 298, 203, 206}

$$\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}-\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}+\frac{2}{3}c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/3 + (a^(3/4)*c*Sqrt[c*Sqrt[a
+ b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(a + b*x^2)^(1/4) - (a^(3/4)*c
*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(a + b*x^2)^(1
/4)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x}\,dx}{\sqrt[4]{a+bx^2}}\\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}\\
&= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}\\
&= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{\left(2ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx,x,\sqrt[4]{a+bx^2}\right)}{b\sqrt[4]{a+bx^2}}\\
&= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx,x,\sqrt[4]{a+bx^2}\right)}{\sqrt[4]{a+bx^2}}+\frac{\left(ac\sqrt{c\sqrt{a+b\right.}}}{}\\
&= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}-\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+}}{\sqrt[4]{}}\right.}{\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.038205, size = 96, normalized size = 0.82

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(3a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)-3a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)+2\left(a+bx^2\right)^{3/4}\right)}{3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] ((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*x
^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a +
 b*x^2)^(3/4))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)
```

Giac [B] time = 1.17565, size = 257, normalized size = 2.2

$$-\frac{1}{12}\left(6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x
^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) + 3*sq
rt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a
) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4))*c^(3/2)
```

3.254 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2}+\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] -(c*Sqrt[a + b*x^2])^(3/2)/(2*x^2) + (3*b*(c*Sqrt[a + b*x^2])^(3/2)*ArcTan[
(1 + (b*x^2)/a)^(1/4)])/(4*a*(1 + (b*x^2)/a)^(3/4)) - (3*b*(c*Sqrt[a + b*x^
2])^(3/2)*ArcTanh[(1 + (b*x^2)/a)^(1/4)])/(4*a*(1 + (b*x^2)/a)^(3/4))
```

Rubi [A] time = 0.162866, antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 266, 47, 63, 298, 203, 206}

$$-\frac{c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{2x^2}+\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}-\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] -(c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/(2*x^2) + (3*b*c*Sqrt[c*Sqrt[a
 + b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^(1/4)*(a + b*x^2)^(1/4))
 - (3*b*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^
(1/4)*(a + b*x^2)^(1/4))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^3}\,dx}{\sqrt[4]{a+bx^2}}\\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^2}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2x^2}+\frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx,x,x^2\right)}{8\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2x^2}+\frac{\left(3c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx,x,\sqrt[4]{a+bx^2}\right)}{2\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2x^2}-\frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx,x,\sqrt[4]{a+bx^2}\right)}{4\sqrt[4]{a+bx^2}}+\frac{\left(3bc\sqrt{c\sqrt{a\right.}}\right.}{}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2x^2}+\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}-\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right.}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.0126223, size = 50, normalized size = 0.38

$$\frac{2b\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{bx^2}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] (2*b*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*Hypergeometric2F1[7/4, 2, 11/4,
1 + (b*x^2)/a])/(7*a^2)
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

Giac [A] time = 1.19769, size = 282, normalized size = 2.12

$$-\frac{1}{16}\left(\frac{6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}-\frac{3\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x
^2 + a)^(1/4))/(-a)^(1/4))/a + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/a - 3*sqrt(2)*(-a)^(3/4
)*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a))/a
+ 3*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^2 + a) + sqrt(-a))/a + 8*(b*x^2 + a)^(3/4)/(b*x^2))*b*c^(3/2)
```

3.255 $\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=152

$$\frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2} E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)}+\frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b}$$

```
[Out] (2*a*x*(c*Sqrt[a + b*x^2])^(3/2))/(15*b) + (2*x^3*(c*Sqrt[a + b*x^2])^(3/2)
)/9 - (4*a^2*x*(c*Sqrt[a + b*x^2])^(3/2))/(15*b*(a + b*x^2)) + (4*a^(3/2)*(
c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b
^(3/2)*(1 + (b*x^2)/a)^(3/4))
```

Rubi [A] time = 0.165506, antiderivative size = 191, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6720, 279, 321, 229, 227, 196}

$$\frac{4a^{5/2}c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt{a+bx^2}}-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2}{9}cx^3\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}+\frac{2acx\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-4*a^2*c*x*Sqrt[c*Sqrt[a + b*x^2]])/(15*b*Sqrt[a + b*x^2]) + (2*a*c*x*Sqrt
[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/(15*b) + (2*c*x^3*Sqrt[c*Sqrt[a + b*x^
2]]*Sqrt[a + b*x^2])/9 + (4*a^(5/2)*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(3/2)*Sqrt[a +
b*x^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^2\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$

$$=\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\int\frac{x^2}{\sqrt[4]{a+bx^2}}dx}{3\sqrt[4]{a+bx^2}}$$

$$=\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(2a^2c\sqrt{c\sqrt{a+bx^2}}\right)\int\frac{1}{\sqrt[4]{a+bx^2}}dx}{15b\sqrt[4]{a+bx^2}}$$

$$=\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(2a^2c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \ldots}{15b\sqrt{a+bx^2}}$$

$$=-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{\left(2a^2c\sqrt{c \ldots}\right.}{\ldots}$$

$$=-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{4a^{5/2}c\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 0.0583909, size = 68, normalized size = 0.45

$$\frac{2x\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}+a+bx^2\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2,
 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int x^2 \left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}cx^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**2*(c*sqrt(a + b*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.256 $\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=119

$$\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2))/5 + (6*a*x*(c*Sqrt[a + b*x^2])^(3/2))/(5*(a
 + b*x^2)) - (6*Sqrt[a]*(c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(1 + (b*x^2)/a)^(3/4))
```

Rubi [A] time = 0.0522481, antiderivative size = 146, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6720, 195, 229, 227, 196}

$$-\frac{6a^{3/2}c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt{a+bx^2}}+\frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}}+\frac{2}{5}cx\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (6*a*c*x*Sqrt[c*Sqrt[a + b*x^2]])/(5*Sqrt[a + b*x^2]) + (2*c*x*Sqrt[c*Sqrt[
a + b*x^2]]*Sqrt[a + b*x^2])/5 - (6*a^(3/2)*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 +
(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*Sq
rt[a + b*x^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \left(a+bx^2\right)^{3/4} dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} + \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}} dx}{5\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} + \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}} dx}{5\sqrt{a+bx^2}}$$

$$= \frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}} + \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} - \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}} dx}{5\sqrt{a+bx^2}}$$

$$= \frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}} + \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} - \frac{6a^{3/2}c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{5\sqrt{b}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0066299, size = 52, normalized size = 0.44

$$\frac{x\left(c\sqrt{a+bx^2}\right)^{3/2} {}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (x*(c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)
])/(1 + (b*x^2)/a)^(3/4)
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}c, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

3.257 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2}-\frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] -((c*Sqrt[a + b*x^2])^(3/2)/x) + (3*b*x*(c*Sqrt[a + b*x^2])^(3/2))/(a + b*x
^2) - (3*Sqrt[b]*(c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(Sqrt[a]*(1 + (b*x^2)/a)^(3/4))
```

Rubi [A] time = 0.137411, antiderivative size = 142, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6720, 277, 229, 227, 196}

$$-\frac{c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{x}+\frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}}-\frac{3\sqrt{a}\sqrt{b}c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] (3*b*c*x*Sqrt[c*Sqrt[a + b*x^2]])/Sqrt[a + b*x^2] - (c*Sqrt[c*Sqrt[a + b*x^
2]]*Sqrt[a + b*x^2])/x - (3*Sqrt[a]*Sqrt[b]*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 +
(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[a + b*x^
2]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^2}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{x} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{x} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{a+bx^2}}$$

$$= \frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{x} - \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2\sqrt{a+bx^2}}$$

$$= \frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{x} - \frac{3\sqrt{a}\sqrt{b}c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.0101591, size = 55, normalized size = 0.48

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} {}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] -(((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a
)])/(x*(1 + (b*x^2)/a)^(3/4)))
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{bx^2 + a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2 + a}\sqrt{\sqrt{bx^2 + a}c}c}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sqrt{a + bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.258 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4} dx$

Optimal. Leaf size=154

$$-\frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}+\frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)}-\frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3}$$

```
[Out] -(c*Sqrt[a + b*x^2])^(3/2)/(3*x^3) - (b*(c*Sqrt[a + b*x^2])^(3/2))/(2*a*x)
+ (b^2*x*(c*Sqrt[a + b*x^2])^(3/2))/(2*a*(a + b*x^2)) - (b^(3/2)*(c*Sqrt[a
+ b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(1
+ (b*x^2)/a)^(3/4))
```

Rubi [A] time = 0.157801, antiderivative size = 193, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6720, 277, 325, 229, 227, 196}

$$\frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}}-\frac{b^{3/2}c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt{a+bx^2}}-\frac{bc\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{2ax}-\frac{c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] (b^2*c*x*Sqrt[c*Sqrt[a + b*x^2]])/(2*a*Sqrt[a + b*x^2]) - (c*Sqrt[c*Sqrt[a
+ b*x^2]]*Sqrt[a + b*x^2])/(3*x^3) - (b*c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a +
b*x^2])/(2*a*x) - (b^(3/2)*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 + (b*x^2)/a)^(1/4)*
EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*Sqrt[a + b*x^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^4}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{3x^3} + \frac{\left(bc\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2ax} + \frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2ax} + \frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,d}{4a\sqrt{a+bx^2}}$$

$$= \frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2ax} - \frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1}\right.}{4a\sqrt{a}}$$

$$= \frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2ax} - \frac{b^{3/2}c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1}}{2\sqrt{}}$$

Mathematica [C] time = 0.0123121, size = 57, normalized size = 0.37

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] -((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^2)/a
)]])/(3*x^3*(1 + (b*x^2)/a)^(3/4))
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(c\sqrt{bx^2+a}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.259 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rubi [A] time = 0.0254128, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1981, 612, 621, 204}

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```


```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```


Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-ab+(a+b)x-x^2}\,dx$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} + \frac{1}{8}(a-b)^2 \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} + \frac{1}{4}(a-b)^2 \operatorname{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x}}\right.$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2 \tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

Mathematica [A] time = 0.157026, size = 106, normalized size = 1.49

$$\frac{(a-x)\left((a-b)^{5/2}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)-(a-x)(b-x)(a+b-2x)\right)}{4(x-a)\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] ((a - x)*(-((a + b - 2*x)*(a - x)*(b - x)) + (a - b)^(5/2)*Sqrt[(a - x)/(a
- b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*(-a + x)*Sqrt[(a -
x)*(-b + x)])
```

Maple [A] time = 0.013, size = 122, normalized size = 1.7

$$-\frac{a+b-2x}{4}\sqrt{-ab+(a+b)x-x^2} - \frac{ab}{4}\arctan\left(\left(x-\frac{b}{2}-\frac{a}{2}\right)\frac{1}{\sqrt{-ab+(a+b)x-x^2}}\right) + \frac{a^2}{8}\arctan\left(\left(x-\frac{b}{2}-\frac{a}{2}\right)\frac{1}{\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*b-1/2*a)/(-a*b+(a
+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*
a^2+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27196, size = 209, normalized size = 2.94

$$-\frac{1}{8}\left(a^2-2ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2x)}{2\left(ab-(a+b)x+x^2\right)}\right) - \frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(-a+x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

Giac [A] time = 1.20003, size = 82, normalized size = 1.15

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}\left(-a+b\right)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.260 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rubi [A] time = 0.0428554, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1988, 1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] &&  !TrinomialMatchQ[u, x]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\left(1-x^2\right)\left(3+x^2\right)}\,dx &= \int \sqrt{3-2x^2-x^4}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx+8\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [C] time = 0.0571395, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(\sqrt{-x^4-2x^2+3}x-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] time = 0.017, size = 114, normalized size = 2.4

$$\frac{x}{3}\sqrt{-x^4-2\,x^2+3}+\frac{2\,\mathrm{EllipticF}\left(x,i/3\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-x^4-2\,x^2+3}}+\frac{2\,\mathrm{EllipticF}\left(x,i/3\sqrt{3}\right)-2\,\mathrm{EllipticE}\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x^2+1)*(x^2+3))^(1/2),x)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2+3\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 - 2x^2 + 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(1 - x^2\right)\left(x^2 + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x**2)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2 + 3\right)\left(x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.261 $\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rubi [A] time = 0.0130767, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1981, 621, 204}

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx &= \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}} \, dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{-4-x^2} \, dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right) \\
&= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.0260302, size = 72, normalized size = 2.25

$$-\frac{2\sqrt{a-b}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[(a - x)/(a - b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[
a - b]])/Sqrt[(a - x)*(-b + x)]
```

Maple [A] time = 0.003, size = 28, normalized size = 0.9

$$\arctan\left(\left(x-\frac{b}{2}-\frac{a}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x)
```

```
[Out] arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34545, size = 111, normalized size = 3.47

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(-a+x)\,(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

Giac [A] time = 1.24874, size = 30, normalized size = 0.94

$$\arcsin\left(\frac{a+b-2x}{a-b}\right)\operatorname{sgn}(-a+b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin((a + b - 2*x)/(a - b))*sgn(-a + b)
```

3.262 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rubi [A] time = 0.0131315, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1988, 1095, 419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]

[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] &&  !TrinomialMatchQ[u, x]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\left(1-x^2\right)\left(3+x^2\right)}}\,dx &= \int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx\\ &= 2\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0157029, size = 18, normalized size = 1.5

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] time = 0.007, size = 43, normalized size = 3.6

$$\frac{\text{EllipticF}\left(x, \frac{i}{3}\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x^2+1)*(x^2+3))^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4-2x^2+3}}{x^4+2x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 - 2*x^2 + 3)/(x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(1-x^2)(x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] Integral(1/sqrt((1 - x**2)*(x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^2+3\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.263 $\int x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=244

$$\frac{(c+dx^2)\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}+\frac{(c+dx^2)^3\,(}{6bd}$$

```
[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)
/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTa
nh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5
/2)*d^(7/2))
```

Rubi [A] time = 0.332508, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 463, 455, 385, 208}

$$\frac{(c+dx^2)\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}+\frac{(c+dx^2)^3\,(}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)
/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTa
nh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5
/2)*d^(7/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)^3}{6bd^2e} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^2\left(-3\left(2a^2d^2e^2-(bce-ade)^2\right)+6bc^2dex^2\right)}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)^3}{6bd^2e} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{3d(bc-ad)(3bc+\cdots}{(be\cdots}\right.}{24\cdots}$$

$$= \frac{\left(11b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\cdots}{6b\cdots}$$

$$= \frac{\left(11b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\cdots}{6b\cdots}$$

Mathematica [A] time = 0.5243, size = 198, normalized size = 0.81

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-b\sqrt{d}\left(c+dx^2\right)\left(3a^2d^2-2abd\left(dx^2-2c\right)+b^2\left(-15c^2+10cdx^2-8d^2x^4\right)\right)-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)(bc-ad)^{3/2}\sqrt{\frac{b(c+d\cdots}{bc-a\cdots}}\cdots}{\sqrt{a+bx^2}}\right.}{48b^3d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2*
a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c -
 a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a
```

```
*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/
(48*b^3*d^(7/2))
```

Maple [B] time = 0.056, size = 527, normalized size = 2.2

$$\frac{dx^2+c}{96\,d^3b^2}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(-12\sqrt{bdx^4+adx^2+bcx^2+acx^2}abd^2\sqrt{bd}-36\sqrt{bdx^4+adx^2+bcx^2+acx^2}cb^2d\sqrt{bd}+3\,d^3\ln\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-12*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*x^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*b*d*x^2+2*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*
b*d^2+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(3/2)*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*a^2*d^2*(b*d)^(1/2)-24*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d
)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/((d*x^2
+c)*(b*x^2+a))^(1/2)/b^2/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5371, size = 1122, normalized size = 4.6

$$\left[-\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,b^2d^3x^4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d)
)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d
```

```
+ a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*
b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2*d^3
*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4
 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d
^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(8*b^2*d^3*x^6 + 15*b^2*c^
3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*b^2*c^2*d
- 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49939, size = 451, normalized size = 1.85

$$\frac{1}{48}\sqrt{bdx^4e + bcx^2e + adx^2e + ace}\left(2\left(\frac{4\,x^2\mathrm{sgn}\left(dx^2+c\right)}{d} - \frac{5\,b^3cd^2\mathrm{sgn}\left(dx^2+c\right) - ab^2d^3\mathrm{sgn}\left(dx^2+c\right)}{b^3d^4}\right)x^2 + \frac{15\,b^3c^2d\mathrm{sg}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*(4*x^2*sgn(d*x^2 +
c)/d - (5*b^3*c*d^2*sgn(d*x^2 + c) - a*b^2*d^3*sgn(d*x^2 + c))/(b^3*d^4))*x
^2 + (15*b^3*c^2*d*sgn(d*x^2 + c) - 4*a*b^2*c*d^2*sgn(d*x^2 + c) - 3*a^2*b*
d^3*sgn(d*x^2 + c))/(b^3*d^4)) + 1/32*(5*b^4*c^3*d*e*sgn(d*x^2 + c) - 3*a*b
^3*c^2*d^2*e*sgn(d*x^2 + c) - a^2*b^2*c*d^3*e*sgn(d*x^2 + c) - a^3*b*d^4*e*
sgn(d*x^2 + c))*sqrt(b*d)*e^(-1/2)*log(abs(-sqrt(b*d)*b*c*e^(1/2) - sqrt(b*
d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*
d*x^2*e + a*c*e))*b*d))/(b^4*d^5)
```

3.264 $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{e}(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{(c+dx^2)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

```
[Out] -((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b*d^2) +
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*(3
*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```

Rubi [A] time = 0.162943, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{(c+dx^2)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```


```
[Out] -((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b*d^2) +
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*(3
*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2(-ae+cx^2)}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{(bc-ad)e+4cdx^2}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4d^2}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8bd^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2} + \frac{((bc-ad)(3bc+ad)e)\,\text{Subst}\left(\int \frac{}{be\cdots}\right.}{8bd^2}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8bd^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2} + \frac{(bc-ad)(3bc+ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\cdots}{\cdots}\right.}{8b^{3/2}d^{5/2}}$$

Mathematica [A] time = 0.38612, size = 149, normalized size = 0.93

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}(c+dx^2)(ad-3bc+2bdx^2)+\frac{(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{8b^2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(c + d*x^2)*(-3*b*c + a*d + 2
*b*d*x^2) + ((b*c - a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*
d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(
8*b^2*d^(5/2))
```

Maple [B] time = 0.012, size = 342, normalized size = 2.1

$$\frac{dx^2+c}{16\,bd^2}\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\left(4\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}x^2bd - d^2\ln\left(\frac{1}{2}\left(2\,bdx^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+a\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*x^2*b*d-d^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2-2*ln(1/2*(2*b*d*x^2+2*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*
d+3*b^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)*a*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/d^2/b/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32398, size = 849, normalized size = 5.27

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e-4\left(2\,b^2d^3x^4+b^2c^2d+abc\right.\right.}{32\,bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4
 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b
^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d
- (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/16*((
3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c
 + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) -
 2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(b*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4132, size = 257, normalized size = 1.6

$$\frac{1}{16}\left(2\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\left(\frac{2x^2}{d}-\frac{3bc-ad}{bd^2}\right)-\frac{\left(3b^2c^2e-2abcde-a^2d^2e\right)\sqrt{bd}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bd}bce^{\frac{1}{2}}-\sqrt{bd}\right.\right.}{b^2d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2/d - (3*b*c -
 a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*sqrt(b*d)*e^(-1/2)
*log(abs(-sqrt(b*d)*b*c*e^(1/2) - sqrt(b*d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*
e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*b*d))/(b^2*d^3))
*sgn(d*x^2 + c)
```

3.265 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

Rubi [A] time = 0.0698071, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 288, 208}

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^2}{\left(be-dx^2\right)^2}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2d}-\frac{((bc-ad)e)\,\text{Subst}\left(\int\frac{1}{be-dx^2}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}$$

$$=\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2d}-\frac{(bc-ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

Mathematica [A] time = 0.284086, size = 143, normalized size = 1.39

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(b\sqrt{d}\left(a+bx^2\right)\left(c+dx^2\right)-\sqrt{a+bx^2}(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{2bd^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(a + b*x^2)*(c + d*x^2) - (b*
c - a*d)^(3/2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(S
qrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(2*b*d^(3/2)*(a + b*x^2))
```

Maple [B] time = 0.006, size = 200, normalized size = 1.9

$$\frac{dx^2+c}{4d}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(a\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)d-b\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bc}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(a*ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d-b*ln(1/2*(2
*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^
2+a))^(1/2)/d/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.58203, size = 656, normalized size = 6.37

$$\left[-\frac{(bc-ad)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,b^2d^3x^4+b^2c^2d+abcd^2+\left(3\,b^2cd^2\right.\right.\right.}{8\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2
)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d
+ a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d, 1/4*
((b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)*sqrt(
(b*e*x^2 + a*e)/(d*x^2 + c)))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3491, size = 201, normalized size = 1.95

$$\frac{1}{4}\left(\frac{(bce-ade)\sqrt{bd}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bd}bce^{\frac{1}{2}}-\sqrt{bd}ade^{\frac{1}{2}}-2\left(\sqrt{bd}x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)bd\right|\right)}{bd^2}+\frac{2\sqrt{bdx^4e+bc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((b*c*e - a*d*e)*sqrt(b*d)*e^(-1/2)*log(abs(-sqrt(b*d)*b*c*e^(1/2) - sq
rt(b*d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e
 + a*d*x^2*e + a*c*e))*b*d))/(b*d^2) + 2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x
^2*e + a*c*e)/d)*sgn(d*x^2 + c)
```

3.266 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b
*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

Rubi [A] time = 0.127791, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 481, 208}

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b
*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$
$$= (ae)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right) + (be)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$
$$= -\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

Mathematica [A] time = 0.178075, size = 173, normalized size = 1.54

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - \sqrt{a}\sqrt{d}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]] - Sqr
t[a]*Sqrt[d]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqr
t[c + d*x^2])]))/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2])
```

Maple [B] time = 0.022, size = 179, normalized size = 1.6

$$\frac{dx^2+c}{2}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)b\sqrt{ac}-a\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x)
```

```
[Out] 1/2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*(a*c)^(1/2)-a
*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*
c)/x^2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.01844, size = 1854, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d
^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 1/4*sqrt(
a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^
2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^
2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*sqrt(-b
*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 + 6*a*b*c
*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2
*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2
*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3
*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))), 1/2*
sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e/d)*arctan(1/2*(2
*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e
*x^2 + a*b*e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x, x)
```

3.267 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$

Optimal. Leaf size=127

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c*(a - (c*(a + b*x^2))/(
c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))
```

Rubi [A] time = 0.086561, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 288, 208}

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c*(a - (c*(a + b*x^2))/(
c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^3}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2c}$$

$$= \frac{(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2c\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{(bc-ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

Mathematica [A] time = 0.100646, size = 133, normalized size = 1.05

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}c^{3/2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqr
t[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2))))/(2*Sqrt[a + b*x^2])
```

Maple [B] time = 0.02, size = 326, normalized size = 2.6

$$-\frac{dx^2+c}{4\,ac^2x^2}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(-2\,bd\sqrt{bdx^4+adx^2+bcx^2+acx^4}\sqrt{ac}-a^2\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x)
```

```
[Out] -1/4*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*x^4*(a*c)^(1/2)-a^2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d*c*x^2+c^2*ln((a*d*x^2+b*c*x^2+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b*a*x^2-2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*
c)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/(a*c)^(1/2)/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.48498, size = 697, normalized size = 5.49

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)+4\left(dx^2+\right.}{8\,cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e
*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*
d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))*sqrt(e/(a*c)))/x^4) + 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e
)) - 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^3, x)
```

3.268 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{e}(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] -((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^2*(a - (c*(a + b*x^
2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c +
3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]
*Sqrt[e])])/(8*a^(3/2)*c^(5/2))
```

Rubi [A] time = 0.169509, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```


```
[Out] -((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^2*(a - (c*(a + b*x^
2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c +
3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]
*Sqrt[e])])/(8*a^(3/2)*c^(5/2))
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^5}\,dx = ((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{x^2\left(be-dx^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{-(bc-ad)e+4cdx^2}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{4c^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{((bc-ad)(bc+3ad)e)\,\mathrm{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x,\right.}{8ac^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^2\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+3ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}$$

Mathematica [A] time = 0.110395, size = 174, normalized size = 0.84

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(x^4\left(-3a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-bcx^2\right)\right)}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*b*
c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])
```

Maple [B] time = 0.023, size = 558, normalized size = 2.7

$$\frac{dx^2+c}{16\,a^2c^3x^4}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(-10\,bd^2\sqrt{bdx^4+adx^2+bcx^2+acx^6}a\sqrt{ac}-2\,b^2d\sqrt{bdx^4+adx^2+bcx^2+acx^6}c\sqrt{ac}-3\,a^3\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x)
```

```
[Out] 1/16*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-10*b*d^2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1/2)-2*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*x^6*c*(a*c)^(1/2)-3*a^3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d*b*a^2*c^2*x^4+c^3*l
n((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)
/x^2)*b^2*a*x^4-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1
/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*c^2*x^4*(a*c)^(1/2)+10*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/
((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/(a*c)^(1/2)/a^2/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.78701, size = 890, normalized size = 4.28

$$\left[-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)x^4\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2}{dx^2}}}{x^4}\right.}{32\,ac^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4
*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d^2)*
x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*
c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))*arcta
n(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(
```

```
a*c))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*
d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^5, x)
```

3.269 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$

Optimal. Leaf size=318

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+\frac{e^2(bc-\ldots}{6ac^2(\ldots}$$

```
[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(8*a*c^3*(a
 - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11
*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(16*a^2*c^3*(a - (c*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/
(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 +
 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

Rubi [A] time = 0.311522, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 463, 455, 385, 208}

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+\frac{e^2(bc-\ldots}{6ac^2(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(8*a*c^3*(a
 - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11
*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(16*a^2*c^3*(a - (c*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/
(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 +
 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
```

```
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = ((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{x^2\left(be-dx^2\right)^2}{\left(-ae+cx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{x^2\left(-3\left(2b^2c^2e^2-(bce-ade)^2\right)+6acd^2ex^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6ac^2}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{3c(bc-ad)(bc+3ad)e^2-24ac^2\ldots}{\left(-ae+cx^2\right)^2}\ldots\right)}{24ac^4}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)\ldots}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)\ldots}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)\ldots}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)\ldots}$$

Mathematica [A] time = 0.17518, size = 222, normalized size = 0.7

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2\left(-8c^2+10cdx^2-15d^2x^4\right)-2abcx^2\left(c-2dx^2\right)+3b^2c^2x^4\right)-3x^6\left(3a^2\right.\right.}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d
^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

Maple [B] time = 0.027, size = 849, normalized size = 2.7

$$-\frac{dx^2+c}{96\,c^4a^3x^6}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(-66\,bd^3\sqrt{bdx^4+adx^2+bcx^2+acx}^8a^2\sqrt{ac}-24\,b^2d^2\sqrt{bdx^4+adx^2+bcx^2+acx}^8ac\sqrt{ac}-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x)
```

```
[Out] -1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-66*b*d^3*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*x^8*a^2*(a*c)^(1/2)-24*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*x^8*a*c*(a*c)^(1/2)-6*b^3*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x
^8*c^2*(a*c)^(1/2)-15*a^4*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d^3*c*x^6+9*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1
/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d^2*b*a^3*c^2*x^6+3*ln(
(a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x
^2)*d*b^2*a^2*c^3*x^6+3*c^4*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b^3*a*x^6-66*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*d^3*a^3*x^6*(a*c)^(1/2)-54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2
*b*a^2*c*x^6*(a*c)^(1/2)-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*d*a*c^2
*x^6*(a*c)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3*c^3*x^6*(a*c)^(1
/2)+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d^2*a^2*x^4*(a*c)^(1/2)+24*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*b*a*c*x^4*(a*c)^(1/2)+6*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(3/2)*b^2*c^2*x^4*(a*c)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(3/2)*d*a^2*c*x^2*(a*c)^(1/2)-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*a*c^
2*x^2*(a*c)^(1/2)+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a^2*c^2*(a*c)^(1/2
))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^4/(a*c)^(1/2)/a^3/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 23.8737, size = 1162, normalized size = 3.65

$$\left[-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)x^6\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2\right.\right.}{x^4}\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/(
a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2
 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 +
 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) -
4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 +
2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2
*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) + 2*((
3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 + 2*a*
b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^7, x)
```

3.270 $\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=357

$$\frac{x\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2}-\frac{\sqrt{c}\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}(4bc-}{}$$

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
15*b^2*d^2) - ((4*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(15*b*d^2) + (x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.519954, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 478, 582, 531, 418, 492, 411}

$$\frac{x\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2}-\frac{\sqrt{c}\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}(4bc-}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
15*b^2*d^2) - ((4*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(15*b*d^2) + (x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{a+bx^2}}$$

$$= \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2(3ac+(4bc-ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{5d\sqrt{a+bx^2}}$$

$$= -\frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{ac(4bc-ad}{}}{15bd^2\sqrt{a+}}$$

$$= -\frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d} + \frac{\left(ac(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right.}{15bd^2\sqrt{a+bx}}$$

$$= \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx\right.}{5d}$$

$$= \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx\right.}{5d}$$

Mathematica [C] time = 0.47942, size = 255, normalized size = 0.71

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+7abcd-8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2+3abcd\right.\right.}{15bd^3\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(
-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqrt[b/a
]*d^3*(a + b*x^2))
```

Maple [A] time = 0.036, size = 552, normalized size = 1.6

$$\frac{dx^2+c}{15\,d^3b}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+4\sqrt{-\frac{b}{a}}x^5abd^3-\sqrt{-\frac{b}{a}}x^5b^2cd^2+\sqrt{-\frac{b}{a}}x^3a^2d^3-4\sqrt{-\frac{b}{a}}x^3b^2c^2d+\sqrt{\frac{bx^2+a}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/15*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+4*
(-b/a)^(1/2)*x^5*a*b*d^3-(-b/a)^(1/2)*x^5*b^2*c*d^2+(-b/a)^(1/2)*x^3*a^2*d^
3-4*(-b/a)^(1/2)*x^3*b^2*c^2*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^2*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+(-b/a)^(1/2
)*x*a^2*c*d^2-4*(-b/a)^(1/2)*x*a*b*c^2*d)/d^3/((d*x^2+c)*(b*x^2+a))^(1/2)/b
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\sqrt{\frac{bex^2+ae}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

3.271 $\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=266

$$-\frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d}-x(2bc$$

```
[Out] -((2*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*b*d) + (x*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt[(
e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.337844, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6719, 478, 531, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d}-x(2bc$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((2*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*b*d) + (x*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt[(
e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$=\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}-\frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{ac+(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}}$$

$$=\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}-\frac{\left(ac\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}}-\frac{\left((2bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c\right.}{3d\sqrt{a}}$$

$$=-\frac{(2bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}-\frac{c^{3/2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{c(}{}$$

$$=-\frac{(2bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}+\frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{3bd^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

Mathematica [C] time = 0.288099, size = 208, normalized size = 0.78

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(a\right.}{3d^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) -
 I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*S
qrt[b/a]*d^2*(a + b*x^2))
```

Maple [A] time = 0.01, size = 356, normalized size = 1.3

$$\frac{dx^2+c}{3d^2}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd-2ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((-b/a)^(1/2)*x^5*b*d^2+(-b/a)^
(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d-2*a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b*c^2+(-b/a)^(1/2)*x*a*c*d)/((d*x^2+c)*(b*x^2+a))^(1/2)/d^2
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\sqrt{\frac{bex^2+ae}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

3.272 $\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.122526, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6719, 422, 418, 492, 411}

$$x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx &= \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}\\
&= \frac{\left(a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}+\frac{\left(b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}\\
&= x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\left(c\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{a+bx^2}}\\
&= x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}-\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}
\end{aligned}
$$

Mathematica [A] time = 0.0554609, size = 86, normalized size = 0.44

$$
\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqr
t[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a])
```

Maple [A] time = 0.009, size = 184, normalized size = 1.

$$
\frac{dx^2+c}{d}\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\left(a\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\mathrm{EllipticE}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] (e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/((d*x
^2+c)*(b*x^2+a))^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\frac{bex^2+ae}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.273 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2} dx$

Optimal. Leaf size=239

$$\frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cx}+\frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c - (Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(c*x) - (Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]) + (b*Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.313869, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cx}+\frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c - (Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(c*x) - (Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]) + (b*Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{}}}{c\sqrt{a+bx^2}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)}{\sqrt{a+bx^2}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\right.\right.}{a\sqrt{d}\sqrt{\frac{c(a+bx}{a(c+dx}}}$$

Mathematica [A] time = 0.251279, size = 111, normalized size = 0.46

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\sqrt{\frac{bx^2}{a}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}(a+bx^2)\sqrt{\frac{dx^2}{c}+1}} - \frac{1}{x}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)*(-x^(-1) + (b*Sqrt[1 + (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*(a + b*
x^2)*Sqrt[1 + (d*x^2)/c])))/c
```

Maple [A] time = 0.02, size = 192, normalized size = 0.8

$$-\frac{dx^2+c}{cx}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(\sqrt{-\frac{b}{a}}x^4bd - bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}x\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right) + \sqrt{-\frac{b}{a}}x^2ad + \sqrt{-\frac{b}{a}}x^2bc + \sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x)
```

```
[Out] -(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((-b/a)^(1/2)*x^4*b*d-b*c*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
```

```
+(-b/a)^(1/2)*x^2*a*d+(-b/a)^(1/2)*x^2*b*c+(-b/a)^(1/2)*a*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

3.274 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx$

Optimal. Leaf size=321

$$\frac{dx(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2}-\frac{(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2x}-\frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(c+dx^2)\sqrt{\cdots}}{3cx^3}$$

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*a*c^2) - (Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (b*Sq
rt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.444654, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 583, 531, 418, 492, 411}

$$\frac{dx(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2}-\frac{(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2x}-\frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(c+dx^2)\sqrt{\cdots}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*a*c^2) - (Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (b*Sq
rt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-2ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{abcd-bd(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3ac^2\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3c\sqrt{a+bx^2}} +$$

$$= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{b\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan\right.}{3a\sqrt{c}\sqrt{\frac{c(}{a(}}}$$

$$= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+b}{c+d}}}{3ac^{3/}}$$

Mathematica [C] time = 0.640265, size = 238, normalized size = 0.74

$$-\frac{\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(a\left(c-2dx^2\right)+bcx^2\right)+ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\right.}{3bc^2x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] -(Sqrt[b/a]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d
*x^2)*(b*c*x^2 + a*(c - 2*d*x^2)) - I*b*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
+ I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c^2*x^3*(a + b*x^2))
```

Maple [A] time = 0.02, size = 444, normalized size = 1.4

$$\frac{dx^2+c}{3\,c^2x^3a}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(2\sqrt{-\frac{b}{a}}x^6abd^2-\sqrt{-\frac{b}{a}}x^6b^2cd+bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac-\sqrt{\frac{bx^2+}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x)
```

```
[Out] 1/3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*(-b/a)^(1/2)*x^6*a*b*d^2-(-b
/a)^(1/2)*x^6*b^2*c*d+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2-2*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*
b*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*x^3*b^2*c^2+2*(-b/a)^(1/2)*x^4*a^2*d^2-(-b/a)^(1/2)*x^4*b^2*c^
2+(-b/a)^(1/2)*x^2*a^2*c*d-2*(-b/a)^(1/2)*x^2*a*b*c^2-(-b/a)^(1/2)*a^2*c^2)
/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/x^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

3.275 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx$

Optimal. Leaf size=424

$$-\frac{dx\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3}+\frac{\left(c+dx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3x}+\frac{\sqrt{d}\left(-8a^2d^2+3abcd+2\right.}{15}$$

```
[Out] -(d*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(15*a^2*c^3) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c*x^5)
- ((b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(15*a*c^2*x
^3) + ((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
]*(c + d*x^2))/(15*a^2*c^3*x) + (Sqrt[d]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*a^2*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) -
(b*Sqrt[d]*(b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^2*c^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.63358, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 583, 531, 418, 492, 411}

$$-\frac{dx\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3}+\frac{\left(c+dx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3x}+\frac{\sqrt{d}\left(-8a^2d^2+3abcd+2\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```


```
[Out] -(d*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(15*a^2*c^3) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c*x^5)
- ((b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(15*a*c^2*x
^3) + ((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
]*(c + d*x^2))/(15*a^2*c^3*x) + (Sqrt[d]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*a^2*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) -
(b*Sqrt[d]*(b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^2*c^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))])
```


Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-4ad-3bdx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{2b^2c^2+3abcd-8a^2d^2+b}{x^2\sqrt{a+bx^2}\sqrt{c+}}}{15ac^2\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} + \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{15a^2c^3x}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} + \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{15a^2c^3x}$$

$$= -\frac{d\left(2b^2c^2+3abcd-8a^2d^2\right)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} + \cdots$$

$$= -\frac{d\left(2b^2c^2+3abcd-8a^2d^2\right)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} + \cdots$$

Mathematica [C] time = 0.568318, size = 302, normalized size = 0.71

$$-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(a^2\left(3c^2-4cdx^2+8d^2x^4\right)+abcx^2\left(c-3dx^2\right)-2b^2c^2x^4\right)-2ibcx^5\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}}\cdots\right.}{15a^2c^3x\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*
b^2*c^2*x^4 + a*b*c*x^2*(c - 3*d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4)
) + I*b*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b*c*
(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^2*Sqrt[b/a]*c^3*
x^5*(a + b*x^2))
```

Maple [A] time = 0.024, size = 708, normalized size = 1.7

$$-\frac{dx^2+c}{15\,c^3x^5a^2}\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\left(8\sqrt{-\frac{b}{a}}x^8a^2bd^3-3\sqrt{-\frac{b}{a}}x^8ab^2cd^2-2\sqrt{-\frac{b}{a}}x^8b^3c^2d+4\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x)
```

```
[Out] -1/15*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(8*(-b/a)^(1/2)*x^8*a^2*b*d^3
-3*(-b/a)^(1/2)*x^8*a*b^2*c*d^2-2*(-b/a)^(1/2)*x^8*b^3*c^2*d+4*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^5*a
^2*b*c*d^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*x^5*a*b^2*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^5*b^3*c^3-8*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^5*a^2*b
*c*d^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*x^5*a*b^2*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^5*b^3*c^3+8*(-b/a)^(1/2)*x^6*a^3
*d^3+(-b/a)^(1/2)*x^6*a^2*b*c*d^2-4*(-b/a)^(1/2)*x^6*a*b^2*c^2*d-2*(-b/a)^(
1/2)*x^6*b^3*c^3+4*(-b/a)^(1/2)*x^4*a^3*c*d^2-3*(-b/a)^(1/2)*x^4*a^2*b*c^2*
d-(-b/a)^(1/2)*x^4*a*b^2*c^3-(-b/a)^(1/2)*x^2*a^3*c^2*d+4*(-b/a)^(1/2)*x^2*
a^2*b*c^3+3*(-b/a)^(1/2)*a^3*c^3)/((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/x^5/a^2/(
-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

3.276 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=282

$$-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}+\frac{e\left(c+dx^2\right)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}+c^2\cdots+\cdots$$

```
[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^4 + ((79*b^2*c^2 -
50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4
8*b*d^4) - ((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^
2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*
e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9
/2))
```

Rubi [A] time = 0.383243, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1960, 463, 455, 1157, 388, 208}

$$-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}+\frac{e\left(c+dx^2\right)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}+c^2\cdots+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^4 + ((79*b^2*c^2 -
50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4
8*b*d^4) - ((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^
2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*
e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9
/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
```

```
IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}\left(c+dx^2\right)^3}{6bd^2e}-\frac{(bc-ad)\,\text{Subst}\left(\int\frac{x^4\left(-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2\right)}{\left(be-dx^2\right)^3}dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6bd^2}$$

$$=-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24d^4}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}\left(c+dx^2\right)^3}{6bd^2e}-\frac{(bc-ad)\,\text{Subst}\left(\int\frac{-bd(bc-ad)(1}{}\right.}{}$$

$$=\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24d^4}+\frac{\left(\frac{e(a+b}{c+dx}\right.}{}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}+\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a}{c}}}{24d}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}+\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a}{c}}}{24d}$$

Mathematica [A] time = 0.569633, size = 294, normalized size = 1.04

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}\sqrt{bc-ad}\left(a^2bd\left(-100c^2-35cdx^2+17d^2x^4\right)+3a^3d^2\left(c+dx^2\right)+ab^2\left(-65c^2dx^2+105c^3-52cd^2x^4+22d\right.\right.\right.}{48b^2d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[b*c - a*d]*(3*a^3*d^2*
(c + d*x^2) + a^2*b*d*(-100*c^2 - 35*c*d*x^2 + 17*d^2*x^4) + b^3*x^2*(105*c
^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6) + a*b^2*(105*c^3 - 65*c^2*d*x
^2 - 52*c*d^2*x^4 + 22*d^3*x^6)) - 3*(b*c - a*d)^2*(35*b^2*c^2 - 10*a*b*c*d
 - a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt
[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^2*d^(9/2)*Sqrt[b*c - a*d]*(a
+ b*x^2))
```

Maple [B] time = 0.032, size = 1027, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/96*(-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*a*b*d^3+60*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*c*d^2+3*ln(1/2*(2*b*
d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2
))*x^2*a^3*d^4+27*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
```

```
b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a^2*b*c*d^3-135*ln(1/2*(2*b*d*x^2+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a*b
^2*c^2*d^2+105*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^3*c^3*d-16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(3/2)*(b*d)^(1/2)*x^2*b*d^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)*x^2*a^2*d^3+108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*
b*c*d^2-54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^2*c^2*d+3*
ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*a^3*c*d^3+27*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c^2*d^2-135*ln(1/2*(2*b*d*
x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*a*b^2*c^3*d+105*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*c^4+96*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d
)^(1/2)*a*b*c^2*d-96*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b^2*c^3-16*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(b*d)^(1/2)*b*c*d-6*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*c*d^2+120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2)*a*b*c^2*d-114*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*
b^2*c^3)/d^4/b*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(b*d)^(1/2)/((d*x^2+
c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 17.1062, size = 1175, normalized size = 4.17

$$\left[\frac{3\left(35\,b^3c^3 - 45\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3\right)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4 + 8\left(b^2cd + abd^2\right)ex^2 + \left(b^2c^2 + 6\,abcd + a^2d^2\right)e - 4\left(2\,b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e/
(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2
 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(8*b^
2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c*d^2
 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d
+ 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(
8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c
*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sq
```

```
rt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^5, x)
```

3.277 $\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=199

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}+\frac{be\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e\left(c+dx^2\right)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc-$$

```
[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^3) - ((9*b*c - 5*a*
d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*d^3) + (b*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^3) + (3*(b*c - a*d)*(5*b*c -
a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(8*Sqrt[b]*d^(7/2))
```

Rubi [A] time = 0.220538, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 455, 1157, 388, 208}

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}+\frac{be\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e\left(c+dx^2\right)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```


```
[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^3) - ((9*b*c - 5*a*
d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*d^3) + (b*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^3) + (3*(b*c - a*d)*(5*b*c -
a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(8*Sqrt[b]*d^(7/2))
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4\left(-ae+cx^2\right)}{\left(be-dx^2\right)^3}dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}+\frac{((bc-ad)e)\,\text{Subst}\left(\int\frac{-b(bc-ad)e^2-4d(bc-ad)ex^2-4cd^2x^4}{\left(be-dx^2\right)^2}dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4d^3}$$

$$=-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}-\frac{(bc-ad)\,\text{Subst}\left(\int\frac{-b(7bc-3ad)e^2}{be-dx}\right.}{8bd^3}$$

$$=-\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}+\frac{\left(3(bc\right.}{}$$

$$=-\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}+\frac{3(bc-}{}$$

Mathematica [A] time = 0.621445, size = 191, normalized size = 0.96

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3\sqrt{bc-ad}\left(a^2d^2-6abcd+5b^2c^2\right)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^2}\left(ad\left(13c+5dx^2\right)+b\left(-15c^2\right.\right.\right.}{8bd^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[a + b*x^2]*(a*d*(13*c
+ 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*Sqrt[b*c - a*d]*(5*b^
2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(8*b*d^(7/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.013, size = 679, normalized size = 3.4

$$\frac{dx^2+c}{16\,d^3\left(bx^2+a\right)}\left(4\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}x^4bd^2+3\ln\left(1/2\,\frac{2\,bdx^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b*d^2+3*ln(1/2*
(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)
^(1/2))*x^2*a^2*d^3-18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a*b*c*d^2+15*ln(1/2*(2*b*d*x^2+2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b
^2*c^2*d+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*d^2-10*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*c*d+3*ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^
2*c*d^2-18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1
/2)+a*d+b*c)/(b*d)^(1/2))*a*b*c^2*d+15*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c^3+10*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*c*d-14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(b*d)^(1/2)*b*c^2+16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d-
16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2)/d^3*(d*x^2+c)*(e*(b*x^2+a
)/(d*x^2+c))^(3/2)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.89172, size = 886, normalized size = 4.45

$$\left[\frac{3\left(5\,b^2c^2-6\,abcd+a^2d^2\right)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,b^2d^3x^4+b^2c^2d\right.\right.}{32\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e*
x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(
2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((
```

```
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*d -
a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(
b*e*x^2 + a*e)) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*b*c^2 -
13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39202, size = 409, normalized size = 2.06

$$\frac{1}{16}\left(2\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\left(\frac{2bx^2}{d^2}-\frac{7b^2cd^5-5abd^6}{bd^8}\right)-\frac{16\left(b^2c^3e-2abc^2de+a^2cd^2e\right)}{\left(\sqrt{bd}ce^{\frac{1}{2}}+\left(\sqrt{bd}x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*b*x^2/d^2 - (7*b
^2*c*d^5 - 5*a*b*d^6)/(b*d^8)) - 16*(b^2*c^3*e - 2*a*b*c^2*d*e + a^2*c*d^2*
e)/((sqrt(b*d)*c*e^(1/2) + (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^
2*e + a*d*x^2*e + a*c*e))*d)*d^3) - 3*(5*sqrt(b*d)*b^2*c^2*e^(1/2) - 6*sqrt
(b*d)*a*b*c*d*e^(1/2) + sqrt(b*d)*a^2*d^2*e^(1/2))*log(abs(-sqrt(b*d)*b*c*e
^(1/2) - sqrt(b*d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e
+ b*c*x^2*e + a*d*x^2*e + a*c*e))*b*d))/(b*d^4))*e*sgn(d*x^2 + c)
```

3.278 $\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=141

$$-\frac{3\sqrt{b}e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*d^2) + (((e*(a + b*x
^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2
*d^(5/2))
```

Rubi [A] time = 0.0899479, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1960, 288, 321, 208}

$$-\frac{3\sqrt{b}e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*d^2) + (((e*(a + b*x
^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2
*d^(5/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int x\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx = ((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4}{\left(be-dx^2\right)^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d}-\frac{(3(bc-ad)e)\,\text{Subst}\left(\int\frac{x^2}{be-dx^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}$$

$$=\frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2}+\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d}-\frac{\left(3b(bc-ad)e^2\right)\text{Subst}\left(\int\frac{1}{be-dx^2}dx, x, \sqrt{\frac{e(a+b}{c+d}}\right.}{2d^2}$$

$$=\frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2}+\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d}-\frac{3\sqrt{b}(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

Mathematica [C] time = 0.0600596, size = 96, normalized size = 0.68

$$\frac{e\left(a+bx^2\right)^2\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{d\left(bx^2+a\right)}{ad-bc}\right)}{5bc-5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*(a + b*x^2)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*
c - a*d)]*Hypergeometric2F1[3/2, 5/2, 7/2, (d*(a + b*x^2))/(-(b*c) + a*d)])
/(5*b*c - 5*a*d)
```

Maple [B] time = 0.012, size = 432, normalized size = 3.1

$$\frac{dx^2+c}{4d^2\left(bx^2+a\right)}\left(3\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^2abd^2-3\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*(3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*x^2*a*b*d^2-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^2*c*d+2*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*d+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^
```

```
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*d-3*b^
2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b
*c)/(b*d)^(1/2))*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c-
4*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d+4*(b*d)^(1/2)*((d*x^2+c)*(b*x
^2+a))^(1/2)*b*c)/d^2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(b*d)^(1/2)/(
(d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.25471, size = 707, normalized size = 5.01

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{be}{d}}e\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,bd^3x^4+bc^2d+acd^2+\left(3\,bcd^2\right.\right.\right.}{8\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d
^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d +
a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 2*(
b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38977, size = 339, normalized size = 2.4

$$\frac{1}{4}\left(\frac{2\sqrt{bdx^4e+bcx^2e+adx^2e+ace}b}{d^2}+\frac{4\left(b^2c^2e-2\,abcde+a^2d^2e\right)}{\left(\sqrt{bd}ce^{\frac{1}{2}}+\left(\sqrt{bd}x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)d\right)d^2}+\frac{3\left(\sqrt{bd}b^2ce^{\frac{1}{2}}-\sqrt{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*b/d^2 + 4*(b^2*c^2*e
 - 2*a*b*c*d*e + a^2*d^2*e)/((sqrt(b*d)*c*e^(1/2) + (sqrt(b*d)*x^2*e^(1/2)
- sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*d)*d^2) + 3*(sqrt(b*d)*b
^2*c*e^(1/2) - sqrt(b*d)*a*b*d*e^(1/2))*log(abs(-sqrt(b*d)*b*c*e^(1/2) - sq
rt(b*d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e
 + a*d*x^2*e + a*c*e))*b*d))/(b*d^3))*e*sgn(d*x^2 + c)
```

3.279 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=151

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

Rubi [A] time = 0.19189, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 479, 522, 208}

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-abe^2+(bc+ad)ex^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{cd}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{c} + \frac{(b^2e^2)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\right.}}{d}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

Mathematica [A] time = 1.28275, size = 193, normalized size = 1.28

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\left(-\frac{a^{3/2}d\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{c^{3/2}\sqrt{a+bx^2}}+\frac{ad}{c}-b\right)+\frac{b\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((b*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2
))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a
+ b*x^2] + Sqrt[d]*(-b + (a*d)/c - (a^(3/2)*d*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(c^(3/2)*Sqrt[a + b*x^2]))
))/d^(3/2)
```

Maple [B] time = 0.015, size = 401, normalized size = 2.7

$$\frac{dx^2+c}{2\,cd\left(bx^2+a\right)}\left(\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{ac}x^2b^2cd-\sqrt{bd}\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*
d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*x^2*b^2*c*d-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a^2*d^2+
ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*c*d+2*(b*d)^(1/2)
*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d-2*(b*d)^(1/2)*(a*c)^(1/2)*((d*
x^2+c)*(b*x^2+a))^(1/2)*b*c)/c/d*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(a
*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.45608, size = 2226, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
 (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (
3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + a*
d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e +
8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3
 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) - 4*
(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*sqrt(-b
*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b
*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)
) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (
b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*
b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(b
*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2*(a*d*sqrt(-a*e/c)
*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2
 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a
*b*e)) - 2*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x, x)
```

3.280 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{3\sqrt{a}e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c^2) + ((b*c - a*d)*
((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))
) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

Rubi [A] time = 0.103718, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 288, 321, 208}

$$-\frac{3\sqrt{a}e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c^2) + ((b*c - a*d)*
((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))
) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{\left(3a(bc-ad)e^2\right)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{3\sqrt{a}(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

Mathematica [A] time = 0.0906491, size = 146, normalized size = 0.88

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{a+bx^2}\left(2bcx^2-a\left(c+3dx^2\right)\right)-3\sqrt{a}x^2\sqrt{c+dx^2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a + b*
x^2])
```

Maple [B] time = 0.015, size = 641, normalized size = 3.9

$$-\frac{dx^2+c}{4\,x^2c^3\left(bx^2+a\right)}\left(-2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{ac}x^6bd^2-3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2\,ac}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b*d^2-3*ln((a*
d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)
*x^4*a^2*c*d^2+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*x^4*a*b*c^2*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*x^4*a*d^2-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^
4*b*c*d-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)+2*a*c)/x^2)*x^2*a^2*c^2*d+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a*b*c^3+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c
)^(1/2)*x^2*a*c*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*b*c
^2+4*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a*c*d-4*(a*c)^(1/2)*((d*x^
2+c)*(b*x^2+a))^(1/2)*x^2*b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2/(a*c)^(1/2)/c^3/(b*x^
2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.50565, size = 748, normalized size = 4.53

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{ae}{c}}\,ex^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(\left(bc^2d+acd^2\right)x^4+2\,ac^3+\left(bc^3+3\,ac^2d\right)x^2\right)\sqrt{\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)-4\left((2\,b\right.}{8\,c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^2)
*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan(1/2*
((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a
*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^3, x)
```

3.281 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=256

$$-\frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}}-\frac{de(bc-$$

```
[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^3) - (a*(b*c - a*d)
^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^3*(a*e - (c*e*(a + b*x^2))/(
c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)])/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*
(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```

Rubi [A] time = 0.217923, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 455, 1157, 388, 208}

$$-\frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}}-\frac{de(bc-$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```


```
[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^3) - (a*(b*c - a*d)
^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^3*(a*e - (c*e*(a + b*x^2))/(
c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)])/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*
(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = ((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{x^4\left(be-dx^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{-a(bc-ad)e^2-4c(bc-ad)ex^2+4c^2dx^4}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{4c^3}$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{-a(3bc-7ad)e^2+8acde\ldots}{-ae+cx^2}\ldots\right.}{8ac^3}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{\left(3(bc-5ad\ldots\right.}{\ldots}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8c^3\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)} - \frac{3(bc-5ad)(\ldots}{\ldots}$$

Mathematica [A] time = 0.124734, size = 186, normalized size = 0.73

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3x^4\sqrt{c+dx^2}\left(5a^2d^2-6abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\left(a\left(2c^2-5cdx^2-15d^2x^4\right)+b\right.\right.}{8\sqrt{a}c^{7/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(b*c
*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*c^2 -
6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2]
)/(Sqrt[a]*Sqrt[c + d*x^2])]))/(8*Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2])
```

Maple [B] time = 0.016, size = 1042, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x)
```

```
[Out] 1/16*(-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b*d^3+6*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^2*c*d^2-15*ln((a*d*x^2+b
*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a^
3*c*d^3+18*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)+2*a*c)/x^2)*x^6*a^2*b*c^2*d^2-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a*b^2*c^3*d-18*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*d^3-26*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b*c*d^2+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^6*b^2*c^2*d-15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^3*c^2*d^2+18*ln((a*d*x^2+b*c*x
^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^2*b*
c^3*d-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)+2*a*c)/x^2)*x^4*a*b^2*c^4+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(
1/2)*x^4*a*d^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b*c*d-
18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-8*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c^2*d+6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*b^2*c^3+16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a
))^(1/2)*x^4*a^2*c*d^2-16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a*b*c
^2*d+14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*c*d-6*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b*c^2-4*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(3/2)*(a*c)^(1/2)*a*c^2)/a*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/
x^4/(a*c)^(1/2)/c^4/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 19.6949, size = 925, normalized size = 3.61

$$\left[\frac{3\left(b^2c^2-6\,abcd+5\,a^2d^2\right)ex^4\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+a}{dx^2+c}}}{x^4}\right.}{32\,c^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt(e/(a*c))*log(((b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2
- 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((13*b*c*d - 15*
a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt
(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) - 2*((13*b*c*d - 15*a*d^2)*e*x^4 + 2
*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^3
*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^5, x)
```

3.282 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=366

$$-\frac{e^2\left(-79a^2d^2+50abcd+5b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}\left(-35a^2d^2+10abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}\cdot$$

```
[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^4 + ((b*c - a*d)^3*
e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/
(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)
*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
])/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2
 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```

Rubi [A] time = 0.368982, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1960, 463, 455, 1157, 388, 208}

$$-\frac{e^2\left(-79a^2d^2+50abcd+5b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}\left(-35a^2d^2+10abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```


```
[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^4 + ((b*c - a*d)^3*
e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/
(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)
*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
])/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2
 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4\left(be-dx^2\right)^2}{\left(-ae+cx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4\left(-6b^2c^2e^2+5(bce-ade)^2+6acd^2ex^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6ac^2}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{ac(bc-ad)(bc+11\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-79a\ldots\right.}{48ac^4\left(ae-\frac{ce(a+bx\ldots}{c+dx^2}\right.}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-a\ldots}{\ldots}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-a\ldots}{\ldots}$$

Mathematica [A] time = 0.19232, size = 245, normalized size = 0.67

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3x^6\sqrt{c+dx^2}\left(-45a^2bcd^2+35a^3d^3+9ab^2c^2d+b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)-\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\left(a^2\left(-14c^2dx^2\ldots\right.\right.\right.}{48a^{3/2}c^{9/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3*
b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) + a
^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 + 9*a
*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*x^6*S
qrt[a + b*x^2])
```

Maple [B] time = 0.018, size = 1498, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x)
```

```
[Out] -1/96*(-105*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)+2*a*c)/x^2)*x^8*a^4*c*d^4-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a
*c)^(1/2)*x^8*a^3*d^4-105*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a^4*c^2*d^3-3*ln((a*d*x^2+b*c*x^2+2*(a
*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a*b^3*c^5+174
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a^2*d^3+6*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^3*c^4-6*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(3/2)*(a*c)^(1/2)*x^4*b^2*c^3+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a
*c)^(1/2)*x^10*b^3*c^2*d^2+135*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^8*a^3*b*c^2*d^3-27*ln((a*d*x^2+b*c*
x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^8*a^2*b
^2*c^3*d^2-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+2*a*c)/x^2)*x^8*a*b^3*c^4*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(a*c)^(1/2)*x^8*b^3*c^3*d+135*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a^3*b*c^3*d^2-27*ln((a*d*x^2+b*c*x
^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a^2*b^
2*c^4*d+96*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^6*a^3*c*d^3-6*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*b^2*c^2*d-174*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^3*c*d^3+114*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(3/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-44*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
(a*c)^(1/2)*x^2*a^2*c^2*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2
)*x^2*a*b*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c^3-17
4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a^2*b*d^4-60*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*b*c^2*d+72*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a*b^2*c*d^3-216*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a^2*b*c*d^3+138*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(a*c)^(1/2)*x^8*a*b^2*c^2*d^2-96*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1
/2)*x^6*a^2*b*c^2*d^2-72*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^
6*a*b*c*d^2-42*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*c^
2*d^2+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b^2*c^3*d)/a
^2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6/(a*c)^(1/2)/c^5/(b*x^2+a)/((
d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 93.0066, size = 1211, normalized size = 3.31

$$\left[\frac{3\left(b^3c^3 + 9\,ab^2c^2d - 45\,a^2bcd^2 + 35\,a^3d^3\right)ex^6\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3\cdots\right.\right.}{x^4}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqr
t(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*
b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*
c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^
4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*e + (
3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^2*d)*e
*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*c^3 + 9
*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))
/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 +
8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3
 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^7, x)
```

3.283 $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=391

$$-\frac{\sqrt{c}e\left(a^2d^2-16abcd+16b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex\left(-\frac{a^2d}{b}+16ac-\frac{16bc^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}+\frac{c^{3/2}e(8bc-7ad}{}$$

```
[Out] -((16*a*c - (16*b*c^2)/d - (a^2*d)/b)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(5*d^2) - (e*x^3*(a + b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d - ((8*b
*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^3) + (6
*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^2) - (Sqrt[c]*
(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b*d^(7/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(8*b*c - 7*a*d)*e*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(5*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.678075, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6719, 467, 581, 582, 531, 418, 492, 411}

$$-\frac{\sqrt{c}e\left(a^2d^2-16abcd+16b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex\left(-\frac{a^2d}{b}+16ac-\frac{16bc^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}+\frac{c^{3/2}e(8bc-7ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((16*a*c - (16*b*c^2)/d - (a^2*d)/b)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(5*d^2) - (e*x^3*(a + b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d - ((8*b
*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^3) + (6
*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^2) - (Sqrt[c]*
(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b*d^(7/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(8*b*c - 7*a*d)*e*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(5*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^4\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^4(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2\sqrt{a+bx^2}(3a+6bx^2)}{\sqrt{c+dx^2}}dx}{d\sqrt{a+bx^2}}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^2}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2(-3a(6bc-5a}{\sqrt{a+b}}}{5d^2\sqrt{a+bx^2}}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^3}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^2}-$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^3}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^2}+$$

$$=\frac{\left(16b^2c^2-16abcd+a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5bd^3}-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c}{5d^3}$$

$$=\frac{\left(16b^2c^2-16abcd+a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5bd^3}-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c}{5d^3}$$

Mathematica [C] time = 0.520487, size = 290, normalized size = 0.74

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(dx\sqrt{\frac{b}{a}}\left(a^2d\left(7c+2dx^2\right)+ab\left(-8c^2+5cdx^2+3d^2x^4\right)+b^2x^2\left(-8c^2-2cdx^2+d^2x^4\right)\right)+8ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\right.}{5d^4\sqrt{\frac{b}{a}}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a^2*d*(7*c + 2*d*x^2)
+ b^2*x^2*(-8*c^2 - 2*c*d*x^2 + d^2*x^4) + a*b*(-8*c^2 + 5*c*d*x^2 + 3*d^2*
x^4)) - I*c*(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*I)*c*(2*b^
2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(5*Sqrt[b/a]*d^4*(a + b*x^2))
```

Maple [B] time = 0.032, size = 933, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/5*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-
b/a)^(1/2)*x^7*b^2*d^3+3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x^5*a*b*d
^3-2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x^5*b^2*c*d^2+2*((d*x^2+c)*(b
*x^2+a))^(1/2)*(-b/a)^(1/2)*x^3*a^2*d^3-3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a
)^(1/2)*x^3*b^2*c^2*d+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^
3*a*b*c*d^2-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*b^2*c^2*
d-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+24*((d*x^2+c)*(b*x^2+a))^(
1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a*b*c^2*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+((d*x^2
+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a*b*c^2*d+16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+2*((d*x^2+c)*(b*x^
2+a))^(1/2)*(-b/a)^(1/2)*x*a^2*c*d^2-3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(
1/2)*x*a*b*c^2*d+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*a^2*c
*d^2-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*a*b*c^2*d)/d^4/(b
*x^2+a)^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bex^6+aex^4)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^6 + a*e*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^2 + c),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

3.284 $\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=310

$$\frac{4bex\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{\sqrt{c}e(4bc-3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}e(8bc-7}{}$$

```
[Out] -((8*b*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*d^2) - (e*x*(a
+ b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d + (4*b*e*x*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2))/(3*d^2) + (Sqrt[c]*(8*b*c - 7*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (Sqrt[c]*(4*b*c
- 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
)
```

Rubi [A] time = 0.438167, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 467, 528, 531, 418, 492, 411}

$$\frac{4bex\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{\sqrt{c}e(4bc-3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}e(8bc-7}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((8*b*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*d^2) - (e*x*(a
+ b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d + (4*b*e*x*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2))/(3*d^2) + (Sqrt[c]*(8*b*c - 7*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (Sqrt[c]*(4*b*c
- 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
)
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}(a+4bx^2)}{\sqrt{c+dx^2}}dx}{d\sqrt{a+bx^2}}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{-a(4bc-3ad)-}{\sqrt{a+bx^2}}}{3d^2\sqrt{a+bx^2}}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} - \frac{\left(b(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)}{3d^2\sqrt{a+bx^2}}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} - \frac{\sqrt{c}(4bc-}{}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} + \frac{\sqrt{c}(8bc-}{}$$

Mathematica [C] time = 0.415328, size = 235, normalized size = 0.76

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,(3a^2d^2-11abcd+8b^2c^2)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}(a+bx^2)\left(3ad-b\left(4c+dx\right.\right.\right.}{3d^3\sqrt{\frac{b}{a}}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*d - b
*(4*c + d*x^2)) + I*b*c*(-8*b*c + 7*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(8*b^2*c^2 - 11*a*
b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/a]*d^3*(a + b*x^2))
```

Maple [B] time = 0.014, size = 738, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(-(-b/a)^(1/2)*((d*x^2+c)*(b*x
^2+a))^(1/2)*x^5*b^2*d^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)
```

```
*x^3*a*b*d^2-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*b^2*c*d
-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*a*b*d^2-(-b/a)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*x^3*b^2*c*d-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^
2+11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d-8*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+
a))^(1/2)*b^2*c^2-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d+8*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d
*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/
a)^(1/2)*x*a^2*d^2-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*a*b
*c*d-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x*a*b*c*d)/(b*x^2+a)^2/d^3/(-
b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bex^4+aex^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^4 + a*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^2 + c),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

3.285 $\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=262

$$\frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{ex(2bc-$$

```
[Out] -(((b*c - a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) + ((2*b*c - a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d) - ((2*b*c - a*d)*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*Sqrt[c
]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.202139, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6719, 413, 531, 418, 492, 411}

$$\frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{ex(2bc-$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```


```
[Out] -(((b*c - a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) + ((2*b*c - a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d) - ((2*b*c - a*d)*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*Sqrt[c
]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```


Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2} dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{d\sqrt{a+bx^2}} + \frac{\left(b(2bc-ad)e\sqrt{\frac{e(a+b}{c+dx}} \right.}{c}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} -$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] time = 0.317002, size = 206, normalized size = 0.79

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left((ad-bc)\left(dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)-2ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-\right.}{cd^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-
(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(Sqrt[b/
a]*c*d^2*(a + b*x^2))
```

Maple [A] time = 0.012, size = 527, normalized size = 2.

$$\frac{dx^2+c}{\left(bx^2+a\right)^2 d^2 c}\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}\left(\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}x^3abd^2-\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}x^3b^2cd+2\sqrt{\frac{bx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] (e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*x^3*a*b*d^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)
*x^3*b^2*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d-2*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+
c)*(b*x^2+a))^(1/2)*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d+2*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*x*a^2*d^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x
*a*b*c*d)/(b*x^2+a)^2/d^2/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bex^2+ae\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.286 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=307

$$\frac{e\left(c+dx^2\right)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2dx}-\frac{ex(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2}+\frac{e(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(bc-ad)\sqrt{\cdots}}{cdx}$$

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x)) - ((b*c - 2*a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^2 + ((b*c - 2*a*d)*e*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(c^2*d*x) + ((b*c - 2*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*e*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.449728, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$\frac{e\left(c+dx^2\right)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2dx}-\frac{ex(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2}+\frac{e(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(bc-ad)\sqrt{\cdots}}{cdx}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x)) - ((b*c - 2*a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^2 + ((b*c - 2*a*d)*e*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(c^2*d*x) + ((b*c - 2*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*e*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^2(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(bc-2ad)-abdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a^2bcd-abd(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx} + \frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{c\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F(}{\sqrt{c}\sqrt{d}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx} + \frac{(bc-2ad)e\sqrt{}}{c^3}$$

Mathematica [C] time = 0.35807, size = 228, normalized size = 0.74

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(d\sqrt{\frac{b}{a}}(a+bx^2)(ac+2adx^2-bcx^2) - ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right) + ibcx\sqrt{\frac{bx^2}{a}+1}}{c^2dx\sqrt{\frac{b}{a}}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -((e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*(a + b*x^2)*(a*c - b*c*
x^2 + 2*a*d*x^2) + I*b*c*(-(b*c) + 2*a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a
*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c^2*d*x*(a + b*x^2)))
```

Maple [A] time = 0.014, size = 670, normalized size = 2.2

$$-\frac{dx^2+c}{(bx^2+a)^2c^2dx}\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}\left(\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^4abd^2 - \sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^4b^2cd + \sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x)
```

```
[Out] -(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*x^4*a*b*d^2-(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*x^4*b^2*c*d+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a*b*d^2+((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*(
(d*x^2+c)*(b*x^2+a))^(1/2)*x*a*b*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x*b
^2*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x*a*b*c*d+((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*
x^2+a))^(1/2)*x*b^2*c^2+(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^
2*a^2*d^2-(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*c*d+(-b/
a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a^2*d^2+(-b/a)^(1/2)*((d*x^2+c)*(b
*x^2+a))^(1/2)*x^2*a*b*c*d+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c*d
)/(b*x^2+a)^2/c^2/d/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bex^2+ae\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^4+cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^4 + c*x^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

3.287 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=383

$$-\frac{e\left(c+dx^2\right)(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3x}+\frac{e\left(c+dx^2\right)(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^2dx^3}+\frac{dex(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3}+\frac{be(3bc-4ad)\sqrt{\cdots}}{3ac\cdots}$$

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^3)) + (d*(7*b*c
- 8*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*c^3) + ((3*b*c - 4*a*d)*
e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^2*d*x^3) - ((7*b*c -
8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^3*x) - (Sqrt[d
]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]) + (b*(3*b*c - 4*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.631784, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$-\frac{e\left(c+dx^2\right)(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3x}+\frac{e\left(c+dx^2\right)(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^2dx^3}+\frac{dex(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3}+\frac{be(3bc-4ad)\sqrt{\cdots}}{3ac\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```


```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^3)) + (d*(7*b*c
- 8*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*c^3) + ((3*b*c - 4*a*d)*
e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^2*d*x^3) - ((7*b*c -
8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^3*x) - (Sqrt[d
]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]) + (b*(3*b*c - 4*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```


Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
```

```
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^4(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(3bc-4ad)+b(2bc-3ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a^2d(7bc-8ad)+}{x^2\sqrt{a+bx}}}{3ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^3x} - $$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^3x} + $$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8}{}$$

Mathematica [C] time = 0.454519, size = 275, normalized size = 0.72

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}\left(a^2\left(c^2-4cdx^2-8d^2x^4\right)+abx^2\left(5c^2+3cdx^2-8d^2x^4\right)+b^2cx^4\left(4c+7dx^2\right)\right)-4ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}}\right.}{3c^3x^3\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(b^2*c*x^4*(4*c + 7*d*x^2
) + a^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*x^2*(5*c^2 + 3*c*d*x^2 - 8*d^2*
x^4))) + I*b*c*(-7*b*c + 8*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*b*c*(-(b*c) + a*d)*
x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)]))/(3*Sqrt[b/a]*c^3*x^3*(a + b*x^2))
```

Maple [A] time = 0.016, size = 791, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x)
```

```
[Out] 1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*x^6*a*b*d^2-3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*x^6*b^2*c*d+5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a*b*d^2
-4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*b^2*c*d+4*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)
*(b*x^2+a))^(1/2)*x^3*a*b*c*d-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*b^2*
c^2-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*a*b*c*d+7*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b
*x^2+a))^(1/2)*x^3*b^2*c^2+3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*x^4*a^2*d^2-3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*a*b*c
*d+5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a^2*d^2-4*(-b/a)^(1/2)*((
d*x^2+c)*(b*x^2+a))^(1/2)*x^4*b^2*c^2+4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^
(1/2)*x^2*a^2*c*d-5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a*b*c^2-(-
b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c^2)/(b*x^2+a)^2/c^3/(-b/a)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bex^2+ae)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^6+cx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^6 + c*x^4),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

3.288 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=480

$$-\frac{e\left(c+dx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4x}+\frac{dex\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4}-\frac{\sqrt{d}e\left(16a^2d^2-16abcd+b^2\right.}{5a}$$

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^5)) + (d*(b^2*c^
2 - 16*a*b*c*d + 16*a^2*d^2)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(5*a*c^
4) + ((5*b*c - 6*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c
^2*d*x^5) - ((7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(5*c^3*x^3) - ((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*(c + d*x^2))/(5*a*c^4*x) - (Sqrt[d]*(b^2*c^2 - 16*a*b*c*d +
16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]) - (b*Sqrt[d]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(5/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.808583, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$-\frac{e\left(c+dx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4x}+\frac{dex\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4}-\frac{\sqrt{d}e\left(16a^2d^2-16abcd+b^2\right.}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```


```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^5)) + (d*(b^2*c^
2 - 16*a*b*c*d + 16*a^2*d^2)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(5*a*c^
4) + ((5*b*c - 6*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c
^2*d*x^5) - ((7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(5*c^3*x^3) - ((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*(c + d*x^2))/(5*a*c^4*x) - (Sqrt[d]*(b^2*c^2 - 16*a*b*c*d +
16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]) - (b*Sqrt[d]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(5/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))])
```


Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^6(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(5bc-6ad)+b(4bc-5ad)x^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{3a^2d(7bc-8ad)+3ab\cdots}{x^4\sqrt{a+bx^2}\sqrt{\cdots}}}{5ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^3x^3} - \frac{\left(e\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2\cdots}{\cdots}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2\cdots}{\cdots}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d\left(b^2c^2-16abcd+16a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5}\cdots$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d\left(b^2c^2-16abcd+16a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5}\cdots$$

Mathematica [C] time = 0.658236, size = 357, normalized size = 0.74

$$-\frac{e\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\left(a^2bx^2\left(-11c^2dx^2+3c^3-8cd^2x^4+16d^3x^6\right)+a^3\left(-2c^2dx^2+c^3+8cd^2x^4+16d^3x^6\right)+ab^2cx^4\left(3c^2-\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -(Sqrt[b/a]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(b^3*c^2*x^6*(c
+ d*x^2) + a*b^2*c*x^4*(3*c^2 - 8*c*d*x^2 - 16*d^2*x^4) + a^2*b*x^2*(3*c^3
- 11*c^2*d*x^2 - 8*c*d^2*x^4 + 16*d^3*x^6) + a^3*(c^3 - 2*c^2*d*x^2 + 8*c*d
^2*x^4 + 16*d^3*x^6)) + I*b*c*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x^5*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - I*b*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(5*b*c^4
*x^5*(a + b*x^2))
```

Maple [B] time = 0.017, size = 1197, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x)
```

```
[Out] -1/5*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(5*(-b/a)^(1/2)*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*x^8*a^2*b*d^3-5*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*x^8*a*b^2*c*d^2+11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^8
*a^2*b*d^3-11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^8*a*b^2*c*d^2+(-b/
a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^8*b^3*c^2*d+8*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x
^2+a))^(1/2)*x^5*a^2*b*c*d^2-9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^5*a*b^2
*c^2*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^5*b^3*c^3-16*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*
(b*x^2+a))^(1/2)*x^5*a^2*b*c*d^2+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^5*
a*b^2*c^2*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^5*b^3*c^3+5*(-b/a)^(1/2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a^3*d^3-5*(-b/a)^(1/2)*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*x^6*a^2*b*c*d^2+11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a)
)^(1/2)*x^6*a^3*d^3-3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a^2*b*c*
d^2-8*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a*b^2*c^2*d+(-b/a)^(1/2)
*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*b^3*c^3+8*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a
))^(1/2)*x^4*a^3*c*d^2-11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a^2*
b*c^2*d+3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a*b^2*c^3-2*(-b/a)^(
1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a^3*c^2*d+3*(-b/a)^(1/2)*((d*x^2+c)*(b
*x^2+a))^(1/2)*x^2*a^2*b*c^3+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^3*c
^3)/a/(b*x^2+a)^2/c^4/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bex^2+ae)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^8+cx^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^8 + c*x^6),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

3.289 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

Rubi [A] time = 0.0233902, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1960, 288, 204}

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)\right)$$
$$= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)$$
$$= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right) - \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

Mathematica [A] time = 0.0313739, size = 86, normalized size = 1.69

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}}\sqrt{x^2+1}\left(\sqrt{x^2+1}\left(x^2-1\right)+2\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)\right)}{2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*Sqrt[1 + x^2]*((-1 + x^2)*Sqrt[1 + x^2] + 2*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x^2]/Sqrt[2]]))/(2*(-1 + x^2))
```

Maple [A] time = 0.02, size = 52, normalized size = 1.

$$\frac{x^2+1}{2}\sqrt{-\frac{x^2-1}{x^2+1}}\left(\sqrt{-x^4+1}+\arcsin\left(x^2\right)\right)\frac{1}{\sqrt{-\left(x^2-1\right)\left(x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-x^2+1)/(x^2+1))^(1/2),x)
```

```
[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*((-x^4+1)^(1/2)+arcsin(x^2))/(-(x^2-1)
*(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

Fricas [A] time = 1.49172, size = 134, normalized size = 2.63

$$\frac{1}{2}\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}} - \arctan\left(\frac{\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 - 1
)/(x^2 + 1)) - 1)/x^2)
```

Sympy [A] time = 36.6693, size = 39, normalized size = 0.76

$$\begin{cases} \frac{\sqrt{1-x^2}\sqrt{x^2+1}}{2} - \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x^2}}{2}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

```
[Out] Piecewise((sqrt(1 - x**2)*sqrt(x**2 + 1)/2 - asin(sqrt(2)*sqrt(1 - x**2)/2)
, (x > -1) & (x < 1)))
```

Giac [A] time = 1.19571, size = 24, normalized size = 0.47

$$\frac{1}{2}\sqrt{-x^4+1} + \frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

3.290 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rubi [A] time = 0.0321963, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1960, 288, 204}

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\left(74\,\text{Subst}\left(\int \frac{x^2}{\left(-7-5x^2\right)^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\right)$$
$$= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right) + \frac{37}{5}\,\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)$$
$$= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right) - \frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}$$

Mathematica [A] time = 0.0465422, size = 104, normalized size = 1.44

$$\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}\sqrt{5x^2+7}\left(35\sqrt{5x^2+7}\left(7x^2-5\right)+74\sqrt{35}\sqrt{5-7x^2}\sin^{-1}\left(\sqrt{\frac{5}{74}}\sqrt{5-7x^2}\right)\right)}{350\left(7x^2-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*Sqrt[7 + 5*x^2]*(35*Sqrt[7 + 5*x^2]*(-5 + 7*
x^2) + 74*Sqrt[35]*Sqrt[5 - 7*x^2]*ArcSin[Sqrt[5/74]*Sqrt[5 - 7*x^2]]))/(35
0*(-5 + 7*x^2))
```

Maple [A] time = 0.021, size = 78, normalized size = 1.1

$$\frac{5\,x^2+7}{350}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\left(37\,\sqrt{35}\arcsin\left(\frac{35\,x^2}{37}+\frac{12}{37}\right)+35\,\sqrt{-35\,x^4-24\,x^2+35}\right)\frac{1}{\sqrt{-\left(7\,x^2-5\right)\left(5\,x^2+7\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x)
```

```
[Out] 1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2+
12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

Maxima [A] time = 1.67384, size = 103, normalized size = 1.43

$$-\frac{37}{175}\,\sqrt{35}\arctan\left(\frac{1}{7}\,\sqrt{35}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\right)-\frac{37\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{5\left(\frac{5\left(7\,x^2-5\right)}{5\,x^2+7}-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

```
[Out] -37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/5
*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

Fricas [A] time = 1.54251, size = 197, normalized size = 2.74

$$\frac{1}{10}\left(5x^2+7\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}-\frac{37}{350}\sqrt{35}\arctan\left(\frac{\sqrt{35}\left(35x^2+12\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{35\left(7x^2-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1/
35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))
```

Sympy [A] time = 126.062, size = 66, normalized size = 0.92

$$\begin{cases}\frac{5\sqrt{35}\left(\frac{\sqrt{25-35x^2}\sqrt{35x^2+49}}{125}-\frac{74\operatorname{asin}\left(\frac{\sqrt{74}\sqrt{25-35x^2}}{74}\right)}{125}\right)}{14} & \text{for}\: x > -\frac{\sqrt{35}}{7}\wedge x < \frac{\sqrt{35}}{7}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] Piecewise((5*sqrt(35)*(sqrt(25 - 35*x**2)*sqrt(35*x**2 + 49)/125 - 74*asin(
sqrt(74)*sqrt(25 - 35*x**2)/74)/125)/14, (x > -sqrt(35)/7) & (x < sqrt(35)/
7)))
```

Giac [A] time = 1.17914, size = 41, normalized size = 0.57

$$\frac{37}{350}\sqrt{35}\arcsin\left(\frac{35}{37}x^2+\frac{12}{37}\right)+\frac{1}{10}\sqrt{-35x^4-24x^2+35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

```
[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 35
)
```

3.291 $\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

Rubi [A] time = 0.0291266, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1960, 288, 204}

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx &= -\left(\frac{4}{3}\operatorname{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx,x,\sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\ &= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{\frac{1-x^3}{1+x^3}}\right)\\ &= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0317817, size = 86, normalized size = 1.62

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\sqrt{x^3+1}\left(\sqrt{x^3+1}\left(x^3-1\right)+2\sqrt{1-x^3}\sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right)\right)}{3\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*((-1 + x^3)*Sqrt[1 + x^3] + 2*Sqrt
[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(3*(-1 + x^3))
```

Maple [A] time = 0.081, size = 68, normalized size = 1.3

$$\frac{x^3+1}{3}\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{\arcsin\left(x^3\right)}{3x^3-3}\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-\left(x^3+1\right)\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1/
2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Fricas [A] time = 1.46181, size = 139, normalized size = 2.62

$$\frac{1}{3}\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{2}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^3
 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17868, size = 30, normalized size = 0.57

$$\frac{1}{3}\left(\sqrt{-x^6+1}+\arcsin\left(x^3\right)\right)\mathrm{sgn}\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

3.292 $\int x^8 \sqrt{\frac{1-x^3}{1+x^3}}\, dx$

Optimal. Leaf size=113

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x
^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

Rubi [A] time = 0.0635612, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1960, 463, 455, 385, 204}

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x
^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx &= -\left(\frac{4}{3}\operatorname{Subst}\left(\int \frac{x^2\left(-1+x^2\right)^2}{\left(-1-x^2\right)^4}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\
&= -\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3-\frac{2}{9}\operatorname{Subst}\left(\int \frac{x^2\left(6-6x^2\right)}{\left(-1-x^2\right)^3}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= -\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3+\frac{1}{18}\operatorname{Subst}\left(\int \frac{12-24x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x\right.\\
&= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0366384, size = 98, normalized size = 0.87

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\sqrt{x^3+1}\left(\sqrt{x^3+1}\left(2x^9-5x^6+7x^3-4\right)+6\sqrt{1-x^3}\sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right)\right)}{18\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*(Sqrt[1 + x^3]*(-4 + 7*x^3 - 5*x^6
 + 2*x^9) + 6*Sqrt[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(18*(-1 + x^3))
```

Maple [A] time = 0.042, size = 80, normalized size = 0.7

$$\frac{\left(2x^6-3x^3+4\right)\left(x^3+1\right)}{18}\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{\arcsin\left(x^3\right)}{6x^3-6}\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-\left(x^3+1\right)\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/18*(2*x^6-3*x^3+4)*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*arcsin(x^3)*(-(x^
3-1)/(x^3+1))^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \sqrt{-\frac{x^3-1}{x^3+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Fricas [A] time = 1.47564, size = 159, normalized size = 1.41

$$\frac{1}{18}\left(2x^9 - x^6 + x^3 + 4\right)\sqrt{-\frac{x^3-1}{x^3+1}} - \frac{1}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}} - 1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
+ 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^8 \sqrt{-\frac{x^3-1}{x^3+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

3.293 $\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}} \, dx$

Optimal. Leaf size=106

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7
+ 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 +
 5*x^5)]])/(875*Sqrt[35])
```

Rubi [A] time = 0.0557393, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1960, 455, 385, 204}

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7
+ 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 +
 5*x^5)]])/(875*Sqrt[35])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx &= -\left(\frac{148}{5}\text{Subst}\left(\int \frac{x^2\left(-5+7x^2\right)}{\left(-7-5x^2\right)^3}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\right)\\ &= \frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{37}{125}\text{Subst}\left(\int \frac{-74+140x^2}{\left(-7-5x^2\right)^2}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\\ &= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2-\frac{2257}{875}\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\\ &= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}\end{aligned}$$

Mathematica [A] time = 0.055097, size = 109, normalized size = 1.03

$$\frac{\sqrt{\frac{5-7x^5}{5x^5+7}}\sqrt{5x^5+7}\left(35\sqrt{5x^5+7}\left(245x^{10}-777x^5+430\right)-4514\sqrt{35}\sqrt{5-7x^5}\sin^{-1}\left(\sqrt{\frac{5}{74}}\sqrt{5-7x^5}\right)\right)}{61250\left(7x^5-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*Sqrt[7 + 5*x^5]*(35*Sqrt[7 + 5*x^5]*(430 - 7
77*x^5 + 245*x^10) - 4514*Sqrt[35]*Sqrt[5 - 7*x^5]*ArcSin[Sqrt[5/74]*Sqrt[5
 - 7*x^5]]))/(61250*(-5 + 7*x^5))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int x^9\sqrt{\frac{-7\,x^5+5}{5\,x^5+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

```
[Out] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

Maxima [A] time = 1.52384, size = 163, normalized size = 1.54

$$\frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)-\frac{37\left(675\left(-\frac{7\,x^5-5}{5\,x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)}{875\left(\frac{25\left(7\,x^5-5\right)^2}{\left(5\,x^5+7\right)^2}-\frac{70\left(7\,x^5-5\right)}{5\,x^5+7}+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

```
[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) - 3
7/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^5
+ 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49)
```

Fricas [A] time = 1.5163, size = 225, normalized size = 2.12

$$\frac{1}{1750}\left(175\,x^{10}-185\,x^{5}-602\right)\sqrt{-\frac{7\,x^{5}-5}{5\,x^{5}+7}}+\frac{2257}{61250}\,\sqrt{35}\arctan\left(\frac{\sqrt{35}\left(35\,x^{5}+12\right)\sqrt{-\frac{7\,x^{5}-5}{5\,x^{5}+7}}}{35\,\left(7\,x^{5}-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/612
50*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7
))/(7*x^5 - 5))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18099, size = 63, normalized size = 0.59

$$\frac{1}{61250}\left(35\,\sqrt{-35\,x^{10}-24\,x^{5}+35}\left(35\,x^{5}-86\right)-2257\,\sqrt{35}\arcsin\left(\frac{35}{37}\,x^{5}+\frac{12}{37}\right)\right)\mathrm{sgn}\left(5\,x^{5}+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

```
[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arcs
in(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

3.294 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqr
t[2]*x)
```

Rubi [A] time = 0.0989364, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6719, 444, 63, 203}

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqr
t[2]*x)
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{x}{\sqrt{-1+x^2}(1+x^2)}\,dx}{x}$$
$$= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}(1+x)}\,dx, x, x^2\right)}{2x}$$
$$= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x^2}\right)}{x}$$
$$= \frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{-1+x^2}\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Mathematica [A] time = 0.0160615, size = 49, normalized size = 0.94

$$\frac{\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[
2]*x)
```

Maple [A] time = 0.011, size = 42, normalized size = 0.8

$$\frac{\sqrt{2}}{2x}\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2-1))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

Fricas [A] time = 1.48377, size = 88, normalized size = 1.69

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

Giac [A] time = 1.16685, size = 55, normalized size = 1.06

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\mathrm{sgn}\left(x^2-1\right)\mathrm{sgn}\left(x\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}i\right)\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sqr
t(2)*arctan(1/2*sqrt(2)*i)*sgn(x)
```

3.295 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{x^2}{-(a+1)x^2-a+1}}\sqrt{(a+1)x^2+a-1}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[
-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rubi [A] time = 0.191046, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6719, 444, 63, 205}

$$\frac{\sqrt{-\frac{x^2}{-(a+1)x^2-a+1}}\sqrt{(a+1)x^2+a-1}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[
-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\int \frac{x}{(1+x^2)\sqrt{-1+a+(1+a)x^2}}\,dx}{x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+a+(1+a)x}}\,dx, x, x^2\right)}{2x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{-1+a}{1+a}+\frac{x^2}{1+a}}\,dx, x, \sqrt{-1+a+(1+a)x^2}\right)}{(1+a)x}$$

$$= \frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Mathematica [A] time = 0.0235464, size = 65, normalized size = 0.96

$$\frac{\sqrt{ax^2+a+x^2-1}\sqrt{\frac{x^2}{(a+1)x^2+a-1}}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-1
 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Maple [A] time = 0.028, size = 60, normalized size = 0.9

$$\frac{\sqrt{2}}{2x}\sqrt{\frac{x^2}{ax^2+x^2+a-1}}\sqrt{ax^2+x^2+a-1}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{ax^2+x^2+a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2*
(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

Fricas [A] time = 1.48598, size = 120, normalized size = 1.76

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\big((a+1)x^2+a-3\big)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
+ a - 1))/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14394, size = 82, normalized size = 1.21

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sgn}\left(ax^2+x^2+a-1\right)\operatorname{sgn}(x)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sgn}(a-1)\operatorname{sgn}(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2 +
 a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*sgn
(x)
```

3.296 $\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=281

$$\frac{(c+dx^2)(5a^2d^2+2abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e}+\frac{(bc-ad)(5a^2d^2+2abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}-\frac{(c+dx^2)^2(5ad+}{24b^2}$$

```
[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x
^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*
d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

Rubi [A] time = 0.290242, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 413, 385, 199, 208}

$$\frac{(c+dx^2)(5a^2d^2+2abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e}+\frac{(bc-ad)(5a^2d^2+2abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}-\frac{(c+dx^2)^2(5ad+}{24b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x
^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*
d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(bc+5ad)e^2+3c(bc+ad)ex^2}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6bd}$$

$$= -\frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{\left((bc-ad)\left(b^2c^2+2abcd\right.\right.}{}$$

$$= \frac{\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{6bd(b}$$

$$= \frac{\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{6bd(b}$$

Mathematica [A] time = 0.388769, size = 224, normalized size = 0.8

$$\frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(15a^2d^2-2abd\left(2c+5dx^2\right)+b^2\left(-3c^2+2cdx^2+8d^2x^4\right)\right)+3\sqrt{bc-ad}\left(5a^2d^2+2ab\right.\right.}{48b^3d^{5/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^
```

```
4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[d]*
Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Maple [B] time = 0.023, size = 527, normalized size = 1.9

$$\frac{bx^2+a}{96\,b^3d^2}\left(-36\,\sqrt{bdx^4+adx^2+bcx^2+ac}x^2abd^2\sqrt{bd}-12\,\sqrt{bdx^4+adx^2+bcx^2+ac}x^2cb^2d\sqrt{bd}-15\,d^3\ln\left(1/2\,\frac{2\,bdx^2+2\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)/b^3*(-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*d^2*(b*
d)^(1/2)-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)-15*
d^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d
+b*c)/(b*d)^(1/2))*a^3+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+3*ln(1/2*(2*b*d*x^2+2*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c^2*
b^2*d+3*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*d*(
b*d)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^2*d^2*(b*d)^(1/2)-24*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2
+c)*(b*x^2+a))^(1/2)/d^2/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82024, size = 1153, normalized size = 4.1

$$\left[-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e-4\left(2\,bd^2x^4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)*
log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d +
```

```
a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b
*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3*d -
 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c
^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d
^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^6 - 3*b^3
*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4
- (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(b^4*d^3*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.297 $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=169

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

```
[Out] -((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^2*d*e)
+ (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a*d
)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.134635, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 385, 199, 208}

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^2*d*e)
+ (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a*d
)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
```

```
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{-ae+cx^2}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{((bc-ad)(bc+3ad)e)\,\text{Subst}\left(\int \frac{1}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4bd}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{((bc-ad)(bc+3ad))\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx\right.}{8b^2d}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.343573, size = 172, normalized size = 1.02

$$\frac{\sqrt{d}\left(a+bx^2\right)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(b\left(c+2dx^2\right)-3ad\right)-\sqrt{a+bx^2}\sqrt{bc-ad}(3ad+bc)\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^2d^{3/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[d]*(a + b*x^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-3*a*d + b*(c + 2*d
*x^2)) - Sqrt[b*c - a*d]*(b*c + 3*a*d)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqr
t[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^2*d^(3/2)*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Maple [B] time = 0.01, size = 342, normalized size = 2.

$$\frac{bx^2+a}{16\,b^2d}\left(4\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}x^2bd+3\,d^2\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)a^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*d+3
*d^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*
d+b*c)/(b*d)^(1/2))*a^2-2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*d-b^2*ln(1/2*(2*b*d*x^2+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2-6*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*
x^2+a))^(1/2)/b^2/d/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68079, size = 886, normalized size = 5.24

$$\left[-\frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\sqrt{bde}\log\left(8\,b^2d^2ex^4 + 8\left(b^2cd + abd^2\right)ex^2 + \left(b^2c^2 + 6\,abcd + a^2d^2\right)e + 4\left(2\,bd^2x^4 + bc^2 + acd + \right.\right.}{32\,b^3d^2e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 +
 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d
^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 +
 a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c
*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1/16*(
(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*
e)) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.298 $\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=106

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

Rubi [A] time = 0.0682991, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 199, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{2be} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{2be} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

Mathematica [A] time = 0.133623, size = 152, normalized size = 1.43

$$\frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}} + \sqrt{bc-ad}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{2b\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
 + Sqrt[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(2*
b*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d
)])
```

Maple [B] time = 0.007, size = 200, normalized size = 1.9

$$\frac{bx^2+a}{4b}\left(-a\ln\left(\frac{1}{2}\left(2\,bdx^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)d+b\ln\left(\frac{1}{2}\left(2\,bdx^2+2\sqrt{bdx^4+adx^2+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(-a*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d+b*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x
^2+a))^(1/2)/b/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59683, size = 679, normalized size = 6.41

$$\left[-\frac{\sqrt{bde}(bc-ad)\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e-4\left(2\,bd^2x^4+bc^2+acd+\left(3\,bcd+ad^2\right)x\right.\right.}{8\,b^2de}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*
e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d
+ (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4
*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4*(sq
rt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2 + b*c
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.299 $\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$

Optimal. Leaf size=112

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rubi [A] time = 0.10582, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 391, 208}

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{1}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= c\,\mathrm{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right) + d\,\mathrm{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}$$

Mathematica [A] time = 0.227685, size = 190, normalized size = 1.7

$$\frac{\sqrt{a+bx^2}\left(\sqrt{a}\sqrt{d}\sqrt{c+dx^2}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - b\sqrt{c}\left(c+dx^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}b\left(c+dx^2\right)^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[(b*(
c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]
 - b*Sqrt[c]*(c + d*x^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c
+ d*x^2])]))/(Sqrt[a]*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^(3/2)
)
```

Maple [B] time = 0.011, size = 179, normalized size = 1.6

$$\frac{bx^2+a}{2}\left(d\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{ac}-c\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\,\sqrt{ac}\sqrt{bdx^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(d*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)-c*ln((a*d*x^2+b*c*x^2+2*(a*c)^(
1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2))/(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.0263, size = 1859, normalized size = 16.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b
^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d +
 a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 1/4*sqrt(
c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2
+ a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*
c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), -1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d
 + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2
 + a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3
*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))),
1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(-d/(b*e))*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e)
)/(b*d*x^2 + a*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x), x)
```

3.300 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=130

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*a*(a*e - (c*e*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])
```

Rubi [A] time = 0.08368, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 199, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*a*(a*e - (c*e*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

Mathematica [A] time = 0.122445, size = 133, normalized size = 1.02

$$\frac{\sqrt{a+bx^2}\left(-\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{c}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax^2}\right)}{2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^2)) - ((-(b*c) +
 a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2
)*Sqrt[c])))/(2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.012, size = 326, normalized size = 2.5

$$-\frac{bx^2+a}{4a^2cx^2}\left(-2bd\sqrt{bdx^4+adx^2+bcx^2+ac}x^4\sqrt{ac}+a^2\ln\left(\frac{1}{x^2}\left(adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*(a*c)^(1/2)+
a^2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2
*a*c)/x^2)*d*c*x^2-c^2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b*a*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1
/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2))/(e*(b*x^2+a)/(d*x^2+
c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/(a*c)^(1/2)/c/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.42144, size = 714, normalized size = 5.49

$$\left[-\frac{\sqrt{ace}(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ace}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)+4\,(acdx^2+ac}{8\,a^2cex^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x
^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2
*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*
e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)
) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*e*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^3), x)
```

3.301 $\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx$

Optimal. Leaf size=218

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

```
[Out] -((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a*c*(a*e - (c*e*(a
+ b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c - a*
d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[
a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.135421, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 385, 199, 208}

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```


```
[Out] -((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a*c*(a*e - (c*e*(a
+ b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c - a*
d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[
a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{be-dx^2}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)(3bc+ad)e)\,\text{Subst}\left(\int \frac{1}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4ac}$$

$$= -\frac{(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx,\right.}{8a^2c}$$

$$= -\frac{(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.156722, size = 173, normalized size = 0.79

$$\frac{\sqrt{a}\sqrt{c}\left(a+bx^2\right)\sqrt{c+dx^2}\left(3bcx^2-a\left(2c+dx^2\right)\right)-x^4\sqrt{a+bx^2}\left(-a^2d^2-2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
- (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.013, size = 558, normalized size = 2.6

$$\frac{bx^2+a}{16\,a^3c^2x^4}\left(-2\,bd^2\sqrt{bdx^4+adx^2+bcx^2+ac}x^6a\sqrt{ac}-10\,b^2d\sqrt{bdx^4+adx^2+bcx^2+ac}x^6c\sqrt{ac}+a^3\ln\left(\frac{1}{x^2}\left(adx^2+bcx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)*(-2*b*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1
/2)-10*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*c*(a*c)^(1/2)+a^3*ln((
a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^
2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+2*a*c)/x^2)*d*b*a^2*c^2*x^4-3*c^3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b^2*a*x^4-2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b
^2*c^2*x^4*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^
(1/2)+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
((d*x^2+c)*(b*x^2+a))^(1/2)/a^3/(a*c)^(1/2)/c^2/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.21647, size = 926, normalized size = 4.25

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{ace}x^4\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\,\left(abc^2+a^2cd\right)ex^2+4\,\left(\left(bcd+ad^2\right)x^4+2\,ac^2+\left(bc^2+3\,acd\right)x^2\right)\sqrt{ace}\sqrt{\frac{bex^2+a}{dx^2+c}}}{x^4}\right.}{32\,a^3c^2ex^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(
(b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*
x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*
c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*arctan
(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
```

```
))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4
- 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c^2*
e*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^5), x)
```

3.302 $\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=403

$$-\frac{x\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)}{15b^3d\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+\right.}{15b^2}$$

```
[Out] ((b*c - 4*a*d)*x*(a + b*x^2))/(15*b^2*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
+ (x^3*(a + b*x^2))/(5*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b^2*c^2 +
 3*a*b*c*d - 8*a^2*d^2)*x*(a + b*x^2))/(15*b^3*d*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*(c + d*x^2)) + (Sqrt[c]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*(a + b*
x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3*d^(3/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)) - (c^(3/2)*(b*c - 4*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.522041, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 478, 582, 531, 418, 492, 411}

$$-\frac{x\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)}{15b^3d\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+\right.}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b*c - 4*a*d)*x*(a + b*x^2))/(15*b^2*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
+ (x^3*(a + b*x^2))/(5*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b^2*c^2 +
 3*a*b*c*d - 8*a^2*d^2)*x*(a + b*x^2))/(15*b^3*d*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*(c + d*x^2)) + (Sqrt[c]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*(a + b*
x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3*d^(3/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)) - (c^(3/2)*(b*c - 4*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
```

```
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{x^2\left(3ac+(-bc+4ad)x^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-ac(bc-4ad)+\left(-2b^2c^2-3abcd+8a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(bc-4ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(\left(-2b^2c^2-3\right.\right.}{}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)x\left(a+bx^2\right)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{c^{3/2}(bc-4ad)\left(a+\right.}{15b^2d^{3/2}\sqrt{\frac{c(a+}{a(c+}}}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)x\left(a+bx^2\right)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\left(2b^2c^2+3abcd\right.}{15b^3d^{3/}}$$

Mathematica [C] time = 0.4799, size = 258, normalized size = 0.64

$$\frac{2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-abcd-b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2d^2-3abcd-2b^2c^2\right)E\left(\right.}{15a^2d^2\left(\frac{b}{a}\right)^{5/2}\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d - b*(c + 3*d*x^2))) - I*c*(
-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b^2*c^2) - a*b
*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Maple [A] time = 0.018, size = 553, normalized size = 1.4

$$\frac{bx^2+a}{15\,b^2d^2}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3 - \sqrt{-\frac{b}{a}}x^5abd^3 + 4\sqrt{-\frac{b}{a}}x^5b^2cd^2 - 4\sqrt{-\frac{b}{a}}x^3a^2d^3 + \sqrt{-\frac{b}{a}}x^3b^2c^2d - 4\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{Ell}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)*(3*(-b/a)^(1/2)*x^7*b^2*d^3-(-b/a)^(1/2)*x^5*a*b*d^3+4*(-b/a
)^(1/2)*x^5*b^2*c*d^2-4*(-b/a)^(1/2)*x^3*a^2*d^3+(-b/a)^(1/2)*x^3*b^2*c^2*d
-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a^2*c*d^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+8*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-
3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*b*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-4*(-b/a)^(1/2)*x*a^2*c*d^2+(-b/a)^(1/2)
*x*a*b*c^2*d)/b^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)
/d^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^6+cx^4\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^2+ae}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^6 + c*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^2 + a*e),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.303 $\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=312

$$\frac{x\left(a+bx^2\right)(bc-2ad)}{3b^2\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a\cdots}{3b\sqrt{\cdots}}$$

```
[Out] (x*(a + b*x^2))/(3*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b*c - 2*a*d)*x*
(a + b*x^2))/(3*b^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[
c]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.338106, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6719, 478, 531, 418, 492, 411}

$$\frac{x\left(a+bx^2\right)(bc-2ad)}{3b^2\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a\cdots}{3b\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (x*(a + b*x^2))/(3*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b*c - 2*a*d)*x*
(a + b*x^2))/(3*b^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[
c]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{ac+(-bc+2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left((-bc+2ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x\left(a+bx^2\right)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\left(c(-bc+2ad)\sqrt{a\cdots\right.}{3b^2\sqrt{\frac{e(a\cdots}{c\cdots}}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x\left(a+bx^2\right)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\sqrt{c}(bc-2ad)\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{c^{3/2}\left(a\cdots\right.}{3b\sqrt{d}\cdots}$$

Mathematica [C] time = 0.272699, size = 212, normalized size = 0.68

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(2ad-bc)E\left(i\sinh\right.}{3bd\sqrt{\frac{b}{a}}\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)]*(c + d*x^2))
```

Maple [A] time = 0.011, size = 358, normalized size = 1.2

$$\frac{bx^2+a}{3\,bd}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((-b/a)^(1/2)*x^5*b*d^2+(-b/a)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x
^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+(-b/a)^(1/2)
*x*a*c*d)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/b/(-b/a
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^4+cx^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^2+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^2 + a*e),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.304 $\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$

Optimal. Leaf size=252

$$\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{b\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] (d*x*(a + b*x^2))/(b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt
[c]*Sqrt[d]*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*(c + d*x^2)) + (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.130102, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6719, 422, 418, 492, 411}

$$\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{b\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (d*x*(a + b*x^2))/(b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt
[c]*Sqrt[d]*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*(c + d*x^2)) + (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{\left(c\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx\left(a+bx^2\right)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx\left(a+bx^2\right)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+\right.}$$

Mathematica [A] time = 0.0569944, size = 86, normalized size = 0.34

$$\frac{\sqrt{\frac{a+bx^2}{a}}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[
-(b/a)]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c])
```

Maple [A] time = 0.007, size = 127, normalized size = 0.5

$$c\left(bx^2+a\right)\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}}\frac{1}{\sqrt{\left(dx^2+c\right)\left(bx^2+a\right)}}\frac{1}{\sqrt{-\frac{b}{a}}}\frac{1}{\sqrt{bdx^4+adx^2+bcx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] (b*x^2+a)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2
)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^2+ae}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^2 + a*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.305 $\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=289

$$-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{a\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] -((a + b*x^2)/(a*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])) + (d*x*(a + b*x^2))/
(a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d]*(a + b
*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
 + (Sqrt[c]*Sqrt[d]*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.311714, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 21, 422, 418, 492, 411}

$$-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{a\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])) + (d*x*(a + b*x^2))/
(a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d]*(a + b
*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
 + (Sqrt[c]*Sqrt[d]*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx\left(a+bx^2\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\left(cd\sqrt{a+bx^2}\right)\int}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx\left(a+bx^2\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A] time = 0.257569, size = 111, normalized size = 0.38

$$\frac{\left(a+bx^2\right)\left(\frac{d\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{bx^2}{a}+1}\left(c+dx^2\right)}-\frac{1}{x}\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((a + b*x^2)*(-x^(-1) + (d*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[1 + (b*x^2)/a]*(c + d*x^2))))/(a*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)])
```

Maple [A] time = 0.013, size = 297, normalized size = 1.

$$-\frac{bx^2+a}{ax}\left(\sqrt{-\frac{b}{a}}x^4bd-\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)xad+bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}x\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] -(b*x^2+a)*((-b/a)^(1/2)*x^4*b*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*a*d+b*c*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-b*c*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+(-b
/a)^(1/2)*x^2*a*d+(-b/a)^(1/2)*x^2*b*c+(-b/a)^(1/2)*a*c)/(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2+c)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^4+aex^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^4 + a*e*x^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

3.306 $\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=372

$$\frac{(a+bx^2)(2bc-ad)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(2bc-ad)}{3a^2c(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{d}(a+bx^2)(2bc-ad}{3a^2\sqrt{c}(c+dx^2)}$$

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((2*b*c - a*d)*(
a + b*x^2))/(3*a^2*c*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(2*b*c - a*d
)*x*(a + b*x^2))/(3*a^2*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (b*Sqrt[c]*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.453529, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 583, 531, 418, 492, 411}

$$\frac{(a+bx^2)(2bc-ad)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(2bc-ad)}{3a^2c(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{d}(a+bx^2)(2bc-ad}{3a^2\sqrt{c}(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((2*b*c - a*d)*(
a + b*x^2))/(3*a^2*c*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(2*b*c - a*d
)*x*(a + b*x^2))/(3*a^2*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (b*Sqrt[c]*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
```

```
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-2bc+ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{abcd+bd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(bd(2bc-ad)\sqrt{a+\right.}{3a^2c\sqrt{\frac{e(a}{c}}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\right.}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{d}(2bc-ad)(a+bx^2)E\left(\right.}{3a^2\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(}{c}}}$$

Mathematica [C] time = 0.403667, size = 238, normalized size = 0.64

$$\frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)\left(a(c+dx^2)-2bcx^2\right)+2ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ibcx^3\sqrt{\frac{bx^2}{a}}}{3a^2cx^3\sqrt{\frac{b}{a}}(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*x^2 + a*(c + d*x^2))) - I*b*c*
(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*a^2*Sqrt[b/a]*c*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Maple [A] time = 0.01, size = 444, normalized size = 1.2

$$-\frac{bx^2+a}{3a^2x^3c}\left(\sqrt{-\frac{b}{a}}x^6abd^2-2\sqrt{-\frac{b}{a}}x^6b^2cd+2bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac-2\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{d}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)*((-b/a)^(1/2)*x^6*a*b*d^2-2*(-b/a)^(1/2)*x^6*b^2*c*d+2*b*d*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*x^3*a*c-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*b*c*d+2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*b^2*c^2+
(-b/a)^(1/2)*x^4*a^2*d^2-2*(-b/a)^(1/2)*x^4*b^2*c^2+2*(-b/a)^(1/2)*x^2*a^2*
c*d-(-b/a)^(1/2)*x^2*a*b*c^2+(-b/a)^(1/2)*a^2*c^2)/(e*(b*x^2+a)/(d*x^2+c))^
(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/x^3/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2+c)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^6+aex^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^6 + a*e*x^4),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

3.307 $\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=354

$$\frac{\left(c+dx^2\right)^3\left(7a^2d^2-2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6b^2de^2(bc-ad)^2}-\frac{a^2\left(c+dx^2\right)^3}{be(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)\left(5ad(2bc-7ad)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}$$

```
[Out] -((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^4*d*e^2) - ((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b^3*d*(b*c - a*d)*e^2) - (a^2*(c + d*
x^2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^2*c^2 -
 2*a*b*c*d + 7*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6
*b^2*d*(b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*
ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16
*b^(9/2)*d^(3/2)*e^(3/2))
```

Rubi [A] time = 0.378598, antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 462, 385, 199, 208}

$$-\frac{a^2\left(c+dx^2\right)^3}{be(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)\left(5ad(2bc-7ad)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}+\frac{\left(c+dx^2\right)^3\left(\frac{c^2}{d}-\frac{a(2bc-7ad)}{b^2}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6e^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^4*d*e^2) - ((c^2/d + (5*a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a +
b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b*(b*c - a*d)*e^2) - (a^2*(c + d*x^
2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((c^2/d - (a*
(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6*(
b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(9/2)
*d^(3/2)*e^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{\left(-ae+cx^2\right)^2}{x^2\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(2bc-7ad)e^2+bc^2ex^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{b}$$

$$= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-2abcd+7a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3}{6b^2d(bc-ad)^2e^2} - \frac{\left((bc-ad)\left(b^2c^2+5ad(2b\right.\right.}{}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^3d(bc-ad)e^2} - \frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-2abcd+7a^2d^2\right)}{6b^2d(bc-}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^4de^2} - \frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^3d(bc-ad)e^2} - \frac{}{b(}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^4de^2} - \frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^3d(bc-ad)e^2} - \frac{}{b(}$$

Mathematica [A] time = 0.501843, size = 247, normalized size = 0.7

$$\frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(5a^2bd\left(7dx^2-20c\right)+105a^3d^2+ab^2\left(3c^2-38cdx^2-14d^2x^4\right)+b^3x^2\left(3c^2+14cdx^2+8d^2x^4\right)\right)-3\sqrt{a+bx^2}}{48b^4d^{3/2}e\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
+ 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 14*
c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^
2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*
b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c -
 a*d)])
```

Maple [B] time = 0.031, size = 1027, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/96*(-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*a*b^2*d^2+12*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^3*c*d-105*ln(1/2*(2*b
*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/
2))*x^2*a^3*b*d^3+135*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a^2*b^2*c*d^2-27*ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^
2*a*b^3*c^2*d-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^4*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(3/2)*(b*d)^(1/2)*x^2*b^2*d+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)*x^2*a^2*b*d^2-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*
a*b^2*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^3*c^2-105
*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*
c)/(b*d)^(1/2))*a^4*d^3+135*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*b*c*d^2-27*ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^
2*b^2*c^2*d-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^3*c^3+96*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)
^(1/2)*a^3*d^2-96*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a^2*b*c*d+16*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(b*d)^(1/2)*a*b*d+114*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(b*d)^(1/2)*a^3*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(b*d)^(1/2)*a^2*b*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b
^2*c^2)/d/b^4*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(
e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 18.5063, size = 1652, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4
*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*lo
g(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d
*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d -
 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6
 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d -
35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^3*c^3 + 9
*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 4
```

```
5*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b
*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b
*d*e)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b
*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*
d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*
d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x
^2 + a*b^5*d^2*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.308 $\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=202

$$\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])
])/(8*b^(7/2)*Sqrt[d]*e^(3/2))
```

Rubi [A] time = 0.238669, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 456, 453, 208}

$$\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])
])/(8*b^(7/2)*Sqrt[d]*e^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{-ae+cx^2}{x^2\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} - \frac{1}{4}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{4a}{b}-\frac{3(bc-ad)x^2}{b^2e}}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{1}{8}((bc-ad)e)\,\text{Subst}\left(\int \frac{-\frac{8a}{b^2e}+\frac{(3bc-7ad)x^2}{b^3e^2}}{x^2\left(be-dx^2\right)}\right.$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{(3(bc-5ad)(bc-ad))\,\text{Su}}{}$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{3(bc-5ad)(bc-ad)\tan}{8b^{7/2}\sqrt{d}e^3}$$

Mathematica [A] time = 0.331849, size = 190, normalized size = 0.94

$$\frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(-15a^2d+ab\left(13c-5dx^2\right)+b^2x^2\left(5c+2dx^2\right)\right)+3\sqrt{a+bx^2}(bc-5ad)\sqrt{bc-ad}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^3\sqrt{d}e\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-15*a^2*d + a*b*(13*c - 5*d*x^2
) + b^2*x^2*(5*c + 2*d*x^2)) + 3*(b*c - 5*a*d)*Sqrt[b*c - a*d]*Sqrt[a + b*x
^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^3*Sqrt[d]*e*Sq
```

```
rt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Maple [B] time = 0.013, size = 679, normalized size = 3.4

$$\frac{bx^2+a}{16\,b^3\left(dx^2+c\right)}\left(4\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}x^4b^2d+15\,\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+b}{\sqrt{bd}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*d+15*ln(1/2
*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d
)^(1/2))*x^2*a^2*b*d^2-18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*a*b^2*c*d+3*ln(1/2*(2*b*d*x^2+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2
*b^3*c^2-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*b*d+10*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^2*c+15*ln(1/2*(2*b*d*x^
2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a
^3*d^2-18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c*d+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c^2-14*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(b*d)^(1/2)*a*b*c-16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d+
16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c)/b^3*(b*x^2+a)/(b*d)^(1/2)
/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.55153, size = 1239, normalized size = 6.13

$$\left[\frac{3\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)x^2\right)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abc\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*
a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x
^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (
3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(2
*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3
)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2 - 6*a^
2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(-b*d*
e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15
*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d
^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 +
 a*b^4*d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.309 $\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=146

$$\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/
(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*
e^(3/2))
```

Rubi [A] time = 0.0983798, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/
(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*
e^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}$$

$$= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3d(bc-ad))\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b^2e}$$

$$= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

Mathematica [C] time = 0.0702331, size = 86, normalized size = 0.59

$$-\frac{\left(a+bx^2\right)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{d(bx^2+a)}{ad-bc}\right)}{b\left(\frac{b(c+dx^2)}{bc-ad}\right)^{3/2}\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((a + b*x^2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x^2))/(-(b*c) +
 a*d)])/(b*((e*(a + b*x^2))/(c + d*x^2))^(3/2)*((b*(c + d*x^2))/(b*c - a*d)
)^(3/2)))
```

Maple [B] time = 0.01, size = 432, normalized size = 3.

$$\frac{bx^2+a}{4\,b^2\left(dx^2+c\right)}\left(-3\,\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc}{\sqrt{bd}}\right)x^2abd^2+3\,\ln\left(1/2\,\frac{2\,bdx^2+2\,\sqrt{bdx^4+ad}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*(-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*x^2*a*b*d^2+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*x^2*b^2*c*d+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*d-3*d^2*ln(1/2*(2*b*d*x^2+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2+3*l
n(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)
/(b*d)^(1/2))*a*c*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d
+4*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d-4*(b*d)^(1/2)*((d*x^2+c)*(b*
x^2+a))^(1/2)*b*c)/b^2*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d
*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.93855, size = 914, normalized size = 6.26

$$\left[-\frac{3\left(\left(b^2c-abd\right)ex^2+\left(abc-a^2d\right)e\right)\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2\,b^2d^2x^4+b^2c^2\cdots\right.\right.}{8\left(b^3e^2x^2+ab^2e^2\right)\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(d/(b*e))*log(8*b^
2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(
2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (
b*c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b
^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(-d/(b*e))
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(
-d/(b*e))/(b*d*x^2 + a*d)) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*
a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^2*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.310 $\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*e^(3
/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(b^(3/2)*e^(3/2))
```

Rubi [A] time = 0.196113, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 480, 522, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*e^(3
/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(b^(3/2)*e^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\ &= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-(bc+ad)e+cdx^2}{\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{abe}\\ &= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{ae} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{be}\\ &= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.342338, size = 253, normalized size = 1.66

$$\frac{-a^{3/2}d^{3/2}\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - b\left(c+dx^2\right)\sqrt{bc-ad}\left(bc^{3/2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right.\right.}{a^{3/2}b^2e\left(c+dx^2\right)^{3/2}\sqrt{bc-ad}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(a^(3/2)*d^(3/2)*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[(b*(
c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]
) - b*Sqrt[b*c - a*d]*(c + d*x^2)*(Sqrt[a]*(-(b*c) + a*d)*Sqrt[c + d*x^2] +
 b*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])]))/(a^(3/2)*b^2*Sqrt[b*c - a*d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*(c + d*x^2)^(3/2))
```

Maple [B] time = 0.016, size = 401, normalized size = 2.6

$$\frac{bx^2+a}{2\,ab\left(dx^2+c\right)}\left(\ln\left(\frac{1}{2}\left(2\,bdx^2+2\,\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{ac}x^2abd^2-\sqrt{bd}\ln\left(\frac{1}{x^2}\left(adx^2+bc\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*
d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*x^2*a*b*d^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*b^2*c^2+
ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b*c^2-2*(b*d)^(1/2)
*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d+2*(b*d)^(1/2)*(a*c)^(1/2)*((d*
x^2+c)*(b*x^2+a))^(1/2)*b*c)/a/b*(b*x^2+a)/(a*c)^(1/2)/(b*d)^(1/2)/((d*x^2+
c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.17323, size = 2674, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6
*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2
 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*s
qrt(d/(b*e))) + (b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d +
 a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), -1/4*(2*(a
*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) - (b^2*c*e*x
^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^
2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2
+ (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)
))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), 1/4*(2*(b^2*c*e*x^2 + a*b*c*e)*sqrt(
```

```
-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + (a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e
))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2
)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 4*(b*c^2 - a*c*d + (b*c*d
 - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^
2), 1/2*((b^2*c*e*x^2 + a*b*c*e)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2
 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c))
 - (a*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 2*(b*
c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^
2*e^2*x^2 + a^2*b*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x), x)
```

3.311 $\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{3\sqrt{c}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/
(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2)
)) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))
```

Rubi [A] time = 0.111482, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{c}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/
(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2)
)) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2(-ae+cx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3c(bc-ad))\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a^2e}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{3\sqrt{c}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

Mathematica [A] time = 0.0848137, size = 148, normalized size = 0.87

$$\frac{3\sqrt{c}x^2\sqrt{a+bx^2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)-\sqrt{a}\sqrt{c+dx^2}\left(a\left(c-2dx^2\right)+3bcx^2\right)}{2a^{5/2}ex^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
- a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d
*x^2])
```

Maple [B] time = 0.016, size = 641, normalized size = 3.8

$$-\frac{bx^2+a}{4a^3x^2\left(dx^2+c\right)}\left(-2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{ac}x^6b^2d+3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^2*d+3*ln((a*
d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)
*x^4*a^2*b*c*d-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*x^4*a*b^2*c^2-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*x^4*a*b*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^
4*b^2*c+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)+2*a*c)/x^2)*x^2*a^3*c*d-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^2*a^2*b*c^2+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c
)^(1/2)*x^2*a^2*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a*b
*c-4*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a^2*d+4*(a*c)^(1/2)*((d*x^
2+c)*(b*x^2+a))^(1/2)*x^2*a*b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*a)*(b*x^2+a)/x^2/(a*c)^(1/2)/a^3/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+
c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.3692, size = 957, normalized size = 5.63

$$\left[-\frac{3\left(\left(b^2c-abd\right)ex^4+\left(abc-a^2d\right)ex^2\right)\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2+6abcd+a^2d^2)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(\left(abcd+a^2d^2\right)x^4+2a^2c^2+\left(abc^2+3a^2cd\right)x^2\right)}{x^4}\right)}{8\left(a^2be^2x^4+a^3e^2x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log(
((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^
2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*((3*b*c*d - 2*a*d^2)
*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a
^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*
d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + 2*((3*b*c*d - 2*a*d^
2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^2*b*e^2*x^4 + a^3*e^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^3), x)
```

3.312 $\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x
^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c -
a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```

Rubi [A] time = 0.22828, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 456, 453, 208}

$$-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```


```
[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x
^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c -
a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```


Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{be-dx^2}{x^2\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{1}{4}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{4b}{a}+\frac{3(bc-ad)x^2}{a^2e}}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{1}{8}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{8b}{a^2e}+\frac{(7bc-3ad)x}{a^3e^2}}{x^2\left(-ae+cx^2\right.}\right.$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)(5bc-ad))\,\text{S}}{}$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-ad)(5bc-ad)\tan}{8a^{7/2}\sqrt{ce}}$$

Mathematica [A] time = 0.120786, size = 189, normalized size = 0.74

$$\frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}\left(-a^2\left(2c+5dx^2\right)+abx^2\left(5c-13dx^2\right)+15b^2cx^4\right)-3x^4\sqrt{a+bx^2}\left(a^2d^2-6abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+dx}}\right.}{8a^{7/2}\sqrt{c}ex^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2) -
 a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a + b*
x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(7/
2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] time = 0.018, size = 1042, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/16*(-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*d^2+18*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^3*c*d+3*ln((a*d*x^2+b
*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a^
3*b*c*d^2-18*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+2*a*c)/x^2)*x^6*a^2*b^2*c^2*d+15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^6*a*b^3*c^3-12*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*d^2+26*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b^2*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(a*c)^(1/2)*x^6*b^3*c^2+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^4*c*d^2-18*ln((a*d*x^2+b*c*x^2
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*x^4*a^3*b*c^
2*d+15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)+2*a*c)/x^2)*x^4*a^2*b^2*c^3+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(
1/2)*x^4*a*b*d-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b^2*c
-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a^3*d^2+8*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a^2*b*c*d+18*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b^2*c^2+16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+
a))^(1/2)*x^4*a^2*b*c*d-16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a*b^
2*c^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*d-14*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*b*c+4*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c)/c*(b*x^2+a)/x^4/(a*c)^(1/2)/a^4/((d*x^2+c
)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 36.7036, size = 1284, normalized size = 5.04

$$\left[\frac{3\left(\left(5\,b^3c^2 - 6\,ab^2cd + a^2bd^2\right)x^6 + \left(5\,ab^2c^2 - 6\,a^2bcd + a^3d^2\right)x^4\right)\sqrt{ace}\,\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(\left(b \cdots\right.\right.}{x^4}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*
b*c*d + a^3*d^2)*x^4)*sqrt(a*c*e)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^
4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*
a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/x^4) + 4*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*
c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4), 1/16*(
3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*b*c*d +
 a^3*d^2)*x^4)*sqrt(-a*c*e)*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) + 2*((15*a*b^
2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^2*b*c^2*d -
 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^5), x)
```

3.313 $\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=453

$$\frac{x\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)}{5b^4e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+bx^2\right.}{5b^3\sqrt{d}}$$

```
[Out] ((7*b*c - 8*a*d)*x*(a + b*x^2))/(5*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
 + (6*d*x^3*(a + b*x^2))/(5*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^
2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x*(a + b*x^2))/(5*b^4*e*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2)) - (x^3*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]) - (Sqrt[c]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(a + b*x^2)*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^4*Sqrt[d]*e*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2)) - (c^(3/2)*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^3*Sqrt[d]*e*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.675178, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6719, 467, 581, 582, 531, 418, 492, 411}

$$\frac{x\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)}{5b^4e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+bx^2\right.}{5b^3\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```


```
[Out] ((7*b*c - 8*a*d)*x*(a + b*x^2))/(5*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
 + (6*d*x^3*(a + b*x^2))/(5*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^
2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x*(a + b*x^2))/(5*b^4*e*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2)) - (x^3*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]) - (Sqrt[c]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(a + b*x^2)*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^4*Sqrt[d]*e*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2)) - (c^(3/2)*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^3*Sqrt[d]*e*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```


Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x^3\left(c+dx^2\right)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}(3c+6dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{6dx^3\left(a+bx^2\right)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3\left(c+dx^2\right)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2(3c(5bc-6ad)+3d(7bc-8ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x\left(a+bx^2\right)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3\left(a+bx^2\right)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3\left(c+dx^2\right)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{3acd(7bc-8ad)-3d(b^2c^2-16abcd+}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15b^3de\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x\left(a+bx^2\right)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3\left(a+bx^2\right)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3\left(c+dx^2\right)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+}}}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x\left(a+bx^2\right)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3\left(a+bx^2\right)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-16abcd+16a^2d^2\right)x\left(a+bx^2\right)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{x^3\left(c+dx^2\right)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$= \frac{(7bc-8ad)x\left(a+bx^2\right)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3\left(a+bx^2\right)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-16abcd+16a^2d^2\right)x\left(a+bx^2\right)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{x^3\left(c+dx^2\right)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Mathematica [C] time = 0.504485, size = 271, normalized size = 0.6

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2d^2-9abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(16a^2d^2-16abc\right.}{5b^3de^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(c + d*x^2)*(-8*a^2*d + a
*b*(7*c - 2*d*x^2) + b^2*x^2*(2*c + d*x^2)) - I*c*(b^2*c^2 - 16*a*b*c*d + 1
6*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
)/(5*b^3*Sqrt[b/a]*d*e^2*(a + b*x^2))
```

Maple [A] time = 0.035, size = 935, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/5*(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x^7*b^2*d^3-2*((d*x
^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x^5*a*b*d^3+3*((d*x^2+c)*(b*x^2+a))^(1/
2)*(-b/a)^(1/2)*x^5*b^2*c*d^2-3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x^
3*a^2*d^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x^3*b^2*c^2*d-5*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*a^2*d^3+5*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*a*b*c*d^2-8*((d*x^2+c)*(b*x^2+a))^(1/2)*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a^2*c*d^2+9*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-((d*x^2+c)*
(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c
*d^2-16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+((d*x^2+c)*(b*x^2+a))^
(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*b^2*c^3-3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x*a^2*c*d^2
+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*x*a*b*c^2*d-5*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*a^2*c*d^2+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(-b/a)^(1/2)*x*a*b*c^2*d)/b^3/d/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2
+c)^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^8 + 2\,cdx^6 + c^2x^4\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^4 + 2\,abe^2x^2 + a^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^8 + 2*c*d*x^6 + c^2*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/
(b^2*e^2*x^4 + 2*a*b*e^2*x^2 + a^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.314 $\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=378

$$\frac{c^{3/2}\left(a+bx^2\right)(3bc-4ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(7bc-8ad)}{3b^3e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(7bc-8}{3b^3e\left(c+dx^2\right)}$$

```
[Out] (4*d*x*(a + b*x^2))/(3*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(7*b*c
 - 8*a*d)*x*(a + b*x^2))/(3*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*
x^2)) - (x*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (Sqrt[c]*
Sqrt[d]*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(3*b^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*(3*b*c - 4*a*d)*(a + b*x^2)
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*
e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(
c + d*x^2))
```

Rubi [A] time = 0.441961, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 467, 528, 531, 418, 492, 411}

$$\frac{c^{3/2}\left(a+bx^2\right)(3bc-4ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(7bc-8ad)}{3b^3e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(7bc-8}{3b^3e\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (4*d*x*(a + b*x^2))/(3*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(7*b*c
 - 8*a*d)*x*(a + b*x^2))/(3*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*
x^2)) - (x*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (Sqrt[c]*
Sqrt[d]*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(3*b^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*(3*b*c - 4*a*d)*(a + b*x^2)
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*
e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(
c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}(c+4dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{c(3bc-4ad)+d(7bc-8ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(c(3bc-4ad)\sqrt{a+bx\right.}{3b^2e\sqrt{\frac{e(a+bx}{c+dx}}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^{3/2}(3bc-4ad)(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{3ab^2\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+d}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(7bc-8ad)(a+bx^2)E\left(\tan^{-1}\left(\right.\right.}{3b^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+d}$$

Mathematica [C] time = 0.389404, size = 219, normalized size = 0.58

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(x\sqrt{\frac{b}{a}}(c+dx^2)(4ad-3bc+bdx^2)-4ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}}\right.}{3a^2e^2\left(\frac{b}{a}\right)^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*x*(c + d*x^2)*(-3*b*c + 4*a*d
 + b*d*x^2) + I*c*(-7*b*c + 8*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*c*(-(b*c) + a*d)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(3*a^2*(b/a)^(5/2)*e^2*(a + b*x^2))
```

Maple [A] time = 0.015, size = 643, normalized size = 1.7

$$\frac{bx^2+a}{3\,b^2\left(dx^2+c\right)^2}\left(\sqrt{-\frac{b}{a}}\sqrt{(dx^2+c)(bx^2+a)}x^5bd^2+3\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}x^3ad^2-3\sqrt{bdx^4+adx^2+bcx^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^5*b*d^2+3*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*a*d^2-3*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*b*c*d+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1
/2)*x^3*a*d^2+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*b*c*d+4*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*(
(d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2
-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^
(1/2)*b*c^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*a*c*d-3*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*b*c^2+(-b/a)^(1/2)*((d*x^2
+c)*(b*x^2+a))^(1/2)*x*a*c*d)/b^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^2x^6+2\,cdx^4+c^2x^2)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^4+2\,abe^2x^2+a^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^4 + c^2*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/
(b^2*e^2*x^4 + 2*a*b*e^2*x^2 + a^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.315 $\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=327

$$-\frac{dx\left(a+bx^2\right)(bc-2ad)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{abe\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2}{c+dx^2}}}$$

```
[Out] ((b*c - a*d)*x)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(b*c - 2*a*d
)*x*(a + b*x^2))/(a*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[c]*Sqrt[d]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*b^2*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*e*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.21972, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6719, 413, 531, 418, 492, 411}

$$-\frac{dx\left(a+bx^2\right)(bc-2ad)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{abe\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(b*c - 2*a*d
)*x*(a + b*x^2))/(a*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[c]*Sqrt[d]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*b^2*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*e*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(d(bc-2ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x\left(a+bx^2\right)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\left(cd(bc-2ad\right.}{ab^2e\sqrt{}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x\left(a+bx^2\right)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/}}{}$$

Mathematica [C] time = 0.464743, size = 203, normalized size = 0.62

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left((bc-ad)\left(x\sqrt{\frac{b}{a}}\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(2ad-bc)E}{a^2e^2\left(\frac{b}{a}\right)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*(b/a)^(3/2)*e
^2*(a + b*x^2))
```

Maple [A] time = 0.013, size = 514, normalized size = 1.6

$$-\frac{bx^2+a}{b\left(dx^2+c\right)^2a}\left(\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}x^3ad^2-\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}x^3bcd+\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] -(b*x^2+a)/b*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*a*d^2-(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x^3*b*c*d+((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*
(b*x^2+a))^(1/2)*a*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2-2*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((
d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2+(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*a*c*d-(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(-b/a)^(1/2)*x*b*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+
c)^2/a/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^4 + 2\,cdx^2 + c^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^4 + 2\,abe^2x^2 + a^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2
*e^2*x^4 + 2*a*b*e^2*x^2 + a^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

3.316 $\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=380

$$\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\left(a+bx^2\right)(2bc-ad)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(2bc-ad)}{a^2be\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(2bc-}{a^2be\left(c+dx^2\right)}$$

```
[Out] (b*c - a*d)/(a*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b*c - a*d)*(a
 + b*x^2))/(a^2*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(2*b*c - a*d)
*x*(a + b*x^2))/(a^2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (
Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a^2*b*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a^2*e*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.470159, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\left(a+bx^2\right)(2bc-ad)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(2bc-ad)}{a^2be\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(2bc-}{a^2be\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b*c - a*d)*(a
 + b*x^2))/(a^2*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(2*b*c - a*d)
*x*(a + b*x^2))/(a^2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (
Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a^2*b*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a^2*e*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(2bc-ad)-bcdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{abc^2d+bcd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ae\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(d(2bc-ad)\sqrt{a+\right.}{a^2e\sqrt{\frac{e(a}{c}}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\right.}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2}{c+dx^2}}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(2bc-ad)(a+bx^2)}{a^2be\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e}{}}}$$

Mathematica [C] time = 0.365372, size = 223, normalized size = 0.59

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)(ac-adx^2+2bcx^2)-2icx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+icx\sqrt{\frac{bx^2}{a}+1}\right.}{a^2e^2x\sqrt{\frac{b}{a}}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(a*c + 2*b*c*x^
2 - a*d*x^2)) + I*c*(-2*b*c + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*x
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(a^2*Sqrt[b/a]*e^2*x*(a + b*x^2))
```

Maple [A] time = 0.016, size = 654, normalized size = 1.7

$$\frac{bx^2+a}{(dx^2+c)^2a^2x}\left(\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+bcx^2+acx^4}ad^2-\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+bcx^2+acx^4}bcd-\sqrt{-\frac{b}{a}}\sqrt{(dx^2+c)(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] (b*x^2+a)*((-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*a*d^2-(-b/a
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*b*c*d-(-b/a)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*x^4*b*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x*a*c*d-
2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x*b*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(
1/2)*x*a*c*d+2*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x+(-b/a)^(1/2)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*c*d-(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*x^2*b*c^2-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a*c*
d-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^2*b*c^2-(-b/a)^(1/2)*((d*x^2+c
)*(b*x^2+a))^(1/2)*a*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^2/(-b
/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^2x^4 + 2\,cdx^2 + c^2)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^6 + 2\,abe^2x^4 + a^2e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2
*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

3.317 $\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=444

$$\frac{(a+bx^2)(8bc-7ad)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(a+bx^2)(4bc-3ad)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(8bc-7ad)}{3a^3e(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}(a+bx^2)(4bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{3a^3e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] (b*c - a*d)/(a*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((4*b*c - 3*a*d
)*(a + b*x^2))/(3*a^2*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((8*b*c
- 7*a*d)*(a + b*x^2))/(3*a^3*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(8
*b*c - 7*a*d)*x*(a + b*x^2))/(3*a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)) + (Sqrt[c]*Sqrt[d]*(8*b*c - 7*a*d)*(a + b*x^2)*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d
]*(4*b*c - 3*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] time = 0.649331, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$\frac{(a+bx^2)(8bc-7ad)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(a+bx^2)(4bc-3ad)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(8bc-7ad)}{3a^3e(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}(a+bx^2)(4bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{3a^3e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((4*b*c - 3*a*d
)*(a + b*x^2))/(3*a^2*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((8*b*c
- 7*a*d)*(a + b*x^2))/(3*a^3*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(8
*b*c - 7*a*d)*x*(a + b*x^2))/(3*a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)) + (Sqrt[c]*Sqrt[d]*(8*b*c - 7*a*d)*(a + b*x^2)*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d
]*(4*b*c - 3*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(4bc-3ad)-d(3bc-2ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-bc^2(8bc-7ad)-bcd(4bc-3ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{abc^2d(4bc-3ad)+b^2c^2d}{\sqrt{a+bx^2}\sqrt{c+d}}}{3a^3bc^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd(8bc-7ad)\sqrt{a+bx^2}\right)\int}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{}}{}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{}}{}$$

Mathematica [C] time = 0.465775, size = 266, normalized size = 0.6

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ix^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-11abcd+8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(a^2\left(c+4dx^2\right)+ab\left(7\right.\right.\right.}{3a^3e^2x^3\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(-8*b^2*c*x^4 +
 a^2*(c + 4*d*x^2) + a*b*(-4*c*x^2 + 7*d*x^4))) - I*b*c*(-8*b*c + 7*a*d)*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3
*a^3*Sqrt[b/a]*e^2*x^3*(a + b*x^2))
```

Maple [A] time = 0.016, size = 866, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/3*(b*x^2+a)*(3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*b*
d^2-3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*b^2*c*d+4*(-b/a)
^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a*b*d^2-5*(-b/a)^(1/2)*((d*x^2+c)*(b
*x^2+a))^(1/2)*x^6*b^2*c*d-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*a^2*d^2
+11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*a*b*c*d-8*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x
^2+a))^(1/2)*x^3*b^2*c^2-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*a*b*c*d+8
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x^3*b^2*c^2+3*(-b/a)^(1/2)*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*x^4*a*b*c*d-3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*x^4*b^2*c^2+4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*a^2*
d^2-5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^4*b^2*c^2+5*(-b/a)^(1/2)*(
(d*x^2+c)*(b*x^2+a))^(1/2)*x^2*a^2*c*d-4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))
^(1/2)*x^2*a*b*c^2+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c^2)/(e*(b*
x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^2x^4 + 2\,cdx^2 + c^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^8 + 2\,abe^2x^6 + a^2e^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2
*e^2*x^8 + 2*a*b*e^2*x^6 + a^2*e^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

3.318 $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=216

$$-\frac{\left(-8a^2c^2+4abc+b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}+\frac{\left(c+dx^2\right)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{6ad^3}$$

```
[Out] -((b^2 + 4*a*b*c - 8*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)])/(16*a^2*d^3) - ((b + 4*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)])/(8*a*d^3) + ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*x^2))^(3
/2))/(6*a*d^3) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(5/2)*d^3)
```

Rubi [A] time = 0.620321, antiderivative size = 259, normalized size of antiderivative = 1.2, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(8a^2c^2+4abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{5/2}d^3\sqrt{a\left(c+dx^2\right)+b}}-\frac{(8ac+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b^2 + 4*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(16*a^2*d
^3) - ((3*b + 8*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)
))/(24*a^2*d^3) + (x^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^
2)))/(6*a*d^2) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a + b/
(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(1
6*a^(5/2)*d^3*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x^2\sqrt{b+ac+adx}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{6ad^2} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{\sqrt{b+ac+adx}\left(-c(b+\ldots\right.}{\sqrt{c+\ldots}}\right.}{6ad^2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3} + \frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\ldots\right.\right.}{6ad^2}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\ldots\right.}{24a^2d^3}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\ldots\right.}{24a^2d^3}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\ldots\right.}{24a^2d^3}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\ldots\right.}{24a^2d^3}$$

Mathematica [A] time = 0.309146, size = 137, normalized size = 0.63

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^2-cdx^2+d^2x^4\right)+2ab\left(dx^2-5c\right)-3b^2\right)+3b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b*
(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c + 8*
a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(5/2)*d^3)
```

Maple [B] time = 0.05, size = 533, normalized size = 2.5

$$\frac{dx^2+c}{96\,a^2d^3}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-48\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}x^2ca^2d\sqrt{ad^2}-12\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-48*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)-12*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)+24*ln(1/2*(2*a*d^2
*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2
)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b
^2*c*a*d+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(1/2)
-36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)+3*l
n(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a^2/(
a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37123, size = 944, normalized size = 4.37

$$\left[\frac{3\left(8\,a^{2}bc^{2}+4\,ab^{2}c+b^{3}\right)\sqrt{a}\log\left(8\,a^{2}d^{2}x^{4}+8\,a^{2}c^{2}+8\left(2\,a^{2}c+ab\right)dx^{2}+8\,abc+b^{2}+4\left(2\,ad^{2}x^{4}+(4\,ac+b)dx^{2}+2\,ac^{2}\right.\right.}{192\,a^{3}d^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2
*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c -
 (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d
^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^
2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 +
a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c
^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(a^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [A] time = 1.68737, size = 408, normalized size = 1.89

$$\frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\left(\frac{4\,x^2\mathrm{sgn}\left(dx^2+c\right)}{d}-\frac{4\,a^3cd^4\mathrm{sgn}\left(dx^2+c\right)-a^2bd^4\mathrm{sgn}\left(dx^2+c\right)}{a^3d^6}\right)x^2+\frac{8\,a^3c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*x^2*sgn(d*
x^2 + c)/d - (4*a^3*c*d^4*sgn(d*x^2 + c) - a^2*b*d^4*sgn(d*x^2 + c))/(a^3*d
^6))*x^2 + (8*a^3*c^2*d^3*sgn(d*x^2 + c) - 10*a^2*b*c*d^3*sgn(d*x^2 + c) -
3*a*b^2*d^3*sgn(d*x^2 + c))/(a^3*d^6)) - 1/32*(8*a^3*b*c^2*d^4*sgn(d*x^2 +
c) + 4*a^2*b^2*c*d^4*sgn(d*x^2 + c) + a*b^3*d^4*sgn(d*x^2 + c))*log(abs(-2*
a^(3/2)*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*a*abs(d) - sqrt(a)*b*d))/(a^(7/2)*d^6*abs(d))
```

3.319 $\int x^3 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^2}$$

```
[Out] ((b - 4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a*d^2) +
 ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*d^2) - (b*(b + 4*
a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2)*d^2
)
```

Rubi [A] time = 0.466895, antiderivative size = 181, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{b(4ac+b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{3/2}d^2\sqrt{a\left(c+dx^2\right)+b}}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4ad^2}-\frac{(4ac+b)\left(c+dx^2\right)\sqrt{a}}{8ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((b + 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*a*d^2) + ((c + d*x^2)
*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*a*d^2) - (b*(b + 4*a*c)*Sq
rt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqr
t[b + a*(c + d*x^2)]])/(8*a^(3/2)*d^2*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x\sqrt{b+ac+adx}}{\sqrt{c+dx}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\left((b+4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{\sqrt{b+ac+adx}}{\sqrt{c+dx}}dx\right.}{8ad\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\left(b(b+4ac)\sqrt{c+d\right.}{}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\left(b(b+4ac)\sqrt{c+d\right.}{}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\left(b(b+4ac)\sqrt{c+d\right.}{}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{b(b+4ac)\sqrt{c+dx^2}}{8a^{3/2}}$$

Mathematica [A] time = 0.173972, size = 97, normalized size = 0.69

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac+2adx^2+b\right)-b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a
*d*x^2) - b*(b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2
)*d^2)
```

Maple [B] time = 0.012, size = 354, normalized size = 2.5

$$\frac{dx^2+c}{16\,ad^2}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(4\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}x^2ad-4\ln\left(1/2\,\frac{2\,ad^2x^2+2\,acd+2\sqrt{ad^2x^4+2\,acd}}{\sqrt{ad}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*d-4*ln(1/2*(2*a*d^2*x^2+2*
a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a*b*c*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)*a*c-ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+
b))^(1/2)/a/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35704, size = 744, normalized size = 5.28

$$\left[\frac{\left(4\,abc + b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4 + 8\,a^2c^2 + 8\left(2\,a^2c + ab\right)dx^2 + 8\,abc + b^2 - 4\left(2\,ad^2x^4 + (4\,ac + b)dx^2 + 2\,ac^2 + bc\right)\sqrt{a}\sqrt{\cdots}\right.}{32\,a^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
 a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 +
a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*
d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*
(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [A] time = 1.36809, size = 219, normalized size = 1.55

$$\frac{1}{16}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{\left(4\,abc+b^2\right)\log\left(\left|-2\,a^{\frac{3}{2}}cd-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,ac}\right.\right.\right.}{a^{\frac{3}{2}}d|d|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (2
*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(-2*a^(3/2)*c*d - 2*(sqrt(a
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d)
 - sqrt(a)*b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

3.320 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=69

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rubi [A] time = 0.0539107, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 47, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{x}}\,dx,x,c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,\frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\frac{1}{c+dx^2}\right)}{4d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.0870507, size = 77, normalized size = 1.12

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}+b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + b*ArcTanh[Sqrt
[a + b/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Maple [B] time = 0.007, size = 180, normalized size = 2.6

$$\frac{dx^2+c}{4\,d}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(b\ln\left(\frac{1}{2}\left(2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd\right)\frac{1}{\sqrt{ad^2}}\right)d+2\,\sqrt{ad^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
```

```
*d^2)^(1/2))*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/d/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41242, size = 608, normalized size = 8.81

$$\left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}{8\,ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*
c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [B] time = 1.3464, size = 171, normalized size = 2.48

$$-\frac{1}{4}\left(\frac{b\log\left(\left|-2\,a^{\frac{3}{2}}cd-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a|d|-\sqrt{ab}d\right|\right)}{\sqrt{a}|d|}-\frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b*log(abs(-2*a^(3/2)*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - sqrt(a)*b*d))/(sqrt(a)*abs(d))
 - 2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 + c
)
```

3.321 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx$

Optimal. Leaf size=96

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b +
a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]
)/Sqrt[c]
```

Rubi [A] time = 0.432868, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c +
d*x^2])/Sqrt[b + a*(c + d*x^2)]])/Sqrt[b + a*(c + d*x^2)] - (Sqrt[b + a*c]*
Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x
^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)}{2\sqrt{b}}$$

$$= \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \sqrt{c+dx^2}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Sub}}{\sqrt{b+a(}}$$

$$= -\frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\right.}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx}}{\sqrt{c}\sqrt{b+a(c+dx}}\right.}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [A] time = 0.132368, size = 80, normalized size = 0.83

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b + a*c]*ArcTanh[(
Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[c]
```

Maple [B] time = 0.02, size = 235, normalized size = 2.5

$$\frac{dx^2+c}{2c}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(\ln\left(\frac{1}{2}\left(2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd\right)\frac{1}{\sqrt{ad^2}}\right)acd-\sqrt{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x,x)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*a*d^2*x^2+2*a*c*
d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a*c*d-(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2
+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*
d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00838, size = 2101, normalized size = 21.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*
b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2
 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c
^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*
c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt
((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(
-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/4*
sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16
*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^
2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(-(a*c + b)/c
)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c +
 b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^
2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqr
t(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)*arctan(1/2*(2*a*
d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^
2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(
a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x, x)
```

3.322 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3} \, dx$

Optimal. Leaf size=104

$$\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}}-\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

```
[Out] -((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*c*x^2) + (b*d*ArcTa
nh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*c^(3/
2)*Sqrt[b + a*c])
```

Rubi [A] time = 0.385875, antiderivative size = 140, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2c^{3/2}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*c*x^2) + (b*d*Sqrt[c + d*x^2]*Sqr
t[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[
b + a*(c + d*x^2)])])/(2*c^(3/2)*Sqrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx &= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}\\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}\\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}\\
&= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}-\frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{4c\sqrt{b+a\left(c+dx^2\right)}}\\
&= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}-\frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{2c\sqrt{b+a\left(c+dx^2\right)}}\\
&= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}+\frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{2c^{3/2}\sqrt{b+ac}\sqrt{b+a\left(c+dx^2\right)}}
\end{aligned}$$

Mathematica [A] time = 0.356297, size = 132, normalized size = 1.27

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{bd\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)}{\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}\left(c+dx^2\right)}{x^2}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((Sqrt[c]*(c + d*x^2))/x^2) + (b*d
*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b +
a*c + a*d*x^2])])/(Sqrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])))/(2*c^(3/2))
```

Maple [B] time = 0.02, size = 454, normalized size = 4.4

$$-\frac{dx^2+c}{4c^2(ac+b)x^2}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-2ad^2\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,x^4\sqrt{c^2a+bc}-\ln\left(\frac{1}{x^2}\left(2acdx^2+bdx^2+2c^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^3,x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x
^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)+2*b*c)/x^2)*x^2*a*b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b
*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*b
^2*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^2/(a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1593, size = 936, normalized size = 9.

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2+4((2ac+b)d^2x^4+2ac^3+(4ac^2+3bc)dx^2+2bc^2)\sqrt{ac^2+bc}\sqrt{\cdots}}{x^4}\right.}{8(ac^3+bc^2)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
```

```
+ b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^3, x)
```

3.323 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{bd^2(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{5/2}(ac+b)^{3/2}}+\frac{d(4ac+5b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2x^4}$$

```
[Out] ((5*b + 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^2*
(b + a*c)*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c
^2*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)])/Sqrt[b + a*c]])/(8*c^(5/2)*(b + a*c)^(3/2))
```

Rubi [A] time = 0.507267, antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$-\frac{bd^2(4ac+3b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{5/2}(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(4ac+3b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(\right.}{4cx^4(ac+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] ((3*b + 4*a*c)*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*c^2*(b + a*c)*x^2)
 - ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*c*(b + a*c)
*x^4) - (b*(3*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTan
h[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^
(5/2)*(b + a*c)^(3/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} - \frac{\left((3b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^2\sqrt{c+dx}}\,dx\right.}{8c(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} + \frac{\left(b(3b+4ac)d^2\sqrt{c+dx\right.}{1}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} + \frac{\left(b(3b+4ac)d^2\sqrt{c+dx\right.}{}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} - \frac{b(3b+4ac)d^2\sqrt{c+dx^2}}{8c^{5/2}(b+}$$

Mathematica [A] time = 0.277231, size = 193, normalized size = 1.11

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(bd^2x^4(4ac+3b)\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)+\sqrt{c}\sqrt{ac+b}\left(c+dx^2\right)\left(2ac\left(c-dx^2\right)+b\left(2c-3dx^2\right)\right)\sqrt{a}\right.}{8c^{5/2}x^4(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[c]*Sqrt[b + a*c]*(c + d*x^2)*
(b*(2*c - 3*d*x^2) + 2*a*c*(c - d*x^2))*Sqrt[b + a*(c + d*x^2)] + b*(3*b +
4*a*c)*d^2*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqr
t[c]*Sqrt[b + a*c + a*d*x^2])]))/(8*c^(5/2)*(b + a*c)^(3/2)*x^4*Sqrt[b + a*
(c + d*x^2)])
```

Maple [B] time = 0.025, size = 923, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^5,x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*a^2*d^3*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*c*(a*c^2+b*c)^(3/2)-4*ln((2*a*c*d*x^
2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)+2*b*c)/x^2)*x^4*a^3*b*c^5*d^2-10*a*d^3*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*x^6*b*(a*c^2+b*c)^(3/2)-11*ln((2*a*c*d*x^2+b*d*x^2+2
*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+
2*b*c)/x^2)*x^4*a^2*b^2*c^4*d^2-20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(3/2)-10*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*
a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c
)/x^2)*x^4*a*b^3*c^3*d^2-28*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*a*c*d^2*b*x^4*(a*c^2+b*c)^(3/2)-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2
+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4
*b^4*c^2*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d^2*x^4
*(a*c^2+b*c)^(3/2)+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d
*x^2*(a*c^2+b*c)^(3/2)+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b
*d*x^2*(a*c^2+b*c)^(3/2)-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*
(a*c^2+b*c)^(3/2)*a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(
a*c^2+b*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^3/(a*c+b)^2/x^4/(
a*c^2+b*c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.841, size = 1218, normalized size = 7.

$$\left[\frac{\left(4\,abc+3\,b^2\right)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left(\left(2\,ac+b\right)d^2x^4+2\,ac^3+\left(4\,ac^2+3\,bc\right)\cdots\right.}{x^4}\right.}{32\left(a^2c^5+2\,\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*
c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*
b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)
*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4
 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)*sqrt(-
a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
(a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 +
 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt
```

```
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^5, x)
```

3.324 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$

Optimal. Leaf size=265

$$-\frac{d^2\left(8a^2c^2+20abc+11b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16c^3x^2(ac+b)^2}+\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{7/2}(ac+b)^{5/2}}+\frac{d(4ac+3b)\left(c\right.}{8c^3}$$

```
[Out] -((11*b^2 + 20*a*b*c + 8*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)])/(16*c^3*(b + a*c)^2*x^2) + ((3*b + 4*a*c)*d*(c + d*x^2)^2*Sqr
t[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^3*(b + a*c)*x^4) - ((c + d*x^2)^3*
((b + a*c + a*d*x^2)/(c + d*x^2))^(3/2))/(6*c^2*(b + a*c)*x^6) + (b*(5*b^2
+ 12*a*b*c + 8*a^2*c^2)*d^3*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)])/Sqrt[b + a*c]])/(16*c^(7/2)*(b + a*c)^(5/2))
```

Rubi [A] time = 0.606903, antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 99, 151, 12, 93, 208}

$$\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{16c^{7/2}(ac+b)^{5/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2(2ac+5b)(4ac+3b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{48c^3x^2(ac+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + ((5*b + 4*a*c)*d*(c + d*
x^2)*Sqrt[a + b/(c + d*x^2)])/(24*c^2*(b + a*c)*x^4) - ((5*b + 2*a*c)*(3*b
+ 4*a*c)*d^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(48*c^3*(b + a*c)^2*x^2)
+ (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x
^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2
)])])/(16*c^(7/2)*(b + a*c)^(5/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^4\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}(5b+4ac)d-2ad^2x}{x^3\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int -\right.}{12c^2(b+ac)}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+dx^2\right)\sqrt{}}{48c^3(b+ac)^2x^2}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+dx^2\right)\sqrt{}}{48c^3(b+ac)^2x^2}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+dx^2\right)\sqrt{}}{48c^3(b+ac)^2x^2}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+dx^2\right)\sqrt{}}{48c^3(b+ac)^2x^2}$$

Mathematica [A] time = 0.459011, size = 259, normalized size = 0.98

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(3bd^3x^6\left(8a^2c^2+12abc+5b^2\right)\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)-\sqrt{c}\sqrt{ac+b}\left(c+dx^2\right)\sqrt{a\left(c+dx^2\right)+b}\left(8\right.\right.}{48c^{7/2}x^6(ac+b)^{5/2}\sqrt{a\left(c+dx^2\right)+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[c]*Sqrt[b + a*c]*(c + d*x^2)
*Sqrt[b + a*(c + d*x^2)]*(8*a^2*c^2*(c^2 - c*d*x^2 + d^2*x^4) + 2*a*b*c*(8*
c^2 - 9*c*d*x^2 + 13*d^2*x^4) + b^2*(8*c^2 - 10*c*d*x^2 + 15*d^2*x^4))) + 3
*b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b +
 a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*c + a*d*x^2])]))/(48*c^(7/2)*(b
+ a*c)^(5/2)*x^6*Sqrt[b + a*(c + d*x^2)])
```

Maple [B] time = 0.028, size = 1518, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^7,x)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-24*ln((2*a*c*d*x^2+b*d*
x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)+2*b*c)/x^2)*x^6*a^5*b*c^8*d^3-96*a^3*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*x^8*c^2*(a*c^2+b*c)^(5/2)-108*ln((2*a*c*d*x^2+b*d*x^2+2*
c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2
*b*c)/x^2)*x^6*a^4*b^2*c^7*d^3-156*a^2*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*x^8*c*b*(a*c^2+b*c)^(5/2)-195*ln((2*a*c*d*x^2+b*d*x^2+2*c^2
*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*
c)/x^2)*x^6*a^3*b^3*c^6*d^3-66*a*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*x^8*b^2*(a*c^2+b*c)^(5/2)-144*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*a^3*c^3*d^3*(a*c^2+b*c)^(5/2)*x^6-177*ln((2*a*c*d*x^2+b*d*x^2+
2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
+2*b*c)/x^2)*x^6*a^2*b^4*c^5*d^3-324*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*a^2*c^2*d^3*b*(a*c^2+b*c)^(5/2)*x^6-81*ln((2*a*c*d*x^2+b*d*x^2+2*
c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2
*b*c)/x^2)*x^6*a*b^5*c^4*d^3-252*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*a*c*d^3*b^2*(a*c^2+b*c)^(5/2)*x^6-15*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+
2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/
x^2)*x^6*b^6*c^3*d^3+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a^2
*c^2*d^2*(a*c^2+b*c)^(5/2)*x^4-66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*b^3*d^3*(a*c^2+b*c)^(5/2)*x^6+156*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*a*c*d^2*b*(a*c^2+b*c)^(5/2)*x^4+66*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(3/2)*b^2*d^2*(a*c^2+b*c)^(5/2)*x^4-48*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a^2*c^3*d-84*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a*b*c^2*d-36*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*b^2*c*d+1
6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^4
+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c
^3+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2
*c^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^4/(a*c+b)^3/x^6/(a*c^2+b*c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.93521, size = 1601, normalized size = 6.04

$$\left[\frac{3\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)\sqrt{ac^2+bc}d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left((2\,ac+b)d^2x^4+\right.}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(
((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 +
 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3
 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*
b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*
a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4
+ b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*
b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5
*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 +
2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 +
 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*
c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^7, x)
```

3.325 $\int x^4 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=368

$$-\frac{x\left(-3a^2c^2+7abc+2b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15a^2d^2}+\frac{\sqrt{c}\left(-3a^2c^2+7abc+2b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{c^{3/2}(b-3ac)\sqrt{\frac{ac+a}{c+}}}{15ad^{5/}}$$

```
[Out] -((2*b^2 + 7*a*b*c - 3*a^2*c^2)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(1
5*a^2*d^2) + ((b - 3*a*c)*x*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)])/(15*a*d^2) + (x^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5
*d) + (Sqrt[c]*(2*b^2 + 7*a*b*c - 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*a^2*d^(5/2
)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (c^(3/2)*(b - 3*
a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], b/(b + a*c)])/(15*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*
(c + d*x^2))])
```

Rubi [A] time = 0.718072, antiderivative size = 478, normalized size of antiderivative = 1.3, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 478, 582, 531, 418, 492, 411}

$$-\frac{x\left(-3a^2c^2+7abc+2b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{a\left(c+dx^2\right)+b}}+\frac{\sqrt{c}\left(-3a^2c^2+7abc+2b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan^{-1}\left(\right.\right.}{15a^2d^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((2*b^2 + 7*a*b*c - 3*a^2*c^2)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d
*x^2)])/(15*a^2*d^2*Sqrt[b + a*(c + d*x^2)]) + ((b - 3*a*c)*x*(c + d*x^2)*S
qrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(15*a*d^2*Sqrt[b + a*(c + d
*x^2)]) + (x^3*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])
/(5*d*Sqrt[b + a*(c + d*x^2)]) + (Sqrt[c]*(2*b^2 + 7*a*b*c - 3*a^2*c^2)*Sqr
t[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(15*a^2*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a
*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) - (c^(3/2)*(b - 3*a*c)*Sqrt[b +
a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], b/(b + a*c)])/(15*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
 d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(3c(b+ac)-(b-3ac)dx^2\right)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}dx}{5d\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(b-3ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{x^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{\cdots\right.}{}$$

$$= \frac{(b-3ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{x^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c\left(\cdots\right.\right.}{}$$

$$= -\frac{\left(2b^2+7abc-3a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{(b-3ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\cdots}}{15ad^2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(2b^2+7abc-3a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{(b-3ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\cdots}}{15ad^2\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] time = 0.821535, size = 293, normalized size = 0.8

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(-3a^2\left(c^2-d^2x^4\right)-2ab\left(c-2dx^2\right)+b^2\right)+ic\left(-3a^2c^2+7abc+2b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}E\cdots\right.}{15ad^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b^2 - 2*a*b*(c - 2*d*x^2) - 3*a^2*(c^2 - d^2*x^4)) + I*c*(2*b^2 + 7*a*b*c
 - 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*b*c*(b + 9*a*c)*S
qrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(15*a*d^2*Sqrt[(a*d)/(b + a*c)]*(b
 + a*(c + d*x^2)))
```

Maple [A] time = 0.034, size = 662, normalized size = 1.8

$$\frac{dx^2+c}{15\,ad^2}\left(3\sqrt{-\frac{ad}{ac+b}}x^7a^2d^3+3\sqrt{-\frac{ad}{ac+b}}x^5a^2cd^2+4\sqrt{-\frac{ad}{ac+b}}x^5abd^2-3\sqrt{-\frac{ad}{ac+b}}x^3a^2c^2d+2\sqrt{-\frac{ad}{ac+b}}x^3abcd+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/15*(3*(-a*d/(a*c+b))^(1/2)*x^7*a^2*d^3+3*(-a*d/(a*c+b))^(1/2)*x^5*a^2*c*d
^2+4*(-a*d/(a*c+b))^(1/2)*x^5*a*b*d^2-3*(-a*d/(a*c+b))^(1/2)*x^3*a^2*c^2*d+
2*(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d+3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3
+(-a*d/(a*c+b))^(1/2)*x^3*b^2*d-3*(-a*d/(a*c+b))^(1/2)*x*a^2*c^3+9*((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*a*b*c^2-7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-2*(
-a*d/(a*c+b))^(1/2)*x*a*b*c^2+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-2*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*b^2*c+(-a*d/(a*c+b))^(1/2)*x*b^2*c)*(d*x^2+c)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)/(-a*d/(a*c+b))^(1/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{dx^2 + c}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \sqrt{\frac{adx^2 + ac + b}{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{dx^2 + c}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

3.326 $\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=282

$$-\frac{c^{3/2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{c}(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3ad}+\frac{x(c+a}{}$$

```
[Out] ((b - a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*a*d) + (x*(c + d*x^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) - (Sqrt[c]*(b - a*c)*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/(3*a*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]
) - (c^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.517184, antiderivative size = 370, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 478, 531, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}(b-ac)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b - a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*a*d*Sqrt[b
 + a*(c + d*x^2)]) + (x*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c +
 d*x^2)])/(3*d*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[c]*(b - a*c)*Sqrt[b + a*c +
 a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(3*a*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))]*Sqrt[b + a*(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/
(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2
)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x
^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^2\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^2\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3d\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left((b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,d}{3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3ad\sqrt{b+a\left(c+dx^2\right)}} + \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} - \frac{c^{3/2}\sqrt{b+ac}}{3d^{3/}}$$

$$= \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3ad\sqrt{b+a\left(c+dx^2\right)}} + \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{c}(b-ac)\sqrt{}}{3}$$

Mathematica [C] time = 0.569786, size = 250, normalized size = 0.89

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac+adx^2+b\right)+2ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+ic(ac-b)\sqrt{}\right.}{3d\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b + a*c + a*d*x^2) + I*c*(-b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c
)] + (2*I)*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*d*Sqrt[(a*d)/(b
 + a*c)]*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.013, size = 406, normalized size = 1.4

$$\frac{dx^2+c}{3\,d}\left(\sqrt{-\frac{ad}{ac+b}}x^5ad^2+2\sqrt{-\frac{ad}{ac+b}}x^3acd+\sqrt{-\frac{ad}{ac+b}}x^3bd-\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*x^5*a*d^2+2*(-a*d/(a*c+b))^(1/2)*x^3*a*c*d+(-a*d/
(a*c+b))^(1/2)*x^3*b*d-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b))^
(1/2)*x*a*c^2-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)/(a*c+b)
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^
(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*x*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1
/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

3.327 $\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=213

$$x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}$$

```
[Out] x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] - (Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
 a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.203157, antiderivative size = 279, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6722, 1974, 422, 418, 492, 411}

$$\frac{x\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a\left(c+dx^2\right)+b}}+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan\right.}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+d\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*(c + d*x^2)]
 - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) + (Sqrt[c]*Sqrt[b + a*c
+ a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]
*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left((b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac}}}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c\sqrt{c+d\right.}}{}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{b+a\left(c+dx^2\right)}} + \frac{\sqrt{c}\sqrt{b+}}{}$$

Mathematica [A] time = 0.0636901, size = 98, normalized size = 0.46

$$\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{ac+adx^2+b}{ac+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin
[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/(Sqrt[-(d/c)]*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)])
```

Maple [A] time = 0.007, size = 199, normalized size = 0.9

$$(dx^2+c)\left(ac\text{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+\text{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)b\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{adx}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] (a*c*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+EllipticF(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b)*((d*x^2+c)/c)^(1/2)*((a*d*x^2+a*c+
b)/(a*c+b))^(1/2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+\frac{b}{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\frac{adx^2+ac+b}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

3.328 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2} dx$

Optimal. Leaf size=265

$$\frac{dx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c} - \frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx} + \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}} - \frac{\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}$$

```
[Out] (d*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((c + d*x^2)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)])/(c*x) - (Sqrt[d]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[c]*Sqrt[d]*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/((b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.509594, antiderivative size = 353, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{a\left(c+dx^2\right)+b}} - \frac{\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{a\left(c+dx^2\right)+b}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*Sqrt[b + a*(c + d*
x^2)]) - ((c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x
*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c
+ d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
 + (a*Sqrt[c]*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[(c*(b + a*c
 + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{acd+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad^2\sqrt{c\right.}}{}$$

$$= \frac{dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{b+ac+adx^2}\sqrt{a+}}{(b+ac)\sqrt{\frac{c(b+ac+adx}{(b+ac)(c+dx}}}$$

$$= \frac{dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{d}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+d}}}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{}}$$

<u>Mathematica</u> [C] time = 0.566084, size = 141, normalized size = 0.53

$$\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{iad\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)}{\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}-\frac{dx}{c}-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-x^(-1) - (d*x)/c - (I*a*d*Sqrt[(b +
 a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*
d)/(b + a*c)]*x], 1 + b/(a*c)])/(Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
)
```

<u>Maple</u> [A] time = 0.02, size = 272, normalized size = 1.

$$-\frac{dx^2+c}{cx}\left(\sqrt{-\frac{ad}{ac+b}}x^4ad^2-adc\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}x\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}x^2acd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^2,x)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*x^4*a*d^2-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2*(
-a*d/(a*c+b))^(1/2)*x^2*a*c*d+(-a*d/(a*c+b))^(1/2)*x^2*b*d+(-a*d/(a*c+b))^(
1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/
x/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

3.329 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

Optimal. Leaf size=362

$$-\frac{d^2x(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2(ac+b)}+\frac{d^{3/2}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{d(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2x(ac+b)}-\frac{ad^{3/2}\cdots}{\cdots}$$

```
[Out] -((2*b + a*c)*d^2*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*(b + a*c)
) - ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c*x^3) + ((2*b +
 a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*(b + a*c)
*x) + ((2*b + a*c)*d^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(b
+ a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (a*d^(3/2)*Sqrt[(b + a*c + a*
d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3
*Sqrt[c]*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.630172, antiderivative size = 472, normalized size of antiderivative = 1.3, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 583, 531, 418, 492, 411}

$$-\frac{d^2x(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(ac+b)\sqrt{a\left(c+dx^2\right)+b}}+\frac{d^{3/2}(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(ac+2b\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -((2*b + a*c)*d^2*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*c^2
*(b + a*c)*Sqrt[b + a*(c + d*x^2)]) - ((c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*
Sqrt[a + b/(c + d*x^2)])/(3*c*x^3*Sqrt[b + a*(c + d*x^2)]) + ((2*b + a*c)*d
*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*c^2*(b + a
*c)*x*Sqrt[b + a*(c + d*x^2)]) + ((2*b + a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^
2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*
c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^
2))]*Sqrt[b + a*(c + d*x^2)]) - (a*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a +
 b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*Sqr
t[c]*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b
 + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(2b+ac)d-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)x\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c-\right.}{}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)x\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(ad^2\right.}{}$$

$$= -\frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)d\left(c\right.}{3c^2(}$$

$$= -\frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)d\left(c\right.}{3c^2(}$$

Mathematica [C] time = 0.971735, size = 314, normalized size = 0.87

$$-\frac{\sqrt{\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c^2-d^2x^4\right)+2ab\left(c^2-cdx^2-d^2x^4\right)+b^2\left(c-2dx^2\right)\right)+iabcd^2x^3\sqrt{\frac{dx^2}{c}+1}\sqrt{\right.}{3ac^2dx^3\left(a\left(c+d\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -(Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(
b + a*c)]*(c + d*x^2)*(b^2*(c - 2*d*x^2) + a^2*c*(c^2 - d^2*x^4) + 2*a*b*(c
^2 - c*d*x^2 - d^2*x^4)) - I*a*c*(2*b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^
2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]
*x], 1 + b/(a*c)] + I*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]
))/(3*a*c^2*d*x^3*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.024, size = 571, normalized size = 1.6

$$\frac{dx^2+c}{(3\,ac+3\,b)\,x^3c^2}\left(\sqrt{-\frac{ad}{ac+b}}x^6a^2cd^3+2\sqrt{-\frac{ad}{ac+b}}x^6abd^3-\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac}{\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^4,x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*x^6*a^2*c*d^3+2*(-a*d/(a*c+b))^(1/2)*x^6*a*b*d^3-
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*x^3*a^2*c^2*d^2+(-a*d/(a*c+b))^(1/2)*x^4*a^2
*c^2*d^2+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x^3*a*b*c*d^2-2*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/
a/c)^(1/2))*x^3*a*b*c*d^2+4*(-a*d/(a*c+b))^(1/2)*x^4*a*b*c*d^2+2*(-a*d/(a*c
+b))^(1/2)*x^4*b^2*d^2-(-a*d/(a*c+b))^(1/2)*x^2*a^2*c^3*d+(-a*d/(a*c+b))^(1
/2)*x^2*b^2*c*d-(-a*d/(a*c+b))^(1/2)*a^2*c^4-2*(-a*d/(a*c+b))^(1/2)*a*b*c^3
-(-a*d/(a*c+b))^(1/2)*b^2*c^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c+b
)/x^3/c^2/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

3.330 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

Optimal. Leaf size=466

$$\frac{d^3x\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3(ac+b)^2}-\frac{d^2\left(3a^2c^2+13abc+8b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3x(ac+b)^2}-\frac{d^{5/2}\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{ac}{}}}{15c^{5/2}(ac+b)^2}$$

```
[Out] ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^3*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/(15*c^3*(b + a*c)^2) - ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/(5*c*x^5) + ((4*b + 3*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)])/(15*c^2*(b + a*c)*x^3) - ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^2*(c + d*
x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(15*c^3*(b + a*c)^2*x) - ((8*b^
2 + 13*a*b*c + 3*a^2*c^2)*d^(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(5/2)*(b + a*c)^2*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*(4*b + 3*a*c)*d^
(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], b/(b + a*c)])/(15*c^(3/2)*(b + a*c)^2*Sqrt[(c*(b + a*c + a*d*x^2))/
((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.811981, antiderivative size = 598, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 583, 531, 418, 492, 411}

$$\frac{d^3x\left(3a^2c^2+13abc+8b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(ac+b)^2\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2\left(3a^2c^2+13abc+8b^2\right)\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3x(ac+b)^2\sqrt{a\left(c+dx^2\right)+b}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^3*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c
 + d*x^2)])/(15*c^3*(b + a*c)^2*Sqrt[b + a*(c + d*x^2)]) - ((c + d*x^2)*Sqr
t[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*c*x^5*Sqrt[b + a*(c + d*x^
2)]) + ((4*b + 3*a*c)*d*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c +
 d*x^2)])/(15*c^2*(b + a*c)*x^3*Sqrt[b + a*(c + d*x^2)]) - ((8*b^2 + 13*a*b
*c + 3*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x
^2)])/(15*c^3*(b + a*c)^2*x*Sqrt[b + a*(c + d*x^2)]) - ((8*b^2 + 13*a*b*c +
 3*a^2*c^2)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(5/2)*(b + a*c)^2*Sqrt[
(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) +
 (a*(4*b + 3*a*c)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(3/2)*(b + a*c)^2
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^
2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(4b+3ac)d-3ad^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b+3ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+\right.}{}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b+3ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(8b^2+\right.}{}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b+3ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(8b^2+\right.}{}$$

$$= \frac{\left(8b^2+13abc+3a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(b+ac)^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b}{}$$

$$= \frac{\left(8b^2+13abc+3a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(b+ac)^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b}{}$$

Mathematica [C] time = 1.06506, size = 402, normalized size = 0.86

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2bc\left(-4c^2dx^2+9c^3+9cd^2x^4+13d^3x^6\right)+3a^3c^2\left(c^3+d^3x^6\right)+ab^2\left(-8c^2dx^2+9c^3+17cd\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)*
(b^3*(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4) + 3*a^3*c^2*(c^3 + d^3*x^6) + a*b^2*(9
*c^3 - 8*c^2*d*x^2 + 17*c*d^2*x^4 + 8*d^3*x^6) + a^2*b*c*(9*c^3 - 4*c^2*d*x
^2 + 9*c*d^2*x^4 + 13*d^3*x^6)) + I*a*c*(8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^3*
x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*I)*a*b*c*(2*b + 3*a*c)*d^3
*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(15*c^3*(b + a*c)^2*Sqrt[(a*
d)/(b + a*c)]*x^5*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.026, size = 955, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^6,x)
```

```
[Out] -1/15*(3*(-a*d/(a*c+b))^(1/2)*x^8*a^3*c^2*d^4+13*(-a*d/(a*c+b))^(1/2)*x^8*a
^2*b*c*d^4-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x^5*a^3*c^3*d^3+8*(-a*d/(a*c+b)
)^(1/2)*x^8*a*b^2*d^4+3*(-a*d/(a*c+b))^(1/2)*x^6*a^3*c^3*d^3+6*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((
a*c+b)/a/c)^(1/2))*x^5*a^2*b*c^2*d^3-13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x^5*
a^2*b*c^2*d^3+22*(-a*d/(a*c+b))^(1/2)*x^6*a^2*b*c^2*d^3+4*((a*d*x^2+a*c+b)/
(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b
)/a/c)^(1/2))*x^5*a*b^2*c*d^3-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x^5*a*b^2*c*
d^3+25*(-a*d/(a*c+b))^(1/2)*x^6*a*b^2*c*d^3+8*(-a*d/(a*c+b))^(1/2)*x^6*b^3*
d^3+5*(-a*d/(a*c+b))^(1/2)*x^4*a^2*b*c^3*d^2+9*(-a*d/(a*c+b))^(1/2)*x^4*a*b
^2*c^2*d^2+3*(-a*d/(a*c+b))^(1/2)*x^2*a^3*c^5*d+4*(-a*d/(a*c+b))^(1/2)*x^4*
b^3*c*d^2+5*(-a*d/(a*c+b))^(1/2)*x^2*a^2*b*c^4*d+(-a*d/(a*c+b))^(1/2)*x^2*a
*b^2*c^3*d+3*(-a*d/(a*c+b))^(1/2)*a^3*c^6-(-a*d/(a*c+b))^(1/2)*x^2*b^3*c^2*
d+9*(-a*d/(a*c+b))^(1/2)*a^2*b*c^5+9*(-a*d/(a*c+b))^(1/2)*a*b^2*c^4+3*(-a*d
/(a*c+b))^(1/2)*b^3*c^3)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c+b)^2/x^
5/c^3/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

3.331 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=249

$$-\frac{\left(-24a^2c^2+60abc+5b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3}-\frac{b\left(-24a^2c^2+12abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}-\frac{bc^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^3}+$$

```
[Out] -((b*c^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d^3) - ((5*b^2 + 60*a*b*c -
 24*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(48*a*d^3)
- ((b + 12*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(24*d^
3) + ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*x^2))^(5/2))/(6*a*d^3) - (b
*(b^2 + 12*a*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]/Sqrt[a]])/(16*a^(3/2)*d^3)
```

Rubi [A] time = 0.730147, antiderivative size = 311, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{24abd^3}-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(16*a*
d^3) - ((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(
b + a*(c + d*x^2)))/(24*a*b*d^3) - (c^2*Sqrt[a + b/(c + d*x^2)]*(b + a*(c +
 d*x^2))^2)/(b*d^3) + ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^
2))^2)/(6*a*d^3) - (b*(b^2 + 12*a*b*c - 24*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a
+ b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]
)/(16*a^(3/2)*d^3*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x^2(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^3}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{(b+ac+adx)^{3/2}\left(-\frac{1}{2}c(b-4a\right.}{\sqrt{c+dx}}\right.}{bd^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^3}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{6ad^3}-\frac{\left(\left(-\frac{3}{2}ac(b\right.\right.}{}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{24abd^3}-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+\right.\right.}{bd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+d}}}{24abd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+d}}}{24abd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+d}}}{24abd^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+d}}}{24abd^3}$$

Mathematica [A] time = 0.342408, size = 142, normalized size = 0.57

$$\frac{\sqrt{a}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^3+d^3x^6\right)-2ab\left(47c^2+16cdx^2-7d^2x^4\right)+3b^2\left(c+dx^2\right)\right)-3b\left(-24a^2c^2+12abc+b^2\right)\tanh^{-1}\left(\right.}{48a^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*(
47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12*a
*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(3/2)*d^
3)
```

Maple [B] time = 0.031, size = 1018, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/96*(48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4
*a^2*c*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
*x^4*a*b*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a^2*b*c^2*d^2+48*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^2*c^2*d+
36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a*b^2*c*d^2-16*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a*d+96*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b*c*d+3*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+
b*d)/(a*d^2)^(1/2))*x^2*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2
*b*c^3*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^
2*b^2*d+36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c^2*d+96*((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c^2-16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))
*b^3*c*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^
2*c)/d^3/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.1079, size = 954, normalized size = 3.83

$$\left[\frac{3\left(24\,a^{2}bc^{2}-12\,ab^{2}c-b^{3}\right)\sqrt{a}\log\left(8\,a^{2}d^{2}x^{4}+8\,a^{2}c^{2}+8\left(2\,a^{2}c+ab\right)dx^{2}+8\,abc+b^{2}+4\left(2\,ad^{2}x^{4}+(4\,ac+b)dx^{2}+2\,ac\right.\right.}{192\,a^{2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a
^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
+ 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*
c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a
^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*
a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^5, x)
```

3.332 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=172

$$\frac{a\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8d^2}+\frac{bc\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^2}+\frac{3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

```
[Out] (b*c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d^2 + ((5*b - 4*a*c)*(c + d*x^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*d^2) + (a*(c + d*x^2)^2*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)])/(4*d^2) + (3*b*(b - 4*a*c)*ArcTanh[Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

Rubi [A] time = 0.535715, antiderivative size = 222, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)^2}{bd^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4bd^2}+\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b - 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*d^2) + ((b - 4*a*c)*
(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*b*d^2) + (c*Sqr
t[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(b*d^2) + (3*b*(b - 4*a*c)*Sqrt
[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[
b + a*(c + d*x^2)]])/(8*Sqrt[a]*d^2*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^2}+\frac{\left((b-4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{(b+ac+adx)^{3/2}}{\sqrt{c+dx}}dx,x\right.}{2bd\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^2}+\frac{\left(3(b-4\right.}{}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+}}{}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+}}{}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+}}{}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+}}{}$$

Mathematica [A] time = 0.207054, size = 104, normalized size = 0.6

$$\frac{\sqrt{a}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^2+2ad^2x^4+13bc+5bdx^2\right)+3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^
2 + 2*a*d^2*x^4) + 3*b*(b - 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]
)/(8*Sqrt[a]*d^2)
```

Maple [B] time = 0.016, size = 593, normalized size = 3.5

$$\frac{1}{16\,d^2}\left(4\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}x^4ad^2-12\,\ln\left(1/2\,\frac{2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c}}{\sqrt{ad^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/16*(4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a
*d^2-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a*b*c*d^2+3*ln(1/2*(2*a*d^
2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/
2)+b*d)/(a*d^2)^(1/2))*x^2*b^2*d^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)*x^2*b*d-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*
b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c
^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*d+10*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b*c+16*(a*d^2)^(1/2)*((d*x^2+c)*(a*
d*x^2+a*c+b))^(1/2)*b*c)/d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49546, size = 757, normalized size = 4.4

$$\left[\frac{3\left(4\,abc-b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\right.}{32\,ad^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
+ 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c +
 b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b
)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 +
```

```
a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] time = 3.18521, size = 598, normalized size = 3.48

$$\frac{1}{16}\left(2\sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\left(\frac{2\,ax^2}{d} - \frac{2\,a^2cd^2 - 5\,abd^2}{ad^4}\right) + \frac{\left(4\,a^{\frac{3}{2}}bc - \sqrt{a}b^2\right)\log\left(\left|-2\,a^{\frac{5}{2}}c^3d - 6\left(\sqrt{ad^2}x^2 - \sqrt{\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2/d -
(2*a^2*c*d^2 - 5*a*b*d^2)/(a*d^4)) + (4*a^(3/2)*b*c - sqrt(a)*b^2)*log(abs(
-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*
x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d - 2*(
sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*
a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
+ b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a*d*abs(d)) + 2*(4*a^(3/2)*b*c*a
bs(d) - sqrt(a)*b^2*abs(d))*log(48*d^2*abs(a))/(a*d^3))*sgn(d*x^2 + c)
```

3.333 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=94

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

```
[Out] (-3*b*Sqrt[a + b/(c + d*x^2)])/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/
2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rubi [A] time = 0.0659592, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1591, 242, 47, 50, 63, 208}

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*b*Sqrt[a + b/(c + d*x^2)])/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/
2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx &= \frac{\text{Subst}\left(\int\left(a+\frac{b}{x}\right)^{3/2}dx,x,c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{x^2}dx,x,\frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3b)\,\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x}dx,x,\frac{1}{c+dx^2}\right)}{4d}\\
&= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3ab)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}dx,x,\frac{1}{c+dx^2}\right)}{4d}\\
&= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3a)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}\\
&= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}+\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.102966, size = 79, normalized size = 0.84

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(a\left(c+dx^2\right)-2b\right)+3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*b
*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Maple [B] time = 0.011, size = 336, normalized size = 3.6

$$\frac{1}{4d}\left(3\ln\left(1/2\,\frac{2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd}{\sqrt{ad^2}}\right)x^2abd^2+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*(3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a*b*d^2+2*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*d+3*ln(1/2*(2*a*d^2*x^2+
2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2)*a*c-4*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b)/d*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20824, size = 618, normalized size = 6.57

$$\left[\frac{3\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right.}{8d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*x^2
+ 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] time = 3.37699, size = 510, normalized size = 5.43

$$-\frac{1}{4}\left(\frac{2\sqrt{a}b|d|\log\left(12|a|^{\frac{3}{2}}|d|\right)}{d^2}+\frac{\sqrt{a}b\log\left(\left|-2a^{\frac{5}{2}}c^3d-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2c^2|d|-6\left(\sqrt{ad^2}\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(a)*b*abs(d)*log(12*abs(a)^(3/2)*abs(d))/d^2 + sqrt(a)*b*log(ab
s(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*
d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4
+ 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d - 2
*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^
3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/abs(d) - 2*sqrt(a*d^2*x^4 + 2*a
*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a/d)*sgn(d*x^2 + c)
```

3.334 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx$

Optimal. Leaf size=126

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c + a^(3/2)*ArcTanh[Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)]/Sqrt[a]] - ((b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/c^(3/2)
```

Rubi [A] time = 0.490115, antiderivative size = 206, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a\left(c+dx^2\right)+b}}-\frac{(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{c^{3/2}\sqrt{a\left(c+dx^2\right)+b}}+\frac{b\sqrt{a+\frac{b}{c+dx}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[a + b/(c + d*x^2)])/c + (a^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*
x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/Sqrt[b +
a*(c + d*x^2)] - ((b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*A
rcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/
(c^(3/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x(c+dx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{1}{2}(b+ac)^2d+\frac{1}{2}a^2cd^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left((b+ac)^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a^2d\sqrt{c\right.}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(a^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left((b+ac)^2\sqrt{c+d\right.}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} - \frac{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+d}}\right.}{}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^{3/2}\sqrt{b+a\left(c\right.}}$$

Mathematica [A] time = 0.233806, size = 118, normalized size = 0.94

$$\frac{a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)+b\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + a^(3/2)*c^(3/2)*ArcTanh[
Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a
+ b/(c + d*x^2)])/Sqrt[b + a*c]])/c^(3/2)
```

Maple [B] time = 0.016, size = 652, normalized size = 5.2

$$\frac{1}{2c^2}\left(\ln\left(\frac{1}{2}\left(2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd\right)\frac{1}{\sqrt{ad^2}}\right)x^2a^2c^2d^2-\sqrt{ad^2}\sqrt{c^2a+bc}\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a^2*c^2*d^2-(a*d^2)^(1/2)*(a*
c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*a*c*d+ln(1/2*(2*a*d
^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2)+b*d)/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c
*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*b*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*c+2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/c^2/((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.82807, size = 2414, normalized size = 19.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log(((
8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c
^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c
, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt((
a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c
^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*
c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2
*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 2*(a*c + b)*sqr
t(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*
```

```
x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c, -1/2*(
sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt(-(a*c + b)/
c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c
+ b^2)) - 2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x, x)
```

3.335 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx$

Optimal. Leaf size=138

$$-\frac{3bd\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c^2}+\frac{3bd\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{5/2}}-\frac{\left(c+dx^2\right)\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{2cx^2}$$

```
[Out] (-3*b*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*c^2) - ((c + d*x^2)*((b +
 a*c + a*d*x^2)/(c + d*x^2))^(3/2))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*ArcTan
h[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*c^(5/2
))
```

Rubi [A] time = 0.525602, antiderivative size = 170, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2}+\frac{3bd\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2c^{5/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*d*Sqrt[a + b/(c + d*x^2)])/(2*c^2) - (Sqrt[a + b/(c + d*x^2)]*(b + a*
(c + d*x^2)))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c
+ d*x^2)])])/(2*c^(5/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^3\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^3\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^2(c+dx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x(c+dx)^{3/2}}\,dx,x,x^2\right)}{4c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \ldots\right.}{4c^2\sqrt{b+a\left(c+dx^2\right.}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\ldots\right.}{2c^2\sqrt{b+a\left(c+\ldots\right.}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} + \frac{3b\sqrt{b+ac}\,d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\ldots\right.}{2c^{5/2}\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [A] time = 0.422294, size = 165, normalized size = 1.2

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(3bdx^2\sqrt{ac+b}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)-\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}\left(ac\left(c+dx^2\right)+b\left(c+3dx^2\right)\right)\right)}{2c^{5/2}x^2\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[c]*Sqrt[b + a*(c + d*x^2)]*(
a*c*(c + d*x^2) + b*(c + 3*d*x^2))) + 3*b*Sqrt[b + a*c]*d*x^2*Sqrt[c + d*x^
2]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*c + a*d*x^2]
)]))/(2*c^(5/2)*x^2*Sqrt[b + a*(c + d*x^2)])
```

Maple [B] time = 0.014, size = 820, normalized size = 5.9

$$-\frac{1}{4\,x^2c^3}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{c^2a+bc}x^6ad^3-3\,\ln\left(\frac{2\,acdx^2+bdx^2+2\,c^2a+2\,\sqrt{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^3,x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a*d^3-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2
*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*
c)/x^2)*x^4*a*b*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(1/2)*x^4*a*c*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c
)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*b^2
*c*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*
x^4*b*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*a*b*c^3*d-4*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*c^2*d-3*ln((2
*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*b^2*c^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*x^2*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b*c*d+4*(a*c^2+b*c)^(1/2)*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*c)/(a*c^2+b*c)^(1/2)/x^2/c^3/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65696, size = 895, normalized size = 6.49

$$\left[\frac{3\,bdx^2\sqrt{\frac{ac+b}{c}}\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left(\left(2\,ac^2+bc\right)d^2x^4+2\,ac^4+2\,bc^3+\left(4\,ac^3+3\,bc^2\right)dx^2\right)\sqrt{\frac{adx}{c}}}{x^4}\right)}{8\,c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4
+ 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*((
a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2
*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2
*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a
^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x^2 + a*c
^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^3, x)
```

3.336 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=205

$$\frac{bd^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^3}-\frac{3bd^2(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{7/2}\sqrt{ac+b}}+\frac{d(4ac+9b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^2}-\frac{(ac+b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3x^4}$$

```
[Out] (b*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c^3 + ((9*b + 4*a*c)*d*(c + d
*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^3*x^2) - ((b + a*c)*(c +
d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c^3*x^4) - (3*b*(5*b + 4
*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(8*c^(7/2)*Sqrt[b + a*c])
```

Rubi [A] time = 0.591398, antiderivative size = 260, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(4ac+5b)\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(ac+b)}-\frac{3bd^2(4ac+5b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{7/2}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(4ac+5b)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c\right.\right.}{8c^2x^2(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (3*b*(5*b + 4*a*c)*d^2*Sqrt[a + b/(c + d*x^2)])/(8*c^3*(b + a*c)) + ((5*b +
 4*a*c)*d*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(8*c^2*(b + a*c)*x^2
) - (Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(4*c*(b + a*c)*x^4) - (
3*b*(5*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt
[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(7/2)*S
qrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^5\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^5\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{4c(b+ac)x^4} - \frac{\left((5b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^2(c+dx)^{3/2}}\,dx, x, x\right.}{8c(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{4c(b+ac)x^4} + \frac{\left(3b(5b+4ac)d^2\sqrt{c+\right.}{16}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+d\right.\right.}{4c(b+ac)x^4}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+d\right.\right.}{4c(b+ac)x^4}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+d\right.\right.}{4c(b+ac)x^4}$$

Mathematica [A] time = 0.28848, size = 202, normalized size = 0.99

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{c}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}\left(2ac\left(c^2-d^2x^4\right)+b\left(2c^2-5cdx^2-15d^2x^4\right)\right)+3bd^2x^4(4ac+5b)\sqrt{c+dx^2}\tanh\right.}{8c^{7/2}x^4\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[c]*Sqrt[b + a*c]*Sqrt[b + a*(
c + d*x^2)]*(b*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4) + 2*a*c*(c^2 - d^2*x^4)) +
3*b*(5*b + 4*a*c)*d^2*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d
*x^2])/(Sqrt[c]*Sqrt[b + a*c + a*d*x^2])]))/(8*c^(7/2)*Sqrt[b + a*c]*x^4*Sq
rt[b + a*(c + d*x^2)])
```

Maple [B] time = 0.018, size = 1653, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^5,x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^2*c*d^4-12*ln((2*a*c*d*x^2+b*d*x^2
+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)+2*b*c)/x^2)*x^6*a^3*b*c^5*d^3-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a*b*d^4-39*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*
(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^
2)*x^6*a^2*b^2*c^4*d^3-32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(3/2)*x^6*a^2*c^2*d^3-42*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^
2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^
6*a*b^3*c^3*d^3-12*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*a^3*b*c^6*d^2-6
2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a*b
*c*d^3-15*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^6*b^4*c^2*d^3-39*ln((2*a*c
*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*a^2*b^2*c^5*d^2-18*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*b^2*d^3-20*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*c^3*d^2-42*ln((2*a*
c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*a*b^3*c^4*d^2+12*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a*c*d^2-44*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*c^2*d^2+16*(a*c^2+b*
c)^(3/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a*b*c^2*d^2-15*ln((2*a*c*d*x
^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)+2*b*c)/x^2)*x^4*b^4*c^3*d^2+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*b*d^2-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*b^2*c*d^2+16*(a*c^2+b*c)^(3/2)*((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*b^2*c*d^2+8*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a*c^2*d+14*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*b*c*d-4*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*c^3-4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b*c^2)/(a*c^2+b*c)^(3/2)/x^4
/(a*c+b)/c^4/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.98366, size = 1191, normalized size = 5.81

$$\left[\frac{3\left(4\,abc+5\,b^2\right)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left((2\,ac+b)d^2x^4+2\,ac^3+\left(4\,ac^2+3\,\right.\right.}{x^4}\right.}{32\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*
b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*
c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b
*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2 -
b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 -
 b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2
 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2
*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^5, x)
```

3.337 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=292

$$-\frac{d^2\left(24a^2c^2+108abc+79b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48c^4x^2(ac+b)}+\frac{bd^3\left(24a^2c^2+60abc+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{9/2}(ac+b)^{3/2}}-\frac{bd^3\sqrt{\frac{ac+}{c}}}{c^4}$$

```
[Out] -((b*d^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c^4) - ((79*b^2 + 108*a*b*c
 + 24*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(48*c
^4*(b + a*c)*x^2) + ((11*b + 12*a*c)*d*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(24*c^4*x^4) - ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*
x^2))^(5/2))/(6*c^2*(b + a*c)*x^6) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^
3*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(
16*c^(9/2)*(b + a*c)^(3/2))
```

Rubi [A] time = 0.725455, antiderivative size = 287, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 98, 151, 152, 12, 93, 208}

$$-\frac{d^3\left(8a^2c^2+110abc+105b^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(ac+b)}+\frac{bd^3\left(24a^2c^2+60abc+35b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{16c^{9/2}(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -((105*b^2 + 110*a*b*c + 8*a^2*c^2)*d^3*Sqrt[a + b/(c + d*x^2)])/(48*c^4*(b
 + a*c)) - ((b + a*c)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + (7*b*d*Sqrt[a +
b/(c + d*x^2)])/(24*c^2*x^4) - (b*(35*b + 32*a*c)*d^2*Sqrt[a + b/(c + d*x^2
)])/(48*c^3*(b + a*c)*x^2) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*Sqrt[c
 + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(
Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(16*c^(9/2)*(b + a*c)^(3/2)*Sqrt[b + a*(
c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^4(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{7}{2}b(b+ac)d+3abd^2x}{x^3(c+dx)^{3/2}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{1}{4}b(b+ac)(35b+32ac)d^2+\cdots}{x^2(c+dx)^{3/2}\sqrt{b+ac\cdots}}\right.}{12c^2(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+32ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{48c^3(b+ac)x^2} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\cdots\right.}{\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+3\cdots}{48c\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+3\cdots}{48c\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+3\cdots}{48c\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+3\cdots}{48c\cdots}$$

Mathematica [A] time = 0.490328, size = 267, normalized size = 0.91

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(3bd^3x^6\left(24a^2c^2+60abc+35b^2\right)\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)-\sqrt{c}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}\left(8a^2c^2\left(c^3\cdots\right.\right.\right.}{48c^{9/2}x^6(ac+b)^{3/2}\sqrt{a\left(c+\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[c]*Sqrt[b + a*c]*Sqrt[b + a*
(c + d*x^2)]*(8*a^2*c^2*(c^3 + d^3*x^6) + 2*a*b*c*(8*c^3 - 7*c^2*d*x^2 + 16
*c*d^2*x^4 + 55*d^3*x^6) + b^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d
^3*x^6))) + 3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*x^6*Sqrt[c + d*x^2]*Ar
```

```
cTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*c + a*d*x^2])]))/
(48*c^(9/2)*(b + a*c)^(3/2)*x^6*Sqrt[b + a*(c + d*x^2)])
```

Maple [B] time = 0.021, size = 2605, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^7,x)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^10*a^3*c^2*d^5-396*ln((2*a*c*d*x^2+b*
d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)+2*b*c)/x^2)*x^8*a^4*b^2*c^7*d^4-861*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+
2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/
x^2)*x^8*a^3*b^3*c^6*d^4-72*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(
1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^6*a^5*b*
c^9*d^3-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/
2)*x^10*a*b^2*d^5-240*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^
2+b*c)^(5/2)*x^8*a^3*c^3*d^4-927*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b
*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^8*a
^2*b^4*c^5*d^4-396*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^6*a^4*b^2*c^8*d^3
-495*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^8*a*b^5*c^4*d^4-861*ln((2*a*c*d
*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)+2*b*c)/x^2)*x^6*a^3*b^3*c^7*d^3-144*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a^3*c^4*d^3-927*ln((2*a*c*d*x^
2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)+2*b*c)/x^2)*x^6*a^2*b^4*c^6*d^3+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^6*a^2*c^2*d^3-495*ln((2*a*c*d*x^2+b*
d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)+2*b*c)/x^2)*x^6*a*b^5*c^5*d^3+96*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(
a*c^2+b*c)^(5/2)*x^6*b^3*c*d^3+48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(3/2)*(a*c^2+b*c)^(5/2)*x^4*a^2*c^3*d^2-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*b^3*c*d^3+114*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^4*b^2*c*d^2-32*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a^2*c^4*d-44*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*b^2*c^2*d-
105*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^8*b^6*c^3*d^4-174*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*b^3*d^4-105*ln((2*a
*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^6*b^6*c^4*d^3+174*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^6*b^2*d^3+32*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^4-72*ln((2*a*c*d*x^2+b
*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)+2*b*c)/x^2)*x^8*a^5*b*c^8*d^4+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^5+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c^3+168*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^4*a*b*c^2*d^2-76*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a*b*c^3*d-816*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a^2*b*c^2*d^4-7
38*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a*
b^2*c*d^4+96*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a^2*b*
c^3*d^3-540*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/
```

```
2)*x^6*a^2*b*c^3*d^3+192*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(5/2
)*x^6*a*b^2*c^2*d^3+276*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*
c^2+b*c)^(5/2)*x^6*a*b*c*d^3-564*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*c^2+b*c)^(5/2)*x^6*a*b^2*c^2*d^3-276*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^10*a^2*b*c*d^5)/(a*c^2+b*c)^(5/2)/x^
6/(a*c+b)^2/c^5/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.6162, size = 1580, normalized size = 5.41

$$\left[\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{ac^2+bc}\,d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left((2\,ac+b)d^2\ldots\right.}{x^4}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*lo
g(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c
^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 21
5*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
+ (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*
b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7
+ 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*
sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*
c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 118*a
^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5
 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^7, x)
```

3.338 $\int x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=405

$$\frac{x\left(a^2c^2-14abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5ad^2}-\frac{\sqrt{c}\left(a^2c^2-14abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{c^{3/2}(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d^{5/2}\sqrt{\frac{c(}{(ac}}}$$

```
[Out] ((b^2 - 14*a*b*c + a^2*c^2)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*a*d
^2) + ((7*b - a*c)*x*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*
d^2) + (6*a*x^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*d) -
(x^3*(b + a*c + a*d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d - (Sqrt[c
]*(b^2 - 14*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a*d^(5/2)*Sqrt[(c*(b + a*c
+ a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (c^(3/2)*(7*b - a*c)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)]
)/(5*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.876041, antiderivative size = 526, normalized size of antiderivative = 1.3, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 467, 581, 582, 531, 418, 492, 411}

$$\frac{x\left(a^2c^2-14abc+b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}\left(a^2c^2-14abc+b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\right.}{5ad^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b^2 - 14*a*b*c + a^2*c^2)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2
)])/(5*a*d^2*Sqrt[b + a*(c + d*x^2)]) + ((7*b - a*c)*x*(c + d*x^2)*Sqrt[b +
 a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*d^2*Sqrt[b + a*(c + d*x^2)]) +
(6*a*x^3*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*d*
Sqrt[b + a*(c + d*x^2)]) - (x^3*(b + a*c + a*d*x^2)^(3/2)*Sqrt[a + b/(c + d
*x^2)])/(d*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[c]*(b^2 - 14*a*b*c + a^2*c^2)*S
qrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], b/(b + a*c)])/(5*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) - (c^(3/2)*(7*b - a*c)*Sqrt[b + a
*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(5*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x
^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{x^3\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\sqrt{b+ac+adx^2}\left(3(b+ac)+6adx^2\right)}{\sqrt{c+dx^2}}d}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{6ax^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x^3\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+d\right.}}{}$$

$$=\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{6ax^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{}{c+}}}{5d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{6ax^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{}{c+}}}{5d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(b^2-14abc+a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(b^2-14abc+a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] time = 0.848245, size = 308, normalized size = 0.76

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\sqrt{\frac{ad}{ac+b}}\left(-a^2\left(c-dx^2\right)\left(c+dx^2\right)^2+3ab\left(2c^2+3cdx^2+d^2x^4\right)+b^2\left(7c+2dx^2\right)\right)-ic\left(a^2c^2-14abc+b^2\right)\right.}{5d^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(-(a^2*(c -
 d*x^2)*(c + d*x^2)^2) + b^2*(7*c + 2*d*x^2) + 3*a*b*(2*c^2 + 3*c*d*x^2 + d
^2*x^4)) - I*c*(b^2 - 14*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c
)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/
```

```
(a*c)] + (8*I)*b*c*(b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(5*d
^2*Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [B] time = 0.033, size = 1098, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/5*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^7*a^2*d^3+((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^5*a^2*c*d^2+3*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^5*a*b*d^2-((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a^2*c^2*d+4*((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d+((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d+2*((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*b^2*d-((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)*(-a*d/(a*c+b))^(1/2)*x*a^2*c^3+8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a
*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))
^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-14*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((
a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b)
)^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*
d/(a*c+b))^(1/2)*x*a*b*c^2-8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((
a*c+b)/a/c)^(1/2))*b^2*c+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)
/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+
b)/a/c)^(1/2))*b^2*c+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*
d/(a*c+b))^(1/2)*x*a*b*c^2+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b
))^(1/2)*x*b^2*c+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a
*c+b))^(1/2)*x*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(adx^6 + (ac + b)x^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*d*x^6 + (a*c + b)*x^4)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d
*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

3.339 $\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=331

$$\frac{\sqrt{c}(3b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{4ax\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d}$$

```
[Out] ((7*b - a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) + (4*a*x*(c + d
*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) - (x*(b + a*c + a*d*x^2)
*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d - (Sqrt[c]*(7*b - a*c)*Sqrt[(b +
a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a
*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (
Sqrt[c]*(3*b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.634819, antiderivative size = 430, normalized size of antiderivative = 1.3, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 467, 528, 531, 418, 492, 411}

$$\frac{\sqrt{c}(3b-ac)\sqrt{a+\frac{b}{c+dx^2}}\sqrt{ac+adx^2+b}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}(7b-ac)\sqrt{a+\frac{b}{c+dx^2}}\sqrt{ac+adx^2+b}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+a}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b - a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*d*Sqrt[b
 + a*(c + d*x^2)]) + (4*a*x*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/
(c + d*x^2)])/(3*d*Sqrt[b + a*(c + d*x^2)]) - (x*(b + a*c + a*d*x^2)^(3/2)*
Sqrt[a + b/(c + d*x^2)])/(d*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[c]*(7*b - a*c)
*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) + (Sqrt[c]*(3*b - a*c)*Sqrt[b + a
*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x
^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}\left(b+ac+4adx^2\right)}{\sqrt{c+dx^2}}dx}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\sqrt{a\right.}}{}$$

$$=\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(a(7b-ac)\sqrt{c\right.}}{}$$

$$=\frac{(7b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac\right.}{d}$$

$$=\frac{(7b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac\right.}{d}$$

Mathematica [C] time = 0.782304, size = 270, normalized size = 0.82

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\sqrt{\frac{ad}{ac+b}}\left(a^2\left(c+dx^2\right)^2-2ab\left(c+dx^2\right)-3b^2\right)+ib(5ac-3b)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right.\right.}{3d\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(-3*b^2 - 2
*a*b*(c + d*x^2) + a^2*(c + d*x^2)^2) + I*a*c*(-7*b + a*c)*Sqrt[(b + a*c +
a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b +
 a*c)]*x], 1 + b/(a*c)] + I*b*(-3*b + 5*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 +
 b/(a*c)]))/(3*d*Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [B] time = 0.014, size = 823, normalized size = 2.5

$$-\frac{1}{3d\left(adx^2+ac+b\right)}\left(-\sqrt{-\frac{ad}{ac+b}}\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}x^5a^2d^2-2\sqrt{-\frac{ad}{ac+b}}\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}x^3a^2cd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/3*(-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a^2*d^2-2
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a^2*c*d+3*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a*b*d-(
-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a*b*d+((a*d*x^2+a
*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),(
(a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2-(-a*d/(a*c+b)
)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a^2*c^2+5*((a*d*x^2+a*c+b)/(a*c
+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/
c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c-7*((a*d*x^2+a*c+b)/(a*c+b
))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)
^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c+3*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x*a*b*c-(-a*d/(a*c+b))^(1/2)*((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a*b*c-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x*b^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d/
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x
^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(adx^4 + (ac+b)x^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*d*x^4 + (a*c + b)*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d
*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

3.340 $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=260

$$-\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}+\frac{bx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}+\frac{a\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}$$

```
[Out] (b*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((b - a*c)*x*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)])/c + ((b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqrt[d]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[c]*Sqrt[(b +
 a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.291417, antiderivative size = 348, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6722, 1974, 413, 531, 418, 492, 411}

$$-\frac{x(b-ac)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{a\left(c+dx^2\right)+b}}+\frac{bx\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{a\left(c+dx^2\right)+b}}+\frac{a\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*Sqrt[b + a*(c + d*
x^2)]) - ((b - a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*S
qrt[b + a*(c + d*x^2)]) + ((b - a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c
+ d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqr
t[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c +
d*x^2)]) + (a*Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c +
 a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{ac(b+ac)d-a(b-ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{(a(\cdots}{\cdots}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{a\sqrt{c}\sqrt{b+ac+adx^2}\cdots}{\sqrt{d}\sqrt{\frac{c(b+ac\cdots}{(b+ac)(\cdots}}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{(b-ac)\sqrt{b+ac+a\cdots}}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(b\cdots}{(b+\cdots}}}$$

Mathematica [C] time = 0.542941, size = 243, normalized size = 0.93

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(bx\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)-2iabc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)-iac(ac-b)\sqrt{\frac{dx^2}{c}+1\cdots}\right)}{c\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c
 + d*x^2)) - I*a*c*(-b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*
I)*a*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(c*Sqrt[(a*d)/(b + a*c)]
*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.013, size = 515, normalized size = 2.

$$\frac{1}{c\left(adx^2+ac+b\right)}\left(\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}x^3abd+\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{-\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] ((a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a
*b*d+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c
^2+2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c
-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c
+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c+(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x*a*b*c+(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x*b^2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)/(-a*d/(a*c+b))^(1/2)/c/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(adx^2 + ac + b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^2 +
 c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral((a + b/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

3.341 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=312

$$\frac{dx(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2}-\frac{(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2x}-\frac{\sqrt{d}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx}$$

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x) + ((2*b + a*c)*d*x*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)])/c^2 - ((2*b + a*c)*(c + d*x^2)*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)])/(c^2*x) - ((2*b + a*c)*Sqrt[d]*Sqrt[(b + a*c + a*
d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(c
^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[d]*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))])
```

Rubi [A] time = 0.663576, antiderivative size = 422, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{dx(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{a\left(c+dx^2\right)+b}}-\frac{(ac+2b)\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c^2x\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{d}(ac+2b)\sqrt{ac+adx^2+b}\sqrt{\cdots}}{c^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] (b*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x*Sqrt[b + a*(c + d*
x^2)]) + ((2*b + a*c)*d*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/
(c^2*Sqrt[b + a*(c + d*x^2)]) - ((2*b + a*c)*(c + d*x^2)*Sqrt[b + a*c + a*d
*x^2]*Sqrt[a + b/(c + d*x^2)])/(c^2*x*Sqrt[b + a*(c + d*x^2)]) - ((2*b + a*
c)*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(c^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/
((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) + (a*Sqrt[d]*Sqrt[b + a*c
 + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(2b+ac)d-a(b+ac)d^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2x\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\right.}{c^2(b}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2x\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a(b+ac)d\sqrt{c+\right.}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+}{c^2x\sqrt{b+a\left(\right.}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+}{c^2x\sqrt{b+a\left(\right.}}$$

Mathematica [C] time = 0.703092, size = 278, normalized size = 0.89

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c+dx^2\right)^2+2ab\left(c+dx^2\right)^2+b^2\left(c+2dx^2\right)\right)-iabcdx\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)\right.\right.}{c^2x\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(2*a*b*(c +
 d*x^2)^2 + a^2*c*(c + d*x^2)^2 + b^2*(c + 2*d*x^2)) + I*a*c*(2*b + a*c)*d*
x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSi
nh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*a*b*c*d*x*Sqrt[(b + a*c + a*d
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*
c)]*x], 1 + b/(a*c)]))/(c^2*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c + d*x^2))))
```

Maple [B] time = 0.016, size = 873, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^2,x)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a^2*c*d^2+(-a*
d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*a*b*d^
2+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a*b*d^2-((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a^2*c^2*d+2*(-
a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a^2*c^2*d+((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a*b*c*d-2*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a*b*c*d+(-a*d/
(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*a*b*c*d+
3*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a*b*c*d+(-a*d/
(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b^2*d+(-
a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*b^2*d+(-a*d/(a*c+b
))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3+2*(-a*d/(a*c+b))^(1/2)*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/x/(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/c^2/(a*d*x^2+a*c+
b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(adx^2 + ac + b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^4 + cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^4 +
 c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

3.342 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=388

$$-\frac{d^2x(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^3}-\frac{ad^{3/2}(ac+4b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{d^{3/2}(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{3c^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}$$

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^3) - ((8*b + a*c)*d^2*x*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^3) - ((4*b + a*c)*(c + d*x^2)*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*x^3) + ((8*b + a*c)*d*(c + d*x^2)*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^3*x) + ((8*b + a*c)*d^(3/2)*Sqr
t[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*c^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)]) - (a*(4*b + a*c)*d^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.868577, antiderivative size = 520, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$-\frac{d^2x(ac+8b)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{a\left(c+dx^2\right)+b}}-\frac{ad^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d^{3/2}(ac+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x^3*Sqrt[b + a*(c +
d*x^2)]) - ((8*b + a*c)*d^2*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2
)])/(3*c^3*Sqrt[b + a*(c + d*x^2)]) - ((4*b + a*c)*(c + d*x^2)*Sqrt[b + a*c
 + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*c^2*x^3*Sqrt[b + a*(c + d*x^2)]) +
((8*b + a*c)*d*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])
/(3*c^3*x*Sqrt[b + a*(c + d*x^2)]) + ((8*b + a*c)*d^(3/2)*Sqrt[b + a*c + a*
d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/(3*c^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*S
qrt[b + a*(c + d*x^2)]) - (a*(4*b + a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sq
rt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(3*c^(3/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*
Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(4b+ac)d-a(3b+ac)d^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a\right.}}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac)d\left(c\right.}{3}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac)d\left(c\right.}{3}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(8b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{}}{3c^2x^3\sqrt{b}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(8b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{}}{3c^2x^3\sqrt{b}}$$

Mathematica [C] time = 0.833336, size = 329, normalized size = 0.85

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c-dx^2\right)\left(c+dx^2\right)^2+ab\left(-3c^2dx^2+2c^3-13cd^2x^4-8d^3x^6\right)+b^2\left(c^2-4cdx^2-8d^2x^4\right)\right)+4\right.}{3c^3x^3\sqrt{\frac{}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(a^2*c*(c -
d*x^2)*(c + d*x^2)^2 + b^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*(2*c^3 - 3*c
^2*d*x^2 - 13*c*d^2*x^4 - 8*d^3*x^6)) - I*a*c*(8*b + a*c)*d^2*x^3*Sqrt[(b +
 a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*
d)/(b + a*c)]*x], 1 + b/(a*c)] + (4*I)*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^
2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]
*x], 1 + b/(a*c)]))/(3*c^3*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

Maple [B] time = 0.016, size = 1039, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^4,x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^2*c*d^3+3
*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*a
*b*d^3+5*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a*b*d^3
-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c
+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a^2*c
^2*d^2+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a^2*c^2*d
^2+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a
*b*c*d^2-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
*x^3*a*b*c*d^2+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*x^4*a*b*c*d^2+10*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b)
)^(1/2)*x^4*a*b*c*d^2+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*x^4*b^2*d^2+5*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*x^4*b^2*d^2-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1
/2)*x^2*a^2*c^3*d+3*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*
x^2*a*b*c^2*d+4*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*
b^2*c*d-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^4-2*(-
a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^3-(-a*d/(a*c+b))
^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/x^3/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))
^(1/2)/c^3/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(adx^2+ac+b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^6+cx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^6 +
 c*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

3.343 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=494

$$\frac{d^3x\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4(ac+b)}-\frac{d^2\left(a^2c^2+16abc+16b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4x(ac+b)}-\frac{d^{5/2}\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{ac\cdots}{\cdots}}}{5c^{7/2}(ac+b)\sqrt{\cdots}}$$

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^5) + ((16*b^2 + 16*a*b*c + a
^2*c^2)*d^3*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^4*(b + a*c)) - ((
6*b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^2*x^5) +
 ((8*b + a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^3*x
^3) - ((16*b^2 + 16*a*b*c + a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(5*c^4*(b + a*c)*x) - ((16*b^2 + 16*a*b*c + a^2*c^2)*d^(5/
2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(5*c^(7/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))]) + (a*(8*b + a*c)*d^(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(5/2)*(b
+ a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 1.07518, antiderivative size = 648, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{d^3x\left(a^2c^2+16abc+16b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4(ac+b)\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2\left(a^2c^2+16abc+16b^2\right)\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4x(ac+b)\sqrt{a\left(c+dx^2\right)+b}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] (b*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x^5*Sqrt[b + a*(c +
d*x^2)]) + ((16*b^2 + 16*a*b*c + a^2*c^2)*d^3*x*Sqrt[b + a*c + a*d*x^2]*Sqr
t[a + b/(c + d*x^2)])/(5*c^4*(b + a*c)*Sqrt[b + a*(c + d*x^2)]) - ((6*b + a
*c)*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*c^2*x^5
*Sqrt[b + a*(c + d*x^2)]) + ((8*b + a*c)*d*(c + d*x^2)*Sqrt[b + a*c + a*d*x
^2]*Sqrt[a + b/(c + d*x^2)])/(5*c^3*x^3*Sqrt[b + a*(c + d*x^2)]) - ((16*b^2
 + 16*a*b*c + a^2*c^2)*d^2*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(
c + d*x^2)])/(5*c^4*(b + a*c)*x*Sqrt[b + a*(c + d*x^2)]) - ((16*b^2 + 16*a*
b*c + a^2*c^2)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(7/2)*(b + a*c)*Sqrt[
(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) +
 (a*(8*b + a*c)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(5/2)*(b + a*c)*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^6\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^6\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(6b+ac)d-a(5b+ac)d^2x^2}{x^6\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\cdots\right.}{5\cdots}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac)d\left(c+d\cdots\right.}{5c^3x^{\cdots}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac)d\left(c+d\cdots\right.}{5c^3x^{\cdots}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac)d\left(c+d\cdots\right.}{5c^3x^{\cdots}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4(b+ac)\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(\cdots\right.}{\cdots}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4(b+ac)\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.09354, size = 430, normalized size = 0.87

$$-\frac{\sqrt{\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{\frac{ad}{ac+b}}\left(a^2bc\left(5c^2d^2x^4+3c^4+24cd^3x^6+16d^4x^8\right)+a^3c^2\left(c^3dx^2+c^4+cd^3x^6+d^4x^8\right)+ab^2\left(13c^2d^2x^4\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -(Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(
b + a*c)]*(b^3*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6) + a^3*c^2*(c^
4 + c^3*d*x^2 + c*d^3*x^6 + d^4*x^8) + a^2*b*c*(3*c^4 + 5*c^2*d^2*x^4 + 24*
c*d^3*x^6 + 16*d^4*x^8) + a*b^2*(3*c^4 - 3*c^3*d*x^2 + 13*c^2*d^2*x^4 + 40*
c*d^3*x^6 + 16*d^4*x^8)) + I*a*c*(16*b^2 + 16*a*b*c + a^2*c^2)*d^3*x^5*Sqrt
[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqr
t[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*a*b*c*(8*b + 7*a*c)*d^3*x^5*Sqrt[(b
 + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(
```

```
a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(5*a*c^4*d*x^5*(b + a*(c + d*x^2)))
```

Maple [B] time = 0.019, size = 1666, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^6,x)
```

```
[Out] -1/5*(11*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^8*a*b^2*d
^4+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^3*c^3*d^3+(
-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a^3*c^5*d+8*(-a*d
/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*b^3*c*d^2-2*(-a*d/(a*
c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*b^3*c^2*d+5*(-a*d/(a*c+b)
)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^8*a*b^2*d^4+5*(-a
*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*b^3*d
^3+11*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*b^3*d^3+3*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^5+3*(-a*d/(a
*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c^4+(-a*d/(a*c+b))^(1/
2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^8*a^3*c^2*d^4+(-a*d/(a*c+b))^(1/2)*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^3*c^6+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*b^3*c^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)*x^5*a^3*c^3*d^3+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a^2*b*c^2*d^3-16*((a*d*x^2+a*c+b)/(a*c+b))^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/
2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a^2*b*c^2*d^3+8*((a*d*x^2+a*c+b)/
(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b
)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a*b^2*c*d^3-16*((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a*b^2*c*d^3+11
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^8*a^2*b*c*d^4+19*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^2*b*c^2*d^3+30
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a*b^2*c*d^3+5*(
-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a^2*b*c^3*d^2+13*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a*b^2*c^2*d^2-3*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a*b^2*c^3*d+5*(-
a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^8*a^2*
b*c*d^4+5*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)*x^6*a^2*b*c^2*d^3+10*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*x^6*a*b^2*c*d^3)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/x^5/(
a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)
/c^4/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(adx^2 + ac + b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^8 + cx^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^8 +
 c*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

3.344 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=225

$$\frac{(8a^2c^2+12abc+5b^2)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^3d^3}-\frac{b(8a^2c^2+12abc+5b^2)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}-\frac{(8ac+5b)(c+dx^2)^2\sqrt{}}{24a^2d^3}$$

```
[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)])/(16*a^3*d^3) - ((5*b + 8*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)])/(24*a^2*d^3) + (x^2*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)])/(6*a*d^2) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(7/2)*d^3)
```

Rubi [A] time = 0.6221, antiderivative size = 267, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{(8a^2c^2+12abc+5b^2)(a(c+dx^2)+b)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b(8a^2c^2+12abc+5b^2)\sqrt{a(c+dx^2)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{16a^{7/2}d^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(8ac+5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^3*d^3*Sqrt[a + b
/(c + d*x^2)]) - ((5*b + 8*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(24*a^2*d^
3*Sqrt[a + b/(c + d*x^2)]) + (x^2*(c + d*x^2)*(b + a*(c + d*x^2)))/(6*a*d^2
*Sqrt[a + b/(c + d*x^2)]) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*Sqrt[b + a*(c
 + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(16*
a^(7/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^2\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-c(b+ac)-\frac{1}{2}(5b+8ac)dx\right)}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{6ad^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(\left(-2ac(b+ac)d^2+\frac{1}{2}(\right.\right.}{}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(\right.}{6ad^2\sqrt{}}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(\right.}{6ad^2\sqrt{}}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(\right.}{6ad^2\sqrt{}}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(\right.}{6ad^2\sqrt{}}$$

Mathematica [A] time = 0.269933, size = 140, normalized size = 0.62

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^2-cdx^2+d^2x^4\right)+2ab\left(13c-5dx^2\right)+15b^2\right)-3b\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.}{48a^{7/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b*
(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b*c
 + 8*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(7/2)*d^3)
```

Maple [B] time = 0.022, size = 533, normalized size = 2.4

$$\frac{dx^2+c}{96\,a^3d^3}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-48\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}x^2ca^2d\sqrt{ad^2}-36\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)-36*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)-24*ln(1/2*(2*a
*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^
(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d-36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*b^2*c*a*d+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(
1/2)+36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)
-15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+30*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45516, size = 961, normalized size = 4.27

$$\left[\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2-4\left(2ad^2x^4+(4ac+b)dx^2+2ac\right.\right.}{192a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*
a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c
 + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
 + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^
2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan(1/2*
(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2
*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 +
26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(a + b/(d*x^2 + c)), x)
```

3.345 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=148

$$-\frac{(4ac+3b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^2d^2}+\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4ad^2}$$

```
[Out] -((3*b + 4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a^2*d
^2) + ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*d^2) + (b*
(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^
(5/2)*d^2)
```

Rubi [A] time = 0.464804, antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{(4ac+3b)\left(a\left(c+dx^2\right)+b\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{b(4ac+3b)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{5/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((3*b + 4*a*c)*(b + a*(c + d*x^2)))/(8*a^2*d^2*Sqrt[a + b/(c + d*x^2)]) +
((c + d*x^2)*(b + a*(c + d*x^2)))/(4*a*d^2*Sqrt[a + b/(c + d*x^2)]) + (b*(3
*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[
b + a*(c + d*x^2)]])/(8*a^(5/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]
)
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{x\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{8ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}}{16a^2d\sqrt{c+dx^2}\sqrt{a}}$$
$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}}{8a^2d^2\sqrt{c+dx^2}\sqrt{a}}$$
$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}}{8a^2d^2\sqrt{c+dx^2}\sqrt{a}}$$
$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\right.}{8a^{5/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx}}}$$

Mathematica [A] time = 0.163918, size = 101, normalized size = 0.68

$$\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2a\left(c-dx^2\right)+3b\right)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (-(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b + 2*a*(c
 - d*x^2))) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a
^(5/2)*d^2)
```

Maple [B] time = 0.013, size = 354, normalized size = 2.4

$$\frac{dx^2+c}{16\,a^2d^2}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(4\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}x^2ad+4\,\ln\left(1/2\,\frac{2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acd}}{\sqrt{ad}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*d+4*ln(1/2*(2*a*d^2*x^2+2*
a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a*b*c*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)*a*c+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d-6*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/a^2/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3725, size = 761, normalized size = 5.14

$$\left[\frac{\left(4\,abc+3\,b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\right.}{32\,a^3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
- 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c
 + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a
*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [A] time = 1.33666, size = 262, normalized size = 1.77

$$\frac{1}{8}\sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\left(\frac{2\,x^2}{ad\mathrm{sgn}\left(dx^2 + c\right)} - \frac{2\,acd\mathrm{sgn}\left(dx^2 + c\right) + 3\,bd\mathrm{sgn}\left(dx^2 + c\right)}{a^2d^3}\right) - \frac{\left(4\,abc + 3\,b^2\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d*sgn(d
*x^2 + c)) - (2*a*c*d*sgn(d*x^2 + c) + 3*b*d*sgn(d*x^2 + c))/(a^2*d^3)) - 1
/16*(4*a*b*c + 3*b^2)*log(abs(-2*a^(3/2)*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*
d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - sqrt(a)*b*d))/(a
^(5/2)*d*abs(d)*sgn(d*x^2 + c))
```

3.346 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rubi [A] time = 0.0520451, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 51, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4ad}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2ad}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 0.0774736, size = 70, normalized size = 0.97

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}} - b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)] - b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Maple [B] time = 0.007, size = 184, normalized size = 2.6

$$\frac{dx^2+c}{4\,ad}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-b\ln\left(\frac{1}{2}\left(2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd\right)\frac{1}{\sqrt{ad^2}}\right)d+2\,\sqrt{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-b*ln(1/2*(2*a*d^2*x^2+2*a
*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(
```

```
a*d^2)^(1/2))*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a/d/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32526, size = 612, normalized size = 8.5

$$\left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}{8\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a
*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
 a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [B] time = 1.42982, size = 190, normalized size = 2.64

$$\frac{b\log\left(\left|-2\,a^{\frac{3}{2}}cd-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a|d|-\sqrt{a}bd\right|\right)}{4\,a^{\frac{3}{2}}|d|\mathrm{sgn}\left(dx^2+c\right)}+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+}}{2\,ad\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*a^(3/2)*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - sqrt(a)*b*d))/(a^(3/2)*abs(d)*sg
n(d*x^2 + c)) + 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(
a*d*sgn(d*x^2 + c))
```

3.347 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

```
[Out] ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*A
rcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt
[b + a*c]
```

Rubi [A] time = 0.414477, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c +
d*x^2)]])/(Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*Sqrt
[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b
 + a*(c + d*x^2)])])/(Sqrt[b + a*c]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]
)
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{}{\sqrt{c+dx}}\right.}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+d}}}$$
$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\right.}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= -\frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a(c+d}}\right.}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$
$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [A] time = 0.120279, size = 80, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*ArcTanh[(Sqrt[c
]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[b + a*c]
```

Maple [B] time = 0.012, size = 312, normalized size = 3.3

$$\frac{dx^2+c}{2\,ac+2\,b}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(\ln\left(\frac{1}{2}\left(2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd\right)\frac{1}{\sqrt{ad^2}}\right)acd+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*a*d^2*x^2+2*a*c*
d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a*c*d+b*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*d-(a*c^2+b*c)^(1/2)
*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87221, size = 2168, normalized size = 22.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*
((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 +
(4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*
a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2
*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4*(a*sqr
t(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*
b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2
 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3
+ 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/
(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)))/a, 1/4*(2*
a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*
c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(a*c + b))*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x), x)
```

3.348 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac+b)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}$$

```
[Out] -((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(b + a*c)*x^2) - (b
*d*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/
(2*Sqrt[c]*(b + a*c)^(3/2))
```

Rubi [A] time = 0.387278, antiderivative size = 148, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{a\left(c+dx^2\right)+b}{2x^2(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{bd\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(b + a*(c + d*x^2))/(2*(b + a*c)*x^2*Sqrt[a + b/(c + d*x^2)]) - (b*d*Sqrt[
b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b
+ a*(c + d*x^2)])])/(2*Sqrt[c]*(b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{bd\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{2\sqrt{c}(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.213945, size = 158, normalized size = 1.46

$$
-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{c}\sqrt{ac+b}\left(c+dx^2\right)\sqrt{a\left(c+dx^2\right)+b}+bdx^2\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)\right)}{2\sqrt{c}x^2(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[c]*Sqrt[b + a*c]*(c + d*x^2)*
Sqrt[b + a*(c + d*x^2)] + b*d*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b + a*c]*Sq
rt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*c + a*d*x^2])]))/(2*Sqrt[c]*(b + a*c)^(3
/2)*x^2*Sqrt[b + a*(c + d*x^2)])
```

Maple [B] time = 0.013, size = 452, normalized size = 4.2

$$-\frac{dx^2+c}{4\,(ac+b)^2\,cx^2}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-2\,ad^2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}x^4\sqrt{c^2a+bc}+\ln\left(\frac{1}{x^2}\left(2\,acdx^2+bdx^2+2\,c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x
^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)+2*b*c)/x^2)*x^2*a*b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b
*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*b
^2*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/c/x^2/(a*c^2+b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76838, size = 972, normalized size = 9.

$$\left[\frac{\sqrt{ac^2+bc}bdx^2\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left((2\,ac+b)d^2x^4+2\,ac^3+\left(4\,ac^2+3\,bc\right)dx^2+2\,bc^2\right)\sqrt{ac^2+bc}\sqrt{}}{x^4}\right.}{8\left(a^2c^3+2\,abc^2+b^2c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
```

```
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^
3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2
*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) -
 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^3), x)
```

3.349 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=177

$$\frac{bd^2(4ac+b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{3/2}(ac+b)^{5/2}}+\frac{d(4ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8cx^2(ac+b)^2}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4cx^4(ac+b)}$$

```
[Out] ((b + 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c*(b +
 a*c)^2*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c*(
b + a*c)*x^4) + (b*(b + 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)])/Sqrt[b + a*c]])/(8*c^(3/2)*(b + a*c)^(5/2))
```

Rubi [A] time = 0.472157, antiderivative size = 218, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{bd^2(4ac+b)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{3/2}(ac+b)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{d(4ac+b)\left(a\left(c+dx^2\right)+b\right)}{8cx^2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4cx^4(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ((b + 4*a*c)*d*(b + a*(c + d*x^2)))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*
x^2)]) - ((c + d*x^2)*(b + a*(c + d*x^2)))/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c
 + d*x^2)]) + (b*(b + 4*a*c)*d^2*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b +
a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(3/2)*(b + a
*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+a\left(c+dx^2\right)}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+ac+adx^2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3\sqrt{b+ac+adx}}dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((b+4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}dx,x,x^2\right)}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subs}}{16c(b+ac)^2\sqrt{c+dx}}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subs}}{8c(b+ac)^2\sqrt{c+c}}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(b+4ac)d^2\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}}{8c^{3/2}(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a}}$$

Mathematica [A] time = 0.324367, size = 191, normalized size = 1.08

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(bd^2x^4(4ac+b)\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)-\sqrt{c}\sqrt{ac+b}\left(c+dx^2\right)\sqrt{a\left(c+dx^2\right)+b}\left(2ac\left(c-dx^2\right)+b\left(2\right.\right.\right.}{8c^{3/2}x^4(ac+b)^{5/2}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[c]*Sqrt[b + a*c]*(c + d*x^2)
*Sqrt[b + a*(c + d*x^2)]*(2*a*c*(c - d*x^2) + b*(2*c + d*x^2))) + b*(b + 4*
a*c)*d^2*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[
c]*Sqrt[b + a*c + a*d*x^2])]))/(8*c^(3/2)*(b + a*c)^(5/2)*x^4*Sqrt[b + a*(c
 + d*x^2)])
```

Maple [B] time = 0.016, size = 922, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*a^2*d^3*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*c*(a*c^2+b*c)^(3/2)+4*ln((2*a*c*d*x^
2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)+2*b*c)/x^2)*x^4*a^3*b*c^5*d^2-2*a*d^3*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*x^6*b*(a*c^2+b*c)^(3/2)+9*ln((2*a*c*d*x^2+b*d*x^2+2*c
^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*
b*c)/x^2)*x^4*a^2*b^2*c^4*d^2-20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(3/2)+6*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2
*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x
^2)*x^4*a*b^3*c^3*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*
c*d^2*b*x^4*(a*c^2+b*c)^(3/2)+ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)
^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*b^4*
c^2*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d^2*x^4*(a*c^
2+b*c)^(3/2)+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d*x^2*(
a*c^2+b*c)^(3/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b*d*x^2*
(a*c^2+b*c)^(3/2)-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+
b*c)^(3/2)*a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b
*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/c^2/x^4/(a*c^2+b
*c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.95101, size = 1251, normalized size = 7.07

$$\left[\frac{\left(4\,abc + b^2\right)\sqrt{ac^2 + bc}d^2x^4 \log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left((2\,ac+b)d^2x^4+2\,ac^3+\left(4\,ac^2+3\,bc\right)d\right.}{x^4}\right.}{32\left(a^3c^5 + 3\,a^2bc^4 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
+ b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*
c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d
*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^
4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b
^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a*b*c +
b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2
*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*a^2*c^3 +
a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*
```

```
x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}\, x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^5), x)
```

3.350 $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=443

$$\frac{x\left(3a^2c^2+13abc+8b^2\right)\left(ac+adx^2+b\right)}{15a^3d^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\left(3a^2c^2+13abc+8b^2\right)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{c^{3/2}(3ac}{15a}$$

```
[Out] -((4*b + 3*a*c)*x*(b + a*c + a*d*x^2))/(15*a^2*d^2*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) + (x^3*(b + a*c + a*d*x^2))/(5*a*d*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)]) + ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*x*(b + a*c + a*d*x^2))/(15*
a^3*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(8*b^
2 + 13*a*b*c + 3*a^2*c^2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], b/(b + a*c)])/(15*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^
(3/2)*(4*b + 3*a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], b/(b + a*c)])/(15*a^2*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.708877, antiderivative size = 498, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 478, 582, 531, 418, 492, 411}

$$\frac{x\left(3a^2c^2+13abc+8b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{15a^3d^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\left(3a^2c^2+13abc+8b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{15a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((4*b + 3*a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(15*a^2*
d^2*Sqrt[a + b/(c + d*x^2)]) + (x^3*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c +
 d*x^2)])/(5*a*d*Sqrt[a + b/(c + d*x^2)]) + ((8*b^2 + 13*a*b*c + 3*a^2*c^2)
*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(15*a^3*d^2*(c + d*x^2)
*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(8*b^2 + 13*a*b*c + 3*a^2*c^2)*Sqrt[b
+ a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(15*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) + (c^(3/2)*(4*b + 3*a*c)
*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], b/(b + a*c)])/(15*a^2*d^(5/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a
*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(3c(b+ac)+(4b+3ac)dx^2\right)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c\right.}}{}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c(b+ac)\right.}{}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(8b^2+13\right.}{}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(8b^2+13\right.}{}$$

Mathematica [C] time = 0.792588, size = 297, normalized size = 0.67

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(3a^2\left(c^2-d^2x^4\right)+ab\left(7c+dx^2\right)+4b^2\right)+ic\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2}{ac+b}}\right.}{15a^2d^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2
)*(4*b^2 + a*b*(7*c + d*x^2) + 3*a^2*(c^2 - d^2*x^4)) + I*c*(8*b^2 + 13*a*b
*c + 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ell
ipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*I)*b*c*(2*b +
3*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(15*a^2*d^2*Sqrt[(a*d)/(b
+ a*c)]*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.019, size = 665, normalized size = 1.5

$$\frac{dx^2+c}{15\,a^2d^2}\left(3\sqrt{-\frac{ad}{ac+b}}x^7a^2d^3+3\sqrt{-\frac{ad}{ac+b}}x^5a^2cd^2-\sqrt{-\frac{ad}{ac+b}}x^5abd^2-3\sqrt{-\frac{ad}{ac+b}}x^3a^2c^2d-8\sqrt{-\frac{ad}{ac+b}}x^3abcd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/15*(3*(-a*d/(a*c+b))^(1/2)*x^7*a^2*d^3+3*(-a*d/(a*c+b))^(1/2)*x^5*a^2*c*d
^2-(-a*d/(a*c+b))^(1/2)*x^5*a*b*d^2-3*(-a*d/(a*c+b))^(1/2)*x^3*a^2*c^2*d-8*
(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d+3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3-4
*(-a*d/(a*c+b))^(1/2)*x^3*b^2*d-3*(-a*d/(a*c+b))^(1/2)*x*a^2*c^3-6*((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*a*b*c^2+13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-7*
(-a*d/(a*c+b))^(1/2)*x*a*b*c^2-4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+8*((a
*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))
^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-4*(-a*d/(a*c+b))^(1/2)*x*b^2*c)*(d*x^2+c)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/a^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^6+cx^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^2+ac+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^6 + c*x^4)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^2 + a
*c + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + \frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

3.351 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=354

$$\frac{\sqrt{c}(ac+2b)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(ac+2b)\left(ac+adx^2+b\right)}{3a^2d\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+\cdots}\right)}{3ad^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2\cdots\right)}{(ac+b)\left(c+\cdots\right)}}}$$

```
[Out] (x*(b + a*c + a*d*x^2))/(3*a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((2
*b + a*c)*x*(b + a*c + a*d*x^2))/(3*a^2*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)]) + (Sqrt[c]*(2*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^2*d^(3/2)*(c + d*x^2)*Sqrt[(b +
 a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d
*x^2))]) - (c^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], b/(b + a*c)])/(3*a*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.520623, antiderivative size = 398, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 478, 531, 418, 492, 411}

$$\frac{\sqrt{c}(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{x(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c^{3/2}\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*a*d*Sqrt[a + b/(c +
d*x^2)]) - ((2*b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/
(3*a^2*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) + (Sqrt[c]*(2*b + a*c)*Sqrt[b
 + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], b/(b + a*c)])/(3*a^2*d^(3/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*c
+ a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*a*d^(3/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b+ac)+(2b+ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((2b+ac)\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c(b+ac)\right.}{}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+ac+ad}}{3ad^{3/2}\left(c\right.}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{c}(2b+ac)\sqrt{b+}}{3a^2d}$$

Mathematica [C] time = 0.556936, size = 253, normalized size = 0.71

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac+adx^2+b\right)-ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+ic(ac+2b)\sqrt{\frac{dx^2}{c}}\right.}{3ad\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b + a*c + a*d*x^2) + I*c*(2*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*
c)] - I*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*a*d*Sqrt[(a*d)/(b
+ a*c)]*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.014, size = 409, normalized size = 1.2

$$\frac{dx^2+c}{3\,ad}\left(\sqrt{-\frac{ad}{ac+b}}x^5ad^2+2\sqrt{-\frac{ad}{ac+b}}x^3acd+\sqrt{-\frac{ad}{ac+b}}x^3bd-\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*x^5*a*d^2+2*(-a*d/(a*c+b))^(1/2)*x^3*a*c*d+(-a*d/
(a*c+b))^(1/2)*x^3*b*d-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b))^
(1/2)*x*a*c^2+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-2*((a*d*x^2+a*c+b)/(a*c+b)
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^
(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*x*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1
/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^4+cx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^2+ac+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^2 + a
*c + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

3.352 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=286

$$\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x\left(ac+adx^2+b\right)}{a\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}$$

```
[Out] (x*(b + a*c + a*d*x^2))/(a*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]) - (Sqrt[c]*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*Sqrt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^(3/2)*(b + a*c +
a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sq
rt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c +
a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.208576, antiderivative size = 319, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6722, 1974, 422, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{x\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{a\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{\cdots}}{a\sqrt{d}\left(c+d\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(a*(c + d*x^2)*Sqrt[a +
 b/(c + d*x^2)]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*Sqrt[d]*(c + d*x^2
)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x
^2)]) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d]*(c + d*x^2)*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)
])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c^{3/2}\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(c\cdots}{\cdots}\\
&= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c^{3/2}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.111982, size = 107, normalized size = 0.37

$$
\frac{\sqrt{\frac{ac+adx^2+b}{ac+b}}E\left(\sin^{-1}\left(\sqrt{-\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)}{\sqrt{\frac{dx^2}{c}+1}\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*EllipticE[ArcSin[Sqrt[-((a*d)/(b + a*c
))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]*Sqrt[1 + (d*x^2)/c])
```

Maple [A] time = 0.01, size = 164, normalized size = 0.6

$$c\left(dx^{2}+c\right)\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)\sqrt{\frac{dx^{2}+c}{c}}\sqrt{\frac{adx^{2}+ac+b}{ac+b}}\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}\frac{1}{\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)/c)^(1/2)*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*c*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{dx^{2}+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^{2}+c\right)\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{adx^{2}+ac+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^2 + a*c +
 b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

3.353 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=343

$$-\frac{ac+adx^2+b}{x(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{dx\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\left(ac+a\right.}{(ac+b)\left(c+\right.}$$

```
[Out] -((b + a*c + a*d*x^2)/((b + a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]))
+ (d*x*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) - (Sqrt[c]*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) +
(Sqrt[c]*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.483361, antiderivative size = 387, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{(ac+b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{x(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}F\left(\text{ta}\right.}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/((b + a*c)*x*Sqrt[a + b
/(c + d*x^2)])) + (d*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/((b
 + a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*Sqrt[d]*Sqrt[b + a*
c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) + (Sqrt[c]*Sqrt[d]*Sqrt[b + a*c +
 a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*
(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+a(c+dx^2)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{(b+ac)d+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(ad^2\sqrt{b+a\right.}}{(b+}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{c}\sqrt{d}\sqrt{b+ac+ad}}{(b+ac)\left(c\right.}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{b+ac+ad}}{(b+ac)\left(c\right.}$$

Mathematica [A] time = 0.175935, size = 151, normalized size = 0.44

$$\frac{d\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}(ac+b)\sqrt{\frac{ac+adx^2+b}{ac+b}}} - \frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x)) + (d*S
qrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin[S
qrt[-(d/c)]*x], (a*c)/(b + a*c)])/((b + a*c)*Sqrt[-(d/c)]*Sqrt[(b + a*c + a
*d*x^2)/(b + a*c)])
```

Maple [A] time = 0.014, size = 345, normalized size = 1.

$$-\frac{dx^2+c}{x\left(ac+b\right)}\left(\sqrt{-\frac{ad}{ac+b}}x^4ad^2-adc\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}x\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}x^2ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*x^4*a*d^2-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2*(
-a*d/(a*c+b))^(1/2)*x^2*a*c*d-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x*b*d+(-a*d/(a
*c+b))^(1/2)*x^2*b*d+(-a*d/(a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(
d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)/((d*x^2+c)*(a*d*x^2+a*c+b))^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^4+(ac+b)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^4 + (a*c
+ b)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

3.354 $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=431

$$\frac{d^2x(b-ac)\left(ac+adx^2+b\right)}{3c(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{a\sqrt{c}d^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{d^{3/2}(b-ac)\left(ac+adx^2+b\right)E\left(\tan^{-1}\right.}{3\sqrt{c}(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[Out] -(b + a*c + a*d*x^2)/(3*(b + a*c)*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
) - ((b - a*c)*d*(b + a*c + a*d*x^2))/(3*c*(b + a*c)^2*x*Sqrt[(b + a*c + a*
d*x^2)/(c + d*x^2)]) + ((b - a*c)*d^2*x*(b + a*c + a*d*x^2))/(3*c*(b + a*c)
^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((b - a*c)*d^(3/2)*
(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3
*Sqrt[c]*(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (a*Sqrt[c]*d^(3/2)*(b
+ a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*(b
 + a*c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*
c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.637087, antiderivative size = 486, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 583, 531, 418, 492, 411}

$$\frac{d^2x(b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{3c(ac+b)^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{a\sqrt{c}d^{3/2}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{d^{3/2}(b-a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*(b + a*c)*x^3*Sqrt[a
+ b/(c + d*x^2)]) - ((b - a*c)*d*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*
x^2)])/(3*c*(b + a*c)^2*x*Sqrt[a + b/(c + d*x^2)]) + ((b - a*c)*d^2*x*Sqrt[
b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*c*(b + a*c)^2*(c + d*x^2)*Sq
rt[a + b/(c + d*x^2)]) - ((b - a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b
+ a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*Sq
rt[c]*(b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))]*Sqrt[a + b/(c + d*x^2)]) - (a*Sqrt[c]*d^(3/2)*Sqrt[b + a*c + a*d*x
^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a
*c)])/(3*(b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c
 + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+a\left(c+dx^2\right)}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+ac+adx^2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{(b-ac)d-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}dx}{3(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right.}}{3c(b+ac)^2}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(ad^2\sqrt{b+a\left(c+\right.}\right.}{3(b+ac)}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)d^2x\sqrt{b+}}{3c(b+ac)}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)d^2x\sqrt{b+}}{3c(b+ac)}$$

Mathematica [C] time = 0.909955, size = 314, normalized size = 0.73

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c^2-d^2x^4\right)+ab\left(2c^2+cdx^2+d^2x^4\right)+b^2\left(c+dx^2\right)\right)+2iabcd^2x^3\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}\right.}{3cx^3(ac+b)^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)
*(b^2*(c + d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + c*d*x^2 + d^2*x^4)
)) + I*a*c*(-b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] + (2*
I)*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*c*(b + a*c)^2
*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.016, size = 596, normalized size = 1.4

$$\frac{dx^2+c}{3\,cx^3\,(ac+b)^2}\left(\sqrt{-\frac{ad}{ac+b}}x^6a^2cd^3-\sqrt{-\frac{ad}{ac+b}}x^6abd^3-\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*x^6*a^2*c*d^3-(-a*d/(a*c+b))^(1/2)*x^6*a*b*d^3-((
a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b)
)^(1/2),((a*c+b)/a/c)^(1/2))*x^3*a^2*c^2*d^2+(-a*d/(a*c+b))^(1/2)*x^4*a^2*c
^2*d^2-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x^3*a*b*c*d^2+((a*d*x^2+a*c+b)/(a*c
+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/
c)^(1/2))*x^3*a*b*c*d^2-2*(-a*d/(a*c+b))^(1/2)*x^4*a*b*c*d^2-(-a*d/(a*c+b))
^(1/2)*x^4*b^2*d^2-(-a*d/(a*c+b))^(1/2)*x^2*a^2*c^3*d-3*(-a*d/(a*c+b))^(1/2
)*x^2*a*b*c^2*d-2*(-a*d/(a*c+b))^(1/2)*x^2*b^2*c*d-(-a*d/(a*c+b))^(1/2)*a^2
*c^4-2*(-a*d/(a*c+b))^(1/2)*a*b*c^3-(-a*d/(a*c+b))^(1/2)*b^2*c^2)*(d*x^2+c)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)/(-a*d/(a*c+b))^(1/2)/c/x^3/(a*c+b)^2/((d*x^2+c)*(a*d*x^2+a*c+b))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^6+(ac+b)x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^6 + (a*c
+ b)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

3.355 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx$

Optimal. Leaf size=310

$$\frac{\left(6a^2c^2+12abc+7b^2\right)\left(c+dx^2\right)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a^2b^2d^3}-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^3bd^3}+\frac{\left(24a^2c^2+60abc+3\right.}{}$$

```
[Out] -(((b + a*c)^2*(c + d*x^2)^3)/(a*b^2*d^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)])) + ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)])/(16*a^4*d^3) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x
^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(24*a^3*b*d^3) + ((7*b^2 + 12*
a*b*c + 6*a^2*c^2)*(c + d*x^2)^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(6*
a^2*b^2*d^3) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(9/2)*d^3)
```

Rubi [A] time = 0.782944, antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(24a^2c^2+60abc+35b^2\right)\left(a\left(c+dx^2\right)+b\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b\left(24a^2c^2+60a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b + a*c)^2*(c + d*x^2)^2)/(a^2*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((35*b^2
+ 60*a*b*c + 24*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^4*d^3*Sqrt[a + b/(c + d
*x^2)]) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*(b + a*(c + d*x^2))
)/(24*a^3*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((c + d*x^2)^2*(b + a*(c + d*x^2
)))/(6*a^2*d^3*Sqrt[a + b/(c + d*x^2)]) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^
2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c
+ d*x^2)]])/(16*a^(9/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(\frac{1}{2}(b+ac)(5b+4ac)d-\frac{1}{2}abd^2x\right)}{\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{a^2bd^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\left(b+a\left(c+dx^2\right)\right)}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(\left(35b^2+60abc+24a^2c^2\right)\sqrt{b+a\left(c+dx\cdots\right.\right.}{12a^2bd^2\sqrt{c+dx^2}\cdots}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\left(b+a\cdots\right.}{6a^2d^3\sqrt{a+\cdots}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(\cdots\right.}{24a^3bd^3\sqrt{\cdots}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(\cdots\right.}{24a^3bd^3\sqrt{\cdots}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(\cdots\right.}{24a^3bd^3\sqrt{\cdots}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(\cdots\right.}{24a^3bd^3\sqrt{\cdots}}$$

Mathematica [C] time = 11.731, size = 1215, normalized size = 3.92

$$b\left(-344c^2\,{}_4F_3\left(\frac{1}{2},2,2,2;1,1,\frac{7}{2};\frac{b}{adx^2+ac}+1\right)\left(a+\frac{b}{dx^2+c}\right)^5 - 192c^2\,{}_5F_4\left(\frac{1}{2},2,2,2,2;1,1,1,\frac{7}{2};\frac{b}{adx^2+ac}+1\right)\left(a+\frac{b}{dx^2+c}\right)^5 - \cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*(2835*a^2*(b + a*c)^2*(a + b/(c + d*x^2)) - 3240*a^2*c*(b + a*c)*(a + b/
(c + d*x^2))^2 + 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2 + 765*a^2*c^2*(a
+ b/(c + d*x^2))^3 + 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^3 - 105*a*c^2*(a
```

```
 + b/(c + d*x^2))^4 + (300*(b + a*c)^2*(a + b/(c + d*x^2))^3*ArcTanh[Sqrt[1
 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*x^2))^(3/2) + (60*c*(b + a*c)*(a
+ b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*
x^2))^(3/2) + (120*a*c^2*(a + b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*
d*x^2)]])/Sqrt[1 + b/(a*c + a*d*x^2)] - 2835*a^3*(b + a*c)^2*Sqrt[1 + b/(a*
c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] - 765*a^3*c^2*(a + b/(c
+ d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]
] - 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*Ar
cTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] - 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^
3*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] + 105*a*
c^2*(a + b/(c + d*x^2))^4*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a
*c + a*d*x^2)]] + 3240*a^4*c*(b + a*c)*(1 + b/(a*c + a*d*x^2))^(3/2)*ArcTan
h[Sqrt[1 + b/(a*c + a*d*x^2)]] - 760*(b + a*c)^2*(a + b/(c + d*x^2))^3*Hype
rgeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 1040*c*
(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/
2}, 1 + b/(a*c + a*d*x^2)] - 344*c^2*(a + b/(c + d*x^2))^5*HypergeometricPF
Q[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 256*(b + a*c)^2*(a
+ b/(c + d*x^2))^3*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 +
 b/(a*c + a*d*x^2)] + 448*c*(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricP
FQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 192*c^2*(a +
 b/(c + d*x^2))^5*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 +
b/(a*c + a*d*x^2)] - 32*(b + a*c)^2*(a + b/(c + d*x^2))^3*HypergeometricPFQ
[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 64*c*(b
+ a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1,
 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 32*c^2*(a + b/(c + d*x^2))^5*Hypergeo
metricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)]))
/(720*a^5*d^3*(a + b/(c + d*x^2))^(5/2))
```

Maple [B] time = 0.036, size = 1240, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^4/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^3*c*d^2-60*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^2*b*d^2-72*ln(1/
2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a^3*b*c^2*d^2-48*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^3*c^2*d-180*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+
b*d)/(a*d^2)^(1/2))*x^2*a^2*b^2*c*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a^2*d-105*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/
2))*x^2*a*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b*c^3*d+54*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b^2*d-252
*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d+96*((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*a^2*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2-285*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))
*a*b^3*c*d+192*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b^2*c+16*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*b+222*(a*d^2
```

```
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^2*c-105*ln(1/2*
(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d
^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^4*d+96*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a
*d^2)^(1/2)*b^3+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2)*b^3)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85877, size = 1488, normalized size = 4.8

$$\left[\frac{3\left(24\,a^3bc^3+84\,a^2b^2c^2+95\,ab^3c+35\,b^4+\left(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3\right)dx^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*
c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2
 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d
*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8
*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 +
(18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*
c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(24*a^3*b*
c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c +
 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^4*d^4
*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*
b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 15
0*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(a + b/(d*x^2 + c))^(3/2), x)
```

3.356 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2d^2}-\frac{(4ac+7b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^3d^2}-\frac{b(ac+b)}{a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{3b(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}$$

```
[Out] -((b*(b + a*c))/(a^3*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((7*b +
4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a^3*d^2) + ((c
 + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a^2*d^2) + (3*b*(5*b
+ 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2)
*d^2)
```

Rubi [A] time = 0.573866, antiderivative size = 242, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{(4ac+5b)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3(4ac+5b)\left(a\left(c+dx^2\right)+b\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{3b(4ac+5b)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{7/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b + a*c)*(c + d*x^2)^2)/(a*b*d^2*Sqrt[a + b/(c + d*x^2)])) - (3*(5*b +
4*a*c)*(b + a*(c + d*x^2)))/(8*a^3*d^2*Sqrt[a + b/(c + d*x^2)]) + ((5*b + 4
*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(4*a^2*b*d^2*Sqrt[a + b/(c + d*x^2)]
) + (3*b*(5*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*
x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*a^(7/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((5b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2abd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(3(5b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)}{8a^2d\sqrt{c+dx^2}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} +$$

Mathematica [A] time = 0.275619, size = 133, normalized size = 0.71

$$\frac{3b(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \sqrt{a}\left(2a^2\left(c^2-d^2x^4\right)+ab\left(17c+5dx^2\right)+15b^2\right)}{8a^{7/2}d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-(Sqrt[a]*(15*b^2 + a*b*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4))) + 3*b*(
5*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(8*a^(7/2)*d^2*Sqrt[a + b/(c + d*x^2)])
```

Maple [B] time = 0.014, size = 783, normalized size = 4.2

$$\frac{dx^2+c}{16\,a^3d^2\left(adx^2+ac+b\right)}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(4\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}x^4a^2d^2+12\,\ln\left(1/2\,\frac{2\,ad^2x^2+2\,acd}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^2*(4*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^2*d^2+12*ln(1/2*(2*a*d
^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2)+b*d)/(a*d^2)^(1/2))*x^2*a^2*b*c*d^2+15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1
/2))*x^2*a*b^2*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b
*a*d*(a*d^2)^(1/2)+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d-4*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2+27*ln(1
/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*a*d-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)+15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/
2))*b^3*d-16*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c-14*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2)-16*(a*d^2)^(1/
2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.6916, size = 1172, normalized size = 6.27

$$\left[\frac{3\left(4\,a^2bc^2+9\,ab^2c+\left(4\,a^2bc+5\,ab^2\right)dx^2+5\,b^3\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sq
rt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b
^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*
```

```
x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x
^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a
*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c +
 a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x
^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*d^3*x^6 +
 (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a
^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] time = 8.96026, size = 767, normalized size = 4.1

$$\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{a^2d\mathrm{sgn}\left(dx^2+c\right)}-\frac{2\,a^6cd^2+7\,a^5bd^2}{a^8d^4\mathrm{sgn}\left(dx^2+c\right)}\right)-\frac{\left(4\,a^{\frac{3}{2}}bc+5\,\sqrt{a}b^2\right)\log\left(\left|-2\,a^{\frac{7}{2}}c^3d-\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sgn
(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/16
*(4*a^(3/2)*b*c + 5*sqrt(a)*b^2)*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d)
 - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4
 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) -
5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^4*d
*abs(d)*sgn(d*x^2 + c)) - 1/8*(4*a^(3/2)*b*c*abs(d) + 5*sqrt(a)*b^2*abs(d))
*log(48*a^4*d^2*abs(a)*abs(sgn(d*x^2 + c)))/(a^4*d^3*sgn(d*x^2 + c))
```

3.357 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}$$

```
[Out] (3*b)/(2*a^2*d*Sqrt[a + b/(c + d*x^2)]) + (c + d*x^2)/(2*a*d*Sqrt[a + b/(c
+ d*x^2)]) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

Rubi [A] time = 0.0733392, antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 51, 63, 208}

$$\frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((c + d*x^2)/(a*d*Sqrt[a + b/(c + d*x^2)])) + (3*(c + d*x^2)*Sqrt[a + b/(c
 + d*x^2)])/(2*a^2*d) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a
^(5/2)*d)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2ad}\\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4a^2d}\\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2a^2d}\\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}
\end{aligned}$$

Mathematica [C] time = 0.0538379, size = 50, normalized size = 0.5

$$\frac{b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{a+\frac{b}{dx^2+c}}{a}\right)}{a^2d\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/(c + d*x^2))/a])/(a^2*d*Sqrt[a +
b/(c + d*x^2)])
```

Maple [B] time = 0.014, size = 478, normalized size = 4.8

$$\frac{dx^2+c}{4\,a^2d\left(adx^2+ac+b\right)}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-3\,\ln\left(1/2\,\frac{2\,ad^2x^2+2\,acd+2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd}{\sqrt{ad^2}}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^2/d*(-3*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+
b*d)/(a*d^2)^(1/2))*x^2*a*b*d^2+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*d^2)^(1/2)*x^2*a*d-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d
+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c-3*ln(1
/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*(a*d^2)^(1/2)*b+4*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13275, size = 871, normalized size = 8.71

$$\left[\frac{3\left(abdx^2+abc+b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right.\right.}{8\left(a^4d^2x^2+\left(a^4c+a^3b\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8
*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2
 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^2*d
^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2 + a*
b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x^4 + a^
2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] time = 9.17871, size = 657, normalized size = 6.57

$$\frac{b|d|\log\left(12\,a^4|d|\left|\mathrm{sgn}\left(dx^2+c\right)\right|\right)}{2\,a^{\frac{5}{2}}d^2\mathrm{sgn}\left(dx^2+c\right)} + \frac{b\log\left(\left|-2\,a^{\frac{7}{2}}c^3d - 6\left(\sqrt{ad^2}x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\right)a^3c^2|d| - 6\left(\sqrt{\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b*abs(d)*log(12*a^4*abs(d)*abs(sgn(d*x^2 + c)))/(a^(5/2)*d^2*sgn(d*x^2
+ c)) + 1/4*b*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d) - 6*(sqrt(a*d^2)*x
^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c*d -
 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*
d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) - 5*(sqrt(a*d^2)*x^2
- sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*d - 4*
a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x
^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^(5/2)*abs(d)*sgn(d*x^2
 + c)) + 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a^2*d*s
gn(d*x^2 + c))
```

3.358 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{b}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[Out] -(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ArcTanh[Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b + a*c)^(3/2)
```

Rubi [A] time = 0.51397, antiderivative size = 214, normalized size of antiderivative = 1.6, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c^{3/2}\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b}{a(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + (Sqrt[b + a*(c + d*x^2)]*ArcTa
nh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(a^(3/2)*Sqrt[c + d*
x^2]*Sqrt[a + b/(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(S
qrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/((b + a*c
)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(b+ac+adx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\frac{1}{2}ac^2d+\frac{1}{2}(b+ac)d^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(\cdots\right.}\right.}{\cdots}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \sqrt{c+dx^2}\right)}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c^2\sqrt{b+a\left(c+d\cdots\right.}\right.}{\cdots}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\cdots}{a\sqrt{c+dx\cdots}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\cdots\right.}{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a\cdots}}$$

Mathematica [A] time = 0.472127, size = 110, normalized size = 0.82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}} - \frac{b}{a(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + ArcTanh[Sqrt[a + b/(c + d*x^2)
]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqr
t[b + a*c]])/(b + a*c)^(3/2)
```

Maple [B] time = 0.017, size = 1014, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*x^2*a^3*c^2*d^2+2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a^2
*b*c*d^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*
(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^
2)*x^2*a^2*c*d+ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*x^2*a*b^2*d^2+ln(1/2*(2
*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2
)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*c^3*d+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2)
)*a^2*b*c^2*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2
*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*
c)/x^2)*a^2*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*a*d-(a*d^2)^(1
/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b*c+ln(1/2*(2*
a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b)
)^(1/2)*a*b*c-2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2)/((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.9179, size = 3229, normalized size = 24.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2
*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4
 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2
*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
+ 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x
^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*(a*b*
d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c
+ a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a*b)*d*x^2
```

```
+ 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2
+ a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2
*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*
c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 +
 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*
b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arct
an(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) + (a^2*c^2 + (a^2*c +
 a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a
^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) - 4*(a*b*d*x^2
+ a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*
b^2 + (a^4*c + a^3*b)*d*x^2), -1/2*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*
c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^
2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 +
 a*c^2 + b*c)) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x), x)
```

3.359 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3bd}{2(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3b\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2(ac+b)^{5/2}}$$

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (c + d*x^2)
/(2*(b + a*c)*x^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (3*b*Sqrt[c]*d*A
rcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*(
b + a*c)^(5/2))
```

Rubi [A] time = 0.449395, antiderivative size = 174, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{3bd}{2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\sqrt{c}d\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2(ac+b)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x^2)/(2*(b + a*c)*
x^2*Sqrt[a + b/(c + d*x^2)]) - (3*b*Sqrt[c]*d*Sqrt[b + a*(c + d*x^2)]*ArcTa
nh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(2*(
b + a*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(3bd\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(3bcd\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+a\cdots}}\cdots\right)}{4(b+ac)^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(3bcd\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2\cdots}\cdots\right)}{2(b+ac)^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+\cdots}}}\\
&= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\sqrt{c}d\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\cdots\right)}{2(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.354509, size = 186, normalized size = 1.27

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{ac+b}\left(c+dx^2\right)\sqrt{a\left(c+dx^2\right)+b}\left(ac\left(c+dx^2\right)+b\left(c-2dx^2\right)\right)+3b\sqrt{c}dx^2\sqrt{c+dx^2}\left(a\left(c+dx^2\right)+b\right)\tan\cdots\right)}{2x^2(ac+b)^{5/2}\left(a\left(c+dx^2\right)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[b + a*c]*(c + d*x^2)*Sqrt[b +
 a*(c + d*x^2)]*(b*(c - 2*d*x^2) + a*c*(c + d*x^2)) + 3*b*Sqrt[c]*d*x^2*Sqr
t[c + d*x^2]*(b + a*(c + d*x^2))*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(S
qrt[c]*Sqrt[b + a*c + a*d*x^2])]))/(2*(b + a*c)^(5/2)*x^2*(b + a*(c + d*x^2
))^(3/2))
```

Maple [B] time = 0.018, size = 1088, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a^2*d^3+3*ln((2*a*c*d*x^2+b
*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)+2*b*c)/x^2)*x^4*a^2*b*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^4*a^2*c*d^2+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^
2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b
*c)/x^2)*x^4*a*b^2*c*d^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*c^2+b*c)^(1/2)*x^4*a*b*d^2+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*
c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*a^
2*b*c^3*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/
2)*x^2*a^2*c^2*d+6*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*a*b^2*c^2*d+2*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*x^2*a*d-6*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*b*c
*d-4*(a*c^2+b*c)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a*b*c*d+3*ln((
2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^2*b^3*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b^2*d-4*(a*c^2+b*c)^(1/2)*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*b^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*a*c+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(3/2)*(a*c^2+b*c)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/(
a*d*x^2+a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.64759, size = 1277, normalized size = 8.75

$$\left[\frac{3\left(abd^2x^4 + \left(abc + b^2\right)dx^2\right)\sqrt{\frac{c}{ac+b}}\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left(\left(2\,a^2c^2+3\,abc+b^2\right)d^2x^4+2\,a^2c^4 \cdots\right.}{x^4}\right.}{8\left(\left(a^3c^2 + 2\,a^2bc + ab^2\right)dx^4 + \left(a^3c^3 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*c
^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2
*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 +
2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((a*c -
 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*
b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*s
qrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c))
 - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*
c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^3), x)
```

3.360 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{abd^2}{(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8\sqrt{c}(ac+b)^{7/2}}-\frac{d(3b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8x^2(ac+b)^3}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4x^4(ac+b)^2}$$

```
[Out] -((a*b*d^2)/((b + a*c)^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((3*b -
4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*(b + a*c)^3*
x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(b + a*c)^2
*x^4) - (3*b*(b - 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)])/Sqrt[b + a*c]])/(8*Sqrt[c]*(b + a*c)^(7/2))
```

Rubi [A] time = 0.584397, antiderivative size = 246, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(b-4ac)}{8c(ac+b)^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8\sqrt{c}(ac+b)^{7/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{d(b-4ac)\left(c+dx^2\right)}{8cx^2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\left(c+dx^2\right)^2}{4cx^4(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*(b - 4*a*c)*d^2)/(8*c*(b + a*c)^3*Sqrt[a + b/(c + d*x^2)]) - ((b - 4*a
*c)*d*(c + d*x^2))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x
^2)^2/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c + d*x^2)]) - (3*b*(b - 4*a*c)*d^2*Sq
rt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt
[b + a*(c + d*x^2)])])/(8*Sqrt[c]*(b + a*c)^(7/2)*Sqrt[c + d*x^2]*Sqrt[a +
b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+ac+adx^2\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(b+ac+adx)^{3/2}}dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((b-4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}dx,x,x^2\right)}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Su}}{16c(b+ac)^2\sqrt{c+d}}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b(b-4ac}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b(b-4ac}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b(b-4ac)}{8}$$

Mathematica [A] time = 0.560356, size = 202, normalized size = 0.95

$$\frac{1}{8}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{8b^2d^2}{(ac+b)^3\left(a\left(c+dx^2\right)+b\right)} - \frac{2c^2}{x^4(ac+b)^2} - \frac{3bd^2(b-4ac)\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{ac+adx^2+b}}\right)}{\sqrt{c}(ac+b)^{7/2}\sqrt{a\left(c+dx^2\right)+b}} - \frac{d^2(13}{(ac}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(((13*b - 2*a*c)*d^2)/(b + a*c)^3)
 - (2*c^2)/((b + a*c)^2*x^4) - (7*b*c*d)/((b + a*c)^3*x^2) + (8*b^2*d^2)/((
b + a*c)^3*(b + a*(c + d*x^2))) - (3*b*(b - 4*a*c)*d^2*Sqrt[c + d*x^2]*ArcT
anh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*c + a*d*x^2])])/(Sq
rt[c]*(b + a*c)^(7/2)*Sqrt[b + a*(c + d*x^2)])))/8
```

Maple [B] time = 0.018, size = 1947, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*ln((2*a*c*d*x^2+b*d*
x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)+2*b*c)/x^2)*x^6*a^4*b*c^5*d^3-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^2*b*d^4-21*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*
a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c
)/x^2)*x^6*a^3*b^2*c^4*d^3+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*(a*c^2+b*c)^(3/2)*x^6*a^3*c^2*d^3-6*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)
*x^6*a^2*b^3*c^3*d^3-12*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)
*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*a^4*b*c^6*
d^2+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^6*a*b^4*c^2*d^3-33*ln((2*a*c*d
*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*a^3*b^2*c^5*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a*b^2*d^3+20*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^3*c^3*d^2-27*ln((2*a*
c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*a^2*b^3*c^4*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*c*d^2-3*ln((2*a*c*d*x^2+
b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)+2*b*c)/x^2)*x^4*a*b^4*c^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*d^2-8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a^2*c^2*d+4*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a^2*c^3+4*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b^2*c+12*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^3*c*d^4+16*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-10*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^
2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a
*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*
x^6*a^2*b*c*d^3+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*
a^2*b*c^2*d^2+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*x^4*b^5*c^2*d^2-6*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*b^3*d^2+6
*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*b^2*
d+8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*b*c
^2)/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^4/x^4/(a*c^2+b*c)^(3/2)/(a*
d*x^2+a*c+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.74492, size = 1997, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2
*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^
4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((
2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^
2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 - 11*a
^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2
*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*
a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c
^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*
c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4
*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arctan(1/2*((2*a*
c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*
((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5
- 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4
 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2
*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2
*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^5), x)
```

3.361 $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=482

$$\frac{x\left(a^2c^2+16abc+16b^2\right)\left(ac+adx^2+b\right)}{5a^4d^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\left(a^2c^2+16abc+16b^2\right)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{c^{3/2}(ac+8b}{5a^3d^{5/2}}$$

```
[Out] -((x^3*(c + d*x^2))/(a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((8*b +
a*c)*x*(b + a*c + a*d*x^2))/(5*a^3*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]) + (6*x^3*(b + a*c + a*d*x^2))/(5*a^2*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]) + ((16*b^2 + 16*a*b*c + a^2*c^2)*x*(b + a*c + a*d*x^2))/(5*a^4*d^2*(c
 + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(16*b^2 + 16*a*
b*c + a^2*c^2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(5*a^4*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^(3/2)*(8*b +
 a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c
)])/(5*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(
c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.906294, antiderivative size = 559, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 467, 581, 582, 531, 418, 492, 411}

$$\frac{x\left(a^2c^2+16abc+16b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{5a^4d^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\left(a^2c^2+16abc+16b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}E\left(\right.}{5a^4d^{5/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x^3*(c + d*x^2)*Sqrt[b + a*(c + d*x^2)])/(a*d*Sqrt[b + a*c + a*d*x^2]*Sq
rt[a + b/(c + d*x^2)])) - ((8*b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a
*(c + d*x^2)])/(5*a^3*d^2*Sqrt[a + b/(c + d*x^2)]) + (6*x^3*Sqrt[b + a*c +
a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(5*a^2*d*Sqrt[a + b/(c + d*x^2)]) + ((16*
b^2 + 16*a*b*c + a^2*c^2)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]
)/(5*a^4*d^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(16*b^2 + 16*a
*b*c + a^2*c^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a^4*d^(5/2)*(c + d*x^2)*Sqrt[(
c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) +
(c^(3/2)*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a^3*d^(5/2)*(c + d*x^2)*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)
])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}\left(3c+6dx^2\right)}{\sqrt{b+ac+adx^2}}\,dx}{ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2(-3\cdots}{\cdots}}{5a^2d^2\sqrt{c+dx\cdots}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx\cdots}}{5a^2d\sqrt{\cdots}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx\cdots}}{5a^2d\sqrt{\cdots}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx\cdots}}{5a^2d\sqrt{\cdots}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx\cdots}}{5a^2d\sqrt{\cdots}}$$

Mathematica [C] time = 0.813272, size = 296, normalized size = 0.61

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2\left(c^2-d^2x^4\right)+ab\left(9c+2dx^2\right)+8b^2\right)+ic\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}E\cdots\right.}{5a^3d^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2
)*(8*b^2 + a*b*(9*c + 2*d*x^2) + a^2*(c^2 - d^2*x^4)) + I*c*(16*b^2 + 16*a*
b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Elli
```

```
pticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*b*c*(8*b + 7*a*c
)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(5*a^3*d^2*Sqrt[(a*d)/(b + a*c)
]*(b + a*(c + d*x^2)))
```

Maple [B] time = 0.037, size = 1158, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/5*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^7*a^2*d^3+((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^5*a^2*c*d^2-2*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^5*a*b*d^2-((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a^2*c^2*d-6*((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d+((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3-5*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d-3*((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*b^2*d-((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)*(-a*d/(a*c+b))^(1/2)*x*a^2*c^3-7*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a
*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))
^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((
a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b)
)^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*b^2*d-4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1
/2)*(-a*d/(a*c+b))^(1/2)*x*a*b*c^2-8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*
d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^
(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d
*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(
1/2),((a*c+b)/a/c)^(1/2))*b^2*c-5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(-a*d/(a*c+b))^(1/2)*x*a*b*c^2-3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(
-a*d/(a*c+b))^(1/2)*x*b^2*c-5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*(-a*d/(a*c+b))^(1/2)*x*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/a^3/
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x
^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^8 + 2\,cdx^6 + c^2x^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^4 + a^2c^2 + 2\left(a^2c + ab\right)dx^2 + 2\,abc + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^8 + 2*c*d*x^6 + c^2*x^4)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*d^2*x^4 + a^2*c^2 + 2*(a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

3.362 $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=409

$$-\frac{c^{3/2}(ac+4b)\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{\sqrt{c}(ac+8b)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{4x\left(ac+\ldots\right.}{3a^2d\sqrt{\ldots}}$$

```
[Out] -((x*(c + d*x^2))/(a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + (4*x*(b +
a*c + a*d*x^2))/(3*a^2*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((8*b + a
*c)*x*(b + a*c + a*d*x^2))/(3*a^3*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]) + (Sqrt[c]*(8*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^3*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c +
 a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]) - (c^(3/2)*(4*b + a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], b/(b + a*c)])/(3*a^2*(b + a*c)*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)])
```

Rubi [A] time = 0.660615, antiderivative size = 475, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 467, 528, 531, 418, 492, 411}

$$-\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{c}(ac+8b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}E\ldots}{3a^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x*(c + d*x^2)*Sqrt[b + a*(c + d*x^2)])/(a*d*Sqrt[b + a*c + a*d*x^2]*Sqrt
[a + b/(c + d*x^2)])) + (4*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)
])/(3*a^2*d*Sqrt[a + b/(c + d*x^2)]) - ((8*b + a*c)*x*Sqrt[b + a*c + a*d*x^
2]*Sqrt[b + a*(c + d*x^2)])/(3*a^3*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) +
 (Sqrt[c]*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^3*d^(3/2)*(c + d*x^2)*
Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2
)]) - (c^(3/2)*(4*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^2*(b + a*c)*d^(3/
2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a
 + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}\left(c+4dx^2\right)}{\sqrt{b+ac+adx^2}}dx}{ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{-c(}{}}{3a^2d^2\sqrt{c+dx^2}}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((8b+ac)\sqrt{b+a\left(c+\right.\right.}{3a^2\sqrt{c+}}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+a}}{3a^3d\left(c+dx^2\right.}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+a}}{3a^3d\left(c+dx^2\right.}$$

Mathematica [C] time = 0.589318, size = 255, normalized size = 0.62

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac+adx^2+4b\right)-4ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+ic(ac+8b)\right.}{3a^2d\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(4*b + a*c + a*d*x^2) + I*c*(8*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(
a*c)] - (4*I)*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*a^2*d*Sqrt[(
a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [A] time = 0.015, size = 667, normalized size = 1.6

$$\frac{1}{3\,a^2d\left(adx^2+ac+b\right)}\left(\sqrt{-\frac{ad}{ac+b}}\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}x^5ad^2+2\sqrt{-\frac{ad}{ac+b}}\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}x^3acd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a*d^2+2*(-a
*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a*c*d+3*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*b*d+(-a*d/(a*
c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*b*d-((a*d*x^2+a*c+b)/(a*c
+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/
c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a*c^2+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*b*c+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*(-a*d/(a*c+b))^(1/2)*x*b*c+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*x*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d/(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^4+c^2x^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^4+a^2c^2+2\left(a^2c+ab\right)dx^2+2\,abc+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^4 + c^2*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*d^2*x^4 + a^2*c^2 + 2*(a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

3.363 $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx$

Optimal. Leaf size=356

$$\frac{x(ac+2b)\left(ac+adx^2+b\right)}{a^2(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}(ac+2b)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\right.\right.}{a\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[Out] -((b*x)/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((2*b + a*c)
*x*(b + a*c + a*d*x^2))/(a^2*(b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) - (Sqrt[c]*(2*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a^2*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(
c + d*x^2))]) + (c^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(a*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.309355, antiderivative size = 411, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6722, 1974, 413, 531, 418, 492, 411}

$$\frac{x(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{a^2(ac+b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}(ac+2b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c^{3/2}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((b*x*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a
 + b/(c + d*x^2)])) + ((2*b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c
+ d*x^2)])/(a^2*(b + a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(
2*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a^2*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
 + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b+ac)d+(2b+ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((2b+ac)d\right.}{a}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a^2(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{c^{3/2}\sqrt{b+ac}}{a(b+}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a^2(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}(2b+ac}{a^2}$$

Mathematica [C] time = 0.507183, size = 241, normalized size = 0.68

$$-\frac{\sqrt{\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(bx\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}} - ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right) + ic(ac+2b)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac}{}}\right.}{a^2d\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[(a*d
)/(b + a*c)]*x*(c + d*x^2) + I*c*(2*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 +
 b/(a*c)] - I*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^2*d*(b + a*(
c + d*x^2))))
```

Maple [A] time = 0.013, size = 466, normalized size = 1.3

$$-\frac{1}{a\left(ac+b\right)\left(adx^2+ac+b\right)}\left(\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}x^3bd - \sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -((a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x^3*
b*d-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(
a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^2+
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c-2*((a*
d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^
(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c+(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*x*b*c)/a*((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/
(-a*d/(a*c+b))^(1/2)/(a*c+b)/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^2x^4 + 2\,cdx^2 + c^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^4 + a^2c^2 + 2\left(a^2c + ab\right)dx^2 + 2\,abc + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d^2*x^4 + a^2*c^2 + 2*(a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + \frac{b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral((a + b/(c + d*x**2))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

3.364 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=410

$$\frac{c^{3/2}\sqrt{d}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{(b-ac)\left(ac+adx^2+b\right)}{ax(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{dx(b-ac)\left(ac+adx^2+b\right)}{a(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\ldots$$

```
[Out] -(b/(a*(b + a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((b - a*c)*(b
+ a*c + a*d*x^2))/(a*(b + a*c)^2*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) -
 ((b - a*c)*d*x*(b + a*c + a*d*x^2))/(a*(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a
*c + a*d*x^2)/(c + d*x^2)]) + (Sqrt[c]*(b - a*c)*Sqrt[d]*(b + a*c + a*d*x^2
)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*(b + a*c)^2*(c +
d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/(
(b + a*c)*(c + d*x^2))]) + (c^(3/2)*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)^2*(c + d*x^2)*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))])
```

Rubi [A] time = 0.680291, antiderivative size = 476, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{d}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{(b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{ax(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{dx(b-ac)\ldots}{a(ac\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((b*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a
 + b/(c + d*x^2)])) + ((b - a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*
x^2)])/(a*(b + a*c)^2*x*Sqrt[a + b/(c + d*x^2)]) - ((b - a*c)*d*x*Sqrt[b +
a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)^2*(c + d*x^2)*Sqrt[a +
 b/(c + d*x^2)]) + (Sqrt[c]*(b - a*c)*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[
b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*
(b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)]*Sqrt[a + b/(c + d*x^2)]) + (c^(3/2)*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt
[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((
b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]*Sqrt[a + b/(c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b-ac)d-acd^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+\ldots}}{a\ldots}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(cd\sqrt{b\ldots}\right.}{\ldots}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac\ldots}{a\ldots}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac\ldots}{a\ldots}$$

Mathematica [C] time = 0.652728, size = 268, normalized size = 0.65

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac\left(c+dx^2\right)+b\left(c-dx^2\right)\right)+2ibcdx\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)\ldots\right.}{x(ac+b)^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)
*(b*(c - d*x^2) + a*c*(c + d*x^2)) + I*c*(-b + a*c)*d*x*Sqrt[(b + a*c + a*d
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*
c)]*x], 1 + b/(a*c)] + (2*I)*b*c*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sq
rt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)
]))/((b + a*c)^2*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c + d*x^2))))
```

Maple [A] time = 0.019, size = 686, normalized size = 1.7

$$\frac{1}{x\left(ac+b\right)^2\left(adx^2+ac+b\right)}\left(-\sqrt{-\frac{ad}{ac+b}}\sqrt{\left(dx^2+c\right)\left(adx^2+ac+b\right)}x^4acd^2+\sqrt{-\frac{ad}{ac+b}}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+c^2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] (-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a*c*d^2+(-a*d/
(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*b*d^2+a*
d*c^2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d
/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x-2*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a*c^2*d+2*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*b*c*d-((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*b*c*d+(-a*d/(a*c
+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b*c*d-(-a*d/
(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*b*c*d-(-a*d/(a*c+b))^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^3-(-a*d/(a*c+b))^(1/2)*((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*b*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/x/(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c+b)^2/(a*
d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^6+2\left(a^2c+ab\right)dx^4+\left(a^2c^2+2\,abc+b^2\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d^2*x^6 + 2*(a^2*c + a*b)*d*x^4 + (a^2*c^2 + 2*a*b*c + b^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

3.365 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=490

$$\frac{d^2x(7b-ac)\left(ac+adx^2+b\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{\sqrt{c}d^{3/2}(3b-ac)\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}d^{3/2}(7b-ac)\left(ac+adx^2+\ldots\right.}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+\ldots}{c\ldots}}}$$

```
[Out] -(b/(a*(b + a*c)*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((3*b - a*c)
*(b + a*c + a*d*x^2))/(3*a*(b + a*c)^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]) - ((7*b - a*c)*d*(b + a*c + a*d*x^2))/(3*(b + a*c)^3*x*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)]) + ((7*b - a*c)*d^2*x*(b + a*c + a*d*x^2))/(3*(b +
 a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(7*b
- a*c)*d^(3/2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (Sqrt[c]*(3*b -
 a*c)*d^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rubi [A] time = 0.866721, antiderivative size = 567, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{d^2x(7b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{3(ac+b)^3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{c}d^{3/2}(3b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((b*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)*x^3*Sqrt[b + a*c + a*d*x^2]*Sqrt
[a + b/(c + d*x^2)])) + ((3*b - a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c
+ d*x^2)])/(3*a*(b + a*c)^2*x^3*Sqrt[a + b/(c + d*x^2)]) - ((7*b - a*c)*d*S
qrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*(b + a*c)^3*x*Sqrt[a + b
/(c + d*x^2)]) + ((7*b - a*c)*d^2*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c +
 d*x^2)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(7
*b - a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
+ (Sqrt[c]*(3*b - a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^
2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*(b + a*c)^3*(c
+ d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(3b-ac)d+(2b-ac)d^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a}}{3a}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(7b-ac}{}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(7b-ac}{}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(7b-ac}{}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(7b-ac}{}$$

Mathematica [C] time = 0.89798, size = 319, normalized size = 0.65

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c^2-d^2x^4\right)+ab\left(2c^2+4cdx^2+7d^2x^4\right)+b^2\left(c+4dx^2\right)\right)+ibd^2x^3(3b-5ac)\sqrt{\frac{dx^2}{c}+1}\right.}{3x^3(ac+b)^3\sqrt{\frac{ad}{ac+b}}\left(a\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)*
(b^2*(c + 4*d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + 4*c*d*x^2 + 7*d^2
*x^4)) - I*a*c*(-7*b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqr
t[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]
 + I*b*(3*b - 5*a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*(
b + a*c)^3*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

Maple [B] time = 0.017, size = 1082, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^2*c*d^3-4
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a*b*d^3-((a*d*x
^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/
2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a^2*c^2*d^2-3
*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*a
*b*d^3+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a^2*c^2*d
^2-5*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a
*b*c*d^2+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
*x^3*a*b*c*d^2-8*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4
*a*b*c*d^2+3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2
))*x^3*b^2*d^2-3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*x^4*a*b*c*d^2-4*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*x^4*b^2*d^2-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x
^2*a^2*c^3*d-6*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a
*b*c^2*d-5*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*b^2*c
*d-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^4-2*(-a*d/(
a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^3-(-a*d/(a*c+b))^(1/2
)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/x^3/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2
)/(a*c+b)^3/(a*d*x^2+a*c+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^8+2\left(a^2c+ab\right)dx^6+\left(a^2c^2+2\,abc+b^2\right)x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d^2*x^8 + 2*(a^2*c + a*b)*d*x^6 + (a^2*c^2 + 2*a*b*c + b^2)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

3.366 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx$

Optimal. Leaf size=75

$$\frac{\sqrt{x^5+1}\sqrt{ax^{23}}}{10x^4}-\frac{3\sqrt{x^5+1}\sqrt{ax^{23}}}{20x^9}+\frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}$$

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10
*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rubi [A] time = 0.0174213, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\sqrt{ax^{23}}}{10x^4}-\frac{3\sqrt{x^5+1}\sqrt{ax^{23}}}{20x^9}+\frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10
*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx}{x^{23/2}}\\
&= \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} - \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{8x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{20x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0199386, size = 49, normalized size = 0.65

$$\frac{\sqrt{ax^{23}}\left(\sqrt{x^5+1}\left(2x^5-3\right)x^{5/2}+3\sinh^{-1}\left(x^{5/2}\right)\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*ArcSinh[x^(5/2)]))/(2
0*x^(23/2))
```

Maple [A] time = 0.055, size = 64, normalized size = 0.9

$$\frac{2x^5-3}{20x^9}\sqrt{x^5+1}\sqrt{ax^{23}}+\frac{3}{20x^{12}}\text{Arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{ax^{23}}\sqrt{ax\left(x^5+1\right)}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/20/x^9*(2*x^5-3)*(x^5+1)^(1/2)*(a*x^23)^(1/2)+3/20/a^(1/2)*arcsinh(x^(5/2
))*(a*x^23)^(1/2)/x^12*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

Fricas [A] time = 1.89538, size = 423, normalized size = 5.64

$$\left[\frac{3\sqrt{a}x^{9}\log\left(-\frac{8\,ax^{19}+8\,ax^{14}+ax^{9}+4\,\sqrt{ax^{23}}\left(2\,x^{5}+1\right)\sqrt{x^{5}+1}\sqrt{a}}{x^{9}}\right)+4\,\sqrt{ax^{23}}\left(2\,x^{5}-3\right)\sqrt{x^{5}+1}}{80\,x^{9}},-\frac{3\,\sqrt{-a}x^{9}\arctan\left(\frac{\sqrt{ax^{23}}\left(2\,x^{5}+1\right)\sqrt{x^{5}+1}}{2\left(ax^{19}+ax^{14}\right)}\right.}{40}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2*
x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5
+ 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqrt(x
^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 +
 1))/x^9]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)\left(x^{4}-x^{3}+x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.367 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x
^(13/2))
```

Rubi [A] time = 0.012267, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x
^(13/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{2x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}
\end{aligned}$$

Mathematica [A] time = 0.0104912, size = 42, normalized size = 0.84

$$\frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{x^5+1}-\sinh^{-1}\left(x^{5/2}\right)\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - ArcSinh[x^(5/2)]))/(5*x^(13/2))
```

Maple [A] time = 0.039, size = 57, normalized size = 1.1

$$\frac{1}{5\,x^4}\sqrt{ax^{13}}\sqrt{x^5+1}-\frac{1}{5\,x^7}\text{Arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{ax^{13}}\sqrt{ax\left(x^5+1\right)}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/5*(a*x^13)^(1/2)*(x^5+1)^(1/2)/x^4-1/5/a^(1/2)*arcsinh(x^(5/2))*(a*x^13)^
(1/2)/x^7*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

Fricas [B] time = 1.86198, size = 381, normalized size = 7.62

$$\left[\frac{\sqrt{a}x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}(2\,x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\sqrt{x^5+1}}{20\,x^4}, \frac{\sqrt{-a}x^4\arctan\left(\frac{\sqrt{ax^{13}}(2\,x^5+1)\sqrt{x^5+1}\sqrt{-a}}{2\,(ax^{14}+ax^9)}\right)+2\,\sqrt{ax^{13}}\sqrt{x}}{10\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^5
 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/10
*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(
a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [A] time = 1.21185, size = 92, normalized size = 1.84

$$\frac{a^{\frac{11}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)}{5\,|a|^5}+\frac{\sqrt{a^6x^5+a^6}\sqrt{ax}x^2}{5\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*a^(11/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs(a)^5 + 1
/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

3.368 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$

Optimal. Leaf size=24

$$\frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rubi [A] time = 0.0078643, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {15, 329, 275, 215}

$$\frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{3/2}}$$
$$= \frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Mathematica [A] time = 0.0056179, size = 24, normalized size = 1.

$$\frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Maple [A] time = 0.03, size = 17, normalized size = 0.7

$$\frac{2}{5}\text{Arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{ax^3}\,x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

Fricas [B] time = 1.69821, size = 252, normalized size = 10.5

$$\left[\frac{1}{10}\sqrt{a}\log\left(-8\,ax^{10} - 8\,ax^5 - 4\left(2\,x^6 + x\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{a} - a\right), -\frac{1}{5}\sqrt{-a}\arctan\left(\frac{\left(2\,x^5+1\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{-a}}{2\left(ax^9 + ax^4\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a*
x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(
a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.369 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5

Rubi [A] time = 0.0043128, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 264}

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]

[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx &= \left(\sqrt{\frac{a}{x^7}}x^{7/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}\end{aligned}$$

Mathematica [A] time = 0.0038984, size = 23, normalized size = 1.

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]

```
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Maple [B] time = 0.004, size = 37, normalized size = 1.6

$$-\frac{2x(1+x)\left(x^4-x^3+x^2-x+1\right)}{5}\sqrt{\frac{a}{x^7}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] -2/5*x*(1+x)*(x^4-x^3+x^2-x+1)*(a/x^7)^(1/2)/(x^5+1)^(1/2)
```

Maxima [B] time = 1.55219, size = 55, normalized size = 2.39

$$-\frac{2\left(\sqrt{a}x^6+\sqrt{a}x\right)}{5\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*x
^(7/2))
```

Fricas [A] time = 1.22383, size = 46, normalized size = 2.

$$-\frac{2}{5}\sqrt{x^5+1}x\sqrt{\frac{a}{x^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [A] time = 1.24845, size = 38, normalized size = 1.65

$$-\frac{2\,a^{3}\left(\frac{\sqrt{a+\frac{a}{x^{5}}}}{a^{2}}-\frac{1}{a^{\frac{3}{2}}}\right)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*a^3*(sqrt(a + a/x^5)/a^2 - 1/a^(3/2))/abs(a)
```

3.370 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} dx$

Optimal. Leaf size=49

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/1
5
```

Rubi [A] time = 0.0093764, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 271, 264}

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/1
5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} dx &= \left(\sqrt{\frac{a}{x^{17}}}x^{17/2}\right)\int \frac{1}{x^{17/2}\sqrt{1+x^5}} dx \\
&= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}-\frac{1}{3}\left(2\sqrt{\frac{a}{x^{17}}}x^{17/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}} dx \\
&= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}x^6\sqrt{1+x^5}
\end{aligned}$$

Mathematica [A] time = 0.0062115, size = 30, normalized size = 0.61

$$-\frac{2}{15}x\left(1-2x^5\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*(1 - 2*x^5)*Sqrt[1 + x^5])/15
```

Maple [A] time = 0.004, size = 44, normalized size = 0.9

$$\frac{2\,x\,(1+x)\left(x^4-x^3+x^2-x+1\right)\left(2\,x^5-1\right)}{15}\sqrt{\frac{a}{x^{17}}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/15*x*(1+x)*(x^4-x^3+x^2-x+1)*(2*x^5-1)*(a/x^17)^(1/2)/(x^5+1)^(1/2)
```

Maxima [A] time = 1.61558, size = 68, normalized size = 1.39

$$\frac{2\left(2\,\sqrt{a}x^{11}+\sqrt{a}x^6-\sqrt{a}x\right)}{15\,\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x +
 1)*sqrt(x + 1)*x^(17/2))
```

Fricas [A] time = 1.30168, size = 61, normalized size = 1.24

$$\frac{2}{15}\left(2\,x^6-x\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [A] time = 1.16706, size = 63, normalized size = 1.29

$$-\frac{2\,a^{3}\left(\frac{2}{a^{\frac{3}{2}}}+\frac{\left(a+\frac{a}{x^{5}}\right)^{\frac{3}{2}}a-3\,\sqrt{a+\frac{a}{x^{5}}}\,a^{2}}{a^{4}}\right)\operatorname{sgn}\left(x\right)}{15\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*a^3*(2/a^(3/2) + ((a + a/x^5)^(3/2)*a - 3*sqrt(a + a/x^5)*a^2)/a^4)*s
gn(x)/abs(a)
```

3.371 $\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0077202, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 298, 203, 206}

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}$$
$$= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} - \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}$$
$$= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] time = 0.0133233, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] -(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/(4*x^3)
```

Maple [A] time = 0.01, size = 28, normalized size = 0.8

$$-\frac{\ln(x-1)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/x/(-x^4+1),x)
```

```
[Out] -1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x))/x^3
```

Maxima [A] time = 1.52823, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log(x+1)-\frac{1}{4}\sqrt{a}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Fricas [A] time = 1.30201, size = 80, normalized size = 2.16

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ax^6}}{x^5 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A] time = 1.13243, size = 39, normalized size = 1.05

$$-\frac{1}{4}\left(2\,\arctan\left(x\right)\mathrm{sgn}\left(x\right) - \log\left(|x + 1|\right)\mathrm{sgn}\left(x\right) + \log\left(|x - 1|\right)\mathrm{sgn}\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

3.372 $\int \frac{\sqrt{ax^6}}{x-x^5} dx$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.012713, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 1584, 298, 203, 206}

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}$$
$$= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}$$
$$= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} - \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}$$
$$= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] time = 0.0045869, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/(4*x^3)
```

Maple [A] time = 0.006, size = 28, normalized size = 0.8

$$-\frac{\ln(x-1)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(-x^5+x),x)
```

```
[Out] -1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x))/x^3
```

Maxima [A] time = 1.68832, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log(x+1)-\frac{1}{4}\sqrt{a}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Fricas [A] time = 1.28832, size = 80, normalized size = 2.16

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ax^6}}{x^5 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A] time = 1.11676, size = 39, normalized size = 1.05

$$-\frac{1}{4}\left(2\,\arctan\left(x\right)\mathrm{sgn}\left(x\right) - \log\left(|x + 1|\right)\mathrm{sgn}\left(x\right) + \log\left(|x - 1|\right)\mathrm{sgn}\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

3.373 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}+\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/
(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0146736, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {15, 302, 212, 206, 203}

$$-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}+\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/
(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^6\right)^{3/2}}{x\left(1-x^4\right)}\,dx &= \frac{\left(a\sqrt{ax^6}\right)\int \frac{x^8}{1-x^4}\,dx}{x^3}\\
&= \frac{\left(a\sqrt{ax^6}\right)\int \left(-1-x^4+\frac{1}{1-x^4}\right)dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^4}\,dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.0154545, size = 44, normalized size = 0.62

$$-\frac{a\sqrt{ax^6}\left(4x^5+20x+5\log(1-x)-5\log(x+1)-10\tan^{-1}(x)\right)}{20x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -(a*Sqrt[a*x^6]*(20*x + 4*x^5 - 10*ArcTan[x] + 5*Log[1 - x] - 5*Log[1 + x]))/(20*x^3)
```

Maple [A] time = 0.009, size = 38, normalized size = 0.5

$$-\frac{4\,x^5+5\,\ln(x-1)-5\,\ln(1+x)-10\,\arctan(x)+20\,x}{20\,x^9}\left(ax^6\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(3/2)/x/(-x^4+1),x)
```

```
[Out] -1/20*(a*x^6)^(3/2)*(4*x^5+5*ln(x-1)-5*ln(1+x)-10*arctan(x)+20*x)/x^9
```

Maxima [A] time = 1.67262, size = 54, normalized size = 0.76

$$-\frac{1}{5}\,a^{\frac{3}{2}}x^5-a^{\frac{3}{2}}x+\frac{1}{2}\,a^{\frac{3}{2}}\arctan(x)+\frac{1}{4}\,a^{\frac{3}{2}}\log(x+1)-\frac{1}{4}\,a^{\frac{3}{2}}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
1) - 1/4*a^(3/2)*log(x - 1)
```

Fricas [A] time = 1.30996, size = 116, normalized size = 1.63

$$-\frac{\sqrt{ax^6}\left(4\,ax^5+20\,ax-10\,a\arctan\left(x\right)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x -
 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(ax^6\right)^{\frac{3}{2}}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

```
[Out] -Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

Giac [A] time = 1.12712, size = 57, normalized size = 0.8

$$-\frac{1}{20}\left(4\,x^5\mathrm{sgn}\left(x\right)+20\,x\mathrm{sgn}\left(x\right)-10\,\arctan\left(x\right)\mathrm{sgn}\left(x\right)-5\,\log\left(|x+1|\right)\mathrm{sgn}\left(x\right)+5\,\log\left(|x-1|\right)\mathrm{sgn}\left(x\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1))
*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

3.374 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right) dx$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0133674, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {212, 206, 203, 15, 298}

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rubi steps

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right) dx = \int \frac{1}{1-x^4}\, dx - \int \frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\, dx$$
$$= \frac{1}{2}\int \frac{1}{1-x^2}\, dx + \frac{1}{2}\int \frac{1}{1+x^2}\, dx - \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\, dx}{x^3}$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\, dx}{2x^3} + \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\, dx}{2x^3}$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] time = 0.0636909, size = 29, normalized size = 0.59

$$\frac{1}{2}\left(\frac{\sqrt{ax^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

Maple [A] time = 0.003, size = 37, normalized size = 0.8

$$\frac{\mathrm{Artanh}\,(x)}{2}+\frac{\arctan\,(x)}{2}+\frac{\ln\,(x-1)-\ln\,(1+x)+2\,\arctan\,(x)}{4\,x^3}\sqrt{ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x)
)/x^3
```

Maxima [A] time = 1.66504, size = 57, normalized size = 1.16

$$\frac{1}{2}\sqrt{a}\arctan\,(x)-\frac{1}{4}\sqrt{a}\log\,(x+1)+\frac{1}{4}\sqrt{a}\log\,(x-1)+\frac{1}{2}\arctan\,(x)+\frac{1}{4}\log\,(x+1)-\frac{1}{4}\log\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] time = 1.50862, size = 618, normalized size = 12.61

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\sqrt{ax^6}(\log(x+1)-}{4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{x^5-x}\,dx-\int-\frac{\sqrt{ax^6}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A] time = 1.13407, size = 65, normalized size = 1.33

$$\frac{1}{4}\left(2\arctan(x)\operatorname{sgn}(x)-\log(|x+1|)\operatorname{sgn}(x)+\log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.375 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0177482, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {212, 206, 203, 15, 1584, 298}

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)dx &= \int\frac{1}{1-x^4}\,dx-\int\frac{\sqrt{ax^6}}{x-x^5}\,dx\\ &= \frac{1}{2}\int\frac{1}{1-x^2}\,dx+\frac{1}{2}\int\frac{1}{1+x^2}\,dx-\frac{\sqrt{ax^6}\int\frac{x^3}{x-x^5}\,dx}{x^3}\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int\frac{x^2}{1-x^4}\,dx}{x^3}\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int\frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{ax^6}\int\frac{1}{1+x^2}\,dx}{2x^3}\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

Mathematica [A] time = 0.0152916, size = 29, normalized size = 0.59

$$\frac{1}{2}\left(\frac{\sqrt{ax^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

Maple [A] time = 0.001, size = 37, normalized size = 0.8

$$\frac{\mathrm{Artanh}\,(x)}{2}+\frac{\arctan(x)}{2}+\frac{\ln(x-1)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x)
)/x^3
```

Maxima [A] time = 1.66714, size = 57, normalized size = 1.16

$$\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log(x+1)+\frac{1}{4}\sqrt{a}\log(x-1)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] time = 1.41112, size = 618, normalized size = 12.61

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\sqrt{ax^6}(\log(x+1)-\log$$

$$4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^5-x}\,dx - \int -\frac{\sqrt{ax^6}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A] time = 1.13266, size = 65, normalized size = 1.33

$$\frac{1}{4}\left(2\arctan(x)\operatorname{sgn}(x)-\log(|x+1|)\operatorname{sgn}(x)+\log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.376 $\int \frac{\sqrt{ax^3}}{x-x^3}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x
^(3/2)
```

Rubi [A] time = 0.0148215, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 1584, 329, 298, 203, 206}

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x
^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3}}{x-x^3}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}\,dx}{x^{3/2}}\\
&= \frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= -\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0091698, size = 30, normalized size = 0.68

$$\frac{\sqrt{ax^3}\left(\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] (Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

Maple [A] time = 0.011, size = 43, normalized size = 1.

$$\frac{1}{x}\sqrt{ax^3}\sqrt{a}\left(\text{Artanh}\left(\sqrt{ax}\frac{1}{\sqrt{a}}\right)-\arctan\left(\sqrt{ax}\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(-x^3+x),x)
```

```
[Out] (a*x^3)^(1/2)*a^(1/2)*(arctanh((a*x)^(1/2)/a^(1/2))-arctan((a*x)^(1/2)/a^(1
/2)))/x/(a*x)^(1/2)
```

Maxima [A] time = 1.71386, size = 43, normalized size = 0.98

$$-\sqrt{a}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(s
qrt(x) - 1)
```

Fricas [A] time = 1.3382, size = 308, normalized size = 7.

$$\left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}x}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right),-\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-ax-2\sqrt{ax^3}\sqrt{-a}}{x^2+x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
```

```
[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x +
2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/(a
*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{ax^3}}{x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)
```

Giac [A] time = 1.12935, size = 51, normalized size = 1.16

$$-\left(\frac{a\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
```

```
[Out] -(a*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + sqrt(a)*arctan(sqrt(a*x)/sqrt(a))
)*sgn(x)
```

3.377 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)
```

Rubi [A] time = 0.0064502, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 321, 215}

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^2}}\,dx}{x^2}\\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x}-\frac{\sqrt{ax^4}\int \frac{1}{\sqrt{1+x^2}}\,dx}{2x^2}\\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x}-\frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0080166, size = 32, normalized size = 0.73

$$\frac{\sqrt{ax^4}\left(x\sqrt{x^2+1}-\sinh^{-1}(x)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] - ArcSinh[x]))/(2*x^2)
```

Maple [A] time = 0.005, size = 26, normalized size = 0.6

$$-\frac{1}{2x^2}\sqrt{ax^4}\left(-x\sqrt{x^2+1}+\operatorname{Arcsinh}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/2*(a*x^4)^(1/2)*(-x*(x^2+1)^(1/2)+arcsinh(x))/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

Fricas [A] time = 1.29491, size = 104, normalized size = 2.36

$$\frac{\sqrt{ax^4}\sqrt{x^2+1}x+\sqrt{ax^4}\log\left(-x+\sqrt{x^2+1}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

Giac [A] time = 1.12858, size = 36, normalized size = 0.82

$$\frac{1}{2}\left(\sqrt{x^2+1}x+\log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

3.378 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^2+1}\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{ax^3}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(1 + x^2)/(
1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0258057, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {15, 321, 329, 220}

$$\frac{2\sqrt{x^2+1}\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{ax^3}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(1 + x^2)/(
1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^2}}\,dx}{x^{3/2}}$$
$$= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx}{3x^{3/2}}$$
$$= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)}{3x^{3/2}}$$
$$= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}$$

Mathematica [C] time = 0.0080856, size = 43, normalized size = 0.52

$$\frac{2\sqrt{ax^3}\left(\sqrt{x^2+1} - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*(Sqrt[1 + x^2] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]))/(3
*x)
```

Maple [C] time = 0.021, size = 76, normalized size = 0.9

$$-\frac{1}{3\,x^2}\sqrt{ax^3}\left(i\sqrt{2}\sqrt{-i\,(x+i)}\sqrt{-i\,(-x+i)}\sqrt{ix}\text{EllipticF}\left(\sqrt{-i\,(x+i)},\frac{\sqrt{2}}{2}\right) - 2\,x^3 - 2\,x\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/3*(a*x^3)^(1/2)/x^2/(x^2+1)^(1/2)*(I*2^(1/2)*(-I*(x+I))^(1/2)*(-I*(-x+I)
)^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))-2*x^3-2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

3.379 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=22

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rubi [A] time = 0.0031639, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 261}

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx &= \frac{\sqrt{ax^2} \int \frac{x}{\sqrt{1+x^2}} \, dx}{x} \\ &= \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x} \end{aligned}$$

Mathematica [A] time = 0.0046102, size = 22, normalized size = 1.

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Maple [A] time = 0.001, size = 19, normalized size = 0.9

$$\frac{1}{x}\sqrt{ax^2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Maxima [A] time = 1.56989, size = 26, normalized size = 1.18

$$\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

Fricas [A] time = 1.27875, size = 39, normalized size = 1.77

$$\frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

Sympy [A] time = 0.401803, size = 20, normalized size = 0.91

$$\frac{\sqrt{a}\sqrt{x^2+1}\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x**2 + 1)*sqrt(x**2)/x
```

Giac [A] time = 1.14934, size = 26, normalized size = 1.18

$$\left(\sqrt{x^2+1}\operatorname{sgn}(x)-\operatorname{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

3.380 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{x^2+1}\sqrt{ax}}{x+1}+\frac{\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1
+ x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2] + (Sqrt[
a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[a*x]/Sqrt[a]],
 1/2])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.0818356, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {329, 305, 220, 1196}

$$\frac{2\sqrt{x^2+1}\sqrt{ax}}{x+1}+\frac{\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1
+ x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2] + (Sqrt[
a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[a*x]/Sqrt[a]],
 1/2])/Sqrt[1 + x^2]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)}{a}$$
$$= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right) - 2\,\text{Subst}\left(\int \frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)$$
$$= \frac{2\sqrt{ax}\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

Mathematica [C] time = 0.005063, size = 27, normalized size = 0.21

$$\frac{2}{3}x\sqrt{ax}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*x*Sqrt[a*x]*Hypergeometric2F1[1/2, 3/4, 7/4, -x^2])/3
```

Maple [C] time = 0.018, size = 81, normalized size = 0.6

$$\frac{\sqrt{2}}{x}\sqrt{ax}\sqrt{-i(x+i)}\sqrt{-i(-x+i)}\sqrt{ix}\left(2\,\text{EllipticE}\left(\sqrt{-i(x+i)},1/2\sqrt{2}\right)-\text{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a*x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^
(1/2)*(2*EllipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-EllipticF((-I*(x+I))^(1/2)
,1/2*2^(1/2)))/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Sympy [C] time = 1.03496, size = 36, normalized size = 0.27

$$\frac{\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle| x^2e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(I*pi))
/(2*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

3.381 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=54

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqr
t[x]], 1/2])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.0206193, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 329, 220}

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqr
t[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx$$
$$= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{\frac{a}{x}}\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

Mathematica [C] time = 0.0046895, size = 27, normalized size = 0.5

$$2x\sqrt{\frac{a}{x}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]
```

Maple [C] time = 0.024, size = 62, normalized size = 1.2

$$i\sqrt{2}\sqrt{\frac{a}{x}}\sqrt{-i(x+i)}\sqrt{-i(-x+i)}\sqrt{ix}\,\text{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] I*(a/x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x
)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

3.382 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=22

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rubi [A] time = 0.0085699, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {15, 266, 63, 207}

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^2}}x\right)\int \frac{1}{x\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}\left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^2\right)\\ &= \left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^2}\right)\\ &= -\sqrt{\frac{a}{x^2}}x\tanh^{-1}\left(\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.003938, size = 22, normalized size = 1.

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Maple [A] time = 0.005, size = 19, normalized size = 0.9

$$-\sqrt{\frac{a}{x^2}}x\text{Artanh}\left(\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

Fricas [A] time = 1.00657, size = 182, normalized size = 8.27

$$\left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right), 2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^2
*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

Giac [A] time = 1.12686, size = 41, normalized size = 1.86

$$-\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

3.383 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=159

$$\frac{2\sqrt{x^2+1}x^2\sqrt{\frac{a}{x^3}}}{x+1}-2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}}+\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2])/(1 + x)
- (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcT
an[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x
^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.0518476, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 325, 329, 305, 220, 1196}

$$\frac{2\sqrt{x^2+1}x^2\sqrt{\frac{a}{x^3}}}{x+1}-2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}}+\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```


```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2])/(1 + x)
- (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcT
an[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x
^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```


Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^2}}\,dx \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\left(\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{1+x^2}}\,dx \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right) \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right)-\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \right. \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\frac{2\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^2}}{1+x}-\frac{2\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}+\frac{\sqrt{\frac{a}{x^3}}x^{3/2}(1+x}{}
\end{aligned}$$

Mathematica [C] time = 0.0060097, size = 27, normalized size = 0.17

$$-2x\sqrt{\frac{a}{x^3}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^2]
```

Maple [C] time = 0.026, size = 116, normalized size = 0.7

$$x\sqrt{\frac{a}{x^3}}\left(2\sqrt{-i(x+i)}\sqrt{2}\sqrt{-i(-x+i)}\sqrt{ix}\text{EllipticE}\left(\sqrt{-i(x+i)},1/2\sqrt{2}\right)-\sqrt{-i(x+i)}\sqrt{2}\sqrt{-i(-x+i)}\sqrt{ix}\text{EllipticF}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a/x^3)^(1/2)*x*(2*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^(1/2)*E
llipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))
^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))-2*x^2-2)/(x^2+1)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

3.384 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0042075, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 264}

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1}{x^2\sqrt{1+x^2}}\,dx\\ &= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}\end{aligned}$$

Mathematica [A] time = 0.0038213, size = 21, normalized size = 1.

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$-x\sqrt{\frac{a}{x^4}}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

Maxima [A] time = 1.5862, size = 31, normalized size = 1.48

$$-\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

Fricas [A] time = 0.982961, size = 65, normalized size = 3.1

$$-x^2\sqrt{\frac{a}{x^4}}-\sqrt{x^2+1}x\sqrt{\frac{a}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

Giac [A] time = 1.15102, size = 30, normalized size = 1.43

$$\frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

3.385 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

```
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rubi [A] time = 0.0040289, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 261}

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^3}}\,dx}{x^2}\\ &= \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}\end{aligned}$$

Mathematica [A] time = 0.004558, size = 25, normalized size = 1.

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Maple [A] time = 0.003, size = 31, normalized size = 1.2

$$\frac{(2+2x)\left(x^2-x+1\right)\sqrt{ax^4}}{3x^2}\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(1+x)*(x^2-x+1)/x^2*(a*x^4)^(1/2)/(x^3+1)^(1/2)
```

Maxima [A] time = 1.82366, size = 38, normalized size = 1.52

$$\frac{2\left(\sqrt{a}x^3+\sqrt{a}\right)}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A] time = 0.992094, size = 47, normalized size = 1.88

$$\frac{2\sqrt{ax^4}\sqrt{x^3+1}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^4}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [A] time = 1.15782, size = 16, normalized size = 0.64

$$\frac{2}{3}\sqrt{x^3+1}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)
```

3.386 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=292

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x}{\left(\left(1+\sqrt{3}\right)\right.}}}{}$$

```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x)) - (3^(1
/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ellipti
cE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(
x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])
*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3
^(1/4)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.231685, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 329, 308, 225, 1881}

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x}{\left(\left(1+\sqrt{3}\right)\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x)) - (3^(1
/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ellipti
cE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(
x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])
*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3
^(1/4)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
```

```
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= -\frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} - \frac{\sqrt[4]{3}\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} - \frac{\left(1-\sqrt{3}\right)\sqrt{\ldots}}{\ldots}
\end{aligned}$$

Mathematica [C] time = 0.0041012, size = 29, normalized size = 0.1

$$\frac{2}{5}x\sqrt{ax^3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] (2*x*Sqrt[a*x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

Maple [C] time = 0.234, size = 1521, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(a*x^3)^(1/2)/x*(x^3+1)^(1/2)*a*(I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^(1/2)
)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*
x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1
+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))
*x^2+2*I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*
x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/
2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+
I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x+I*3^(1/2)*((3+I*3^(1/2))*x
/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*(
(I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+
I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(
1/2)))^(1/2))-2*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x
-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2
)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I
*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x^2+3*((3+I*3^(1/2))*x/(1+I*3
^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1
/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/
2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^
(1/2))*x^2-I*3^(1/2)*x^3-4*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*
3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/
(1+x))^(1/2)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1
/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x+6*((3+I*3^(1/2))
*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)
*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(
1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3
^(1/2)))^(1/2))*x+I*3^(1/2)*x^2-2*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/
2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^
(1/2))/(1+x))^(1/2)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),(
(I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))+3*((3+I*3^(
1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^
(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2)
)*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(
3+I*3^(1/2)))^(1/2))-I*3^(1/2)*x-3*x^3+3*x^2-3*x)/(x*(x^3+1)*a)^(1/2)/(3+I*
3^(1/2))/(-a*x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

3.387 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=260

$$\frac{2\sqrt{x^3+1}\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}E}{x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}$$

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(x*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.0607094, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {15, 303, 218, 1877}

$$\frac{2\sqrt{x^3+1}\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}E}{x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(x*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^3}}\,dx}{x}$$
$$= \frac{\sqrt{ax^2}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{x} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{ax^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{x}$$
$$= \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2}\sqrt{ax^2}(1+}{}$$

Mathematica [C] time = 0.0037469, size = 29, normalized size = 0.11

$$\frac{1}{2}x\sqrt{ax^2}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (x*Sqrt[a*x^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [A] time = 0.021, size = 270, normalized size = 1.

$$\frac{i\sqrt{3}-3}{2x}\sqrt{ax^2}\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\left(i\text{EllipticE}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)\sqrt{3}-i\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 1/2*(a*x^2)^(1/2)*(I*3^(1/2)-3)*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-
2*x+1)/(3+I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)*(I*Elli
pticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(3+I*3^(1/2)))^(1/2))*
```

```
3^(1/2)-I*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(3+I*3^(
1/2)))^(1/2))*3^(1/2)+3*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/
2)-3)/(3+I*3^(1/2)))^(1/2))-EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3
^(1/2)-3)/(3+I*3^(1/2)))^(1/2)))/x/(x^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax^2}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

3.388 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{a}\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rubi [A] time = 0.015498, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {329, 275, 215}

$$\frac{2}{3}\sqrt{a}\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^6}{a^3}}}\,dx, x, \sqrt{ax}\right)}{a}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^3}}}\,dx, x, (ax)^{3/2}\right)}{3a}\\
&= \frac{2}{3}\sqrt{a}\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0048701, size = 22, normalized size = 0.96

$$\frac{2\sqrt{ax}\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x]*ArcSinh[x^(3/2)])/(3*Sqrt[x])
```

Maple [C] time = 0.063, size = 321, normalized size = 14.

$$-4\frac{\sqrt{ax}\sqrt{x^3+1}a\left(1+i\sqrt{3}\right)(1+x)^2}{\sqrt{x\left(x^3+1\right)a}\left(3+i\sqrt{3}\right)\sqrt{-ax(1+x)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(1+x)}}\sqrt{\frac{i\sqrt{3}+2x-1}{\left(i\sqrt{3}-1\right)(1+x)}}\sqrt{\frac{i}{(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(a*x)^(1/2)*(x^3+1)^(1/2)*a*(1+I*3^(1/2))*((3+I*3^(1/2))*x/(1+I*3^(1/2))
/(1+x))^(1/2)*(1+x)^2*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(
1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*(EllipticF(((3+I*3^(1/2))*x/(1+I*3^(
1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2))
)^(1/2))-EllipticPi(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),(1+I*3^(1/2
))/(3+I*3^(1/2)),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^
(1/2)))/(x*(x^3+1)*a)^(1/2)/(3+I*3^(1/2))/(-a*x*(1+x)*(I*3^(1/2)+2*x-1)*(I*
3^(1/2)-2*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

Fricas [B] time = 1.25422, size = 225, normalized size = 9.78

$$\left[\frac{1}{6}\sqrt{a}\log\left(-8ax^6-8ax^3-4\left(2x^4+x\right)\sqrt{x^3+1}\sqrt{ax}\sqrt{a}-a\right),-\frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\sqrt{x^3+1}\sqrt{ax}\sqrt{-ax}}{2ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x)
*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/(2
*a*x^3 + a))]
```

Sympy [A] time = 1.05586, size = 14, normalized size = 0.61

$$\frac{2\sqrt{a}\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a)*asinh(x**(3/2))/3
```

Giac [B] time = 1.17155, size = 47, normalized size = 2.04

$$-\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}a^{\frac{3}{2}}x+\sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

3.389 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=116

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.073026, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 329, 225}

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx$$
$$= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{\frac{a}{x}}x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.005088, size = 27, normalized size = 0.23

$$2x\sqrt{\frac{a}{x}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/6, 1/2, 7/6, -x^3]
```

Maple [C] time = 0.105, size = 232, normalized size = 2.

$$4\,\frac{x\sqrt{x^3+1}\left(1+i\sqrt{3}\right)(1+x)^2}{\sqrt{x\left(x^3+1\right)}\left(3+i\sqrt{3}\right)\sqrt{-x(1+x)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}\sqrt{\frac{a}{x}}\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(1+x)}}\sqrt{\frac{i\sqrt{3}+2x-1}{\left(i\sqrt{3}-1\right)(1+x)}}\sqrt{\frac{i\sqrt{3}}{(1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 4*(a/x)^(1/2)*x*(x^3+1)^(1/2)*(1+I*3^(1/2))*((3+I*3^(1/2))*x/(1+I*3^(1/2))/
(1+x))^(1/2)*(1+x)^2*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1
/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/
2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^
(1/2))/(x*(x^3+1))^(1/2)/(3+I*3^(1/2))/(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/
2)-2*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

3.390 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=24

$$-\frac{2}{3} x \sqrt{\frac{a}{x^2}} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rubi [A] time = 0.0087115, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {15, 266, 63, 207}

$$-\frac{2}{3} x \sqrt{\frac{a}{x^2}} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]

[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x^2}}x\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx \\
&= \frac{1}{3}\left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) \\
&= \frac{1}{3}\left(2\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right) \\
&= -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\tanh^{-1}\left(\sqrt{1+x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.0039839, size = 24, normalized size = 1.

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Maple [A] time = 0.006, size = 19, normalized size = 0.8

$$-\frac{2x}{3}\text{Artanh}\left(\sqrt{x^3+1}\right)\sqrt{\frac{a}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

Fricas [A] time = 1.03697, size = 177, normalized size = 7.38

$$\left[\frac{1}{3}x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right), \frac{2}{3}\sqrt{-a}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arcta
n(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [A] time = 1.16676, size = 42, normalized size = 1.75

$$-\frac{1}{3}\sqrt{a}\left(\log\left(\sqrt{x^3+1}+1\right)-\log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

3.391 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=312

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}-2\sqrt{x^3+1}x\sqrt{\frac{a}{x^3}}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2*Sqrt[1 +
x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3]
)*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
+ (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 +
Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.227796, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 325, 329, 308, 225, 1881}

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}-2\sqrt{x^3+1}x\sqrt{\frac{a}{x^3}}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2*Sqrt[1 +
x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3]
)*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
+ (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 +
Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^3}}\,dx\\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx\\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\left(4\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}-\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)+\left(2\left(-1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\Bigg(\\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x}-\frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\right.}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.0057306, size = 27, normalized size = 0.09

$$
-2x\sqrt{\frac{a}{x^3}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-x^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/6, 1/2, 5/6, -x^3]
```

Maple [C] time = 0.069, size = 1784, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(a/x^3)^(1/2)*x/(x^3+1)^(1/2)*(-2*I*3^(1/2)*(x*(x^3+1))^(1/2)*x^3+2*I*3^
(1/2)*(x*(x^3+1))^(1/2)*x^2-2*I*3^(1/2)*(x*(x^3+1))^(1/2)*x-4*((3+I*3^(1/2)
)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2
)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticF(((3+I*3^(1/2))*x/
(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*
3^(1/2)))^(1/2))*(x*(x^3+1))^(1/2)*x^2+6*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+
x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/
(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^
(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*(x*(
x^3+1))^(1/2)*x^2+2*I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*(
(I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2
))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3
^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*(x*(x^3+1))^(1/
2)*x^2+I*3^(1/2)*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)*x^3-8
*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-
1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticF(((3
+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^
(1/2)-1)/(3+I*3^(1/2)))^(1/2))*(x*(x^3+1))^(1/2)*x+12*((3+I*3^(1/2))*x/(1+I
*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^
(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(
1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2))
)^(1/2))*(x*(x^3+1))^(1/2)*x+2*I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+
x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/
(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^
(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*(x*(
x^3+1))^(1/2)-4*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x
-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2
)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I
*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*(x*(x^3+1))^(1/2)+6*((3+I*3^(
1/2))*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^
(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2)
)*x/(1+I*3^(1/2))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(
3+I*3^(1/2)))^(1/2))*(x*(x^3+1))^(1/2)+4*I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^
(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(1+x))^(1/2)*((I*3^(1/
2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2
))/(1+x))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(
1/2))*(x*(x^3+1))^(1/2)*x+3*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^
(1/2)*x^3-6*(x*(x^3+1))^(1/2)*x^3+I*3^(1/2)*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*
3^(1/2)-2*x+1))^(1/2)+6*(x*(x^3+1))^(1/2)*x^2-6*(x*(x^3+1))^(1/2)*x+3*(-x*(
1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2))/(3+I*3^(1/2))/(-x*(1+x)*(I
*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

3.392 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}-\sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)}{}$$

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
 x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
+ (Sqrt[2]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.0730181, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 325, 303, 218, 1877}

$$\frac{\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}-\sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
 x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
+ (Sqrt[2]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx\\
&= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{1}{2}\left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{x}{\sqrt{1+x^3}}\,dx\\
&= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{1}{2}\left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx+\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1}{\sqrt{1+x^3}}\,dx\\
&= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{\sqrt{\frac{a}{x^4}}x^2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.0049953, size = 27, normalized size = 0.1

$$x\left(-\sqrt{\frac{a}{x^4}}\right){}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])
```

Maple [A] time = 0.02, size = 353, normalized size = 1.3

$$\frac{x}{2}\sqrt{\frac{a}{x^4}}\left(i\sqrt{3}\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}x-6\sqrt{\frac{i\sqrt{3}-2x+}{3+i\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 1/2*(a/x^4)^(1/2)*x*(I*3^(1/2)*((I*3^(1/2)-2*x+1)/(3+I*3^(1/2)))^(1/2)*((I*
3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2
),(-(I*3^(1/2)-3)/(3+I*3^(1/2)))^(1/2))*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*x-6*
((I*3^(1/2)-2*x+1)/(3+I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(
1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(3+I*3^(1/2))
)^(1/2))*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*x+3*((I*3^(1/2)-2*x+1)/(3+I*3^(1/2)
))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(
1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(3+I*3^(1/2)))^(1/2))*(-2*(1+x)/(I*3^(1/2)-3
))^(1/2)*x-2*x^3-2)/(x^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

3.393 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=37

$$\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Rubi [A] time = 0.0122416, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {15, 364}

$$\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx\\ &= \frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0119701, size = 37, normalized size = 1.

$$\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \sqrt{a x^{2 n}}\frac{1}{\sqrt{1+x^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x^{2 n}}}{\sqrt{x^{n}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x^{2n}}}{\sqrt{x^{n}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

3.394 $\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=48

$$\frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2
 + n)
```

Rubi [A] time = 0.0141593, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 364}

$$\frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n/2}\sqrt{ax^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.0105009, size = 40, normalized size = 0.83

$$\frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/
(2 + n)
```

Maple [A] time = 0.04, size = 35, normalized size = 0.7

$$2\,\frac{x\,{}_2F_1(1/2,1/2+n^{-1};\,3/2+n^{-1};\,-x^n)\sqrt{ax^n}}{2+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^n)^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] 2*x*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

3.395 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=52

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n)/4, -x^n]
)/(4 + n)
```

Rubi [A] time = 0.0148895, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {15, 364}

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n)/4, -x^n]
)/(4 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n/4}\sqrt{ax^{n/2}}\right)\int \frac{x^{n/4}}{\sqrt{1+x^n}}\,dx\\ &= \frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{4+n}\end{aligned}$$

Mathematica [A] time = 0.0115588, size = 44, normalized size = 0.85

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n};\frac{5}{4}+\frac{1}{n};-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n^(-1), -x^
n])/(4 + n)
```

Maple [A] time = 0.064, size = 37, normalized size = 0.7

$$4\,\frac{x_2F_1(1/2,1/4+n^{-1};\,5/4+n^{-1};\,-x^n)\sqrt{ax^{n/2}}}{4+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] 4*x*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{\frac{n}{2}}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

3.396 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx$

Optimal. Leaf size=34

$$\frac{2x^{1-n}\sqrt{x^n+1}\sqrt{ax^{2n}}}{n+2}$$

```
[Out] (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Sqrt[1 + x^n])/(2 + n)
```

Rubi [C] time = 0.0295837, antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15, 364, 245}

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1
 + x^n]),x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(
-1), -x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx = \frac{2\int \frac{x^{-n}\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx}{2+n}+\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$$
$$= \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx+\frac{\left(2x^{-n}\sqrt{ax^{2n}}\right)\int \frac{1}{\sqrt{1+x^n}}\,dx}{2+n}$$
$$= \frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}+\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{2+n}$$

Mathematica [A] time = 0.0292111, size = 33, normalized size = 0.97

$$\frac{2ax^{n+1}\sqrt{x^n+1}}{(n+2)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*
Sqrt[1 + x^n]),x]
```

```
[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

Maple [A] time = 0.023, size = 30, normalized size = 0.9

$$2\,\frac{x\sqrt{1+x^n}\sqrt{a\left(x^n\right)^2}}{(2+n)\,x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n)
^(1/2),x)
```

```
[Out] 2*x*(1+x^n)^(1/2)/(2+n)*(a*(x^n)^2)^(1/2)/(x^n)
```

Maxima [A] time = 1.77577, size = 24, normalized size = 0.71

$$\frac{2\sqrt{a}\sqrt{x^n+1}x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx + \int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx + \int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(
x**n)/(1+x**n)**(1/2),x)
```

```
[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*n
))/sqrt(x**n + 1), x) + Integral(2*x**(-n)*sqrt(a*x**(2*n))/sqrt(x**n + 1),
 x))/(n + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}} + \frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x
^n + 1)*x^n), x)
```

3.397 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]*S
qrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rubi [A] time = 0.0561957, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {114, 113}

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]*S
qrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\right)\int \frac{\sqrt{-\frac{ex}{d}}}{\sqrt{d+ex}\sqrt{\frac{e^2}{e^2-df}+\frac{efx}{e^2-df}}}dx}{\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$
$$= \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Mathematica [C] time = 0.204797, size = 106, normalized size = 0.93

$$-\frac{2ie\sqrt{ax}\sqrt{\frac{fx}{e}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(e*x)/d]],
(d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/e^2]))/(f*Sqrt[(e*x)
/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

Maple [A] time = 0.055, size = 191, normalized size = 1.7

$$-2\,\frac{d\sqrt{fx+e}\sqrt{ex+d}\sqrt{ax}}{e^2fx\left(efx^2+dfx+e^2x+de\right)}\left(e^2\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)+\text{EllipticE}\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)df-\text{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -2*(e^2*EllipticF(((e*x+d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))+EllipticE(((e*x+
d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))*d*f-EllipticE(((e*x+d)/d)^(1/2),(d*f/(d*
f-e^2))^(1/2))*e^2)*(-e*x/d)^(1/2)*(-(f*x+e)*e/(d*f-e^2))^(1/2)*((e*x+d)/d)
^(1/2)*d*(f*x+e)^(1/2)*(e*x+d)^(1/2)*(a*x)^(1/2)/f/e^2/x/(e*f*x^2+d*f*x+e^2
*x+d*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax}\sqrt{ex+d}\sqrt{fx+e}}{efx^2+de+\left(e^2+df\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x)*sqrt(e*x + d)*sqrt(f*x + e)/(e*f*x^2 + d*e + (e^2 + d*f)
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

3.398 $\int (ax^m)^r \, dx$

Optimal. Leaf size=16

$$\frac{x (ax^m)^r}{mr + 1}$$

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rubi [A] time = 0.0046085, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x (ax^m)^r}{mr + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (ax^m)^r \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} \, dx \\ &= \frac{x (ax^m)^r}{1 + mr} \end{aligned}$$

Mathematica [A] time = 0.0031748, size = 16, normalized size = 1.

$$\frac{x (ax^m)^r}{mr + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Maple [A] time = 0.002, size = 17, normalized size = 1.1

$$\frac{x\,(a x^{m})^{r}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x)
```

```
[Out] x*(a*x^m)^r/(m*r+1)
```

Maxima [A] time = 1.31621, size = 23, normalized size = 1.44

$$\frac{a^{r}x(x^{m})^{r}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="maxima")
```

```
[Out] a^r*x*(x^m)^r/(m*r + 1)
```

Fricas [A] time = 1.00261, size = 53, normalized size = 3.31

$$\frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.14182, size = 27, normalized size = 1.69

$$\frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

3.399 $\int (ax^m)^r (bx^n)^s \, dx$

Optimal. Leaf size=26

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rubi [A] time = 0.0091378, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} (bx^n)^s \, dx \\
&= \left(x^{-mr-ns} (ax^m)^r (bx^n)^s\right) \int x^{mr+ns} \, dx \\
&= \frac{x (ax^m)^r (bx^n)^s}{1 + mr + ns}
\end{aligned}$$

Mathematica [A] time = 0.0073769, size = 26, normalized size = 1.

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Maple [A] time = 0.002, size = 27, normalized size = 1.

$$\frac{x\,(a x^{m})^{r}\,(b x^{n})^{s}}{m r + n s + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

Maxima [A] time = 1.50721, size = 43, normalized size = 1.65

$$\frac{a^{r} b^{s} x e^{\left(r \log\left(x^{m}\right) + s \log\left(x^{n}\right)\right)}}{m r + n s + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
```

```
[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

Fricas [A] time = 1.01956, size = 93, normalized size = 3.58

$$\frac{x e^{\left(m r \log\left(x\right) + n s \log\left(x\right) + r \log\left(a\right) + s \log\left(b\right)\right)}}{m r + n s + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.17513, size = 43, normalized size = 1.65

$$\frac{x e^{\left(m r \log\left(x\right) + n s \log\left(x\right) + r \log\left(a\right) + s \log\left(b\right)\right)}}{m r + n s + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")

[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

3.400 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

Optimal. Leaf size=36

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rubi [A] time = 0.0155787, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} (bx^n)^s (cx^p)^t \, dx \\
&= \left(x^{-mr-ns} (ax^m)^r (bx^n)^s\right) \int x^{mr+ns} (cx^p)^t \, dx \\
&= \left(x^{-mr-ns-pt} (ax^m)^r (bx^n)^s (cx^p)^t\right) \int x^{mr+ns+pt} \, dx \\
&= \frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{1 + mr + ns + pt}
\end{aligned}$$

Mathematica [A] time = 0.012172, size = 36, normalized size = 1.

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Maple [A] time = 0.001, size = 37, normalized size = 1.

$$\frac{x\left(a x^{m}\right)^{r}\left(b x^{n}\right)^{s}\left(c x^{p}\right)^{t}}{m r+n s+p t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Maxima [A] time = 1.44166, size = 59, normalized size = 1.64

$$\frac{a^{r} b^{s} c^{t} x e^{\left(r \log\left(x^{m}\right)+s \log\left(x^{n}\right)+t \log\left(x^{p}\right)\right)}}{m r+n s+p t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
```

```
[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t + 1
)
```

Fricas [A] time = 0.989281, size = 134, normalized size = 3.72

$$\frac{x e^{\left(m r \log\left(x\right)+n s \log\left(x\right)+p t \log\left(x\right)+r \log\left(a\right)+s \log\left(b\right)+t \log\left(c\right)\right)}}{m r+n s+p t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15558, size = 59, normalized size = 1.64

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c)\right)}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

3.401 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3
*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*
b^3*(a - c))
```

Rubi [A] time = 0.132343, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2104, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```


```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3
*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*
b^3*(a - c))
```


Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= -\frac{b\int x^2\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x^2\sqrt{c+bx}\,dx}{-ab+bc}\\ &= -\frac{b\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}+\frac{b\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)}+\frac{4c(c+bx)^{5/2}}{5b^3(a-c)}-\frac{2(c+bx)^{7/2}}{7b^3(a-c)}\end{aligned}$$

Mathematica [A] time = 0.170526, size = 140, normalized size = 0.95

$$\frac{2\left(8a^3\sqrt{a+bx}-4a^2bx\sqrt{a+bx}+15b^3x^3\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+3ab^2x^2\sqrt{a+bx}-3b^2cx^2\sqrt{bx+c}-8c^3\sqrt{bx+c}+4bc}{105b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b*
x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c + b
*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

Maple [A] time = 0.004, size = 90, normalized size = 0.6

$$2\,\frac{1/7\,(bx+a)^{7/2}-2/5\,a\,(bx+a)^{5/2}+1/3\,a^2\,(bx+a)^{3/2}}{(a-c)\,b^3}-2\,\frac{1/7\,(bx+c)^{7/2}-2/5\,c\,(bx+c)^{5/2}+1/3\,c^2\,(bx+c)^{3/2}}{(a-c)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-2
/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A] time = 0.927076, size = 201, normalized size = 1.37

$$\frac{2\left(\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}-\left(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3\right)\sqrt{bx+c}\right)}{105\left(ab^3-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*b
^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.402 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Rubi [A] time = 0.0812099, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2104, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= -\frac{b\int x\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x\sqrt{c+bx}\,dx}{-ab+bc}\\
&= -\frac{b\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{-ab+bc}+\frac{b\int\left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{-ab+bc}\\
&= -\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}
\end{aligned}$$

Mathematica [A] time = 0.0979928, size = 95, normalized size = 1.

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Maple [A] time = 0.004, size = 66, normalized size = 0.7

$$2\,\frac{1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+a)^{3/2}}{(a-c)\,b^2}-2\,\frac{1/5\,(bx+c)^{5/2}-1/3\,c\,(bx+c)^{3/2}}{(a-c)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-2/(a-c)/b^2*(1/5*(b*x+c
)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A] time = 0.940223, size = 149, normalized size = 1.57

$$\frac{2\left(\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}-\left(3\,b^2x^2+bcx-2\,c^2\right)\sqrt{bx+c}\right)}{15\left(ab^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c^
2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.403 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}} dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(bx+c)^{3/2}}{3b(a-c)}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

Rubi [A] time = 0.0466709, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6689}

$$\frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}} dx &= \frac{\int \left(\sqrt{a+bx}-\sqrt{c+bx}\right) dx}{a-c} \\
&= \frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(c+bx)^{3/2}}{3b(a-c)}
\end{aligned}$$

Mathematica [A] time = 0.0480739, size = 35, normalized size = 0.74

$$\frac{2\left((a+bx)^{3/2}-(bx+c)^{3/2}\right)}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

Maple [A] time = 0.003, size = 40, normalized size = 0.9

$$\frac{2}{3\,b\,(a-c)}(bx+a)^{\frac{3}{2}} - \frac{2}{3\,b\,(a-c)}(bx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A] time = 0.966095, size = 72, normalized size = 1.53

$$\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

Sympy [A] time = 0.647716, size = 136, normalized size = 2.89

$$\begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
+ b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) +
 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)), Ne(
b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.404 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]
)/(a - c)
```

Rubi [A] time = 0.103954, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2104, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]
)/(a - c)
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int \frac{\sqrt{a+bx}}{x}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x}\,dx}{-ab+bc}$$
$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{a-c}-\frac{c\int \frac{1}{x\sqrt{c+bx}}\,dx}{a-c}$$
$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(a-c)}-\frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx,\right.}{b(a-c)}$$
$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

Mathematica [A] time = 0.0721256, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{bx+c}+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
+ Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]))/(a - c)
```

Maple [A] time = 0.006, size = 73, normalized size = 0.8

$$\frac{1}{a-c}\left(2\sqrt{bx+a}-2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-\frac{1}{a-c}\left(2\sqrt{bx+c}-2\sqrt{c}\text{Artanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)*
(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Fricas [A] time = 1.02143, size = 794, normalized size = 8.19

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},-\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
- 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a
 - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x + 2
*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a -
c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x - 2*sqr
t(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c),
2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(b*x + c
)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.405 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}$$

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)*Sqrt[c])
```

Rubi [A] time = 0.102121, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2104, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)*Sqrt[c])
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int \frac{\sqrt{a+bx}}{x^2}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x^2}\,dx}{-ab+bc}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(a-c)}-\frac{b\int \frac{1}{x\sqrt{c+bx}}\,dx}{2(a-c)}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a-c}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{c\right.}}{a-c}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

Mathematica [A] time = 0.278175, size = 99, normalized size = 0.96

$$\frac{\frac{bx\sqrt{\frac{bx}{c}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c}{\sqrt{bx+c}}-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{a+bx}}}{x(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (-((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*
x]) + (c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]])/Sqrt[c +
 b*x])/((a - c)*x)
```

Maple [A] time = 0.013, size = 88, normalized size = 0.9

$$2\,\frac{b}{a-c}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-2\,\frac{b}{a-c}\left(-1/2\,\frac{\sqrt{bx+c}}{bx}-1/2\,\frac{1}{\sqrt{c}}\text{Artanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1
/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Fricas [A] time = 1.06672, size = 980, normalized size = 9.51

$$\left[-\frac{\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+ab\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2\left(a^2c-ac^2\right)x},-\frac{2ab\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-}}{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sqr
t(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c -
 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan(sq
rt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x),
 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - a*b*sqrt(c)*x*log
((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*sqrt(b*
x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(
-a)/a) - a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) - sqrt(b*x + a)*a*
c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.406 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=228

$$-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{}}{32b^3(a-}$$

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2
)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a
 + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2
)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))
/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c +
b*x]])/(32*b^3)
```

Rubi [A] time = 0.373396, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6689, 90, 80, 50, 63, 217, 206}

$$-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{}}{32b^3(a-}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2
)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a
 + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2
)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))
/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c +
b*x]])/(32*b^3)
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x^2+2bx^3-2x^2\sqrt{a+bx}\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{2\int x^2\sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}-\frac{\int \sqrt{a+bx}\sqrt{c+bx}\left(-ac-\frac{5}{2}b(a+c)x\right)d}{2b^2(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}+\frac{\left(4ac-5\right.}{}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(c+bx}{12b^3(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/}}{16b^3(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/}}{16b^3(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/}}{16b^3(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/}}{16b^3(a-c)^2}$$

Mathematica [A] time = 1.36713, size = 361, normalized size = 1.58

$$\frac{3\sqrt{b}(c-a)^3\left(5a^2+6ac+5c^2\right)\sqrt{-\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b(c-a)}\sqrt{bx+c}}+a^2\sqrt{a+bx}\sqrt{bx+c}(10bx+7c)-15a^3\sqrt{a+bx}\sqrt{bx+c}-a\left(8b^2x^2\sqrt{a+bx}\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (48*b^4*x^4 - 15*a^3*Sqrt[a + b*x]*Sqrt[c + b*x] - 15*c^3*Sqrt[a + b*x]*Sqr
t[c + b*x] + 10*b*c^2*x*Sqrt[a + b*x]*Sqrt[c + b*x] - 8*b^2*c*x^2*Sqrt[a +
b*x]*Sqrt[c + b*x] + a^2*Sqrt[a + b*x]*Sqrt[c + b*x]*(7*c + 10*b*x) - 16*b^
3*x^3*(-2*c + 3*Sqrt[a + b*x]*Sqrt[c + b*x]) - a*(-32*b^3*x^3 - 7*c^2*Sqrt[
a + b*x]*Sqrt[c + b*x] + 4*b*c*x*Sqrt[a + b*x]*Sqrt[c + b*x] + 8*b^2*x^2*Sq
rt[a + b*x]*Sqrt[c + b*x]) + (3*Sqrt[b]*(-a + c)^3*(5*a^2 + 6*a*c + 5*c^2)*
Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]
])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(96*b^3*(a - c)^2)
```

Maple [C] time = 0.019, size = 604, normalized size = 2.7

$$\frac{ax^3}{3\,(a-c)^2}+\frac{cx^3}{3\,(a-c)^2}+\frac{bx^4}{2\,(a-c)^2}-\frac{\operatorname{csgn}(b)}{192\,(a-c)^2\,b^3}\sqrt{bx+a}\sqrt{bx+c}\left(96\operatorname{csgn}(b)\,x^3b^3\sqrt{b^2x^2+abx+bcx+ac}+16\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(a-c)^2*a+1/3*x^3/(a-c)^2*c+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16
*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*(b^
2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a
^2*b+8*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b*c-20*csgn(b)*(b^2*x^2+
a*b*x+b*c*x+a*c)^(1/2)*x*b*c^2+30*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a
^3-14*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2*c-14*csgn(b)*(b^2*x^2+a*b
*x+b*c*x+a*c)^(1/2)*a*c^2+30*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*c^3-15
*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^4+
12*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^
3*c+6*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))
*a^2*c^2+12*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*cs
gn(b))*a*c^3-15*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c
)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Fricas [A] time = 1.01601, size = 437, normalized size = 1.92

$$\frac{96 b^4x^4 + 64 \left(ab^3 + b^3c\right)x^3 - 2 \left(48 b^3x^3 + 15 a^3 - 7 a^2c - 7 ac^2 + 15 c^3 + 8 \left(ab^2 + b^2c\right)x^2 - 2 \left(5 a^2b - 2 abc + 5 bc^2\right)x\right)\sqrt{bx}}{192 \left(a^2b^3 - 2 ab^3c + b^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^2
*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*b*
c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4*a*
c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^3
- 2*a*b^3*c + b^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a + bx} + \sqrt{bx + c}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.407 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=165

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}$$

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b
*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x]
)/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2)
 - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)
```

Rubi [A] time = 0.21001, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6689, 80, 50, 63, 217, 206}

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b
*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x]
)/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2)
 - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x+2bx^2-2x\sqrt{a+bx}\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2\int x\sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)\int\sqrt{a+bx}\sqrt{c+bx}\,dx}{b(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\int\frac{\sqrt{}}{\sqrt{}}}{4b(a-}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3}}{3b^2(}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3}}{3b^2(}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3}}{3b^2(}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3}}{3b^2(}$$

Mathematica [A] time = 0.871075, size = 229, normalized size = 1.39

$$\frac{3a^2\sqrt{a+bx}\sqrt{bx+c}-2a\left(bx\sqrt{a+bx}\sqrt{bx+c}+c\sqrt{a+bx}\sqrt{bx+c}-3b^2x^2\right)+(4bx+3c)\left(-2bx\sqrt{a+bx}\sqrt{bx+c}+c\sqrt{a+b}\right.}{12b^2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (3*a^2*Sqrt[a + b*x]*Sqrt[c + b*x] + (3*c + 4*b*x)*(2*b^2*x^2 + c*Sqrt[a +
b*x]*Sqrt[c + b*x] - 2*b*x*Sqrt[a + b*x]*Sqrt[c + b*x]) - 2*a*(-3*b^2*x^2 +
 c*Sqrt[a + b*x]*Sqrt[c + b*x] + b*x*Sqrt[a + b*x]*Sqrt[c + b*x]))/(12*b^2*
(a - c)^2) - (Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(
```

```
Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]]])/(4*b^(5/2)*Sqrt[c + b*x])
```

Maple [C] time = 0.01, size = 431, normalized size = 2.6

$$\frac{ax^2}{2\,(a-c)^2}+\frac{cx^2}{2\,(a-c)^2}+\frac{2\,bx^3}{3\,(a-c)^2}-\frac{\operatorname{csgn}(b)}{24\,(a-c)^2\,b^2}\sqrt{bx+a}\sqrt{bx+c}\left(16\operatorname{csgn}(b)\,x^2b^2\sqrt{b^2x^2+abx+bcx+ac}+4\operatorname{csg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)^
(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4*c
sgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b+4*csgn(b)*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*x*b*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2+4*csgn(b)*(
b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)*c^2+3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b
))*a^3-3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(
b))*a^2*c-3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*cs
gn(b))*a*c^2+3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)
*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Fricas [A] time = 0.975902, size = 332, normalized size = 2.01

$$\frac{16\,b^3x^3+12\left(ab^2+b^2c\right)x^2-2\left(8\,b^2x^2-3\,a^2+2\,ac-3\,c^2+2\,(ab+bc)x\right)\sqrt{bx+a}\sqrt{bx+c}+3\left(a^3-a^2c-ac^2+c^3\right)\log}{24\left(a^2b^2-2\,ab^2c+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a*c
^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2 - 2
*a*b^2*c + b^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.408 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}$$

```
[Out] (a - c)^2/(8*b*(Sqrt[a + b*x] + Sqrt[c + b*x])^4) + ArcTanh[Sqrt[a + b*x]/S
qrt[c + b*x]]/(2*b)
```

Rubi [A] time = 0.0985642, antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6689, 50, 63, 217, 206}

$$\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{bx+c}}{2b(a-c)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{x(a+c)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x])/(
2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a - c)^2) + ArcTanh[Sqrt
[a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)dx}{(a-c)^2}$$
$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{2\int \sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2}$$
$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{2(a-c)}$$
$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{1}{4}\int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx$$
$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx,x,\right.}{2b}$$
$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right.}{2b}$$
$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}$$

Mathematica [B] time = 0.569051, size = 179, normalized size = 2.84

$$\frac{2bx\left(bx-\sqrt{a+bx}\sqrt{bx+c}\right)+a\left(2bx-\sqrt{a+bx}\sqrt{bx+c}\right)+c\left(2bx-\sqrt{a+bx}\sqrt{bx+c}\right)+\frac{\sqrt{b}(c-a)^3\sqrt{\frac{bx+c}{c-a}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b(c-a)}\sqrt{bx+c}}+2c^2}{2b(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] (2*c^2 + 2*b*x*(b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + a*(2*b*x - Sqrt[a + b*
x]*Sqrt[c + b*x]) + c*(2*b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[b]*(-a
+ c)^3*Sqrt[(c + b*x)/(-a + c)]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a
+ c)]])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(2*b*(a - c)^2)
```

Maple [B] time = 0.006, size = 377, normalized size = 6.

$$\frac{ax}{(a-c)^2}+\frac{cx}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{1}{(a-c)^2 b}\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}-\frac{a}{2\,(a-c)^2 b}\sqrt{bx+c}\sqrt{bx+a}+\frac{c}{2\,(a-c)^2 b}\sqrt{bx+c}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] x/(a-c)^2*a+x/(a-c)^2*c+b*x^2/(a-c)^2-1/(a-c)^2/b*(b*x+a)^(1/2)*(b*x+c)^(3/
2)-1/2/(a-c)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*a+1/2/(a-c)^2/b*(b*x+c)^(1/2)*
```

```
(b*x+a)^(1/2)*c+1/4/(a-c)^2*((b*x+c)*(b*x+a))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2)
)/(b^2)^(1/2)*a^2-1/2/(a-c)^2*((b*x+c)*(b*x+a))^(1/2)/(b*x+c)^(1/2)/(b*x+a)
^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/
2))/(b^2)^(1/2)*a*c+1/4/(a-c)^2*((b*x+c)*(b*x+a))^(1/2)/(b*x+c)^(1/2)/(b*x+
a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(
1/2))/(b^2)^(1/2)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

Fricas [B] time = 0.993924, size = 247, normalized size = 3.92

$$\frac{4\,b^2x^2 - 2\,(2\,bx + a + c)\sqrt{bx+a}\sqrt{bx+c} + 4\,(ab+bc)x - \left(a^2 - 2\,ac + c^2\right)\log\left(-2\,bx + 2\,\sqrt{bx+a}\sqrt{bx+c} - a - c\right)}{4\left(a^2b - 2\,abc + bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b + b
*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a
- c))/(a^2*b - 2*a*b*c + b*c^2)
```

Sympy [A] time = 0.992041, size = 318, normalized size = 5.05

$$\begin{cases} \frac{a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{2ab+4b^2x+2bc+4b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{2ab+4b^2x+2bc+4b\sqrt{a+bx}\sqrt{bx+c}} + \frac{c\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{2ab+4b^2x+2bc+4b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2\sqrt{a+bx}\sqrt{bx+c}\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{2ab+4b^2x+2bc+4b\sqrt{a+bx}\sqrt{bx+c}} - \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a*log(sqrt(a + b*x) + sqrt(b*x + c))/(2*a*b + 4*b**2*x + 2*b*c +
 4*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x*log(sqrt(a + b*x) + sqrt(b*x + c)
)/(2*a*b + 4*b**2*x + 2*b*c + 4*b*sqrt(a + b*x)*sqrt(b*x + c)) + c*log(sqrt
(a + b*x) + sqrt(b*x + c))/(2*a*b + 4*b**2*x + 2*b*c + 4*b*sqrt(a + b*x)*sq
rt(b*x + c)) + 2*sqrt(a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x +
 c))/(2*a*b + 4*b**2*x + 2*b*c + 4*b*sqrt(a + b*x)*sqrt(b*x + c)) - sqrt(a
+ b*x)*sqrt(b*x + c)/(2*a*b + 4*b**2*x + 2*b*c + 4*b*sqrt(a + b*x)*sqrt(b*x
 + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**2, True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.409 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTan
h[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Lo
g[x])/(a - c)^2
```

Rubi [A] time = 0.229698, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {6689, 101, 157, 63, 217, 206, 93, 208}

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTan
h[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Lo
g[x])/(a - c)^2
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx &= \frac{\int \left(2b+\frac{a\left(1+\frac{c}{a}\right)}{x}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x}\right)dx}{(a-c)^2}\\
&= \frac{2bx}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x}\,dx}{(a-c)^2}\\
&= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}+\frac{2\int \frac{-ac-\frac{1}{2}b(a+c)x}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}\\
&= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(2ac)\int \frac{1}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}-\frac{(b(a+c))\int \cdots}{(a-\cdots}\\
&= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(4ac)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}-\frac{(2\cdots}{\cdots}\\
&= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(2(a+c))\,\text{S}\cdots}{\cdots}\\
&= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2}+\cdots
\end{aligned}
$$

Mathematica [A] time = 1.0643, size = 195, normalized size = 1.47

$$\frac{\sqrt{b}\left(-2\left(c\sqrt{a+bx}+bx\left(\sqrt{a+bx}-\sqrt{bx+c}\right)\right)+(a+c)\log(x)\sqrt{bx+c}+4\sqrt{a}\sqrt{c}\sqrt{bx+c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)\right)-2(a+c)}{\sqrt{b}(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-2*Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sq
rt[a + b*x])/Sqrt[b*(-a + c)]] + Sqrt[b]*(-2*(c*Sqrt[a + b*x] + b*x*(Sqrt[a
 + b*x] - Sqrt[c + b*x])) + 4*Sqrt[a]*Sqrt[c]*Sqrt[c + b*x]*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])] + (a + c)*Sqrt[c + b*x]*Log[x]))/
(Sqrt[b]*(a - c)^2*Sqrt[c + b*x])
```

Maple [C] time = 0.013, size = 258, normalized size = 1.9

$$\frac{a\ln(x)}{(a-c)^2}+\frac{c\ln(x)}{(a-c)^2}+2\frac{bx}{(a-c)^2}+\frac{\operatorname{csgn}(b)}{(a-c)^2}\sqrt{bx+a}\sqrt{bx+c}\left(2\operatorname{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\sqrt{b^2x^2+abx+bcx+ac}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(
b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+
a*c)^(1/2)+2*a*c)/x)*a*c-2*csgn(b)*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1
/2)-ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*(
a*c)^(1/2)*a-ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*c
sgn(b))*(a*c)^(1/2)*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Fricas [A] time = 1.05974, size = 730, normalized size = 5.49

$$\left[\frac{2bx+(a+c)\log\left(-2bx+2\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+(a+c)\log(x)+2\sqrt{ac}\log\left(\frac{2a^2c+2ac^2+2\left(2ac+\sqrt{ac}(a+c)\right)\sqrt{bx+a}\sqrt{bx-}}{}\right.}{a^2-2ac+c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
+ c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(a
+ c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a*c
+ (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*
c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a -
 c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sq
rt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2
*a*c + c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.410 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) -
(4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (2*b*(a + c)*ArcTanh
[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[
c]) + (2*b*Log[x])/(a - c)^2
```

Rubi [A] time = 0.215897, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {6689, 97, 157, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) -
(4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (2*b*(a + c)*ArcTanh
[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[
c]) + (2*b*Log[x])/(a - c)^2
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx &= \frac{\int \left(\frac{a\left(1+\frac{c}{a}\right)}{x^2}+\frac{2b}{x}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x^2}\right)dx}{(a-c)^2}\\
&= -\frac{a+c}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x^2}\,dx}{(a-c)^2}\\
&= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\frac{1}{2}b(a+c)+b^2x}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}\\
&= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}-\frac{(b(a+c))\int \frac{}{x\sqrt{}}}{(a-}\\
&= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx,x,\sqrt{a+bx}\right)}{(a-c)^2}\\
&= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}}+\frac{2b\log(x)}{(a-c)^2}-\frac{(4b)\,\text{Subs}}{}\\
&= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}}+
\end{aligned}$$

Mathematica [A] time = 0.980618, size = 205, normalized size = 1.45

$$\frac{\frac{a\left(-\sqrt{bx+c}\right)+2c\sqrt{a+bx}+2bx\sqrt{a+bx}-c\sqrt{bx+c}+2bx\log(x)\sqrt{bx+c}}{x}-4\sqrt{b}\sqrt{b(c-a)}\sqrt{-\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)+\frac{2b(a+c)\sqrt{bx+c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}}}{(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-4*Sqrt[b]*Sqrt[b*(-a + c)]*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sq
rt[a + b*x])/Sqrt[b*(-a + c)]] + (2*b*(a + c)*Sqrt[c + b*x]*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*Sqrt[c]) + (2*c*Sqrt[a
+ b*x] + 2*b*x*Sqrt[a + b*x] - a*Sqrt[c + b*x] - c*Sqrt[c + b*x] + 2*b*x*Sq
rt[c + b*x]*Log[x])/x)/((a - c)^2*Sqrt[c + b*x])
```

Maple [C] time = 0.013, size = 274, normalized size = 1.9

$$-\frac{a}{x(a-c)^2}-\frac{c}{x(a-c)^2}+2\frac{b\ln(x)}{(a-c)^2}+\frac{\operatorname{csgn}(b)}{x(a-c)^2}\sqrt{bx+a}\sqrt{bx+c}\left(\operatorname{csgn}(b)\ln\left(\frac{1}{x}\left(abx+bcx+2\sqrt{ac}\sqrt{b^2x^2+abx+bc}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x
+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)+2*a*c)/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+
b*c*x+a*c)^(1/2)+2*a*c)/x)*x*b*c-2*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a
*c)^(1/2)+2*b*x+a+c)*csgn(b))*x*b*(a*c)^(1/2)+2*csgn(b)*(a*c)^(1/2)*(b^2*x^
2+a*b*x+b*c*x+a*c)^(1/2))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Fricas [A] time = 1.15903, size = 873, normalized size = 6.19

$$\left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+2\,abcx\log(x)+2\,abcx+(ab+bc)\sqrt{ac}\,x\log\left(\frac{2\,a^2c+2\,ac^2+2\left(2\,ac+\sqrt{ac}(a+c)\right.}{\ldots}\right.}{\left(a^3c-2\,a^2c^2+ac^3\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x
*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*(2
*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c +
b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sqrt(b
*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c*x*lo
g(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x) + 2*a*
b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sqr
t(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*c - a^2*
c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.411 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=375

$$\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}$$

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (4*a*(
a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (24*a*(a + b*x)^(7/2))/(7*b^3
*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x
)^(9/2))/(9*b^3*(a - c)^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2)
)/(5*b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3) + (
24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*
b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c)^3)
```

Rubi [A] time = 0.37164, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6689, 43}

$$\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (4*a*(
a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (24*a*(a + b*x)^(7/2))/(7*b^3
*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x
)^(9/2))/(9*b^3*(a - c)^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2)
)/(5*b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3) + (
24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*
b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c)^3)
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x^2\sqrt{a+bx}+4bx^3\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x^2\sqrt{c+bx}-4bx^3\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$= \frac{(4b)\int x^3\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^3\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} + \frac{(a+3c)\int x^2\sqrt{a}}{(a-c)^3}$$

$$= \frac{(4b)\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx}{(a-c)^3} - \frac{(4b)\int\left(-\frac{c^3\sqrt{c+bx}}{b^3}+\frac{3c^2(c+bx)^{3/2}}{b^3}\right.}{(a-c}$$

$$= -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{24a}{7}$$

Mathematica [A] time = 0.412515, size = 282, normalized size = 0.75

$$\frac{2\left(4a^3\sqrt{a+bx}(5bx+18c)-3a^2bx\sqrt{a+bx}(5bx+12c)-40a^4\sqrt{a+bx}+a\left(27b^2cx^2\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+5b^3x^3\left(13\sqrt{a}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(-40*a^4*Sqrt[a + b*x] - 3*a^2*b*x*Sqrt[a + b*x]*(12*c + 5*b*x) + 4*a^3*
Sqrt[a + b*x]*(18*c + 5*b*x) + a*(-72*c^3*Sqrt[c + b*x] + 36*b*c^2*x*Sqrt[c
 + b*x] + 5*b^3*x^3*(13*Sqrt[a + b*x] - 27*Sqrt[c + b*x]) + 27*b^2*c*x^2*(S
qrt[a + b*x] - Sqrt[c + b*x])) + 5*(8*c^4*Sqrt[c + b*x] - 4*b*c^3*x*Sqrt[c
+ b*x] + 3*b^2*c^2*x^2*Sqrt[c + b*x] + b^3*c*x^3*(27*Sqrt[a + b*x] - 13*Sqr
t[c + b*x]) + 28*b^4*x^4*(Sqrt[a + b*x] - Sqrt[c + b*x]))))/(315*b^3*(a - c
)^3)
```

Maple [A] time = 0.003, size = 294, normalized size = 0.8

$$2\,\frac{a\left(1/7\,(bx+a)^{7/2}-2/5\,a\,(bx+a)^{5/2}+1/3\,a^2\,(bx+a)^{3/2}\right)}{(a-c)^3\,b^3}+6\,\frac{c\left(1/7\,(bx+a)^{7/2}-2/5\,a\,(bx+a)^{5/2}+1/3\,a^2\,(bx+a)^{3/2}\right)}{(a-c)^3\,b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2
))+6/(a-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(
3/2))-6/(a-c)^3*a/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c
)^(3/2))-2/(a-c)^3*c/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*
x+c)^(3/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*(b*x+a)^(7/2)*a+3/5*a^2*(b
*x+a)^(5/2)-1/3*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*c*(
b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Fricas [A] time = 0.945516, size = 452, normalized size = 1.21

$$\frac{2\left(\left(140\,b^4x^4-40\,a^4+72\,a^3c+5\left(13\,ab^3+27\,b^3c\right)x^3-3\left(5\,a^2b^2-9\,ab^2c\right)x^2+4\left(5\,a^3b-9\,a^2bc\right)x\right)\sqrt{bx+a}-\left(140\,b^4x\right.\right.}{315\left(a^3b^3-3\,a^2b^3c+3\,ab^3c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*(
5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (14
0*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2*c
- 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a
^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.412 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=261

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+c}{7b^2(a-}$$

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))
/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c
)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3
) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3
/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a -
c)^3)
```

Rubi [A] time = 0.236346, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6689, 43}

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+c}{7b^2(a-}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))
/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c
)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3
) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3
/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a -
c)^3)
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x\sqrt{a+bx}+4bx^2\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x\sqrt{c+bx}-4bx^2\sqrt{c+bx}\right)dx}{(a-c)^3}$$
$$= \frac{(4b)\int x^2\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x\sqrt{c+bx}\,dx}{(a-c)^3} + \frac{(a+3c)\int x\sqrt{}}{(a-c}$$
$$= \frac{(4b)\int \left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3} - \frac{(4b)\int \left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3}$$
$$= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{8(a+}{7b^2(}$$

Mathematica [A] time = 0.269916, size = 214, normalized size = 0.82

$$\frac{2\left(-a^2\sqrt{a+bx}(3bx+14c)+6a^3\sqrt{a+bx}+a\left(b^2x^2\left(11\sqrt{a+bx}-21\sqrt{bx+c}\right)+7bcx\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+14c^2\sqrt{bx}\right.\right.}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(6*a^3*Sqrt[a + b*x] - 6*c^3*Sqrt[c + b*x] + 3*b*c^2*x*Sqrt[c + b*x] - a
^2*Sqrt[a + b*x]*(14*c + 3*b*x) + b^2*c*x^2*(21*Sqrt[a + b*x] - 11*Sqrt[c +
 b*x]) + 20*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x]) + a*(14*c^2*Sqrt[c + b*
x] + b^2*x^2*(11*Sqrt[a + b*x] - 21*Sqrt[c + b*x]) + 7*b*c*x*(Sqrt[a + b*x]
 - Sqrt[c + b*x]))))/(35*b^2*(a - c)^3)
```

Maple [A] time = 0.003, size = 222, normalized size = 0.9

$$2\,\frac{a\left(1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+a)^{3/2}\right)}{(a-c)^3\,b^2}+6\,\frac{c\left(1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+a)^{3/2}\right)}{(a-c)^3\,b^2}-6\,\frac{a\left(1/5\,(bx+c)^{5/2}-1/3\,c\,(bx+c)^{3/}\right.}{(a-c)^3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))+6/(a-c)^3*c/b^2*(1/
5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-6/(a-c)^3*a/b^2*(1/5*(b*x+c)^(5/2)-1/3
*c*(b*x+c)^(3/2))-2/(a-c)^3*c/b^2*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))+8
/(a-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-
8/(a-c)^3/b^2*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Fricas [A] time = 0.998509, size = 343, normalized size = 1.31

$$\frac{2\left(\left(20\,b^3x^3 + 6\,a^3 - 14\,a^2c + \left(11\,ab^2 + 21\,b^2c\right)x^2 - \left(3\,a^2b - 7\,abc\right)x\right)\sqrt{bx + a} - \left(20\,b^3x^3 - 14\,ac^2 + 6\,c^3 + \left(21\,ab^2 + 11\,b^2c\right.\right.}{35\left(a^3b^2 - 3\,a^2b^2c + 3\,ab^2c^2 - b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*b
 - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2 +
 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*b^2
*c + 3*a*b^2*c^2 - b^2*c^3)
```

Sympy [A] time = 2.07677, size = 942, normalized size = 3.61

$$\begin{cases} \frac{12a^2}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b^2c\sqrt{a+bx}+35b^2c\sqrt{bx+c}} + \frac{54abx}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b} \\ \frac{x^2}{2\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 14
0*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x
) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b*
*2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 10
5*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(
a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x
*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 36*a
*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
 + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sq
rt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*sqrt(a + b
*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/
(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(
b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 1
05*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x +
c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b*
*2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 14
0*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)
) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b
**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne(b, 0)), (x**2/(2*(sqrt(a
) + sqrt(c))**3), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.413 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^5}-\frac{1}{2b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}$$

```
[Out] (a - c)^2/(10*b*(Sqrt[a + b*x] + Sqrt[c + b*x])^5) - 1/(2*b*(Sqrt[a + b*x]
+ Sqrt[c + b*x]))
```

Rubi [B] time = 0.0946979, antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6689, 43}

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\frac{\int \left(a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}+4bx\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}-4bx\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$=\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3}+\frac{(4b)\int x\sqrt{a+bx}\,dx}{(a-c)^3}-\frac{(4b)\int x\sqrt{c+bx}\,dx}{(a-c)^3}$$

$$=\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3}+\frac{(4b)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(a-c)^3}-\frac{(4b)\int\left(-\right.}{}$$

$$=-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}+\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{8c(c+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3}}{3b(a-c)^3}$$

Mathematica [B] time = 0.151964, size = 151, normalized size = 2.36

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

Maple [B] time = 0.003, size = 146, normalized size = 2.3

$$\frac{2a}{3b(a-c)^3}(bx+a)^{\frac{3}{2}}+2\frac{c(bx+a)^{3/2}}{b(a-c)^3}-2\frac{a(bx+c)^{3/2}}{b(a-c)^3}-\frac{2c}{3b(a-c)^3}(bx+c)^{\frac{3}{2}}+8\frac{1/5(bx+a)^{5/2}-1/3a(bx+a)^{3/2}}{b(a-c)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)/b/(a-c)^3+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x+
c)^(3/2)/b-2/3*c*(b*x+c)^(3/2)/b/(a-c)^3+8/(a-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3
*a*(b*x+a)^(3/2))-8/(a-c)^3/b*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

Fricas [B] time = 0.988146, size = 228, normalized size = 3.56

$$\frac{2\left(\left(4b^2x^2-a^2+5ac+(3ab+5bc)x\right)\sqrt{bx+a}-\left(4b^2x^2+5ac-c^2+(5ab+3bc)x\right)\sqrt{bx+c}\right)}{5\left(a^3b-3a^2bc+3abc^2-bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*x
^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c + 3
*a*b*c^2 - b*c^3)
```

Sympy [A] time = 1.91095, size = 384, normalized size = 6.

$$\begin{cases} -\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}} - \frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+}} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(
a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x
+ c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*
x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a
+ b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**3, True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.414 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=157

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\tan}{(a-}$$

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) -
 (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3)
- (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[
c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

Rubi [A] time = 0.23279, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6689, 50, 63, 208}

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\tan}{(a-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```


```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) -
 (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3)
- (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[
c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```


Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(4b\sqrt{a+bx}+\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x}-4b\sqrt{c+bx}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$= \frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(c(3a+c))\int \frac{1}{x\sqrt{c}}}{(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(2c(3a+c))\,\mathrm{Sub}}{}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}(a+3c)\tanh}{(a-c)}$$

Mathematica [A] time = 0.196875, size = 142, normalized size = 0.9

$$\frac{2\left(-9a\sqrt{bx+c}+9c\sqrt{a+bx}-3\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+3\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+7a\sqrt{a+bx}+4bx\sqrt{a+bx}-\right.}{3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(7*a*Sqrt[a + b*x] + 9*c*Sqrt[a + b*x] + 4*b*x*Sqrt[a + b*x] - 9*a*Sqrt[
c + b*x] - 7*c*Sqrt[c + b*x] - 4*b*x*Sqrt[c + b*x] - 3*Sqrt[a]*(a + 3*c)*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sq
rt[c]]))/(3*(a - c)^3)
```

Maple [A] time = 0.003, size = 181, normalized size = 1.2

$$\frac{a}{(a-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+\frac{8}{3(a-c)^3}(bx+a)^{\frac{3}{2}}-\frac{8}{3(a-c)^3}(bx+c)^{\frac{3}{2}}+3\frac{c}{(a-c)^3}\left(2\sqrt{bx+a}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3*
(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1/2
)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)-2*
c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*c^(1
/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Fricas [A] time = 1.041, size = 1283, normalized size = 8.17

$$\left[-\frac{3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(3\,a+c)\sqrt{c}\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+a}+2\,(4\,bx+9}{3\left(a^3-3\,a^2c+3\,ac^2-c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3
*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*x
+ 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*
a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*sqr
t(-c)/c) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c
))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan(sqrt(b
*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c
) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7*c)*s
qrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a + 3*c)*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*
sqrt(-c)/c) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a + 7*c)*sqrt(
b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.415 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=162

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{3b(a+3c)\tanh^{-1}\left(\sqrt{b\ldots}\right.}{\sqrt{c}(c-a)^3}$$

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (
8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (3
*b*(3*a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) - (3*b*(a
+ 3*c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(Sqrt[c]*(-a + c)^3)
```

Rubi [A] time = 0.273317, antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6689, 47, 63, 208, 50}

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{8\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (
8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (8
*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 - (b*(a + 3*c)*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c
+ b*x]/Sqrt[c]])/(a - c)^3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)^3*Sqrt[c])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x^2}+\frac{4b\sqrt{a+bx}}{x}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x^2}-\frac{4b\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$= \frac{(4b)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}-\frac{(4b)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x^2}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(a-c)^3}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(4ab)\int \frac{1}{x\sqrt{a+bx}}\,dx}{(a-c)^3}-$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(8a)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{(a-c}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{8\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{(a-c)^3}$$

Mathematica [A] time = 0.596263, size = 187, normalized size = 1.15

$$\frac{b\left(-\frac{(a+3c)\left(bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{bx\sqrt{a+bx}}+\frac{(3a+c)\left(bx\sqrt{\frac{bx}{c}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c\right)}{bx\sqrt{bx+c}}+8\sqrt{a+bx}-8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-8\sqrt{bx+c}\right.}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x + b*
x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]))/(b*x*Sqrt[a + b*x]) + ((3*
a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b*x*S
qrt[c + b*x])))/(a - c)^3
```

Maple [A] time = 0.003, size = 252, normalized size = 1.6

$$2\,\frac{ab}{(a-c)^3}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+6\,\frac{bc}{(a-c)^3}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-6\,\frac{a}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arctanh((b*
x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arct
anh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh(
(b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x
+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Fricas [A] time = 1.11474, size = 1558, normalized size = 9.62

$$\left[-\frac{3\left(3\,abc+bc^2\right)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\left(a^2b+3\,abc\right)\sqrt{c}x\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\left(8\,abcx-a^2c-3\,ac^2\right)\sqrt{bx\cdots}}{2\left(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2
*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) +
 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3
*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x),
 -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*
(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*
c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*(6*(3*
a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a
*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a*b*c*x
 - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*
x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c + b*c^2)*s
qrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(-c)*x
*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x
+ a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3
*a^2*c^3 - a*c^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.416 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0059096, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2106, 30, 32}

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx &= -\int \sqrt{x}\,dx+\int \sqrt{1+x}\,dx\\ &= -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0189383, size = 21, normalized size = 1.

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$-\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{3}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Fricas [A] time = 0.941612, size = 45, normalized size = 2.14

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [B] time = 0.356942, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x+1}}+\frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

Giac [A] time = 1.15683, size = 18, normalized size = 0.86

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

3.417 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi [A] time = 0.00695, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2106, 30, 32}

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx &= -\int \sqrt{-1+x}\,dx+\int \sqrt{x}\,dx\\ &= -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}\end{aligned}$$

Mathematica [A] time = 0.0201221, size = 21, normalized size = 1.

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+x^(1/2)),x)
```

```
[Out] -2/3*(x-1)^(3/2)+2/3*x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

Fricas [A] time = 1.09429, size = 46, normalized size = 2.19

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [B] time = 0.373593, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x-1}}-\frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

Giac [A] time = 1.12708, size = 18, normalized size = 0.86

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.418 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

```
[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Rubi [A] time = 0.0225083, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6689}

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]

[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx &= -\left(\frac{1}{2}\int \left(\sqrt{-1+x}-\sqrt{1+x}\right)dx\right)\\ &= -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0202374, size = 23, normalized size = 1.

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]

[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$-\frac{1}{3}(x-1)^{\frac{3}{2}}+\frac{1}{3}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -1/3*(x-1)^(3/2)+1/3*(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Fricas [A] time = 1.22195, size = 53, normalized size = 2.3

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Sympy [B] time = 0.3948, size = 51, normalized size = 2.22

$$\frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}} + \frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x -
 1) + 3*sqrt(x + 1))
```

Giac [A] time = 1.1578, size = 20, normalized size = 0.87

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

3.419 $\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal. Leaf size=38

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rubi [A] time = 0.113624, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6742, 266, 43}

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx &= \int \left(2x^3+2x^3\sqrt{1-x^2}\right) dx\\
&= \frac{x^4}{2}+2\int x^3\sqrt{1-x^2}\, dx\\
&= \frac{x^4}{2}+\text{Subst}\left(\int \sqrt{1-x}x \, dx, x, x^2\right)\\
&= \frac{x^4}{2}+\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right) dx, x, x^2\right)\\
&= \frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0536561, size = 38, normalized size = 1.

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Maple [A] time = 0.004, size = 33, normalized size = 0.9

$$\frac{x^4}{2}+\frac{\left(2x^2-2\right)\left(3x^2+2\right)}{15}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] 1/2*x^4+2/15*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Maxima [A] time = 1.69087, size = 42, normalized size = 1.11

$$\frac{1}{2}x^4-\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 1.24945, size = 80, normalized size = 2.11

$$\frac{1}{2}x^4+\frac{2}{15}\left(3x^4-x^2-2\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11482, size = 76, normalized size = 2.

$$\frac{1}{2}(x+1)^4 - 2(x+1)^3 + \frac{2}{15}((3(x+1)(x-3)+17)(x+1)-10)(x+1)^{\frac{3}{2}}\sqrt{-x+1} + 3(x+1)^2 - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)^4 - 2*(x + 1)^3 + 2/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)*
(x + 1)^(3/2)*sqrt(-x + 1) + 3*(x + 1)^2 - 2*x - 2
```

3.420 $\int x^2 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal. Leaf size=48

$$\frac{1}{2}\sqrt{1-x^2}x^3+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{4}\sin^{-1}(x)$$

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Rubi [A] time = 0.0895353, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6742, 279, 321, 216}

$$\frac{1}{2}\sqrt{1-x^2}x^3+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \int \left(2x^2+2x^2\sqrt{1-x^2}\right) dx$$
$$= \frac{2x^3}{3}+2\int x^2\sqrt{1-x^2}\,dx$$
$$= \frac{2x^3}{3}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx$$
$$= \frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}(x)$$

Mathematica [A] time = 0.0431275, size = 42, normalized size = 0.88

$$\frac{1}{12}\left(\left(6\sqrt{1-x^2}+8\right)x^3-3\sqrt{1-x^2}x+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (-3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 3*ArcSin[x])/12
```

Maple [A] time = 0.007, size = 59, normalized size = 1.2

$$\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x}\sqrt{1+x}\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] 2/3*x^3+1/4*(1+x)^(1/2)*(1-x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+
arcsin(x))/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.6985, size = 46, normalized size = 0.96

$$\frac{2}{3}x^3-\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{1}{4}\sqrt{-x^2+1}x+\frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

Fricas [A] time = 1.22624, size = 134, normalized size = 2.79

$$\frac{2}{3}x^3+\frac{1}{4}\left(2x^3-x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x + 1
)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11715, size = 84, normalized size = 1.75

$$\frac{2}{3}(x+1)^3 - 2(x+1)^2 + \frac{1}{4}((2(x+1)(x-2)+5)(x+1)-1)\sqrt{x+1}\sqrt{-x+1} + 2x + \frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^3 - 2*(x + 1)^2 + 1/4*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sqr
t(x + 1)*sqrt(-x + 1) + 2*x + 1/2*arcsin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

3.421 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal. Leaf size=19

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rubi [A] time = 0.0541514, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6742, 261}

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx &= \int \left(2x+2x\sqrt{1-x^2}\right) \, dx\\ &= x^2+2\int x\sqrt{1-x^2} \, dx\\ &= x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0206148, size = 19, normalized size = 1.

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Maple [A] time = 0.003, size = 24, normalized size = 1.3

$$x^2 + \frac{2x^2 - 2}{3}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] x^2+2/3*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)
```

Maxima [A] time = 1.60683, size = 20, normalized size = 1.05

$$x^2 - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] x^2 - 2/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 1.25989, size = 62, normalized size = 3.26

$$x^2 + \frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [A] time = 79.1557, size = 110, normalized size = 5.79

$$-\frac{x^3}{3} - x + \frac{(x+1)^3}{3} - 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) + 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] -x**3/3 - x + (x + 1)**3/3 - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asi
n(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 4*Piecewise((x*sqrt(1 -
 x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x +
 1)/2)/2, (x >= -1) & (x < 1))) - 1
```

Giac [A] time = 1.11003, size = 36, normalized size = 1.89

$$\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1} + (x+1)^2 - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + (x + 1)^2 - 2*x - 2
```

3.422 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal. Leaf size=19

$$\sqrt{1-x^2}x+2x+\sin^{-1}(x)$$

```
[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0249363, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6742, 195, 216}

$$\sqrt{1-x^2}x+2x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx &= \int \left(2+2\sqrt{1-x^2}\right) dx\\ &= 2x+2\int \sqrt{1-x^2}\,dx\\ &= 2x+x\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= 2x+x\sqrt{1-x^2}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0148542, size = 18, normalized size = 0.95

$$x\left(\sqrt{1-x^2}+2\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x*(2 + Sqrt[1 - x^2]) + ArcSin[x]
```

Maple [B] time = 0.005, size = 58, normalized size = 3.1

$$2x-\sqrt{1+x}\,(1-x)^{\frac{3}{2}}+\sqrt{1-x}\sqrt{1+x}+\arcsin(x)\sqrt{(1-x)(1+x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] 2*x-(1+x)^(1/2)*(1-x)^(3/2)+(1-x)^(1/2)*(1+x)^(1/2)+((1-x)*(1+x))^(1/2)/(1-
x)^(1/2)/(1+x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.90388, size = 23, normalized size = 1.21

$$\sqrt{-x^2+1}x+2x+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

Fricas [B] time = 1.22892, size = 107, normalized size = 5.63

$$\sqrt{x+1}x\sqrt{-x+1}+2x-2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/
x)
```

Sympy [A] time = 24.3767, size = 44, normalized size = 2.32

$$2x+4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\: x\geq -1\wedge x<1\end{cases}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*x + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2
)/2, (x >= -1) & (x < 1))) + 2
```

Giac [A] time = 1.16092, size = 43, normalized size = 2.26

$$\sqrt{x+1}x\sqrt{-x+1}+2\,x+2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

3.423 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx$

Optimal. Leaf size=32

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rubi [A] time = 0.0887197, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 266, 50, 63, 206}

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx &= \int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx\\ &= 2\log(x)+2\int \frac{\sqrt{1-x^2}}{x}\,dx\\ &= 2\log(x)+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right)\\ &= 2\sqrt{1-x^2}+2\log(x)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\ &= 2\sqrt{1-x^2}+2\log(x)-2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= 2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0284104, size = 32, normalized size = 1.

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Maple [A] time = 0.009, size = 51, normalized size = 1.6

$$2\ln(x)+2\,\frac{\sqrt{1-x}\sqrt{1+x}\left(\sqrt{-x^2+1}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x)
```

```
[Out] 2*ln(x)+2*(1+x)^(1/2)*(1-x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/
(-x^2+1)^(1/2)))
```

Maxima [A] time = 1.7694, size = 55, normalized size = 1.72

$$2\sqrt{-x^2+1}+2\log(x)-2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 1.27879, size = 109, normalized size = 3.41

$$2\sqrt{x+1}\sqrt{-x+1}+2\log(x)+2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.424 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

```
[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rubi [A] time = 0.080425, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6742, 277, 216}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} dx &= \int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right) dx \\
&= -\frac{2}{x}+2\int \frac{\sqrt{1-x^2}}{x^2} dx \\
&= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\int \frac{1}{\sqrt{1-x^2}} dx \\
&= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0267812, size = 22, normalized size = 0.85

$$-\frac{2\left(\sqrt{1-x^2}+x\sin^{-1}(x)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] (-2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x
```

Maple [B] time = 0.012, size = 50, normalized size = 1.9

$$-2\,x^{-1}+2\,\frac{\left(-\arcsin(x)\,x-\sqrt{-x^2+1}\right)\sqrt{1+x}\sqrt{1-x}}{x\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x)
```

```
[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1+x)^(1/2)*(1-x)^(1/2)/x/(-x^2+1)^(1/
2)
```

Maxima [A] time = 1.51806, size = 32, normalized size = 1.23

$$-\frac{2\sqrt{-x^2+1}}{x}-\frac{2}{x}-2\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)
```

Fricas [A] time = 1.12653, size = 112, normalized size = 4.31

$$\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] 2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
- 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)
```

Giac [B] time = 1.15333, size = 201, normalized size = 7.73

$$-2\pi-\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}-\frac{2}{x}-4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
- sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-
x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

3.425 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0911825, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 266, 47, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx &= \int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\ &= -\frac{1}{x^2}+2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\ &= -\frac{1}{x^2}+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\ &= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0407227, size = 45, normalized size = 1.32

$$-\frac{1}{x^2\sqrt{1-x^2}}+\frac{1}{\sqrt{1-x^2}}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) + 1/Sqrt[1 - x^2] - 1/(x^2*Sqrt[1 - x^2]) + ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.012, size = 58, normalized size = 1.7

$$-x^{-2}+\frac{1}{x^2}\sqrt{1-x}\sqrt{1+x}\left(\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x^2-\sqrt{-x^2+1}\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x)
```

```
[Out] -1/x^2+(1+x)^(1/2)*(1-x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49132, size = 73, normalized size = 2.15

$$-\sqrt{-x^2+1}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2}-\frac{1}{x^2}+\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

Fricas [A] time = 1.27848, size = 109, normalized size = 3.21

$$-\frac{x^2 \log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + \sqrt{x+1}\sqrt{-x+1} + 1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] -(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{1-x} + \sqrt{x+1}\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.426 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Rubi [A] time = 0.121838, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2103, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int x^2\sqrt{a+bx}\,dx}{b-c}-\frac{\int x^2\sqrt{a+cx}\,dx}{b-c}\\ &= \frac{\int \left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{b-c}-\frac{\int \left(\frac{a^2\sqrt{a+cx}}{c^2}-\frac{2a(a+cx)^{3/2}}{c^2}+\frac{(a+cx)^{5/2}}{c^2}\right)dx}{b-c}\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3}+\frac{4a(a+cx)^{5/2}}{5(b-c)c^3}-\frac{2(a+cx)^{7/2}}{7(b-c)c^3}\end{aligned}$$

Mathematica [A] time = 0.218664, size = 147, normalized size = 1.

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Maple [A] time = 0.004, size = 90, normalized size = 0.6

$$2\,{\frac{1/7\,(bx+a)^{7/2}-2/5\,a\,(bx+a)^{5/2}+1/3\,a^2\,(bx+a)^{3/2}}{(b-c)\,b^3}}-2\,{\frac{1/7\,(cx+a)^{7/2}-2/5\,a\,(cx+a)^{5/2}+1/3\,a^2\,(cx+a)^{3/2}}{(b-c)\,c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-2
/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 1.20086, size = 250, normalized size = 1.7

$$\frac{2\left(\left(15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3\right)\sqrt{bx+a}-\left(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3\right)\sqrt{cx+a}\right)}{105\left(b^4c^3-b^3c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt(
b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)*s
qrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.427 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)
) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*
c^2)
```

Rubi [A] time = 0.100534, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2103, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)
) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*
c^2)
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int x\sqrt{a+bx}\,dx}{b-c}-\frac{\int x\sqrt{a+cx}\,dx}{b-c}\\ &= \frac{\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{b-c}-\frac{\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{b-c}\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}\end{aligned}$$

Mathematica [A] time = 0.180861, size = 70, normalized size = 0.74

$$\frac{2\left(\frac{3(a+bx)^{5/2}}{b^2}-\frac{5a(a+bx)^{3/2}}{b^2}-\frac{3(a+cx)^{5/2}}{c^2}+\frac{5a(a+cx)^{3/2}}{c^2}\right)}{15(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*((-5*a*(a + b*x)^(3/2))/b^2 + (3*(a + b*x)^(5/2))/b^2 + (5*a*(a + c*x)^(
3/2))/c^2 - (3*(a + c*x)^(5/2))/c^2))/(15*(b - c))
```

Maple [A] time = 0.003, size = 66, normalized size = 0.7

$$2\,{\frac{1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+a)^{3/2}}{(b-c)\,b^{2}}}-2\,{\frac{1/5\,(cx+a)^{5/2}-1/3\,a\,(cx+a)^{3/2}}{(b-c)\,c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-2/(b-c)/c^2*(1/5*(c*x+a
)^(5/2)-1/3*a*(c*x+a)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 1.10644, size = 186, normalized size = 1.96

$$\frac{2\left(\left(3\,b^{2}c^{2}x^{2}+abc^{2}x-2\,a^{2}c^{2}\right)\sqrt{bx+a}-\left(3\,b^{2}c^{2}x^{2}+ab^{2}cx-2\,a^{2}b^{2}\right)\sqrt{cx+a}\right)}{15\left(b^{3}c^{2}-b^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x^
2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.428 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Rubi [A] time = 0.0550301, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2103, 32}

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int \sqrt{a+bx}\,dx}{b-c}-\frac{\int \sqrt{a+cx}\,dx}{b-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}\end{aligned}$$

Mathematica [A] time = 0.0760489, size = 39, normalized size = 0.83

$$\frac{2\left(\frac{(a+bx)^{3/2}}{b}-\frac{(a+cx)^{3/2}}{c}\right)}{3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*((a + b*x)^(3/2)/b - (a + c*x)^(3/2)/c))/(3*(b - c))
```

Maple [A] time = 0.002, size = 40, normalized size = 0.9

$$\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3b\left(b-c\right)}-\frac{2\left(cx+a\right)^{\frac{3}{2}}}{\left(3b-3c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 1.20339, size = 109, normalized size = 2.32

$$\frac{2\left(\left(bcx+ac\right)\sqrt{bx+a}-\left(bcx+ab\right)\sqrt{cx+a}\right)}{3\left(b^{2}c-bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b*
c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.429 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(b - c)
```

Rubi [A] time = 0.0708231, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6690, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(b - c)
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right)dx}{b-c}$$
$$= \frac{\int \frac{\sqrt{a+bx}}{x}\,dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x}\,dx}{b-c}$$
$$= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{b-c}-\frac{a\int \frac{1}{x\sqrt{a+cx}}\,dx}{b-c}$$
$$= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(b-c)}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\right.}{(b-c)c}$$
$$= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Mathematica [A] time = 0.0483586, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+cx}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[a + c*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
+ Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(b - c)
```

Maple [A] time = 0.003, size = 73, normalized size = 0.8

$$\frac{1}{b-c}\left(2\sqrt{bx+a}-2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-\frac{1}{b-c}\left(2\sqrt{cx+a}-2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)*
(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 1.29619, size = 393, normalized size = 4.05

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-a}\,\text{ar}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
- 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/(b
 - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.430 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(
Sqrt[a]*(b - c))
```

Rubi [A] time = 0.0945461, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2103, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(
Sqrt[a]*(b - c))
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{\int \frac{\sqrt{a+bx}}{x^2}\,dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^2}\,dx}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)} - \frac{c\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b-c} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a}\right.}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

Mathematica [A] time = 0.198739, size = 135, normalized size = 1.31

$$\frac{-\frac{a}{\sqrt{a+bx}} - \frac{bx}{\sqrt{a+bx}} - \frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{a+bx}} + \frac{a}{\sqrt{a+cx}} + \frac{cx}{\sqrt{a+cx}} + \frac{cx\sqrt{\frac{cx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)}{\sqrt{a+cx}}}{bx-cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-(a/Sqrt[a + b*x]) - (b*x)/Sqrt[a + b*x] + a/Sqrt[a + c*x] + (c*x)/Sqrt[a
+ c*x] - (b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x] +
 (c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 + (c*x)/a]])/Sqrt[a + c*x])/(b*x - c
*x)
```

Maple [A] time = 0.004, size = 88, normalized size = 0.9

$$2\,\frac{b}{b-c}\left(-1/2\,\frac{\sqrt{bx+a}}{bx} - 1/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right) - 2\,\frac{c}{b-c}\left(-1/2\,\frac{\sqrt{cx+a}}{cx} - 1/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Fricas [A] time = 1.20451, size = 450, normalized size = 4.37

$$\left[-\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}cx\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a-2\sqrt{cx+a}a}{2(ab-ac)x}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c
*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*sqr
t(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/
a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a + sqrt
(c*x + a)*a)/((a*b - a*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.431 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=171

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

```
[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt[a
 + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/Sq
rt[a]])/(4*a^(3/2)*(b - c))
```

Rubi [A] time = 0.112481, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2103, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt[a
 + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/Sq
rt[a]])/(4*a^(3/2)*(b - c))
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}dx = \frac{\int \frac{\sqrt{a+bx}}{x^3}dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^3}dx}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4(b-c)}-\frac{c\int \frac{1}{x^2\sqrt{a+cx}}dx}{4(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}dx}{8a(b-c)}+\frac{c^2\int \frac{1}{x\sqrt{a+cx}}dx}{8a(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+bx}\right)}{4a(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\right.}{4a^{3/2}(b-}$$

Mathematica [C] time = 0.0892831, size = 75, normalized size = 0.44

$$\frac{2c^2(a+cx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{a}+1\right)-2b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^2
*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b - c
))
```

Maple [A] time = 0.009, size = 120, normalized size = 0.7

$$2\,\frac{b^2}{b-c}\left(\frac{1}{b^2x^2}\left(-1/8\,\frac{(bx+a)^{3/2}}{a}-1/8\,\sqrt{bx+a}\right)+1/8\,\frac{1}{a^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-2\,\frac{c^2}{b-c}\left(\frac{1}{c^2x^2}\left(-1/8\,\frac{(cx+a)^{3/2}}{a}-1/8\,\sqrt{cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/b^2/x^2+1/8/a^(3/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+
```

```
a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Fricas [A] time = 1.35671, size = 570, normalized size = 3.33

$$\left[-\frac{\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}c^2x^2\log\left(\frac{cx+2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(abx+2a^2\right)\sqrt{bx+a}-2\left(acx+2a^2\right)\sqrt{cx+a}}{8\left(a^2b-a^2c\right)x^2},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(
a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^2)
*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2), -1
/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^2*ar
ctan(sqrt(c*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*x + 2
*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.432 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}$$

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x
]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a +
c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c
)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x
])])/(4*b^(5/2)*c^(5/2))
```

Rubi [A] time = 0.3493, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6690, 80, 50, 63, 217, 206}

$$\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x
]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a +
c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c
)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x
])])/(4*b^(5/2)*c^(5/2))
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2ax+b\left(1+\frac{c}{b}\right)x^2-2x\sqrt{a+bx}\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2\int x\sqrt{a+bx}\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{(a(b+c))\int \sqrt{a+bx}\sqrt{a+cx}\,dx}{b(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{\left(a^2(b+\right.}{4b}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+}{}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+}{}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+}{}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+}{}$$

Mathematica [A] time = 0.676149, size = 238, normalized size = 1.22

$$\frac{b\sqrt{c}\left(a^2\left(3b^2-2bc+3c^2\right)\sqrt{a+bx}\sqrt{a+cx}+4b^2c^2x^2\left(-2\sqrt{a+bx}\sqrt{a+cx}+bx+cx\right)-2abcx\left(b\sqrt{a+bx}\sqrt{a+cx}+c\sqrt{}\right.}{12b^3c^{5/2}(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*Sqrt[c]*(a^2*(3*b^2 - 2*b*c + 3*c^2)*Sqrt[a + b*x]*Sqrt[a + c*x] + 4*b^2
*c^2*x^2*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - 2*a*b*c*x*(-6*b*c*x
+ b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + (3*a^4*
(-b + c)^3*(b + c)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a
+ b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c)]*Sqrt[a + c*x]))/(12*b^3*(b - c)^
2*c^(5/2))
```

Maple [B] time = 0.014, size = 517, normalized size = 2.7

$$\frac{bx^3}{3\,(b-c)^2}+\frac{cx^3}{3\,(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{1}{24\,(b-c)^2\,b^2c^2}\sqrt{bx+a}\sqrt{cx+a}\left(16\,x^2b^2c^2\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}+3\,\ln\left(1/2\,\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(b-c)^2*b+1/3*x^3/(b-c)^2*c+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/2
)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)
+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(
b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/2*(2*b
*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^
3*c^3+4*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*b^2*c+4*(b*c)^(1/2)
*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*b*c^2-6*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c
*x+a^2)^(1/2)*a^2*b^2+4*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2*b*c
-6*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2*c^2)/(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A] time = 1.34945, size = 1015, normalized size = 5.21

$$\left[\frac{24\,ab^3c^3x^2+8\left(b^4c^3+b^3c^4\right)x^3+3\left(a^3b^3-a^3b^2c-a^3bc^2+a^3c^3\right)\sqrt{bc}\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc-\sqrt{bc}(b+c)\right)\sqrt{bx}\right.}{24\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*
c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c -
 sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(
2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^
2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x +
 a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4*c^3 +
 b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-b*c)*ar
ctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (8*
b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*
b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.433 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$

Optimal. Leaf size=142

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a +
 c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a
^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(
3/2))
```

Rubi [A] time = 0.227759, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6690, 50, 63, 217, 206}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a +
 c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a
^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(
3/2))
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2a+b\left(1+\frac{c}{b}\right)x-2\sqrt{a+bx}\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{2\int \sqrt{a+bx}\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}-\frac{a\int \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\,dx}{2b(b-c)}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}}\,dx}{4bc}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ac}{b}+\cdots}}\right.}{2b^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1-\frac{cx^2}{b}}\,dx\right.}{2b^2c}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

Mathematica [A] time = 0.579419, size = 177, normalized size = 1.25

$$\frac{b\sqrt{c}\left(bcx\left(-2\sqrt{a+bx}\sqrt{a+cx}+bx+cx\right)-a\left(b\sqrt{a+bx}\sqrt{a+cx}+c\sqrt{a+bx}\sqrt{a+cx}-4bcx\right)\right)+\frac{(a(b-c))^{5/2}\sqrt{\frac{b(a+cx)}{a(b-c)}}\sinh^{-1}\left(\cdots\right.}{\sqrt{a+cx}}}{2b^2c^{3/2}(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*Sqrt[c]*(b*c*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - a*(-4*b*c*x
 + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + ((a*(b
 - c))^(5/2)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a + b*x]
)/Sqrt[a*(b - c)]])/Sqrt[a + c*x])/(2*b^2*(b - c)^2*c^(3/2))
```

Maple [B] time = 0.008, size = 385, normalized size = 2.7

$$\frac{bx^2}{2\,(b-c)^2}+\frac{cx^2}{2\,(b-c)^2}+2\,\frac{ax}{(b-c)^2}-\frac{1}{(b-c)^2\,c}\sqrt{bx+a}\,(cx+a)^{\frac{3}{2}}+\frac{a}{2\,(b-c)^2\,c}\sqrt{cx+a}\sqrt{bx+a}-\frac{a}{2\,(b-c)^2\,b}\sqrt{cx+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-1/(b-c)^2/c*(b*x+a)^(1/2)
*(c*x+a)^(3/2)+1/2/(b-c)^2/c*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a-1/2/(b-c)^2/b*(c
*x+a)^(1/2)*(b*x+a)^(1/2)*a+1/4/(b-c)^2/c*((c*x+a)*(b*x+a))^(1/2)/(c*x+a)^(
1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c
)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2*b-1/2/(b-c)^2*((c*x+a)*(b*x+a))^(1/2)/(c*x+
a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b
+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2+1/4/(b-c)^2*c/b*((c*x+a)*(b*x+a))^(1/2)
/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^
2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A] time = 1.36518, size = 813, normalized size = 5.73

$$\left[\frac{8\,ab^2c^2x+2\left(b^3c^2+b^2c^3\right)x^2+\left(a^2b^2-2\,a^2bc+a^2c^2\right)\sqrt{bc}\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc+\sqrt{bc}(b+c)\right)\sqrt{bx+a}\sqrt{cx+a}\right.}{4\left(b^4c^2-2\,b^3c^3+b^2c^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a^
2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b + c)
)*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b + a*
c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x
+ a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 + b^2*
c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b*c)*sq
rt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x + a*b^2*c
 + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.434 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal. Leaf size=135

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (
2*a*Log[x])/(b - c)^2
```

Rubi [A] time = 0.181448, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6690, 101, 157, 63, 217, 206, 93, 208}

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (
2*a*Log[x])/(b - c)^2
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(b\left(1+\frac{c}{b}\right)+\frac{2a}{x}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{2\int \frac{-a^2-\frac{1}{2}a(b+c)x}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{\left(2a^2\right)\int \frac{1}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(b+c))\int \frac{}{\sqrt{a+}}}{(b-c)}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{\left(4a^2\right)\mathrm{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{(2a}{}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{(2a(b+c))\,\mathrm{Subst}\Big(}{b(b-}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}(b-c)^2\sqrt{c}}+\frac{2a}{(}$$

Mathematica [A] time = 0.843061, size = 195, normalized size = 1.44

$$\frac{\frac{2(b+c)\sqrt{a(b-c)}(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{c}\sqrt{\frac{b(a+cx)}{a(b-c)}}}-(b-c)\left(-2cx\sqrt{a+bx}+bx\sqrt{a+cx}+4a\sqrt{a+cx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)-2a\sqrt{a+bx}+cx\sqrt{a+\cdots}\right.}{(c-b)^3\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((2*Sqrt[a*(b - c)]*(b + c)*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[
a*(b - c)]])/(Sqrt[c]*Sqrt[(b*(a + c*x))/(a*(b - c))]) - (b - c)*(-2*a*Sqrt
[a + b*x] - 2*c*x*Sqrt[a + b*x] + b*x*Sqrt[a + c*x] + c*x*Sqrt[a + c*x] + 4
*a*Sqrt[a + c*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]] + 2*a*Sqrt[a + c*x]*L
og[x]))/((-b + c)^3*Sqrt[a + c*x])
```

Maple [C] time = 0.013, size = 266, normalized size = 2.

$$\frac{bx}{(b-c)^2}+\frac{cx}{(b-c)^2}+2\frac{a\ln(x)}{(b-c)^2}-\frac{\mathrm{csgn}(a)}{(b-c)^2}\sqrt{bx+a}\sqrt{cx+a}\left(\mathrm{csgn}(a)\ln\left(\frac{1}{2}\left(2bcx+2\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}+ab+\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(
1/2)*(csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)
+a*b+a*c)/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a
^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a*c+2*csgn(a)*(b*c)^(1/2)*(b*c*
x^2+a*b*x+a*c*x+a^2)^(1/2)-2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)+b*x+c*x+2*a)/x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b
*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A] time = 1.45708, size = 829, normalized size = 6.14

$$\left[\frac{2abc\log(x)-2abc\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)-2\sqrt{bx+a}\sqrt{cx+a}bc+(ab+ac)\sqrt{bc}\log\left(ab^2+2abc+ac^2+2\left(2bc\cdots\right.\right.}{b^3c-2b^2c^2+bc^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
+ 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log(a
*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c
*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b^2*c
 + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*log(-((
b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c
*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqr
t(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2
 + b*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.435 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal. Leaf size=138

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(
b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b
+ c)*Log[x])/(b - c)^2
```

Rubi [A] time = 0.113665, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6690, 97, 157, 63, 217, 206, 93, 208}

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(
b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b
+ c)*Log[x])/(b - c)^2
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx &= \frac{\int \left(\frac{2a}{x^2}+\frac{b\left(1+\frac{c}{b}\right)}{x}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int\frac{\sqrt{a+bx}\sqrt{a+cx}}{x^2}\,dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int\frac{\frac{1}{2}a(b+c)+bcx}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(2bc)\int\frac{1}{\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(b+c))\ldots}{(\ldots}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(4c)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\,dx,x,\sqrt{a-\ldots}\right.}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(4c)\,\mathrm{Sul}\ldots}{\ldots}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\ldots\right.}{(b-c)^2}
\end{aligned}
$$

Mathematica [A] time = 0.721369, size = 178, normalized size = 1.29

$$\frac{2c\sqrt{a+bx}+\frac{2a\left(\sqrt{a+bx}-\sqrt{a+cx}\right)}{x}+(b+c)\log(x)\sqrt{a+cx}-\frac{4b\sqrt{c}(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a(b-c)}\sqrt{\frac{b(a+cx)}{a(b-c)}}}+2(b+c)\sqrt{a+cx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (2*c*Sqrt[a + b*x] + (2*a*(Sqrt[a + b*x] - Sqrt[a + c*x]))/x - (4*b*Sqrt[c]
*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c
)]*Sqrt[(b*(a + c*x))/(a*(b - c))]) + 2*(b + c)*Sqrt[a + c*x]*ArcTanh[Sqrt[
a + b*x]/Sqrt[a + c*x]] + (b + c)*Sqrt[a + c*x]*Log[x])/((b - c)^2*Sqrt[a +
 c*x])
```

Maple [C] time = 0.012, size = 272, normalized size = 2.

$$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-2\frac{a}{(b-c)^2x}-\frac{\mathrm{csgn}(a)}{(b-c)^2x}\sqrt{bx+a}\sqrt{cx+a}\left(2\,\mathrm{csgn}(a)\ln\left(1/2\,\frac{2\,bcx+2\,\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}+}{\sqrt{bc}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x-1/(b-c)^2*(b*x+a)^(1/2)*(
c*x+a)^(1/2)*(2*csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(
b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*x*b*c-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x
+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*b*(b*c)^(1/2)-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+
a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*c*(b*c)^(1/2)-2*csgn(a)*(b*c)^(1/2)*(b*c
*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x/(b*c
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

Fricas [A] time = 1.38796, size = 810, normalized size = 5.87

$$\left[\frac{2(b+c)x\log(x)-2(b+c)x\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+4\sqrt{bc}x\log\left(ab^2+2abc+ac^2+2\left(2bc-\sqrt{bc}(b+c)\right)\sqrt{bx+}\right.}{2\left(b^2-2bc+c^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sq
rt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*
c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x -
2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x +
 a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*l
og(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*c)*x*a
rctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b
 + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.436 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x]
)/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - Arc
Tanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rubi [A] time = 0.201447, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6690, 94, 93, 208}

$$\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x]
)/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - Arc
Tanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^3}+\frac{b\left(1+\frac{c}{b}\right)}{x^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^3}\right)dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3}\,dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\int \frac{\sqrt{a+cx}}{x^2\sqrt{a+bx}}\,dx}{2(b-c)}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}+\frac{1}{4}\int \frac{1}{x\sqrt{a+bx}\sqrt{}}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{}{-a}\right.$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}$$

Mathematica [A] time = 0.147289, size = 109, normalized size = 0.89

$$\frac{-2a^2-x^2(b-c)^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)+2a\left(\sqrt{a+bx}\sqrt{a+cx}-bx-cx\right)+x(b+c)\sqrt{a+bx}\sqrt{a+cx}}{2ax^2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt[
a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]
])/(2*a*(b - c)^2*x^2)
```

Maple [C] time = 0.015, size = 313, normalized size = 2.5

$$-\frac{b}{x(b-c)^2}-\frac{c}{x(b-c)^2}-\frac{a}{(b-c)^2x^2}+\frac{\operatorname{csgn}(a)}{4(b-c)^2ax^2}\sqrt{bx+a}\sqrt{cx+a}\left(-\ln\left(\frac{a}{x}\left(2\operatorname{csgn}(a)\sqrt{bcx^2+abx+acx+a^2}+b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a*(-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)
*x^2*b^2+2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*
x^2*b*c-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^2
*c^2+2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b+2*csgn(a)*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/
(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Fricas [A] time = 1.30788, size = 308, normalized size = 2.5

$$\frac{4\left(b^2-2\,bc+c^2\right)x^2\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+\left(b^2+2\,bc+c^2\right)x^2+8\,((b+c)x+2\,a)\sqrt{bx+a}\sqrt{cx+a}-16\,a^2-16\,(ab}{16\left(ab^2-2\,abc+ac^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
+ a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x + a
)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*x^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.437 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)^2}$$

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*
x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2
))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b -
c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rubi [A] time = 0.223245, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6690, 96, 94, 93, 208}

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```


```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*
x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2
))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b -
c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)
```


Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^4}+\frac{b\left(1+\frac{c}{b}\right)}{x^3}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^4}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^4}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3}\,dx}{a(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b}{}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(}{}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(}{}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(}{}$$

Mathematica [A] time = 0.179233, size = 153, normalized size = 0.88

$$\frac{a^2\left(8\sqrt{a+bx}\sqrt{a+cx}-6bx-6cx\right)-8a^3+x^2\left(-3b^2+2bc-3c^2\right)\sqrt{a+bx}\sqrt{a+cx}+3x^3(b-c)^2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)+}{12a^2x^3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c - 3*c
^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a + b*x]
*Sqrt[a + c*x]) + 3*(b - c)^2*(b + c)*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*
x]])/(12*a^2*(b - c)^2*x^3)
```

Maple [C] time = 0.011, size = 457, normalized size = 2.6

$$-\frac{b}{2\,x^2\,(b-c)^2}-\frac{c}{2\,x^2\,(b-c)^2}-\frac{2\,a}{3\,(b-c)^2\,x^3}-\frac{\mathrm{csgn}\,(a)}{24\,(b-c)^2\,a^2x^3}\sqrt{bx+a}\sqrt{cx+a}\left(-3\ln\left(\frac{a\left(2\,\mathrm{csgn}\,(a)\sqrt{bcx^2+abx+acx}\right.}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a)
^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)+b*x+c*x+2*a)/x)*x^3*b^3+3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
+b*x+c*x+2*a)/x)*x^3*b^2*c+3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)+b*x+c*x+2*a)/x)*x^3*b*c^2-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/
2)+b*x+c*x+2*a)/x)*x^3*c^3+6*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b^
2-4*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b*c+6*csgn(a)*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*x^2*c^2-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b-
4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*c-16*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Fricas [A] time = 1.3039, size = 408, normalized size = 2.34

$$-\frac{12\left(b^3-b^2c-bc^2+c^3\right)x^3\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+\left(5b^3+3b^2c+3bc^2+5c^3\right)x^3+64a^3+8\left(\left(3b^2-2bc+3\right.\right.}{96\left(a^2b^2-2a^2bc+a^2c^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a^3
 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x + a)*
sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.438 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

Optimal. Leaf size=277

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(}{5b^3(b}$$

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b +
 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7
*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b +
 c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a
 + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rubi [A] time = 0.319216, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6690, 43}

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(}{5b^3(b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b +
 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7
*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b +
 c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a
 + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(4ax\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x^2\sqrt{a+bx}-4ax\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x^2\sqrt{a+cx}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int x\sqrt{a+bx}\,dx}{(b-c)^3} - \frac{(4a)\int x\sqrt{a+cx}\,dx}{(b-c)^3} - \frac{(3b+c)\int x^2\sqrt{a+cx}\,dx}{(b-c)^3} + \frac{(b+3c)\int x^2\sqrt{a+}}{(b-c)^3}$$

$$= \frac{(4a)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(b-c)^3} - \frac{(4a)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3} - \frac{(3b+c)\int\left(\frac{a^2\sqrt{a+cx}}{c^2}-\right.}{(b}$$

$$= -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} + \frac{2(b+}{7}$$

Mathematica [A] time = 0.389686, size = 271, normalized size = 0.98

$$\frac{2\left(8a^3\left(b^4\left(-\sqrt{a+cx}\right)+2b^3c\sqrt{a+cx}-2bc^3\sqrt{a+bx}+c^4\sqrt{a+bx}\right)+4a^2bcx\left(b^3\sqrt{a+cx}-2b^2c\sqrt{a+cx}+2bc^2\sqrt{a+bx}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(5*b^3*c^3*x^3*(b*Sqrt[a + b*x] + 3*c*Sqrt[a + b*x] - 3*b*Sqrt[a + c*x]
- c*Sqrt[a + c*x]) + 4*a^2*b*c*x*(2*b*c^2*Sqrt[a + b*x] - c^3*Sqrt[a + b*x]
 + b^3*Sqrt[a + c*x] - 2*b^2*c*Sqrt[a + c*x]) + 8*a^3*(-2*b*c^3*Sqrt[a + b*
x] + c^4*Sqrt[a + b*x] - b^4*Sqrt[a + c*x] + 2*b^3*c*Sqrt[a + c*x]) + a*b^2
*c^2*x^2*(3*c^2*Sqrt[a + b*x] - 3*b^2*Sqrt[a + c*x] + 29*b*c*(Sqrt[a + b*x]
 - Sqrt[a + c*x]))))/(35*b^3*(b - c)^3*c^3)
```

Maple [A] time = 0.003, size = 246, normalized size = 0.9

$$2\,\frac{1/7\,(bx+a)^{7/2}-2/5\,a\,(bx+a)^{5/2}+1/3\,a^2\,(bx+a)^{3/2}}{(b-c)^3\,b^2}+8\,\frac{a\left(1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+a)^{3/2}\right)}{(b-c)^3\,b^2}-8\,\frac{a\left(1/5\,(cx+a)^{5/2}-1/\right.}{(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))
+8/(b-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-8/(b-c)^3*a/c^2*(1
/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2))+6/(b-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/
5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/2)
-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7/2
)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 1.26517, size = 452, normalized size = 1.63

$$-\frac{2\left(\left(16\,a^3bc^3-8\,a^3c^4-5\left(b^4c^3+3\,b^3c^4\right)x^3-\left(29\,ab^3c^3+3\,ab^2c^4\right)x^2-4\left(2\,a^2b^2c^3-a^2bc^4\right)x\right)\sqrt{bx+a}+\left(8\,a^3b^4-16\,a\right.\right.}{35\left(b^6c^3-3\,b^5c^4+3\,b^4c^5-b^3c^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3*
c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) + (
8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 + 29*
a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 -
 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.439 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*
b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a
+ c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^
3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)
```

Rubi [A] time = 0.217549, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6690, 43}

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*
b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a
+ c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^
3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx &= \frac{\int \left(4a\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x\sqrt{a+bx}-4a\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x\sqrt{a+cx}\right)dx}{(b-c)^3}\\
&= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int x\sqrt{a+cx}\,dx}{(b-c)^3}+\frac{(b+3c)\int x\sqrt{a+bx}\,dx}{(b-c)^3}\\
&= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3}+\frac{(b+3c)\int \left(-\frac{a\sqrt{a+bx}}{b}\right.}{(b-c}\\
&= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}+\frac{2a(3b+c)}{3(b-}
\end{aligned}$$

Mathematica [A] time = 0.435509, size = 120, normalized size = 0.74

$$\frac{2\left(\frac{3(b+3c)(a+bx)^{5/2}}{b^2}-\frac{5a(b+3c)(a+bx)^{3/2}}{b^2}-\frac{3(3b+c)(a+cx)^{5/2}}{c^2}+\frac{5a(3b+c)(a+cx)^{3/2}}{c^2}+\frac{20a(a+bx)^{3/2}}{b}-\frac{20a(a+cx)^{3/2}}{c}\right)}{15(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*((20*a*(a + b*x)^(3/2))/b - (5*a*(b + 3*c)*(a + b*x)^(3/2))/b^2 + (3*(b
+ 3*c)*(a + b*x)^(5/2))/b^2 - (20*a*(a + c*x)^(3/2))/c + (5*a*(3*b + c)*(a
+ c*x)^(3/2))/c^2 - (3*(3*b + c)*(a + c*x)^(5/2))/c^2))/(15*(b - c)^3)
```

Maple [A] time = 0.003, size = 172, normalized size = 1.1

$$2\,\frac{1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+a)^{3/2}}{(b-c)^3\,b}+\frac{8\,a}{3\,(b-c)^3\,b}(bx+a)^{\frac{3}{2}}-\frac{8\,a}{3\,(b-c)^3\,c}(cx+a)^{\frac{3}{2}}+6\,\frac{c\left(1/5\,(bx+a)^{5/2}-1/3\,a\,(bx+\right.}{(b-c)^3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))+8/3*a*(b*x+a)^(3/2)/b/(
b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(1/5*(b*x+a)^(5/2)-1/3
*a*(b*x+a)^(3/2))-6/(b-c)^3*b/c^2*(1/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2))-2
/(b-c)^3/c*(1/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 1.30071, size = 331, normalized size = 2.03

$$\frac{2\left(\left(6\,a^2bc^2-2\,a^2c^3+\left(b^3c^2+3\,b^2c^3\right)x^2+\left(7\,ab^2c^2+abc^3\right)x\right)\sqrt{bx+a}+\left(2\,a^2b^3-6\,a^2b^2c-\left(3\,b^3c^2+b^2c^3\right)x^2-\left(ab^3c+\right.\right.\right.}{5\left(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c^3
```

```
)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 + 3*
b^3*c^4 - b^2*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.440 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

Optimal. Leaf size=155

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)}{3c(b-c)^3}$$

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^
3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b -
c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)
*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rubi [A] time = 0.202475, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6690, 50, 63, 208}

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^
3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b -
c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)
*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}+\frac{4a\sqrt{a+bx}}{x}-3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}-\frac{4a\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{(4a)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{\left(4a^2\right)\int \frac{1}{x\sqrt{a+bx}}\,c}{(b-c)^3}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{\left(8a^2\right)\mathrm{Subst}\left(\int\right.}{b}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}-\frac{8a^{3/2}\tanh^{-1}\left(\sqrt{\right.}{(b-c)^3}$$

Mathematica [A] time = 0.270552, size = 119, normalized size = 0.77

$$\frac{2\left(-12a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+12a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+\frac{(b+3c)(a+bx)^{3/2}}{b}-\frac{(3b+c)(a+cx)^{3/2}}{c}+12a\sqrt{a+bx}-12a\sqrt{a+cx}\right)}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(12*a*Sqrt[a + b*x] + ((b + 3*c)*(a + b*x)^(3/2))/b - 12*a*Sqrt[a + c*x]
 - ((3*b + c)*(a + c*x)^(3/2))/c - 12*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
] + 12*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(3*(b - c)^3)
```

Maple [A] time = 0.004, size = 148, normalized size = 1.

$$\frac{2}{3\,(b-c)^3}(bx+a)^{\frac{3}{2}}+2\,\frac{c\,(bx+a)^{3/2}}{(b-c)^3\,b}-2\,\frac{b\,(cx+a)^{3/2}}{(b-c)^3\,c}-\frac{2}{3\,(b-c)^3}(cx+a)^{\frac{3}{2}}+4\,\frac{a}{(b-c)^3}\left(2\,\sqrt{bx+a}-2\,\sqrt{a}\mathrm{Artanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^(
3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4/(b-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))-4/(b-c)^3*a*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh
((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 1.31923, size = 755, normalized size = 4.87

$$\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)-\left(13\,abc+3\,ac^{2}+\left(b^{2}c+3\,bc^{2}\right)x\right)\sqrt{bx+a}+\left(3\,ab^{2}\cdots\right.\right.}{3\left(b^{4}c-3\,b^{3}c^{2}+3\,b^{2}c^{3}-bc^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3/
2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2 +
 (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^
2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*sqrt(
-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x
+ a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3
*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.441 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\right.}{(b-c)^3}$$

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) -
 (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x)
- (6*Sqrt[a]*(b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (6*Sqrt[a]
*(b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rubi [A] time = 0.217252, antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6690, 47, 63, 208, 50}

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{2\sqrt{a}(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{4\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) -
 (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x)
- (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 - (2*Sqrt[a]*(b +
3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(b - c)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(b - c)^3
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^2}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x}-\frac{4a\sqrt{a+cx}}{x^2}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}+\frac{(b+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(2ab)\int \frac{1}{x\sqrt{a+bx}}\,dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(4a)\,\mathrm{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.}{(b}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-\frac{4\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{}}\right.}{(b-c)^3}$$

Mathematica [A] time = 0.812005, size = 192, normalized size = 1.22

$$\frac{2\left(-(3b+c)\sqrt{a+cx}+(b+3c)\sqrt{a+bx}+\sqrt{a}(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)-\sqrt{a}(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{2a\left(bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\right.}{x\sqrt{a+}}\right.}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*((b + 3*c)*Sqrt[a + b*x] - (3*b + c)*Sqrt[a + c*x] - Sqrt[a]*(b + 3*c)*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]] - (2*a*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTa
nh[Sqrt[1 + (b*x)/a]]))/(x*Sqrt[a + b*x]) + Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]] + (2*a*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 +
(c*x)/a]]))/(x*Sqrt[a + c*x])))/(b - c)^3
```

Maple [A] time = 0.003, size = 237, normalized size = 1.5

$$\frac{b}{(b-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+8\frac{ab}{(b-c)^3}\left(-1/2\frac{\sqrt{bx+a}}{bx}-1/2\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-8\frac{ac}{(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/(b
-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-8/(b-c)^3*a*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/
a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 1.39099, size = 641, normalized size = 4.08

$$\left[-\frac{3\,\sqrt{a}(b+c)x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,\sqrt{a}(b+c)x\log\left(\frac{cx-2\,\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)-2\left((b+3\,c)x-2\,a\right)\sqrt{bx+a}+2\left((3\,b+c)x-\right.}{\left(b^3-3\,b^2c+3\,bc^2-c^3\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqr
t(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c)*
x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(b*x + a)*sqrt(-
a)/a) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) + ((b + 3*c)*
x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c
 + 3*b*c^2 - c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")

[Out] Timed out
```

3.442 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

Optimal. Leaf size=164

$$-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{(2b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{(3b+2c)\sqrt{a+cx}}{x(b-c)^3}-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}+\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - ((2*b + 3*c)*Sqrt[a + b*x])/((b - c)
^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2) + ((3*b + 2*c)*Sqrt[a + c*x])/(
(b - c)^3*x) - (3*b*c*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) +
 (3*b*c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3)
```

Rubi [A] time = 0.1781, antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6690, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}-\frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}+\frac{(3b+}{x(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)^3*x) - ((
b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2)
 + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b + c)*Sqrt[a + c*x])/((b - c)^3*x
) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a
+ c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(Sqrt[a]*(b - c)^3)
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^3}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x^2}-\frac{4a\sqrt{a+cx}}{x^3}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^3}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^3}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}+\frac{(b+3c)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x}+\frac{(ab)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+cx}}{(b-c)^3x}+\frac{(3b+c)\sqrt{a-}}{(b-c)^3x}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+cx}}{(b-c)^3x}+\frac{(3b+c)\sqrt{a-}}{(b-c)^3x}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+cx}}{(b-c)^3x}+\frac{(3b+c)\sqrt{a-}}{(b-c)^3x}$$

Mathematica [C] time = 0.260628, size = 182, normalized size = 1.11

$$\frac{-\frac{8b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{a^2}+\frac{8c^2(a+cx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{a}+1\right)}{a^2}-\frac{3(b+3c)\left(bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{x\sqrt{a+bx}}+\frac{3(3b+c)\left(cx\sqrt{\frac{cx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)\right.}{x\sqrt{a+cx}}}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] ((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])
)/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh
[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hypergeome
tric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hypergeomet
ric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

Maple [B] time = 0.003, size = 300, normalized size = 1.8

$$2\,{\frac{{b}^{2}}{ \left(b-c \right) ^{3}} \left(-1/2\,{\frac{\sqrt{bx+a}}{bx}}-1/2\,{\frac{1}{\sqrt{a}}{\rm Artanh} \left({\frac{\sqrt{bx+a}}{\sqrt{a}}}\right)} \right) }+8\,{\frac{a{b}^{2}}{ \left(b-c \right) ^{3}} \left({\frac{1}{{b}^{2}{x}^{2}} \left(-1/8\,{\frac{ \left(bx+a \right) ^{3/2}}{a}}-1/8\,\sqrt{bx+a} \right) }+1/8\,{\frac{1}{{a}^{3/2}}{\rm Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+8/(b-c)^3*a*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/b^2/x^2+1/
8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8/a*(c*x+a)^
(3/2)-1/8*(c*x+a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^
(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((
c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

Fricas [A] time = 1.32698, size = 703, normalized size = 4.29

$$\left[-\frac{3\,\sqrt{a}bcx^{2}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,\sqrt{a}bcx^{2}\log\left(\frac{cx-2\,\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)+2\left(2\,a^{2}+(2\,ab+3\,ac)x\right)\sqrt{bx+a}-2\left(2\,a^{2}+(3\,ab+\right.}{2\left(ab^{3}-3\,ab^{2}c+3\,abc^{2}-ac^{3}\right)x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*s
qrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 + (2
*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a
))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*arcta
n(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*sqrt(
-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*a*
c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.443 $\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=31

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0461493, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6688, 195, 216}

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx &= \int \left(1-x+\sqrt{1-x^2}\right)\,dx\\
&= x-\frac{x^2}{2}+\int \sqrt{1-x^2}\,dx\\
&= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.015452, size = 31, normalized size = 1.

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Maple [B] time = 0.003, size = 63, normalized size = 2.

$$x-\frac{x^2}{2}-\frac{1}{2}\sqrt{1+x}\,(1-x)^{\frac{3}{2}}+\frac{1}{2}\sqrt{1-x}\sqrt{1+x}+\frac{\arcsin(x)}{2}\sqrt{(1-x)(1+x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] x-1/2*x^2-1/2*(1+x)^(1/2)*(1-x)^(3/2)+1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1-x
)*(1+x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.57816, size = 31, normalized size = 1.

$$-\frac{1}{2}x^2+\frac{1}{2}\sqrt{-x^2+1}x+x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

Fricas [A] time = 1.29629, size = 122, normalized size = 3.94

$$-\frac{1}{2}x^2+\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}+x-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x)
```

Sympy [A] time = 2.49646, size = 48, normalized size = 1.55

$$-\frac{(1-x)^2}{2}-2\left(\begin{cases}-\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x}}{2}\right)}{2} & \text{for } x\le 1\wedge x>-1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(1 - x)**2/2 - 2*Piecewise((-x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sq
rt(1 - x)/2)/2, (x <= 1) & (x > -1)))
```

Giac [A] time = 1.19635, size = 51, normalized size = 1.65

$$-\frac{1}{2}\,(x-1)^2+\frac{1}{2}\,\sqrt{x+1}x\sqrt{-x+1}-\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(x - 1)^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arcsin(1/2*sqrt(2)*sqrt(-
x + 1))
```

3.444 $\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Optimal. Leaf size=38

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rubi [A] time = 0.324781, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6742, 266, 43}

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx = -\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$$
$$= -\int \left(2x^3+2x^3\sqrt{1-x^2}\right)dx$$
$$= -\frac{x^4}{2}-2\int x^3\sqrt{1-x^2}\,dx$$
$$= -\frac{x^4}{2}-\text{Subst}\left(\int \sqrt{1-x}x\,dx,x,x^2\right)$$
$$= -\frac{x^4}{2}-\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right)dx,x,x^2\right)$$
$$= -\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}$$

Mathematica [A] time = 0.0417771, size = 38, normalized size = 1.

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Maple [A] time = 0.002, size = 33, normalized size = 0.9

$$-\frac{x^4}{2}-\frac{\left(2x^2-2\right)\left(3x^2+2\right)}{15}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -1/2*x^4-2/15*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Maxima [A] time = 1.6875, size = 42, normalized size = 1.11

$$-\frac{1}{2}x^4+\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2+\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 1.22415, size = 81, normalized size = 2.13

$$-\frac{1}{2}x^4-\frac{2}{15}\left(3x^4-x^2-2\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19235, size = 76, normalized size = 2.

$$-\frac{1}{2}(x+1)^4+2(x+1)^3-\frac{2}{15}((3(x+1)(x-3)+17)(x+1)-10)(x+1)^{\frac{3}{2}}\sqrt{-x+1}-3(x+1)^2+2x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -1/2*(x + 1)^4 + 2*(x + 1)^3 - 2/15*((3*(x + 1)*(x - 3) + 17)*(x + 1) - 10)
*(x + 1)^(3/2)*sqrt(-x + 1) - 3*(x + 1)^2 + 2*x + 2
```

3.445 $\int x^2 \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Optimal. Leaf size=48

$$-\frac{1}{2}\sqrt{1-x^2}x^3-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{4}\sin^{-1}(x)$$

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Rubi [A] time = 0.241919, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {6688, 6742, 279, 321, 216}

$$-\frac{1}{2}\sqrt{1-x^2}x^3-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx \\
&= -\int\left(2x^2+2x^2\sqrt{1-x^2}\right)dx \\
&= -\frac{2x^3}{3}-2\int x^2\sqrt{1-x^2}\,dx \\
&= -\frac{2x^3}{3}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx \\
&= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}dx \\
&= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0403239, size = 43, normalized size = 0.9

$$\frac{1}{12}\left(-\left(6\sqrt{1-x^2}+8\right)x^3+3\sqrt{1-x^2}x-3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 3*ArcSin[x])/12
```

Maple [A] time = 0.001, size = 59, normalized size = 1.2

$$-\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x}\sqrt{1+x}\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2/3*x^3-1/4*(1+x)^(1/2)*(1-x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.50632, size = 46, normalized size = 0.96

$$-\frac{2}{3}x^3+\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{1}{4}\sqrt{-x^2+1}x-\frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Fricas [A] time = 1.05418, size = 135, normalized size = 2.81

$$-\frac{2}{3}x^3 - \frac{1}{4}\left(2x^3 - x\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21582, size = 84, normalized size = 1.75

$$-\frac{2}{3}(x+1)^3 + 2(x+1)^2 - \frac{1}{4}((2(x+1)(x-2)+5)(x+1)-1)\sqrt{x+1}\sqrt{-x+1} - 2x - \frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -2/3*(x + 1)^3 + 2*(x + 1)^2 - 1/4*((2*(x + 1)*(x - 2) + 5)*(x + 1) - 1)*sq
rt(x + 1)*sqrt(-x + 1) - 2*x - 1/2*arcsin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

3.446 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Optimal. Leaf size=21

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rubi [A] time = 0.11288, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6688, 6742, 261}

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx\\
&= -\int \left(2x+2x\sqrt{1-x^2}\right)dx\\
&= -x^2-2\int x\sqrt{1-x^2}\,dx\\
&= -x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0175248, size = 21, normalized size = 1.

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Maple [A] time = 0.002, size = 26, normalized size = 1.2

$$-x^2 - \frac{2x^2 - 2}{3}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -x^2-2/3*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)
```

Maxima [A] time = 1.53081, size = 23, normalized size = 1.1

$$-x^2 + \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] -x^2 + 2/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 0.945034, size = 63, normalized size = 3.

$$-x^2 - \frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [A] time = 76.971, size = 110, normalized size = 5.24

$$\frac{x^3}{3} + x - \frac{(x+1)^3}{3} + 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x \geq -1 \wedge x < 1 \end{cases}\right) - 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**3/3 + x - (x + 1)**3/3 + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin
(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 4*Piecewise((x*sqrt(1 -
```

```
x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x +
1)/2)/2, (x >= -1) & (x < 1))) + 1
```

Giac [A] time = 1.11091, size = 39, normalized size = 1.86

$$-\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}-(x+1)^2+2x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] -2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) - (x + 1)^2 + 2*x + 2
```

3.447 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Optimal. Leaf size=22

$$-\sqrt{1-x^2}x-2x-\sin^{-1}(x)$$

```
[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rubi [A] time = 0.0545297, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6688, 6742, 195, 216}

$$-\sqrt{1-x^2}x-2x-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx \\
&= -\int \left(2+2\sqrt{1-x^2}\right)dx \\
&= -2x-2\int \sqrt{1-x^2}\,dx \\
&= -2x-x\sqrt{1-x^2}-\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -2x-x\sqrt{1-x^2}-\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0154404, size = 21, normalized size = 0.95

$$-x\left(\sqrt{1-x^2}+2\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -(x*(2 + Sqrt[1 - x^2])) - ArcSin[x]
```

Maple [B] time = 0.003, size = 59, normalized size = 2.7

$$-2x+\sqrt{1+x}\,(1-x)^{\frac{3}{2}}-\sqrt{1-x}\sqrt{1+x}-\arcsin(x)\sqrt{(1-x)(1+x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2*x+(1+x)^(1/2)*(1-x)^(3/2)-(1-x)^(1/2)*(1+x)^(1/2)-((1-x)*(1+x))^(1/2)/(1
-x)^(1/2)/(1+x)^(1/2)*arcsin(x)
```

Maxima [A] time = 1.53986, size = 27, normalized size = 1.23

$$-\sqrt{-x^2+1}x-2x-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

Fricas [B] time = 1.01104, size = 108, normalized size = 4.91

$$-\sqrt{x+1}x\sqrt{-x+1}-2x+2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [A] time = 30.6975, size = 46, normalized size = 2.09

$$-2x-4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x\geq -1\wedge x<1\end{cases}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/
2)/2, (x >= -1) & (x < 1))) - 2
```

Giac [A] time = 1.14039, size = 45, normalized size = 2.05

$$-\sqrt{x+1}x\sqrt{-x+1} - 2\,x - 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

3.448 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=32

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rubi [A] time = 0.195786, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6688, 6742, 266, 50, 63, 206}

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]

[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx\\
&= -\int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx\\
&= -2\log(x)-2\int \frac{\sqrt{1-x^2}}{x}\,dx\\
&= -2\log(x)-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right)\\
&= -2\sqrt{1-x^2}-2\log(x)-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= -2\sqrt{1-x^2}-2\log(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0295521, size = 32, normalized size = 1.

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Maple [A] time = 0.002, size = 51, normalized size = 1.6

$$-2\,\ln(x)-2\,\frac{\sqrt{1-x}\sqrt{1+x}\left(\sqrt{-x^2+1}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x)
```

```
[Out] -2*ln(x)-2*(1+x)^(1/2)*(1-x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

Maxima [A] time = 1.46965, size = 55, normalized size = 1.72

$$-2\sqrt{-x^2+1}-2\,\log(x)+2\,\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.919679, size = 111, normalized size = 3.47

$$-2\sqrt{x+1}\sqrt{-x+1} - 2\log(x) - 2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="fricas")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2}{x}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.449 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

```
[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rubi [A] time = 0.205232, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6742, 277, 216}

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

```
[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx \\ &= -\int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx \\ &= \frac{2}{x}-2\int \frac{\sqrt{1-x^2}}{x^2}\,dx \\ &= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0253906, size = 22, normalized size = 0.85

$$\frac{2\left(\sqrt{1-x^2}+x\sin^{-1}(x)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

```
[Out] (2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x
```

Maple [B] time = 0.002, size = 50, normalized size = 1.9

$$2\,x^{-1}-2\,{\frac{\left(-\arcsin\left(x\right)x-\sqrt{-x^2+1}\right)\sqrt{1+x}\sqrt{1-x}}{x\sqrt{-x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x)
```

```
[Out] 2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1+x)^(1/2)*(1-x)^(1/2)/x/(-x^2+1)^(1/2
)
```

Maxima [A] time = 1.65774, size = 32, normalized size = 1.23

$$\frac{2\sqrt{-x^2+1}}{x}+\frac{2}{x}+2\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

Fricas [A] time = 0.968857, size = 113, normalized size = 4.35

$$-\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="fricas")
```

```
[Out] -2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2}{x^2}\,dx-\int\frac{2\sqrt{1-x}\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

```
[Out] -Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

Giac [B] time = 1.19291, size = 201, normalized size = 7.73

$$2\,\pi+\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}+\frac{2}{x}+4\,\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="giac")
```

```
[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x
 + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

3.450 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.215426, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6688, 6742, 266, 47, 63, 206}

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx \\
&= -\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx \\
&= \frac{1}{x^2}-2\int \frac{\sqrt{1-x^2}}{x^3}\,dx \\
&= \frac{1}{x^2}-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right) \\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right) \\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right) \\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0395861, size = 46, normalized size = 1.39

$$\frac{1}{x^2\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) - 1/Sqrt[1 - x^2] + 1/(x^2*Sqrt[1 - x^2]) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.001, size = 57, normalized size = 1.7

$$x^{-2}-\frac{1}{x^2}\sqrt{1-x}\sqrt{1+x}\left(\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x^2-\sqrt{-x^2+1}\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x)
```

```
[Out] 1/x^2-(1+x)^(1/2)*(1-x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2)
)/x^2/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.67616, size = 69, normalized size = 2.09

$$\sqrt{-x^2+1}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2}+\frac{1}{x^2}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x)
 + 2/abs(x))
```

Fricas [A] time = 0.979822, size = 108, normalized size = 3.27

$$\frac{x^2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="fricas")
```

```
[Out] (x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/
x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2}{x^3}\,dx-\int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

```
[Out] -Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.451 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=28

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rubi [A] time = 0.320949, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {2103, 6688, 14, 266, 50, 63, 206}

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx &= \frac{1}{2}\int \frac{\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx+\frac{1}{2}\int \frac{\sqrt{1+x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx\\
&= \frac{1}{2}\int \frac{1-x+\sqrt{1-x^2}}{x}\,dx+\frac{1}{2}\int \frac{1+x+\sqrt{1-x^2}}{x}\,dx\\
&= \frac{1}{2}\int \left(-1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx+\frac{1}{2}\int \left(1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx\\
&= \log(x)+2\left(\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x}\,dx\right)\\
&= \log(x)+2\left(\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right)\right)\\
&= \log(x)+2\left(\frac{\sqrt{1-x^2}}{2}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\right)\\
&= \log(x)+2\left(\frac{\sqrt{1-x^2}}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\right)\\
&= 2\left(\frac{\sqrt{1-x^2}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)\right)+\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.158765, size = 48, normalized size = 1.71

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)+2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\sin^{-1}(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] + 2*ArcSin[Sqrt[1 - x]/Sqrt[2]] + ArcSin[x] - ArcTanh[Sqrt[1
- x^2]] + Log[x]
```

Maple [A] time = 0.005, size = 48, normalized size = 1.7

$$\ln(x)+\sqrt{1-x}\sqrt{1+x}\left(\sqrt{-x^2+1}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] ln(x)+(1+x)^(1/2)*(1-x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x^
2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}+\sqrt{-x+1}}{\sqrt{x+1}-\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

Fricas [A] time = 0.923263, size = 101, normalized size = 3.61

$$\sqrt{x+1}\sqrt{-x+1}+\log\left(x\right)+\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx-\int \frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x + 1
)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.452 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{x-1}\sqrt{x+1}x + \frac{1}{2}\cosh^{-1}(x)$$

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rubi [A] time = 0.142982, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2104, 6742, 38, 52}

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{x-1}\sqrt{x+1}x + \frac{1}{2}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\left(\frac{1}{2}\int \sqrt{-1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)dx\right)+\frac{1}{2}\int \sqrt{1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)dx$$
$$= \frac{1}{2}\int \left(1+x-\sqrt{-1+x}\sqrt{1+x}\right)dx-\frac{1}{2}\int \left(1-x+\sqrt{-1+x}\sqrt{1+x}\right)dx$$
$$= \frac{x^2}{2}-2\left(\frac{1}{2}\int \sqrt{-1+x}\sqrt{1+x}\,dx\right)$$
$$= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}x\sqrt{1+x}-\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right)$$
$$= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}x\sqrt{1+x}-\frac{1}{4}\cosh^{-1}(x)\right)$$

Mathematica [A] time = 0.161114, size = 58, normalized size = 1.76

$$\frac{1}{2}\left(x^2-\sqrt{x-1}\sqrt{x+1}x+\frac{2(x-1)\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{\sqrt{-(x-1)^2}}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] (1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] + (2*(-1 + x)*ArcSin[Sqrt[1 - x]/Sqrt
[2]])/Sqrt[-(-1 + x)^2])/2
```

Maple [B] time = 0.004, size = 62, normalized size = 1.9

$$-\frac{1}{2}\sqrt{x-1}\,(1+x)^{\frac{3}{2}}+\frac{1}{2}\sqrt{x-1}\sqrt{1+x}+\frac{1}{2}\sqrt{(x-1)(1+x)}\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{x-1}}\frac{1}{\sqrt{1+x}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-1)^(1/2)+(1+x)^(1/2))/((x-1)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -1/2*(x-1)^(1/2)*(1+x)^(3/2)+1/2*(x-1)^(1/2)*(1+x)^(1/2)+1/2*((x-1)*(1+x))^
(1/2)/(1+x)^(1/2)/(x-1)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Fricas [A] time = 0.958409, size = 109, normalized size = 3.3

$$-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}x+\frac{1}{2}x^2-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
- x)
```

Sympy [A] time = 25.2528, size = 226, normalized size = 6.85

$$-\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4}+2\left(\begin{cases}\frac{(x+1)^2}{8}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}-\frac{\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1\\ \frac{(x+1)^2}{8}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}+\frac{i\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(x
 - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + 2*Piecewise(((x + 1)**2/8 + acosh(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(5/2)/(8*sqrt(x - 1)) + 3*(x + 1)**(3/2)/
(8*sqrt(x - 1)) - sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)*
*2/8 - I*asin(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(5/2)/(8*sqrt(1 - x)) -
 3*I*(x + 1)**(3/2)/(8*sqrt(1 - x)) + I*sqrt(x + 1)/(4*sqrt(1 - x)), True))
 + asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

Giac [A] time = 1.1937, size = 57, normalized size = 1.73

$$\frac{1}{2}(x+1)^2-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}x-x-\log\left(\left|-\sqrt{x+1}+\sqrt{x-1}\right|\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(abs(-sqrt(x + 1) +
sqrt(x - 1))) - 1
```

3.453 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=121

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*f^2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n,
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*d^2*e*(1 + n))
```

Rubi [A] time = 0.10316, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2117, 947, 64}

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*f^2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n,
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*d^2*e*(1 + n))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\text{Subst}\left(\int \frac{x^n(d^2+af^2-2dx+x^2)}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(x^n+\frac{af^2x^n}{(d-x)^2}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{(af^2)\,\text{Subst}\left(\int \frac{x^n}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{d+ex+f}{}\right.}{2d^2e(1+n)}$$

Mathematica [A] time = 0.117929, size = 86, normalized size = 0.71

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\left(af^2\,{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)+d^2\right)}{2d^2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*(d^2 + a*f^2*Hypergeometric2
F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d]))/(2*d^2*e*(1
+ n))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.454 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=175

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}+\frac{ad}{}$$

```
[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2)
/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log[e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rubi [A] time = 0.133334, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2117, 893}

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}+\frac{ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2)
/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log[e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x^3(d^2+af^2-2dx+x^2)}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(2adf^2+\frac{ad^3f^2}{(d-x)^2}-\frac{3ad^2f^2}{d-x}+af^2x+x^3\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}{4e}+$$

Mathematica [A] time = 0.32916, size = 158, normalized size = 0.9

$$\frac{-\frac{4ad^3f^2}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+12ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4+2af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2+8adf^2\left(f\right.}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] ((-4*a*d^3*f^2)/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 8*a*d*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]) + 2*a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2 + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4 + 12*a*d^2*f^2*Log[e*x + f*Sqrt[a + (
e^2*x^2)/f^2]])/(8*e)
```

Maple [A] time = 0.01, size = 175, normalized size = 1.

$$f^3x\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}+e^3x^4+2\,de^2x^3+\frac{3\,f^2aex^2}{2}+3\,f^2adx+2\,\frac{df^3}{e}\left(\frac{e^2x^2+af^2}{f^2}\right)^{3/2}+\frac{3\,fd^2x}{2}\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{3\,fd^2a}{2}\ln\left(\frac{e^2}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] f^3*x*(a+e^2*x^2/f^2)^(3/2)+e^3*x^4+2*d*e^2*x^3+3/2*f^2*a*e*x^2+3*f^2*a*d*x
+2*d/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+3/2*f*d^2*x*(a+e^2*x^2/f^2)^(1/2)+3/
2*f*d^2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/
2)+3/2*e*x^2*d^2+x*d^3+1/4*d^4/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.997505, size = 333, normalized size = 1.9

$$\frac{2\,e^4x^4+4\,de^3x^3-3\,ad^2f^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\left(ae^2f^2+d^2e^2\right)x^2+2\left(3\,adef^2+d^3e\right)x+\left(2\,e^3fx^3+4\,de^2fx^2+4\,a\cdots\right.}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^2 + d^3*e)*x + (2*e^
3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*f^3 + 3*d^2*e*f)*x)*sqrt((e^2*
x^2 + a*f^2)/f^2))/e
```

Sympy [A] time = 9.46108, size = 279, normalized size = 1.59

$$\frac{a^{\frac{3}{2}}f^3x\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{a^{\frac{3}{2}}f^3x}{2\sqrt{1+\frac{e^2x^2}{af^2}}}+\frac{3\sqrt{a}d^2fx\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{3\sqrt{a}e^2fx^3}{2\sqrt{1+\frac{e^2x^2}{af^2}}}+\frac{3ad^2f^2\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}+3adf^2x+\frac{3aef^2x^2}{2}+d^3x\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] a**(3/2)*f**3*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a**(3/2)*f**3*x/(2*sqrt(1
+ e**2*x**2/(a*f**2))) + 3*sqrt(a)*d**2*f*x*sqrt(1 + e**2*x**2/(a*f**2))/2
+ 3*sqrt(a)*e**2*f*x**3/(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*a*d**2*f**2*as
inh(e*x/(sqrt(a)*f))/(2*e) + 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + d**3*x + 3*
d**2*e*x**2/2 + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2,
 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True)) + e**3*x**4 + e**4
*x**5/(sqrt(a)*f*sqrt(1 + e**2*x**2/(a*f**2)))
```

Giac [A] time = 1.23474, size = 220, normalized size = 1.26

$$-\frac{3}{2}\,ad^2f\left|f\right|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+\frac{3}{2}\,af^2x^2e+3\,adf^2x+x^4e^3+2\,dx^3e^2+\frac{3}{2}\,d^2x^2e+d^3x+\frac{1}{2}\left(4\,adf\left|f\right|e^{(-1)}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*d^2*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + 3/2*a*f
^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 1/
2*(4*a*d*f*abs(f)*e^(-1) + (2*(x*abs(f)*e^2/f + 2*d*abs(f)*e/f)*x + (2*a*f^
4*abs(f)*e^4 + 3*d^2*f^2*abs(f)*e^4)*e^(-4)/f^3)*x)*sqrt(a*f^2 + x^2*e^2)
```

3.455 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=136

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

```
[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*e)
 + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Rubi [A] time = 0.103329, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2117, 893}

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*e)
 + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2(d^2+af^2-2dx+x^2)}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \left(af^2+\frac{ad^2f^2}{(d-x)^2}-\frac{2adf^2}{d-x}+x^2\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$
$$= -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}{6e} + \frac{adf^2 \text{lo}}{}$$

Mathematica [A] time = 0.201769, size = 128, normalized size = 0.94

$$\frac{\frac{ad^2f^2}{f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex}+\frac{1}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3+2adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] ((a*d^2*f^2)/(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(e*x + f*Sqrt[a +
 (e^2*x^2)/f^2]) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/3 + 2*a*d*f^2*Lo
g[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Maple [A] time = 0.003, size = 126, normalized size = 0.9

$$f^2ax+\frac{2e^2x^3}{3}+\frac{2f^3}{3e}\left(\frac{e^2x^2+af^2}{f^2}\right)^{\frac{3}{2}}+fdx\sqrt{a+\frac{e^2x^2}{f^2}}+dfa\ln\left(\frac{e^2x}{f^2}\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}+ex^2d+xd^2+\frac{d^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] f^2*a*x+2/3*e^2*x^3+2/3/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+f*d*x*(a+e^2*x^2/
f^2)^(1/2)+f*d*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f
^2)^(1/2)+e*x^2*d+x*d^2+1/3*d^3/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.01359, size = 239, normalized size = 1.76

$$\frac{2\,e^3x^3+3\,de^2x^2-3\,adf^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\left(aef^2+d^2e\right)x+\left(2\,e^2fx^2+2\,af^3+3\,defx\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2
)/f^2)) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^3 + 3*d*e*f*x)*sqrt(
(e^2*x^2 + a*f^2)/f^2))/e
```

Sympy [A] time = 3.70792, size = 116, normalized size = 0.85

$$\sqrt{a}dfx\sqrt{1+\frac{e^2x^2}{af^2}}+\frac{adf^2\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{e}+af^2x+d^2x+dex^2+\frac{2e^2x^3}{3}+2ef\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } e^2=0\\ \frac{f^2\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] sqrt(a)*d*f*x*sqrt(1 + e**2*x**2/(a*f**2)) + a*d*f**2*asinh(e*x/(sqrt(a)*f)
)/e + a*f**2*x + d**2*x + d*e*x**2 + 2*e**2*x**3/3 + 2*e*f*Piecewise((sqrt(
a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True))
```

Giac [A] time = 1.17823, size = 139, normalized size = 1.02

$$-adf|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+af^2x+\frac{2}{3}x^3e^2+dx^2e+d^2x+\frac{1}{3}\left(2\,af|f|e^{(-1)}+\left(\frac{2\,x|f|e}{f}+\frac{3\,d|f|}{f}\right)x\right)\sqrt{af^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -a*d*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + a*f^2*x + 2/3
*x^3*e^2 + d*x^2*e + d^2*x + 1/3*(2*a*f*abs(f)*e^(-1) + (2*x*abs(f)*e/f + 3
*d*abs(f)/f)*x)*sqrt(a*f^2 + x^2*e^2)
```

3.456 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right) dx$

Optimal. Leaf size=68

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f
*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rubi [A] time = 0.0339407, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {195, 217, 206}

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f
*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)dx = dx+\frac{ex^2}{2}+f\int \sqrt{a+\frac{e^2x^2}{f^2}}dx$$
$$= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\int \frac{1}{\sqrt{a+\frac{e^2x^2}{f^2}}}dx$$
$$= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\,\text{Subst}\left(\int \frac{1}{1-\frac{e^2x^2}{f^2}}dx, x, \frac{x}{\sqrt{a+\frac{e^2x^2}{f^2}}}\right)$$
$$= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

Mathematica [A] time = 0.0449172, size = 81, normalized size = 1.19

$$\frac{1}{2}fx\sqrt{\frac{af^2+e^2x^2}{f^2}}+\frac{af^2\log\left(ef\sqrt{\frac{af^2+e^2x^2}{f^2}}+e^2x\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[(a*f^2 + e^2*x^2)/f^2])/2 + (a*f^2*Log[e^2*x +
e*f*Sqrt[(a*f^2 + e^2*x^2)/f^2]])/(2*e)
```

Maple [A] time = 0.003, size = 75, normalized size = 1.1

$$dx+\frac{ex^2}{2}+\frac{fx}{2}\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af}{2}\ln\left(\frac{e^2x}{f^2}\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*x*(a+e^2*x^2/f^2)^(1/2)+1/2*f*a*ln(e^2*x/f^2/(e^2/f^2)^
(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.984576, size = 153, normalized size = 2.25

$$\frac{e^2x^2 - af^2\log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + efx\sqrt{\frac{e^2x^2+af^2}{f^2}} + 2\,dex}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + e*f*x*sqrt
((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e
```

Sympy [A] time = 2.15634, size = 54, normalized size = 0.79

$$dx + \frac{ex^2}{2} + f\left(\frac{\sqrt{a}x\sqrt{1 + \frac{e^2x^2}{af^2}}}{2} + \frac{af\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*f*asinh(e*
x/(sqrt(a)*f))/(2*e))
```

Giac [A] time = 1.15571, size = 88, normalized size = 1.29

$$\frac{1}{2}\,x^2e + dx - \frac{\left(af^2e^{(-1)}\log\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right) - \sqrt{af^2 + x^2e^2}x\right)|f|}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x - 1/2*(a*f^2*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2)))
- sqrt(a*f^2 + x^2*e^2)*x)*abs(f)/f
```

3.457 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=117

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

```
[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqr
t[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*e)
```

Rubi [A] time = 0.0941509, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2117, 893}

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqr
t[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d(d-x)^2}+\frac{af^2}{d^2(d-x)}+\frac{d^2+af^2}{d^2x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e} + \frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+f\sqrt{a+\frac{e^2}{f}}\right.}{2e}$$

Mathematica [A] time = 0.142102, size = 109, normalized size = 0.93

$$\frac{-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^2}+\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)+\frac{af^2}{d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] ((a*f^2)/(d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt
[a + (e^2*x^2)/f^2]])/d^2 + (1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]])/(2*e)
```

Maple [B] time = 0.036, size = 1325, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x)
```

```
[Out] -1/4*f/d/e*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1
/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)-1/4*f/d^2*ln(
(1/2*e*(a*f^2-d^2)/f^2/d+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(e^2/f^2)^(1/2)+
(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)
/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)*a+1/4/f
*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(e^2/f^2)^(1
/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+
d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)+1/4
*f^3/d^3/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*
f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4
*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2
/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/d/e/((a^2*f^4+2*a*d^2*f^2+
d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)
/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/
2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f
^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/
```

```
d/e))*a+1/4/f*d/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^
4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2
*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^
2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d
^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2*ln(a*f^2-2*d*e*x-d^2)/e+1
/2/d*x+1/4/d^2/e*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4/e*ln(-a*f^2+2*d*e*x+d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ex + \sqrt{\frac{e^2x^2}{f^2} + af} + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 1.06278, size = 394, normalized size = 3.37

$$\frac{2\,dex - 2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}} + \left(af^2 + d^2\right)\log\left(af^2 - dex + df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + \left(af^2 + d^2\right)\log\left(-af^2 + 2\,dex + d^2\right) - \left(af^2 + d^2\right.}{4\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*log(a*f^2
- d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2*d
*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) +
 (a*f^2 - d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)))/(d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, +\infty, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

3.458 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}$$

```
[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^
2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d
^3*e)
```

Rubi [A] time = 0.112668, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2117, 893}

$$-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^
2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d
^3*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^2(d-x)^2}+\frac{2af^2}{d^3(d-x)}+\frac{d^2+af^2}{d^2x^2}+\frac{2af^2}{d^3x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e} + \cdots$$

Mathematica [A] time = 0.290328, size = 141, normalized size = 0.93

$$-\frac{\frac{af^2}{d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\frac{af^2}{d^2}+1}{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}+\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3}-\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -((a*f^2)/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (1 + (a*f^2)/d^2)/(d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/
f^2]])/d^3 - (2*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^3)/(2*e)
```

Maple [B] time = 0.023, size = 4136, normalized size = 27.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] -1/4*f^5/e/d^2/(a^2*f^4+2*a*d^2*f^2+d^4)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/
f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4
)/f^2/d^2)^(1/2)*a^2-1/4*f^5/d^3/(a^2*f^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*f^2
-d^2)/f^2/d+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(
-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^
2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)*a^3-f^3/d/(a^2*f^4+2
*a*d^2*f^2+d^4)*(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+
1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*x*a+1/4*
f^3/e/d^4/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^
2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f
^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*
e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/
d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/e/d^2/((a^2*f^4+2*a*d^2*f^2
+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2
)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1
/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*
f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)
```

```
/d/e))*a+d*f^3/e^2/(a^2*f^4+2*a*d^2*f^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d/e)*(e^2
*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e
)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(3/2)-3/4/d*f^3/(a^2*f^4+2*a*d^2*f
^2+d^4)*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(e^2/
f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(
-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1
/2)*a^2-3/4*d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2*(
x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^
2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2
+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d^4/f/e/(a^2*f^4+2*a*d^2*f^2+d^
4)/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d
^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*
d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^
2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1
/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/4*f^7/e/d^4/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^
2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2
/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2
+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/
f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x
+1/2*(-a*f^2+d^2)/d/e))*a^4-1/4*f/e/d^2*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f
^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)
/f^2/d^2)^(1/2)+1/4/f/d*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2*(x+1/2*(-a*f^2+d^2)
/d/e)/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2
)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/
2))/(e^2/f^2)^(1/2)+1/4/f/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1
/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2
)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+
d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a
*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2/d^2*x+1/2/d^2*f
^5/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln
((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+
d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f
^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+
2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^3-1/2*d^2*f/e/
(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2
*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/
d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^
2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a+f^5/e^2/d/(a^2*f^4+
2*a*d^2*f^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d/e)*(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/
f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d
^4)/f^2/d^2)^(3/2)*a-1/2*a*f^2/(-a*f^2+2*d*e*x+d^2)/d/e+1/4*d^2*f/e/(a^2*f^
4+2*a*d^2*f^2+d^4)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^
2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)-1/4*d
^3/f/(a^2*f^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2*(x+1/2*(-a*f
^2+d^2)/d/e)/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*
f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d
^2)^(1/2))/(e^2/f^2)^(1/2)-d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*(e^2*(x+1/2*(-a*f^
2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4
+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*x-1/4*f/d^3*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^
2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/
e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*
f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4/e/d^3/(-a*f^2+2*d*e*x+d^2)*a
^2*f^4+1/2/e/d^3*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4*d/(-a*f^2+2*d*e*x+d^2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

Fricas [B] time = 1.12494, size = 579, normalized size = 3.83

$$\frac{a^2f^4-2\,d^2e^2x^2+ad^2f^2-2\,d^3ex+\left(a^2f^4-2\,adef^2x-ad^2f^2\right)\log\left(-aef^2x+2\,de^2x^2+adf^2+\left(af^3-2\,defx\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right.}{2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 - 2*a*d*e*f
^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f
*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*lo
g(-a*f^2 + 2*d*e*x + d^2) - (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-e*x
+ f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*d*f^3 - d^2*e*f*x)*sqrt((e^2*x^
2 + a*f^2)/f^2))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.459 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+$$

```
[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(4
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f*S
qrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(
2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rubi [A] time = 0.127463, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2117, 893}

$$-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(4
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f*S
qrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(
2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^3}\,dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^3(d-x)^2}+\frac{3af^2}{d^4(d-x)}+\frac{d^2+af^2}{d^2x^3}+\frac{2af^2}{d^3x^2}+\frac{3af^2}{d^4x}\right)dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} -$$

Mathematica [A] time = 0.574181, size = 180, normalized size = 0.93

$$\frac{\frac{af^2}{d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)} - \frac{2af^2}{d^3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)} - \frac{\frac{af^2}{d^2}+1}{2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2} - \frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^4} + \frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] ((a*f^2)/(d^3*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2*
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (2*a*f^2)/(d^3*(d + e*x + f*Sqrt
[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4
+ (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4)/(2*e)
```

Maple [B] time = 0.036, size = 9721, normalized size = 50.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

Fricas [B] time = 2.33707, size = 1081, normalized size = 5.6

$$\frac{5\,a^{3}f^{6}+8\,d^{3}e^{3}x^{3}-6\,a^{2}d^{2}f^{4}-3\,ad^{4}f^{2}+2\left(ad^{2}e^{2}f^{2}+5\,d^{4}e^{2}\right)x^{2}-2\left(7\,a^{2}def^{4}+ad^{3}ef^{2}-2\,d^{5}e\right)x+3\left(a^{3}f^{6}+4\,ad^{2}e^{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*(a*d^2*e^2
*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2 - 2*d^5*e)*x + 3*(a^
3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 -
a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x
)*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d
^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*d*e*x
+ d^2) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(
a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d)
 - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3 - 3*(3*a*d^2*e*f^3 - d^4*
e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d
^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^{2}x^{2}}{f^{2}}}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.460 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=225

$$-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d\right.}{3e}$$

```
[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rubi [A] time = 0.185237, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1810, 206}

$$-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-ad^2f^2-2adf^2x^2-2af^2x^4+2dx^6-2x^8}{d-x^2}\,dx, x, \sqrt{d\right.}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(4adf^2+2af^2x^2+2x^6-\frac{5ad^2f^2}{d-x^2}\right) dx, x\right.}{2e}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex+f\sqrt{\right.}}{3e}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex+f\sqrt{\right.}}{3e}$$

Mathematica [A] time = 0.343763, size = 213, normalized size = 0.95

$$\frac{-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)+\frac{2}{7}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}+\frac{2}{3}af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (4*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d^2*f^2*Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*
f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (2*(d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2])^(7/2))/7 - 5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt
[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Fricas [A] time = 1.39386, size = 946, normalized size = 4.2

$$\left[\frac{105\,ad^{\frac{3}{2}}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(24\,e^3x^3+36\,de^2\right.}{84}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x +
f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*
d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d
*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/e, 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*sqrt((
e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (24*e^3*x^3 + 36*d*e^2*x^2 + 116*a
*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*
d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2
+ a*f^2)/f^2) + d))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.461 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=183

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[
Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rubi [A] time = 0.149687, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1810, 206}

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[
Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2} dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2} dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{adf^2+2af^2x^2-2dx^4+2x^6}{d-x^2} dx, x, \sqrt{d+ex+f\sqrt{\ldots}}\right.}{2e}$$

$$= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \left(-2af^2-2x^4+\frac{3adf^2}{d-x^2}\right) dx, x, \sqrt{d+ex+\ldots}\right.}{2e}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\ldots\right.}{5e}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\ldots\right.}{5e}$$

Mathematica [A] time = 0.233944, size = 175, normalized size = 0.96

$$\frac{\frac{2}{5}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}+2af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (2*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d*f^2*Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/5 - 3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d +
 e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\,f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Fricas [A] time = 1.35343, size = 752, normalized size = 4.11

$$\left[\frac{15\,a\sqrt{d}f^2\log\left(af^2 - 2\,dex + 2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}} + 2\left(\sqrt{d}ex - \sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}} + d}\right) + 2\left(4\,e^2x^2 + 12\,af^2 + \right.}{20\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/
f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*
```

```
d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x
^2 + a*f^2)/f^2) + d))/e, 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt(
(e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (4*e^2*x^2 + 12*a*f^2 + 9*d*e*x +
 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.462 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=147

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

```
[Out] -(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (
e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^
2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

Rubi [A] time = 0.122284, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1153, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (
e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^
2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
```

```
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-af^2+2dx^2-2x^4}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(2x^2-\frac{af^2}{d-x^2}\right)dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{\left(af^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx,\right.}{2}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+}}}{\sqrt{d}}\right.}{2\sqrt{d}e}$$

Mathematica [A] time = 0.329926, size = 139, normalized size = 0.95

$$-\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-\frac{2}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*
x^2)/f^2]) - (2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(2*e)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 1.36777, size = 664, normalized size = 4.52

$$\left[\frac{3\,a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(5\,dex-df\sqrt{\frac{e^2x^2+a}{f^2}}\right.}{12\,de}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f
^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e), 1/6*(3
*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-
d)/d) + (5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sq
rt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.463 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=147

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rubi [A] time = 0.108602, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1157, 388, 206}

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2\sqrt{x}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d^2-af^2+2dx^2}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2de}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(af^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{\quad}\right.}{2de}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

Mathematica [A] time = 0.250127, size = 143, normalized size = 0.97

$$\frac{\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)} + \sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]])/(2*d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)))/e
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 1.33248, size = 659, normalized size = 4.48

$$\left[\frac{a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right.}{4\,d^2e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e), -1/2*(a*s
qrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d
) - (d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^
```

```
2*x^2 + a*f^2)/f^2) + d))/(d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.464 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}+\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2
*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2
*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/(2*d^(5/2)*e)
```

Rubi [A] time = 0.155884, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1259, 453, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}+\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2
*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2
*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/(2*d^(5/2)*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{3/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^2\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d\left(d^2+af^2\right)+\left(2d^2-af^2\right)x^2}{x^2\left(d-x^2\right)}\,dx, x, \sqrt{d+ex+f\sqrt{\ldots}}\right.}{2d^2e}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(3af^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\ldots\right.}{\ldots}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{3af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\ldots}}{\sqrt{\ldots}}\right)}{2d^{5/2}e}$$

Mathematica [A] time = 0.408244, size = 167, normalized size = 1.06

$$\frac{\frac{-2d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)-af^2\left(3f\sqrt{a+\frac{e^2x^2}{f^2}}+d+3ex\right)}{d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}+\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{d^{5/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] ((-2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - a*f^2*(d + 3*e*x + 3*f*Sqrt[a
+ (e^2*x^2)/f^2]))/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d + e*x + f*
Sqrt[a + (e^2*x^2)/f^2]]) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]]/Sqrt[d]])/d^(5/2))/(2*e)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Fricas [A] time = 1.38619, size = 1037, normalized size = 6.56

$$\left[\frac{3\left(a^2f^4-2\,adef^2x-ad^2f^2\right)\sqrt{d}\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)}{4\left(ad^3ef^2-2\,d^4e^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*e*x +
 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x
^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) - 2*(2*d^2
*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d
^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e), -1/2*(3*(a^2*f^4 -
2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2) + d)*sqrt(-d)/d) + (2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e
*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/
f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e
^2*x - d^5*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.465 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+e\ldots}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

```
[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2*a
*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2
*d^(7/2)*e)
```

Rubi [A] time = 0.176685, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1259, 1261, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+e\ldots}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2*a
*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2
*d^(7/2)*e)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{5/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^4(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{2d^2(d^2+af^2)-2d(d^2-af^2)x^2+af^2x^4}{x^4(d-x^2)}\,dx, x, \sqrt{d+ex+\cdots}\right.}{2d^3e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(\frac{2(d^3+adf^2)}{x^4}+\frac{4af^2}{x^2}+\frac{5af^2}{d-x^2}\right)dx, x, \sqrt{d+ex+\cdots}\right.}{2d^3e}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\cdots}}}{2d^3e\left(ex+f\sqrt{a+\cdots}\right.}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\cdots}}}{2d^3e\left(ex+f\sqrt{a+\cdots}\right.}$$

Mathematica [A] time = 0.613331, size = 186, normalized size = 0.93

$$-\frac{\frac{2d(af^2+d^2)}{3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}} + \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex} + \frac{4af^2}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}} - \frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2d^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -((2*d*(d^2 + a*f^2))/(3*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) + (4*
a*f^2)/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x + f*
Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (5*a*f^2*ArcT
anh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(2*d^3*e)
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Fricas [B] time = 1.55992, size = 1683, normalized size = 8.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a
^2*d*e*f^4 - a*d^3*e*f^2)*x)*sqrt(d)*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*
x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))
*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(12*d^3*e^3*x^3 + 10*a^
2*d^2*f^4 - 16*a*d^4*f^2 - 2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*
a^2*d*e*f^4 - 46*a*d^3*e*f^2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2
- 22*a*d^3*f^3 - d^5*f - 8*(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f
^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4
*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x), -1/6
*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*
e*f^4 - a*d^3*e*f^2)*x)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)
/f^2) + d)*sqrt(-d)/d) - (12*d^3*e^3*x^3 + 10*a^2*d^2*f^4 - 16*a*d^4*f^2 -
2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*a^2*d*e*f^4 - 46*a*d^3*e*f^
2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2 - 22*a*d^3*f^3 - d^5*f - 8*
(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt
((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^
2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.466 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

```
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rubi [A] time = 0.0163519, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2117, 14}

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - Sqrt[-4 + x^2]],x]
```

```
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x-\sqrt{-4+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-4+x^2}{x^{3/2}}\,dx, x, x-\sqrt{-4+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4}{x^{3/2}}+\sqrt{x}\right)dx, x, x-\sqrt{-4+x^2}\right)\\ &= \frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0138785, size = 40, normalized size = 0.98

$$\frac{2x^2-2\sqrt{x^2-4}x+8}{3\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]
```

```
[Out] (8 + 2*x^2 - 2*x*Sqrt[-4 + x^2])/(3*Sqrt[x - Sqrt[-4 + x^2]])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2-4)^(1/2))^(1/2),x)
```

```
[Out] int((x-(x^2-4)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

Fricas [A] time = 0.977727, size = 69, normalized size = 1.68

$$\frac{2}{3}\left(2\,x+\sqrt{x^2-4}\right)\sqrt{x-\sqrt{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))
```

Sympy [A] time = 0.345232, size = 42, normalized size = 1.02

$$\frac{4x\sqrt{x-\sqrt{x^2-4}}}{3}+\frac{2\sqrt{x-\sqrt{x^2-4}}\sqrt{x^2-4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2-4)**(1/2))**(1/2),x)
```

```
[Out] 4*x*sqrt(x - sqrt(x**2 - 4))/3 + 2*sqrt(x - sqrt(x**2 - 4))*sqrt(x**2 - 4)/
3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

3.467 $\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=69

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Rubi [A] time = 0.0570967, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2117, 14}

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2c+x^2}{x^{3/2}}\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2c}{x^{3/2}}+\sqrt{x}\right)dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$
$$= -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}} + \frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

Mathematica [A] time = 0.0574548, size = 67, normalized size = 0.97

$$\frac{2\left(abx\sqrt{\frac{a^2x^2}{b^2}+c}+a^2x^2+b^2(-c)\right)}{3a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] (2*(-(b^2*c) + a^2*x^2 + a*b*x*Sqrt[c + (a^2*x^2)/b^2]))/(3*a*Sqrt[a*x + b*
Sqrt[c + (a^2*x^2)/b^2]])
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

Fricas [A] time = 0.99495, size = 120, normalized size = 1.74

$$\frac{2\left(2\,ax - b\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)\sqrt{ax + b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b^
2*c)/b^2))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + b\sqrt{\frac{a^2x^2}{b^2} + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

3.468 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=45

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

```
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Rubi [A] time = 0.009672, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2129}

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx=-\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

Mathematica [A] time = 0.0659507, size = 35, normalized size = 0.78

$$\frac{2x\left(\sqrt{1-x^2}+2\right)}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

Maple [C] time = 0.027, size = 60, normalized size = 1.3

$$\frac{\frac{i}{8}}{\sqrt{\pi}}\left(\frac{32\,i}{3}\sqrt{\pi}\sqrt{2}x^3\cos\left(\frac{3\arcsin(x)}{2}\right)-8\,i\sqrt{\pi}\sqrt{2}\left(-\frac{4\,x^4}{3}+\frac{2\,x^2}{3}+\frac{2}{3}\right)\sin\left(\frac{3\arcsin(x)}{2}\right)\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-x^2+1)^(1/2))^(1/2),x)
```

```
[Out] 1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2)
*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Fricas [A] time = 1.07101, size = 80, normalized size = 1.78

$$\frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

Sympy [B] time = 1.13682, size = 413, normalized size = 9.18

$$\begin{cases}\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}+12\pi\sqrt{i\sqrt{x^2-1}+1}}-\frac{3\sqrt{2}ix\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}+12\pi\sqrt{i\sqrt{x^2-1}+1}}-\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}+12\pi\sqrt{i\sqrt{x^2-1}+1}} & \text{for } \left|x^2\right|>1\\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}}-\frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}}-\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*I*pi*sqrt(x**2 - 1)*sqrt
(I*sqrt(x**2 - 1) + 1) + 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) - 3*sqrt(2)*I*x*
sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x
**2 - 1) + 1) + 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)
```

```
*gamma(1/4)/(12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) + 12*pi*sqrt
(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamma(1/
4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**
2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1
- x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*sqrt
(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1)
 + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

3.469 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}+\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0073905, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}+\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]],x]

[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx=\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

Mathematica [A] time = 0.0572284, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}+2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]

[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]]*(2 + Sqrt[1 + x^2]))/(3*x)
```

Maple [C] time = 0.017, size = 55, normalized size = 1.3

$$-\frac{1}{8\sqrt{\pi}}\left(-\frac{32\sqrt{\pi}\sqrt{2}x^3}{3}\cosh\left(\frac{3\,\mathrm{Arcsinh}\,(x)}{2}\right)-8\,\frac{\sqrt{\pi}\sqrt{2}\left(-4/3\,x^4-2/3\,x^2+2/3\right)\sinh\left(3/2\,\mathrm{Arcsinh}\,(x)\right)}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2
^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Fricas [A] time = 1.08139, size = 77, normalized size = 1.88

$$\frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

Sympy [B] time = 1.08258, size = 197, normalized size = 4.8

$$-\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}x\sqrt{x^2+1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)**(1/2)+1)**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 +
 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gam
ma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*
sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(
x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

3.470 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}+\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}$$

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Rubi [A] time = 0.0074054, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}+\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

Mathematica [A] time = 0.0566115, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+25}-5\right)\sqrt{\sqrt{x^2+25}+5}\left(\sqrt{x^2+25}+10\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*(-5 + Sqrt[25 + x^2])*Sqrt[5 + Sqrt[25 + x^2]]*(10 + Sqrt[25 + x^2]))/(3
*x)
```

Maple [C] time = 0.016, size = 64, normalized size = 1.6

$$-\frac{5\sqrt{5}}{8\sqrt{\pi}}\left(-\frac{32\sqrt{\pi}\sqrt{2}x^3}{375}\cosh\left(\frac{3}{2}\mathrm{Arcsinh}\left(\frac{x}{5}\right)\right)-8\frac{\sqrt{\pi}\sqrt{2}\sinh\left(3/2\,\mathrm{Arcsinh}\left(x/5\right)\right)}{\sqrt{1/25\,x^2+1}}\left(-\frac{4\,x^4}{1875}-\frac{2\,x^2}{75}+2/3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+(x^2+25)^(1/2))^(1/2),x)
```

```
[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x)
)-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/(1
/25*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Fricas [A] time = 1.18357, size = 84, normalized size = 2.05

$$\frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

Sympy [B] time = 1.11623, size = 197, normalized size = 4.8

$$-\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}-\frac{15\sqrt{2}x\sqrt{x^2+25}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}-\frac{}{12\pi\sqrt{x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
+ 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 + 25
)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) +
 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25)
+ 5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

3.471 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

Optimal. Leaf size=66

$$\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}+\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*
Sqrt[a^2/b^2 + c*x^2]]
```

Rubi [A] time = 0.0329764, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2129}

$$\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}+\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*
Sqrt[a^2/b^2 + c*x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

Mathematica [A] time = 0.217532, size = 55, normalized size = 0.83

$$\frac{2bx\sqrt{\frac{a^2}{b^2}+cx^2}+4ax}{3\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (4*a*x + 2*b*x*Sqrt[a^2/b^2 + c*x^2])/(3*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}\,b+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Fricas [A] time = 1.82889, size = 144, normalized size = 2.18

$$\frac{2\left(b^2cx^2+ab\sqrt{\frac{b^2cx^2+a^2}{b^2}}-a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}}+a}}{3\,b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c*
x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{cx^2 + \frac{a^2}{b^2}}b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

3.472 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=166

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)\left(2de-bf^2\right)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+\right.}{2e(n+1)}$$

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (f^2*(4
*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e - b*f^2)^2*(1 + n))
```

Rubi [A] time = 0.179684, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2116, 947, 64}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)\left(2de-bf^2\right)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+\right.}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (f^2*(4
*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e - b*f^2)^2*(1 + n))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
```

```
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = 2\,\text{Subst}\left(\int\frac{x^n\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx,x,d+ex+f\sqrt{a+bx\right.$$

$$=2\,\text{Subst}\left(\int\left(\frac{x^n}{4e}+\frac{\left(4ae^2f^2-b^2f^4\right)x^n}{4e\left(2de-bf^2-2ex\right)^2}\right)dx,x,d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$=\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{\left(4ae^2f^2-b^2f^4\right)\text{Subst}\left(\int\frac{x^n}{\left(2de-bf^2-2ex\right)^2}dx,x,d\right.}{2e}$$

$$=\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+}}{2e\left(2de-bf^2\right.}$$

Mathematica [A] time = 0.302087, size = 134, normalized size = 0.81

$$\frac{\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{n+1}\left(\left(4ae^2f^2-b^2f^4\right){}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{a+x\left(\frac{xe^2}{f^2}+b\right)}\right)}{2de-bf^2}\right)+\left(bf^2-2de\right)^2\right)}{2e(n+1)\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(1 + n)*((-2*d*e + b*f^2)^2 +
(4*a*e^2*f^2 - b^2*f^4)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)]))/(2*d*e - b*f^2)]))/(2*e*(-2*d*e + b*f^2)^
2*(1 + n))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.473 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=303

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}-\frac{f^2\left(4ae^2-\right.}{32e^5\left(2e\left(f\sqrt{a\right.\right.}}$$

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*
x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e
) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2
 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
32*e^5)
```

Rubi [A] time = 0.384399, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}-\frac{f^2\left(4ae^2-\right.}{32e^5\left(2e\left(f\sqrt{a\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```


```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*
x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e
) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2
 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
32*e^5)
```


Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = 2\,\text{Subst}\left(\int \frac{x^3\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx,x,d+ex+f\sqrt{a+\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)}{16e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)x}{16e^3}+\frac{x^3}{4e}+\frac{f^2\left(2a\right.}{32e}\right.\right.$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(d+\right.}{1}$$

Mathematica [A] time = 0.570442, size = 276, normalized size = 0.91

$$\frac{2e^2f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^2+4ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-\frac{f^2(}{2e\left(f\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*
x)/f^2)]) + 2*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*(b + (e^2
*x)/f^2)])^2 + 4*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^4 - (f^2*(
-2*d*e + b*f^2)^3*(-4*a*e^2 + b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)])) - 3*(-4*a*e^2 + b^2*f^2)*(-2*d*e*f + b*f^3)^2*Log[-(b*f^
2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(32*e^5)
```

Maple [B] time = 0.01, size = 685, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] 1/4*d^4/e+f^3*(a+b*x+e^2*x^2/f^2)^(3/2)*x+3/2*e*x^2*d^2+x*d^3+e^3*x^4+3/4*d
^2/e^2*f^3*(a+b*x+e^2*x^2/f^2)^(1/2)*b+3/2*f*d^2*ln((1/2*b+e^2*x/f^2)/(e^2/
f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a-3/4*d/e^3*f^5*b^2*(
a+b*x+e^2*x^2/f^2)^(1/2)+3/8*f^5/e^2*b^2*(a+b*x+e^2*x^2/f^2)^(1/2)*x-3/32*f
^7/e^4*b^4*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/
(e^2/f^2)^(1/2)-3/2*d/e*f^3*b*(a+b*x+e^2*x^2/f^2)^(1/2)*x+3/8*d/e^3*f^5*b^3
*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^
(1/2)-3/8*d^2/e^2*f^3*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f
^2)^(1/2))/(e^2/f^2)^(1/2)*b^2+2*d/e*f^3*(a+b*x+e^2*x^2/f^2)^(3/2)+3/2*f*d^
2*(a+b*x+e^2*x^2/f^2)^(1/2)*x+f^2*x^3*b*e+3/2*f^2*x^2*b*d-1/2*f^5/e^2*(a+b*
x+e^2*x^2/f^2)^(3/2)*b+3/16*f^7/e^4*b^3*(a+b*x+e^2*x^2/f^2)^(1/2)+3/2*f^2*a
```

```
*e*x^2+3*f^2*a*d*x+2*d*e^2*x^3+3/8*f^5/e^2*a*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)
^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*b^2-3/2*d/e*f^3*b*ln((1/2
*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8759, size = 717, normalized size = 2.37

$$\frac{32\,e^{8}x^{4} + 32\,\left(be^{6}f^{2} + 2\,de^{7}\right)x^{3} + 48\,\left(d^{2}e^{6} + \left(bde^{5} + ae^{6}\right)f^{2}\right)x^{2} + 32\,\left(3\,ade^{5}f^{2} + d^{3}e^{5}\right)x + 3\,\left(b^{4}f^{8} - 16\,ad^{2}e^{4}f^{2} - 4\,\left(b^{3}de\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (b*d*e^5 +
a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b^4*f^8 - 16*a*d^2*e^
4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*lo
g(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(3*b^
3*e*f^7 + 16*e^7*f*x^3 - 4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*e^3 +
 8*a*d*e^4)*f^3 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12*d^2*e
^5*f - 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
/e^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^{2}x^{2}}{f^{2}}}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**3, x)
```

Giac [A] time = 1.17376, size = 504, normalized size = 1.66

$$bf^{2}x^{3}e + \frac{3}{2}\,bdf^{2}x^{2} + \frac{3}{2}\,af^{2}x^{2}e + 3\,adf^{2}x + x^{4}e^{3} + 2\,dx^{3}e^{2} + \frac{3}{2}\,d^{2}x^{2}e + d^{3}x + \frac{3}{32}\,\left(b^{4}f^{7}|f| - 4\,b^{3}df^{5}|f|e - 4\,ab^{2}f^{5}|f|e^{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] b*f^2*x^3*e + 3/2*b*d*f^2*x^2 + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2
*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*f^5*abs
(f)*e - 4*a*b^2*f^5*abs(f)*e^2 + 4*b^2*d^2*f^3*abs(f)*e^2 + 16*a*b*d*f^3*ab
s(f)*e^3 - 16*a*d^2*f*abs(f)*e^4)*e^(-5)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f
^2*x + a*f^2 + x^2*e^2))*e)) + 1/16*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*(
2*x*abs(f)*e^2/f + (b*f^4*abs(f)*e^6 + 4*d*f^2*abs(f)*e^7)*e^(-6)/f^3)*x -
(b^2*f^6*abs(f)*e^4 - 4*b*d*f^4*abs(f)*e^5 - 8*a*f^4*abs(f)*e^6 - 12*d^2*f^
2*abs(f)*e^6)*e^(-6)/f^3)*x + (3*b^3*f^8*abs(f)*e^2 - 12*b^2*d*f^6*abs(f)*e
^3 - 8*a*b*f^6*abs(f)*e^4 + 12*b*d^2*f^4*abs(f)*e^4 + 32*a*d*f^4*abs(f)*e^5
)*e^(-6)/f^3)
```

3.474 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=237

$$-\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{8e^4}+\frac{f^2(4$$

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) +
 (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)
^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 +
e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)
```

Rubi [A] time = 0.239352, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{8e^4}+\frac{f^2(4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) +
 (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)
^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 +
e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2dx = 2\,\text{Subst}\left(\int\frac{x^2\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx, x, d+ex+f\sqrt{a+\right.$$
$$= 2\,\text{Subst}\left(\int\left(\frac{4ae^2f^2-b^2f^4}{16e^3}+\frac{x^2}{4e}+\frac{\left(4ae^2-b^2f^2\right)\left(2def-bf^3\right)^2}{16e^3\left(2de-bf^2-2ex\right)^2}-\frac{f^2\left(2de-\right.}{8e^3\left(2d\right.}\right.\right.$$
$$= \frac{f^2\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}{6e}-\frac{}{16}$$

Mathematica [A] time = 0.33063, size = 213, normalized size = 0.9

$$\frac{\frac{3\left(b^2f^2-4ae^2\right)\left(bf^3-2def\right)^2}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+6f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)+6ef^2\left(4ae^2-b^2f^2\right.}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (6*e*f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 8*e^
3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^3 + (3*(-4*a*e^2 + b^2*f^2)*(
-2*d*e*f + b*f^3)^2)/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))
+ 6*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*S
qrt[a + x*(b + (e^2*x)/f^2)])])/(48*e^4)
```

Maple [A] time = 0.005, size = 409, normalized size = 1.7

$$f^2ax+\frac{f^2x^2b}{2}+\frac{2\,e^2x^3}{3}+\frac{2\,f^3}{3\,e}\left(a+bx+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}-\frac{bf^3x}{2\,e}\sqrt{a+bx+\frac{e^2x^2}{f^2}}-\frac{b^2f^5}{4\,e^3}\sqrt{a+bx+\frac{e^2x^2}{f^2}}-\frac{bf^3a}{2\,e}\ln\left(\left(\frac{b}{2}+\frac{e^2x}{f^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] f^2*a*x+1/2*f^2*x^2*b+2/3*e^2*x^3+2/3*(a+b*x+e^2*x^2/f^2)^(3/2)/e*f^3-1/2*b
/e*f^3*(a+b*x+e^2*x^2/f^2)^(1/2)*x-1/4*b^2/e^3*f^5*(a+b*x+e^2*x^2/f^2)^(1/2
)-1/2*b/e*f^3*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2
))/(e^2/f^2)^(1/2)*a+1/8*b^3/e^3*f^5*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(
a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)+f*d*(a+b*x+e^2*x^2/f^2)^(1/2)*x+1
/2*d/e^2*f^3*(a+b*x+e^2*x^2/f^2)^(1/2)*b+f*d*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)
^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d/e^2*f^3*ln((1/2*b
+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*b^2+
e*x^2*d+x*d^2+1/3*d^3/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51377, size = 460, normalized size = 1.94

$$\frac{16\,e^{6}x^{3}+12\left(be^{4}f^{2}+2\,de^{5}\right)x^{2}+24\left(ae^{4}f^{2}+d^{2}e^{4}\right)x-3\left(b^{3}f^{6}+8\,ade^{3}f^{2}-2\left(b^{2}de+2\,abe^{2}\right)f^{4}\right)\log\left(-bf^{2}-2\,e^{2}x+2\,ef\right.}{24\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 + d^2*e^4)*
x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*log(-b*f^2 -
2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(3*b^2*e*f^5 - 8
*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)
```

Giac [A] time = 1.1727, size = 302, normalized size = 1.27

$$\frac{1}{2}\,bf^{2}x^{2}+af^{2}x+\frac{2}{3}\,x^{3}e^{2}+dx^{2}e+d^{2}x-\frac{1}{8}\left(b^{3}f^{5}|f|-2\,b^{2}df^{3}|f|e-4\,abf^{3}|f|e^{2}+8\,adf|f|e^{3}\right)e^{(-4)}\log\left(\left|-bf^{2}-2\left(xe-\sqrt{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x - 1/8*(b^3*f^5*abs(
f) - 2*b^2*d*f^3*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*d*f*abs(f)*e^3)*e^(-
4)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 1/12*sq
rt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*x*abs(f)*e/f + (b*f^3*abs(f)*e^3 + 6*d*
f*abs(f)*e^4)*e^(-4)/f^2)*x - (3*b^2*f^5*abs(f)*e - 6*b*d*f^3*abs(f)*e^2 -
8*a*f^3*abs(f)*e^3)*e^(-4)/f^2)
```

3.475 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx$

Optimal. Leaf size=118

$$\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^
2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])])/(8*e^3)
```

Rubi [A] time = 0.0625189, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {612, 621, 206}

$$\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^
2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])])/(8*e^3)
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)dx = dx+\frac{ex^2}{2}+f\int \sqrt{a+bx+\frac{e^2x^2}{f^2}}dx$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{8}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\int \frac{1}{\sqrt{a+bx+\frac{e^2x^2}{f^2}}}$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{4}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{4e^2}{f^2}-x^2}\right.$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2}{f}}}\right.}{8e^3}$$

Mathematica [A] time = 0.196747, size = 120, normalized size = 1.02

$$\frac{1}{8}\left(\frac{\left(4ae^2f^2-b^2f^4\right)\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{e^3}+\frac{2bf^3\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{e^2}+4fx\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+8dx+4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] (8*d*x + 4*e*x^2 + (2*b*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)])/e^2 + 4*f*x*Sqrt
[a + x*(b + (e^2*x)/f^2)] + ((4*a*e^2*f^2 - b^2*f^4)*Log[b*f^2 + 2*e*(e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e^3)/8
```

Maple [A] time = 0.005, size = 173, normalized size = 1.5

$$dx+\frac{ex^2}{2}+\frac{fx}{2}\sqrt{a+bx+\frac{e^2x^2}{f^2}}+\frac{f^3b}{4e^2}\sqrt{a+bx+\frac{e^2x^2}{f^2}}+\frac{af}{2}\ln\left(\left(\frac{b}{2}+\frac{e^2x}{f^2}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}-\frac{f^3b^2}{8e^2}\ln\left(\left(\frac{b}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*(a+b*x+e^2*x^2/f^2)^(1/2)*x+1/4/e^2*f^3*(a+b*x+e^2*x^2/
f^2)^(1/2)*b+1/2*f*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)
^(1/2))/(e^2/f^2)^(1/2)*a-1/8/e^2*f^3*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+
(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27721, size = 258, normalized size = 2.19

$$\frac{4\,e^4x^2 + 8\,de^3x + \left(b^2f^4 - 4\,ae^2f^2\right)\log\left(-bf^2 - 2\,e^2x + 2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right) + 2\left(bef^3 + 2\,e^3fx\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}{8\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*e^4*x^2 + 8*d*e^3*x + (b^2*f^4 - 4*a*e^2*f^2)*log(-b*f^2 - 2*e^2*x +
 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(b*e*f^3 + 2*e^3*f*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2), x)
```

Giac [A] time = 1.18502, size = 150, normalized size = 1.27

$$\frac{1}{2}x^2e + dx + \frac{\left(\left(b^2f^4 - 4\,af^2e^2\right)e^{(-3)}\log\left(\left|-bf^2 - 2\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right)e\right|\right) + 2\sqrt{bf^2x + af^2 + x^2e^2}\left(bf^2e^{(-2)} + 2\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x + 1/8*((b^2*f^4 - 4*a*f^2*e^2)*e^(-3)*log(abs(-b*f^2 - 2*(x
*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 2*sqrt(b*f^2*x + a*f^2 + x^2*e^
2)*(b*f^2*e^(-2) + 2*x))*abs(f)/f
```

3.476 $\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=215

$$-\frac{f^2(4ae^2-b^2f^2)}{2e(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{2e(2de-bf^2)^2}+\frac{2(aef^2}{}$$

```
[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[
a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*
x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 - b
^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*e*
(2*d*e - b*f^2)^2)
```

Rubi [A] time = 0.192901, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{f^2(4ae^2-b^2f^2)}{2e(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{2e(2de-bf^2)^2}+\frac{2(aef^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[
a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*
x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 - b
^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*e*
(2*d*e - b*f^2)^2)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2}{f}}\right.$$

$$= 2\,\text{Subst}\left(\int\left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x}+\frac{4ae^2f^2-b^2f^4}{2\left(2de-bf^2\right)\left(2de-bf^2-2ex\right)^2}+\frac{4a}{2\left(2de-bf\right.}\right.\right.$$

$$= -\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{2\left(d^2e-bdf^2+aef^2\right)\log\left(\right.}{\left(2de-\right.}$$

Mathematica [A] time = 0.220899, size = 187, normalized size = 0.87

$$\frac{-\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{e\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(b^2f^2-4ae^2\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)}{e}+4\left(aef^2-bdf^2+d^2e\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right.}{2\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (-((f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(e*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)])))) + 4*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + (f^2*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2)
 - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e)/(2*(-2*d*e + b*f^2)^2)
```

Maple [B] time = 0.056, size = 4918, normalized size = 22.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x)
```

```
[Out] f/(b*f^2-2*d*e)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*
f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+
(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^
2)/f^2/(b*f^2-2*d*e)^2)^(1/2)+1/2*f^3/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*
a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2*(x+(a*f^2-d^2)/(b*f^
2-2*d*e))/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b
^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/
(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d
^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)*b^2-f/(b*f^2-
2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)+e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2
-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2
/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2
*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))
/(e^2/f^2)^(1/2)*a*e^2-f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b
```

```
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))/f^2
)/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^
2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)
)+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*
e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)*b*d*e+1/f/(b*f^2-2*d*e)^2*
ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2
*(x+(a*f^2-d^2)/(b*f^2-2*d*e))/f^2)/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*
f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+
2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2
)^(1/2)*e^2*d^2-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4
+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a
^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)
/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^
2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^
2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^
2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d
^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*
d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e
)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*a^2*e^2+2*f^3/(b*f^2-2*d*e)^3/((
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2
*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2
*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))
+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^
4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-
(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^
2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*
b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*
e)))*a*b*d*e-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*
a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*
e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^
2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f
^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(
x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*
e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e
*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2
)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b^2*d^2-2*f/(b*f^2-2*d*e)^3/((a^2*e
^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2
/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^
2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+
2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2
*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b
*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3
*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*
a*d^2*e^2+2*f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d
^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2
*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(
b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*
e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+
2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(
a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2
)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^
4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(
1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b*d^3*e-1/f/(b*f^2-2*d*e)^3/((a^2*e^2*
f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b
```

```
*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e
^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2
*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f
^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^
4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^
2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*
f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*d^4
*e^2-d*ln((b*f^2-2*d*e)*x+a*f^2-d^2)/(b*f^2-2*d*e)-e/(b*f^2-2*d*e)*x+e/(b*f
^2-2*d*e)^2*ln(b*f^2*x+a*f^2-2*d*e*x-d^2)*a*f^2-e/(b*f^2-2*d*e)^2*ln(b*f^2*
x+a*f^2-2*d*e*x-d^2)*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 20.7166, size = 756, normalized size = 3.52

$$-\frac{2\left(be^2f^2 - 2\,de^3\right)x - 2\left(d^2e^2 - \left(bde - ae^2\right)f^2\right)\log\left((bd - 2\,ae)f^2 - \left(bef^2 - 2\,de^2\right)x + \left(bf^3 - 2\,def\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right) - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log((b*
d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e*f)*sqrt((b*f^2*x +
e^2*x^2 + a*f^2)/f^2)) - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log(a*f^2 - d^2
+ (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*log(-b
*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(d^2*e^2
- (b*d*e - a*e^2)*f^2)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) -
 d) - 2*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(b^2*e
*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, +\infty, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

3.477 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2-b\right.}{}$$

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 -
(2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^
2*x))/f^2])])/(2*d*e - b*f^2)^3
```

Rubi [A] time = 0.232046, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 -
(2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^
2*x))/f^2])])/(2*d*e - b*f^2)^3
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^2\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2}{f}}\right.$$

$$= 2\,\text{Subst}\left(\int\left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^2}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x}+\frac{4ae^3f^2-b^2ef^4}{\left(2de-bf^2\right)^2\left(2de-bf^2-2ex\right.}\right.\right.$$

$$= -\frac{2\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\sqrt{a+\right.\right.}}$$

Mathematica [A] time = 0.340708, size = 237, normalized size = 0.89

$$-\frac{\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+2f^2\left(b^2f^2-4ae^2\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)-2f^2\left(b^2f^2-4ae^2\right)\log\left(-2e\left(f\sqrt{a+x\left(b\right.}\right.\right.}{\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -((((2*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)]) + (f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(b*f^2 +
2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 2*f^2*(-4*a*e^2 + b^2*f^2)*L
og[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - 2*f^2*(-4*a*e^2 + b^2*f^2)*
Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(2*d*e - b*f^2
)^3)
```

Maple [B] time = 0.036, size = 58067, normalized size = 218.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

Fricas [B] time = 12.9004, size = 1673, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b*d^3*e - 4
*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (b^3*f^
6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3*e^3)*x - 2*(a*b^2*f^6 + 4*a*d
^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*
d*e + 2*a*b*e^2)*f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*
e^3*f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2
*f^5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2)) - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2
*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*log(
a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 4*((b^2*d -
2*a*b*e)*f^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4
*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a*b^3*f^8 + 8*d^5*e^
3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*d^2*e^2)*f^4 - 4*(3*
b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b^3*d*e*f^6 + 24*b^2*d^2*e^2*f^
4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.478 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=330

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{2ef^2\left(\right.}{\left(2de-bf^2\right)^3\left(2e\left(f\right.\right.}$$

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((
2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])))
 + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rubi [A] time = 0.289473, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{2ef^2\left(\right.}{\left(2de-bf^2\right)^3\left(2e\left(f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((
2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])))
 + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^3\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx+\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^3}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x^2}+\frac{3\left(4ae^3f^2-b^2ef^4\right)}{\left(2de-bf^2\right)^4x}+\frac{}{(2d}\right.\right.$$

$$= -\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(d+ex+f\sqrt{a+b\right.}}$$

Mathematica [A] time = 0.746729, size = 300, normalized size = 0.91

$$-\frac{\frac{2f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}+\frac{2ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)+6ef^2\left(4ae^2-\right.}{\left(bf^2-2de\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x
*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d +
 e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*e^2
 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*e*f^
2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + 6*e*
f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])])/(-2*d*e + b*f^2)^4)
```

Maple [B] time = 0.094, size = 295147, normalized size = 894.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

Fricas [B] time = 96.9478, size = 3945, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] ((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2
- 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^2*d^2*e^3)*f^4 - 4*(
b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*f^2 - 8*d^3*e^6)*x^3 + 2*(b*d^
5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 - 2*(1
1*b^2*d^2*e^3 - 4*a*b*d*e^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^2 + (
16*d^5*e^4 + (3*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^2 - 28
*a^2*b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*f^4 - 8
*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*
(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^
8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a
*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^
2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*
f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4
)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 - 4*(b*d*e -
 a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f
^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b
^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b
^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6
+ 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(a*f^2 - d^2 + (
b*f^2 - 2*d*e)*x) + 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2
*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5
*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x
^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*
b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(-e*x
 + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*b^3*f^9 - 3*(a*b^2*d
*e + 2*a^2*b*e^2)*f^7 - 3*(b^2*d^3*e - 4*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^5 + 2
*(3*b*d^4*e^2 - 10*a*d^3*e^3)*f^3 - 2*(b^3*e^2*f^7 - 6*b^2*d*e^3*f^5 + 12*b
*d^2*e^4*f^3 - 8*d^3*e^5*f)*x^2 + (b^4*f^9 + 12*d^4*e^4*f - 3*(b^3*d*e + 3*
a*b^2*e^2)*f^7 + 3*(b^2*d^2*e^2 + 12*a*b*d*e^3)*f^5 - 4*(2*b*d^3*e^3 + 9*a*
d^2*e^4)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a^2*b^4*f^12 + 16*
d^8*e^4 - 2*(a*b^4*d^2 + 4*a^2*b^3*d*e)*f^10 + (b^4*d^4 + 16*a*b^3*d^3*e +
24*a^2*b^2*d^2*e^2)*f^8 - 8*(b^3*d^5*e + 6*a*b^2*d^4*e^2 + 4*a^2*b*d^3*e^3)
*f^6 + 8*(3*b^2*d^6*e^2 + 8*a*b*d^5*e^3 + 2*a^2*d^4*e^4)*f^4 - 32*(b*d^7*e^
3 + a*d^6*e^4)*f^2 + (b^6*f^12 - 12*b^5*d*e*f^10 + 60*b^4*d^2*e^2*f^8 - 160
*b^3*d^3*e^3*f^6 + 240*b^2*d^4*e^4*f^4 - 192*b*d^5*e^5*f^2 + 64*d^6*e^6)*x^
2 + 2*(a*b^5*f^12 + 32*d^7*e^5 - (b^5*d^2 + 10*a*b^4*d*e)*f^10 + 10*(b^4*d^
3*e + 4*a*b^3*d^2*e^2)*f^8 - 40*(b^3*d^4*e^2 + 2*a*b^2*d^3*e^3)*f^6 + 80*(b
^2*d^5*e^3 + a*b*d^4*e^4)*f^4 - 16*(5*b*d^6*e^4 + 2*a*d^5*e^5)*f^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.479 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}\,dx$

Optimal. Leaf size=370

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4}-\frac{f^2\left(4ae^2-\right.}{1}$$

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a +
 (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/Sqrt[2*d*e - b*f^2]])/(16*Sqrt[2]*e^(9/2))
```

Rubi [A] time = 0.600375, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4}-\frac{f^2\left(4ae^2-\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a +
 (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/Sqrt[2*d*e - b*f^2]])/(16*Sqrt[2]*e^(9/2))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = 2\,\text{Subst}\left(\int \frac{x^{5/2}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2} dx, x, d+ex+f\sqrt{a+\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^6\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2} dx, x, \sqrt{d+ex+f\sqrt{\right.$$

$$= -\frac{f^2\left(2de-bf^2\right)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{-ef^2(2de\right.}{}$$

$$= -\frac{f^2\left(2de-bf^2\right)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int\left(-4ef^2\right.\right.}{}$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2\left(4ae^2-b^2f^2\right)\left(d\right.}{}$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2\left(4ae^2-b^2f^2\right)\left(d\right.}{}$$

Mathematica [A] time = 1.10635, size = 357, normalized size = 0.96

$$\frac{\frac{4}{3}e^2f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}+4ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}-\frac{(4}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)]] + (4*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)])^(3/2))/3 + (16*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])^(7/2))/7 - ((-2*d*e + b*f^2)^2*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e
*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)])) - (5*Sqrt[e]*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*f^2
)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])
/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(16*e^5)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Fricas [A] time = 2.61643, size = 2033, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^
4)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2
*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 -
 2*d*e)/e))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*
e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(105*b^3*f^6
+ 192*e^6*x^3 + 48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e
^2 + 232*a*d*e^3)*f^2 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 +
156*d^2*e^4 - 4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*
x^2 + 12*d^2*e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^
4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2) + d))/e^4, -1/336*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^
3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(
1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2
- 2*d*e)/e)/(b*f^2 - 2*d*e)) - (105*b^3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56
*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^3)*f^2 + 144*(b*
e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4 - 4*(3*b*d*e^3 - 32
*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x^2 + 12*d^2*e^3*f - 4*(21*b*d*
e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^
4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.480 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=302

$$\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^3}-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{3f^2\left(4ae^2\cdots\right.}{\cdots}$$

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(
4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*
d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) -
(3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt
[2]*e^(7/2))
```

Rubi [A] time = 0.41462, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^3}-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{3f^2\left(4ae^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(
4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*
d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) -
(3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt
[2]*e^(7/2))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}dx = 2\,\text{Subst}\left(\int\frac{x^{3/2}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx,x,d+ex+f\sqrt{\cdots}\right.$$

$$=4\,\text{Subst}\left(\int\frac{x^4\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2}dx,x,\sqrt{d+ex+\cdots}\right.$$

$$=-\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}-\frac{\text{Subst}\left(\int\frac{-ef^2(\cdots}{\cdots}\right.}{\cdots}$$

$$=-\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}-\frac{\text{Subst}\left(\int\left(-2e\cdots\right.\right.}{\cdots}$$

$$=\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5\cdots}}{5e}$$

$$=\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5\cdots}}{5e}$$

Mathematica [A] time = 0.577791, size = 291, normalized size = 0.96

$$\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}-\frac{\left(4ae^3f^2-b^2ef^4\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-\frac{3\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\,\text{ta}\cdots}{\cdots}}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
] + (8*e^3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(5/2))/5 - ((2*d*e -
 b*f^2)*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/
f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - (3*Sqrt[e]*
f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d
 + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(
8*e^4)
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Fricas [A] time = 2.59244, size = 1439, normalized size = 4.76

$$\left[-\frac{15\sqrt{\frac{1}{2}}\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-\frac{bf^2-2\,de}{e}}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2\,de^3\right)x+4\left(2\sqrt{\frac{1}{2}}e^2f\sqrt{-\frac{bf^2-2\,de}{e}}\sqrt{\frac{bf^2x+e^2\cdots}{f^2}}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2
)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sq
rt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 - 2*d*e)/e))*sqrt(e*x + f*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x
+ e^2*x^2 + a*f^2)/f^2)) + 2*(15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*
d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x
 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e
^2*f^2)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*sqrt((b*f^2
*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2 - 2*d*e)/e)/(b*f^2 - 2*d*e))
- (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*
e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*sqrt((b*f^2*x
 + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2
```

```
) + d))/e^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.481 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=233

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+\right.}{3e}$$

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*
f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])
```

Rubi [A] time = 0.304588, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1153, 208}

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```



```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*
f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])
```


Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{x}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}\,dx,x,d+ex+f\sqrt{a+\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^2\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2}\,dx,x,\sqrt{d+ex+f\right.$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}-\frac{\text{Subst}\left(\int\frac{-ef^2\left(4ae^2-b^2f^2\right)+4e^2\left(2de-b\right.}{-2de+bf^2+2ex^2}\right.}{}$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}-\frac{\text{Subst}\left(\int\left(-4e^2x^2-\frac{ef^2\left(4ae^2-b^2f^2\right.}{-2de+bf^2+2ex}\right.\right.}{}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}+$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}-$$

Mathematica [A] time = 0.401768, size = 223, normalized size = 0.96

$$\frac{\frac{\left(b^2ef^4-4ae^3f^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-\frac{\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{4de-2bf^2}}+\frac{4}{3}e^2\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((4*e^2*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2))/3 + ((-4*a*e^3*f
^2 + b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2
*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - (Sqrt[e]*f^2*(4*a*e^2 - b^2*f
^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
])/Sqrt[2*d*e - b*f^2]])/Sqrt[4*d*e - 2*b*f^2])/(4*e^3)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 2.61705, size = 1472, normalized size = 6.32

$$\left[-\frac{3\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-2\,bef^2+4\,de^2}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2\,de^3\right)x-2\left(2\sqrt{-2\,bef^2+4\,de^2}ef\sqrt{\frac{bf^2x}{}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 +
 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(-2*b*e*f^2 +
 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d
*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)
 + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*
(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(
b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4), 1/24*(3*
(b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a
*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 -
 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^
2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.482 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=244

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\sqrt{f\sqrt{a+bx\cdots}}$$

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)
/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*(2*d*e - b*f^2)*(b*
f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b
^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rubi [A] time = 0.291653, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1157, 388, 208}

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\sqrt{f\sqrt{a+bx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)
/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*(2*d*e - b*f^2)*(b*
f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b
^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{\sqrt{x}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx+\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\right.$$

$$= -\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-8d^2e+8bdf^2-4\right.}{-2d}\right.}{}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}$$

Mathematica [A] time = 0.590883, size = 238, normalized size = 0.98

$$\frac{f^2\left(b^2f^2-4ae^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+x}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]/e + (f^2*(-4*a*e^2 + b^2*f^
2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*e*(2*d*e - b*f^2)*(b
*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (f^2*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 2.68942, size = 1508, normalized size = 6.18

$$\left[\frac{\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-2\,bef^2+4\,de^2}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2\,de^3\right)x+2\left(2\sqrt{-2\,bef^2+4\,de^2}ef\sqrt{\frac{bf^2x+e^2}{f}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(
b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 2*(2*sqrt(-2*b*e*f^2 + 4*d
*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2
)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d
) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4*(b^2
*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f
^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e
^4), 1/4*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt
(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e
^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*
x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(b^2*e*f^4 - 6*b*d*e
^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.483 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{4\,(a}{\left(2de-bf^2\right)^2}$$

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a
+ (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]
*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rubi [A] time = 0.367633, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1259, 453, 208}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{4\,(a}{\left(2de-bf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```


```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a
+ (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]
*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))
```


Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^{3/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{x^2\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a}}\right.$$

$$= -\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2\left(2de-bf^2\right)\left(d^2e-\right.}{}\right.}{}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\right.\right.}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\right.\right.}$$

Mathematica [A] time = 0.481623, size = 257, normalized size = 0.96

$$\frac{-\frac{2e^2(4ae^2f^2-b^2f^4)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{3e^{3/2}f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{de-\frac{bf^2}{2}}}-\frac{8e^2(aef^2-bdf^2+d^2e)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}}{2e^2\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] ((-8*e^2*(d^2*e - b*d*f^2 + a*e*f^2))/Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2
*x)/f^2)]] - (2*e^2*(4*a*e^2*f^2 - b^2*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + (
3*e^(3/2)*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[d*e - (b*f^2)/2
])/(2*e^2*(-2*d*e + b*f^2)^2)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Fricas [B] time = 3.45926, size = 2992, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6
 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(-2*b*e*f^2 + 4*d*e^
2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*
sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt
(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 +
a*f^2)/f^2)) + 4*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e^2)*f^4
 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d
^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)*f^4 + 2*
(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3*a*b*e^2)
*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2
*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*
x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^2*e + 6*a
*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4*e^3 + 2*
a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d
^3*e^4*f^2 + 16*d^4*e^5)*x), -1/2*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b
*e*f^2 - 2*d*e^2)*x)) - 2*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d
*e^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*
f^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)
*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3
*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^
2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4
*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4
 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.484 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{4f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2\left(4ae^2-b\right.}{}$$

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a +
 b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2
)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 -
b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^
3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*S
qrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sq
rt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)
```

Rubi [A] time = 0.500263, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1259, 1261, 208}

$$-\frac{4f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2\left(4ae^2-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a +
 b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2
)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 -
b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^
3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*S
qrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sq
rt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^{5/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{x^4\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a}}\right.$$

$$= -\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2\left(2de-bf^2\right)^2\left(d^2e\right.}{}\right.}{}$$

$$= -\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8e^2\left(2de-bf^2\right)\left(a\right.}{x}\right.\right.}{}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2\left(4ae^2-b^2f\right.}{\left(2de-bf^2\right)^3\sqrt{d+ex+f\sqrt{}}}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2\left(4ae^2-b^2f\right.}{\left(2de-bf^2\right)^3\sqrt{d+ex+f\sqrt{}}}$$

Mathematica [A] time = 0.846368, size = 315, normalized size = 0.94

$$\frac{\frac{8f^2\left(b^2f^2-4ae^2\right)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}} - \frac{4\left(4ae^3f^2-b^2ef^4\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2} + \frac{10\sqrt{2}\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2de-bf^2}} - \frac{8\left(2de-bf^2\right)\left(aef\right.}{3\left(f\sqrt{a+x\left(b+\frac{e^2}{f}\right.}\right.}}{2\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] ((-8*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(d + e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)])^(3/2)) + (8*f^2*(-4*a*e^2 + b^2*f^2))/Sqrt[d + e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - (4*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*
x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b +
 (e^2*x)/f^2)])) + (10*Sqrt[2]*Sqrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqr
t[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e -
b*f^2]])/Sqrt[2*d*e - b*f^2])/(2*(2*d*e - b*f^2)^3)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Fricas [B] time = 6.47634, size = 5181, normalized size = 15.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*d^2 + 2*a^3*e^2
)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^
3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*x^2 + 2*(a*b^3*
f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2)*f^6 + 2*(b^2*
d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(2)*
(b*e*f^3 - 2*d*e^2*f)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*
f^2)/f^2) - sqrt(2)*(b^2*f^4 - 2*b*d*e*f^2 + 2*(b*e^2*f^2 - 2*d*e^3)*x)*sqr
t(-e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) +
 d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*(4
*d^5*e^2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*
d*e^2)*f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e -
 32*a*d^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)
*f^4 - 4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^
2*e)*f^6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 -
23*a*d^2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e +
15*a^2*e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d
*e^3*f^3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4
*(b^2*d*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(
```

```
e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e
^3 - 2*(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2
*b*d^2*e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*
(3*b*d^6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e
^2*f^6 - 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4
*f^10 - 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d
^2*e^2)*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d
^4*e^4)*f^2)*x), 1/3*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*
d^2 + 2*a^3*e^2)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*
e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*
x^2 + 2*(a*b^3*f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2
)*f^6 + 2*(b^2*d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(e/(b*f^2 -
 2*d*e))*arctan(1/2*(sqrt(2)*(b*f^3 - 2*d*e*f)*sqrt(e/(b*f^2 - 2*d*e))*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2)*(b*d*f^2 - 2*d^2*e + (b*e*f^2 -
 2*d*e^2)*x)*sqrt(e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2) + d)/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(4*d^5*e^
2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*d*e^2)*
f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e - 32*a*d
^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)*f^4 -
4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^2*e)*f^
6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 - 23*a*d^
2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e + 15*a^2*
e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4*(b^2*d
*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e^3 - 2*
(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2*b*d^2*
e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*(3*b*d^
6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e^2*f^6
- 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4*f^10 -
 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d^2*e^2)
*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d^4*e^4)
*f^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.485 $\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=164

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right)^{n+}}{32(n+3)}$$

```
[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rubi [A] time = 0.110497, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 270}

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right)^{n+}}{32(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5 \, dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right) dx, x, x\right.\\
&= -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+\frac{5a^2\left(x\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.342343, size = 138, normalized size = 0.84

$$\frac{1}{32}\left(\sqrt{a+x^2}+x\right)^{n-5}\left(\frac{10a^2\left(\sqrt{a+x^2}+x\right)^6}{n+1}+\frac{10a^3\left(\sqrt{a+x^2}+x\right)^4}{n-1}+\frac{5a^4\left(\sqrt{a+x^2}+x\right)^2}{n-3}+\frac{a^5}{n-5}+\frac{\left(\sqrt{a+x^2}+x\right)^{10}}{n+5}+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a + x^2
])^10/(5 + n)))/32
```

Maple [C] time = 0.044, size = 216, normalized size = 1.3

$$\frac{2^n x^{5+n}}{5+n}{}_3F_2\left(-\frac{n}{2},\frac{1}{2}-\frac{n}{2},-\frac{5}{2}-\frac{n}{2};1-n,-\frac{3}{2}-\frac{n}{2};-\frac{a}{x^2}\right)+\frac{2^{1+n}ax^{3+n}}{3+n}{}_3F_2\left(-\frac{n}{2},\frac{1}{2}-\frac{n}{2},-\frac{3}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^2}\right)+\frac{n}{4\sqrt{\pi}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,1/2-1/2*n,-5/2-1/2*n],[1-n,-3/2-1/2*n],
-a/x^2)+2^(1+n)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,1/2-1/2*n,-3/2-1/2*n],[1-
n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1
+n)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(
1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1
+n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2\left(x+\sqrt{x^2+a}\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.3776, size = 363, normalized size = 2.21

$$-\frac{\left(5\left(n^4-10n^2+9\right)x^5+10\left(an^4-16an^2+15a\right)x^3+5\left(a^2n^4-22a^2n^2+45a^2\right)x-\left(a^2n^5-30a^2n^3+\left(n^5-10n^3+9n\right)\cdots\right.\right.}{n^6-35n^4+259n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

3.486 $\int \left(a+x^2\right) \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=108

$$-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3 a^2 \left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3 a \left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

```
[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rubi [A] time = 0.0626827, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2122, 270}

$$-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3 a^2 \left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3 a \left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right) \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{8} \text{Subst}\left(\int x^{-4+n} \left(a+x^2\right)^3 \, dx,x,x+\sqrt{a+x^2}\right)\\
&= \frac{1}{8} \text{Subst}\left(\int \left(a^3 x^{-4+n}+3 a^2 x^{-2+n}+3 a x^n+x^{2+n}\right) \, dx,x,x+\sqrt{a+x^2}\right)\\
&= -\frac{a^3 \left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3 a^2 \left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3 a \left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.12373, size = 92, normalized size = 0.85

$$\frac{1}{8}\left(\sqrt{a+x^2}+x\right)^{n-3}\left(\frac{3a^2\left(\sqrt{a+x^2}+x\right)^2}{n-1}+\frac{a^3}{n-3}+\frac{\left(\sqrt{a+x^2}+x\right)^6}{n+3}+\frac{3a\left(\sqrt{a+x^2}+x\right)^4}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(3
+ n)))/8
```

Maple [C] time = 0.01, size = 167, normalized size = 1.6

$$\frac{2^n x^{3+n}}{3+n}\,{}_3F_2(-\frac{n}{2},\frac{1}{2}-\frac{n}{2},-\frac{3}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^2})+\frac{n}{4\sqrt{\pi}}a^{\frac{3}{2}+\frac{n}{2}}\left(8\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n\,(-2+2\,n)}\left(\frac{an}{x^2}+n-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(3+n)*x^(3+n)*hypergeom([-1/2*n,1/2-1/2*n,-3/2-1/2*n],[1-n,-1/2-1/2*n],
-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/
2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x
^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.35338, size = 174, normalized size = 1.61

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3\,a\right)x-\left(an^3+\left(n^3-n\right)x^2-7\,an\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^4-10\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2 + a)\left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.487 $\int \left(x+\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rubi [A] time = 0.0211651, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(x+\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^{-2+n}\left(a+x^2\right)\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(ax^{-2+n}+x^n\right)\,dx,x,x+\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0346207, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n-1}\left((n-1)x\left(\sqrt{a+x^2}+x\right)+an\right)}{n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1 +
 n^2)
```

Maple [B] time = 0.007, size = 120, normalized size = 2.3

$$\frac{n}{4\sqrt{\pi}}a^{\frac{1}{2}+\frac{n}{2}}\left(8\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n\,(-2+2\,n)}\left(\frac{an}{x^2}+n-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+n}+4\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n}\sqrt{1+\frac{a}{x^2}}\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a/
x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x^(1+n)*a
^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.38304, size = 74, normalized size = 1.42

$$\frac{\left(\sqrt{x^2+a}n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [B] time = 3.51579, size = 2149, normalized size = 41.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a
)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2)
 + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) +
a**(9/2)*a**(n/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2
*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**(7/2)*a**(n/2)*n**2*x**3*sqrt
(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + a**(7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x/s
qrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 -
 n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2
)) + 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(
7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**5*a*
*(n/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(
1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2
)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*ga
mma(1 - n/2)) + 4*a**4*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) - 2*a**4*a**(n/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) - 2*a**4*a**(n/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2
*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)
*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 -
n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*
a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*
a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*
x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9
/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*
gamma(1 - n/2)), Abs(x**2)/Abs(a) > 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 +
x**2/a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5
/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) + a**(5/2)*a**(n/2)*n*
x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*
a**(5/2)*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2)*x*sqrt(1 + x**2/a)*sinh(n*as
inh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1
- n/2) - 2*a**(5/2)*gamma(1 - n/2)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*s
inh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**
(5/2)*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x
/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamm
a(1 - n/2)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 -
 n/2)) + 2*a**2*a**(n/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*g
amma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)),
 True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

3.488 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$

Optimal. Leaf size=59

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rubi [A] time = 0.0679521, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 364}

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^n}{a+x^2}\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0249493, size = 61, normalized size = 1.03

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+a}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)

[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.489 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx$

Optimal. Leaf size=59

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rubi [A] time = 0.0661979, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 364}

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx &= 8\,\text{Subst}\left(\int \frac{x^{2+n}}{\left(a+x^2\right)^3} dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0302536, size = 61, normalized size = 1.03

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+3}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+a\right)^2}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

```
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.490 $\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=176

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}$$

```
[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rubi [A] time = 0.108888, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5 \, dx, x, x-\sqrt{a+x^2}\right)\\ &= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right) dx, x, x\right.\\ &= -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+\frac{5a^2\left(x\right.}{}\end{aligned}$$

Mathematica [A] time = 0.330408, size = 150, normalized size = 0.85

$$\frac{1}{32}\left(x-\sqrt{a+x^2}\right)^{n-5}\left(\frac{10a^2\left(x-\sqrt{a+x^2}\right)^6}{n+1}+\frac{10a^3\left(x-\sqrt{a+x^2}\right)^4}{n-1}+\frac{5a^4\left(x-\sqrt{a+x^2}\right)^2}{n-3}+\frac{a^5}{n-5}+\frac{\left(x-\sqrt{a+x^2}\right)^{10}}{n+5}+\frac{5\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a + x^2
])^10/(5 + n)))/32
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.40468, size = 363, normalized size = 2.06

$$-\frac{\left(5\left(n^4-10\,n^2+9\right)x^5+10\left(an^4-16\,an^2+15\,a\right)x^3+5\left(a^2n^4-22\,a^2n^2+45\,a^2\right)x+\left(a^2n^5-30\,a^2n^3+\left(n^5-10\,n^3+9\,n\right.\right.\right.}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

3.491 $\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=116

$$-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

```
[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rubi [A] time = 0.0638217, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 270}

$$-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{8}\text{Subst}\left(\int x^{-4+n}\left(a+x^2\right)^3\,dx,x,x-\sqrt{a+x^2}\right)\\ &= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right)dx,x,x-\sqrt{a+x^2}\right)\\ &= -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}\end{aligned}$$

Mathematica [A] time = 0.118305, size = 100, normalized size = 0.86

$$\frac{1}{8}\left(x-\sqrt{a+x^2}\right)^{n-3}\left(\frac{3a^2\left(x-\sqrt{a+x^2}\right)^2}{n-1}+\frac{a^3}{n-3}+\frac{\left(x-\sqrt{a+x^2}\right)^6}{n+3}+\frac{3a\left(x-\sqrt{a+x^2}\right)^4}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(3
+ n)))/8
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.36644, size = 174, normalized size = 1.5

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3\,a\right)x+\left(an^3+\left(n^3-n\right)x^2-7\,an\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^4-10\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.492 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rubi [A] time = 0.0228994, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(x-\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right) \, dx, x, x-\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right) \, dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0598987, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{n-1}\left(\frac{\left(x-\sqrt{a+x^2}\right)^2}{n+1}+\frac{a}{n-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-1 + n)*(a/(-1 + n) + (x - Sqrt[a + x^2])^2/(1 + n)))
/2
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^{2}+a}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^{2}+a}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.33845, size = 76, normalized size = 1.36

$$-\frac{\left(\sqrt{x^{2}+a}n+x\right)\left(x-\sqrt{x^{2}+a}\right)^{n}}{n^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{a+x^{2}}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{x^2 + a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

3.493 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$

Optimal. Leaf size=63

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rubi [A] time = 0.0721823, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 364}

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx &= 2 \operatorname{Subst}\left(\int \frac{x^n}{a+x^2} \, dx,x,x-\sqrt{a+x^2}\right)\\ &= \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0257508, size = 65, normalized size = 1.03

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.494 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rubi [A] time = 0.0650891, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 364}

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx &= 8\,\text{Subst}\left(\int \frac{x^{2+n}}{\left(a+x^2\right)^3}\,dx,x,x-\sqrt{a+x^2}\right)\\ &= \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\,{}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0253371, size = 65, normalized size = 1.03

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+3}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+a\right)^2}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.495 $\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=187

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)}$$

```
[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sqrt[a + x^2
])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 -
n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2
 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x +
 Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rubi [A] time = 0.117178, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sqrt[a + x^2
])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 -
n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2
 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x +
 Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n dx &= \frac{1}{64}\operatorname{Subst}\left(\int x^{-7+n}\left(a+x^2\right)^6 dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{1}{64}\operatorname{Subst}\left(\int \left(a^6x^{-7+n}+6a^5x^{-5+n}+15a^4x^{-3+n}+20a^3x^{-1+n}+15a^2x^{1+n}+6ax^{3+n}+\right.\right.\\ &= -\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}-\frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}-\frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}+\frac{5a^3\left(x\right.}{}\end{aligned}$$

Mathematica [A] time = 0.328109, size = 157, normalized size = 0.84

$$\frac{1}{64}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^6}{(n-6)\left(\sqrt{a+x^2}+x\right)^6}+\frac{6a^5}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}+\frac{15a^4}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{15a^2\left(\sqrt{a+x^2}+x\right)}{n+2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) +
 (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[a
+ x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a + x^
2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n)))/64
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.34751, size = 459, normalized size = 2.45

$$\frac{\left(a^3n^6-50\,a^3n^4+\left(n^6-20\,n^4+64\,n^2\right)x^6+544\,a^3n^2+3\left(an^6-30\,an^4+104\,an^2\right)x^4-720\,a^3+3\left(a^2n^6-40\,a^2n^4+2\right.\right.}{n^7-56\,n^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^
6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2
*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x
^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a
```

```
))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2+a)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

3.496 $\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=131

$$-\frac{a^4 \left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a \left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

```
[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt[a + x^2])
^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt
[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rubi [A] time = 0.094264, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^4 \left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a \left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt[a + x^2])
^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt
[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{16} \text{Subst}\left(\int x^{-5+n} \left(a+x^2\right)^4 \, dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{1}{16} \text{Subst}\left(\int \left(a^4 x^{-5+n}+4a^3 x^{-3+n}+6a^2 x^{-1+n}+4a x^{1+n}+x^{3+n}\right) dx, x, x+\sqrt{a+x}\right.\\
&= -\frac{a^4 \left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}-\frac{a^3 \left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{3a^2 \left(x+\sqrt{a+x^2}\right)^{n}}{8n}+\frac{a \left(x+\sqrt{\right.}}{4(2}
\end{aligned}$$

Mathematica [A] time = 0.222189, size = 111, normalized size = 0.85

$$\frac{1}{16}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^4}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}+\frac{4a^3}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{6a^2}{n}+\frac{4a\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{\left(\sqrt{a+x^2}+x\right)^4}{n+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) +
(4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2 +
 n) + (x + Sqrt[a + x^2])^4/(4 + n)))/16
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int\left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.3603, size = 244, normalized size = 1.86

$$\frac{\left(a^2n^4+\left(n^4-4\,n^2\right)x^4-16\,a^2n^2+2\left(an^4-10\,an^2\right)x^2+24\,a^2-4\left(\left(n^3-4\,n\right)x^3+\left(an^3-10\,an\right)x\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)}{n^5-20\,n^3+64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a
^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2
+ a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+x^2\right)^{\frac{3}{2}}\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

3.497 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

```
[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a + x^2])^n)
/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rubi [A] time = 0.0755965, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a + x^2])^n)
/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{4}\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right)\,dx,x,x+\sqrt{a+x^2}\right)\\ &= -\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n}+\frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0705863, size = 65, normalized size = 0.87

$$\frac{1}{4}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^2}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{2a}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (x
 + Sqrt[a + x^2])^2/(2 + n)))/4
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.55937, size = 109, normalized size = 1.45

$$\frac{\left(n^2x^2+an^2-2\sqrt{x^2+a}\,anx-2\,a\right)\left(x+\sqrt{x^2+a}\right)^n}{n^3-4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 -
4*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.498 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Rubi [A] time = 0.0536436, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \text{Subst}\left(\int x^{-1+n}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

Mathematica [A] time = 0.0061938, size = 17, normalized size = 1.

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^n \frac{1}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

Fricas [A] time = 1.45115, size = 34, normalized size = 2.

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + a))^n/n
```

Sympy [B] time = 3.74217, size = 313, normalized size = 18.41

$$\begin{cases} -\frac{\sqrt{a}a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}}-\frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)}+\frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}n}-\frac{a^{\frac{n}{2}}x\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\cdots\right.}{\sqrt{a}n\sqrt{\frac{a}{x^2}+1}} \\ -\frac{a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}}-\frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)}-\frac{a^{\frac{n}{2}}x^2\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}}+\frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{as}\cdots\right.}{\sqrt{a}n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
n*x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/
(n**2*gamma(-n/2)) + a**(n/2)*x*cosh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*n) - a**(n/2)*x*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*n*sqrt(a/x**2 + 1)), Abs(x**2)/Abs(a) > 1), (-a**(n/2)*sinh(-n*asinh(
x/sqrt(a)) + asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asi
nh(x/sqrt(a)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) - a**(n/2)*x**2*sinh(-n*as
inh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh
(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

3.499 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rubi [A] time = 0.0717657, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 364}

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx &= 4\,\text{Subst}\left(\int \frac{x^{1+n}}{\left(a+x^2\right)^2}\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}\,{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0245902, size = 61, normalized size = 1.03

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+2}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^n \left(x^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

```
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.500 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=59

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rubi [A] time = 0.0715277, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 364}

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx &= 16\,\text{Subst}\left(\int \frac{x^{3+n}}{\left(a+x^2\right)^4}\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}\,{}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.0325427, size = 61, normalized size = 1.03

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+4}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n)
/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int\left(x+\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n}{x^6+3\,ax^4+3\,a^2x^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)

[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.501 $\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=201

$$\frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)}-$$

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2]
)^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n
)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2
+ n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x -
Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rubi [A] time = 0.112134, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 270}

$$\frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2]
)^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n
)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2
+ n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x -
Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n dx = -\left(\frac{1}{64} \text{Subst}\left(\int x^{-7+n} \left(a+x^2\right)^6 dx, x, x-\sqrt{a+x^2}\right)\right)$$

$$= -\left(\frac{1}{64} \text{Subst}\left(\int \left(a^6 x^{-7+n} + 6a^5 x^{-5+n} + 15a^4 x^{-3+n} + 20a^3 x^{-1+n} + 15a^2 x^{1+n} + 6ax\right.\right.\right.$$

$$= \frac{a^6 \left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} + \frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} + \frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)} - \frac{5a^3 (}{}$$

Mathematica [A] time = 0.426202, size = 173, normalized size = 0.86

$$\frac{1}{64} \left(x-\sqrt{a+x^2}\right)^n \left(-\frac{a^6}{(n-6) \left(x-\sqrt{a+x^2}\right)^6} - \frac{6a^5}{(n-4) \left(x-\sqrt{a+x^2}\right)^4} - \frac{15a^4}{(n-2) \left(x-\sqrt{a+x^2}\right)^2} - \frac{15a^2 \left(x-\sqrt{a+x^2}\right)}{n+2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2])^6)
- (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt[a
 + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a + x
^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n)))/64
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.41196, size = 460, normalized size = 2.29

$$-\frac{\left(a^3 n^6 - 50\, a^3 n^4 + \left(n^6 - 20\, n^4 + 64\, n^2\right) x^6 + 544\, a^3 n^2 + 3\left(a n^6 - 30\, a n^4 + 104\, a n^2\right) x^4 - 720\, a^3 + 3\left(a^2 n^6 - 40\, a^2 n^4 +\right.\right.}{n^7 - 56\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n
^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^
2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*
x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 +
a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

3.502 $\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=141

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^
(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[
a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rubi [A] time = 0.0948329, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 270}

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^
(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[
a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx &= -\left(\frac{1}{16} \operatorname{Subst}\left(\int x^{-5+n} \left(a+x^2\right)^4 \, dx, x, x-\sqrt{a+x^2}\right)\right)\\
&= -\left(\frac{1}{16} \operatorname{Subst}\left(\int \left(a^4 x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right) dx, x, x-\sqrt{a\right.\right.\\
&= \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a\right.}{4(2+}
\end{aligned}$$

Mathematica [A] time = 0.222726, size = 123, normalized size = 0.87

$$\frac{1}{16}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^4}{(n-4)\left(x-\sqrt{a+x^2}\right)^4}-\frac{4a^3}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{6a^2}{n}-\frac{4a\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{\left(x-\sqrt{a+x^2}\right)^4}{n+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4) -
 (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(2
+ n) - (x - Sqrt[a + x^2])^4/(4 + n)))/16
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.14681, size = 246, normalized size = 1.74

$$-\frac{\left(a^2n^4+\left(n^4-4\,n^2\right)x^4-16\,a^2n^2+2\left(an^4-10\,an^2\right)x^2+24\,a^2+4\left(\left(n^3-4\,n\right)x^3+\left(an^3-10\,an\right)x\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^5-20\,n^3+64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*
a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2
 + a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+x^2\right)^{\frac{3}{2}} \left(x-\sqrt{a+x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

3.503 $\int \sqrt{a+x^2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=81

$$\frac{a^2 \left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a \left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/
(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rubi [A] time = 0.0750473, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 270}

$$\frac{a^2 \left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a \left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/
(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+x^2} \left(x-\sqrt{a+x^2}\right)^n \, dx &= -\left(\frac{1}{4} \text{Subst}\left(\int x^{-3+n} \left(a+x^2\right)^2 \, dx, x, x-\sqrt{a+x^2}\right)\right)\\ &= -\left(\frac{1}{4} \text{Subst}\left(\int \left(a^2 x^{-3+n}+2 a x^{-1+n}+x^{1+n}\right) \, dx, x, x-\sqrt{a+x^2}\right)\right)\\ &= \frac{a^2 \left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{a \left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0699229, size = 73, normalized size = 0.9

$$\frac{1}{4}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^2}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{2a}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) - (
x - Sqrt[a + x^2])^2/(2 + n)))/4
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 1.06071, size = 111, normalized size = 1.37

$$-\frac{\left(n^2x^2+an^2+2\sqrt{x^2+a}\,anx-2\,a\right)\left(x-\sqrt{x^2+a}\right)^n}{n^3-4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3 -
 4*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.504 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rubi [A] time = 0.0565753, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} dx &= -\text{Subst}\left(\int x^{-1+n} dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

Mathematica [A] time = 0.0068217, size = 20, normalized size = 1.

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^{2}+a}\right)^{n} \frac{1}{\sqrt{x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{\sqrt{x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

Fricas [A] time = 0.990437, size = 35, normalized size = 1.75

$$-\frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x - sqrt(x^2 + a))^n/n
```

Sympy [A] time = 1.81814, size = 36, normalized size = 1.8

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^{2}}\right)^{n}}{n} & \text{for}\: n \neq 0 \\ \begin{cases} \operatorname{asinh}{\left(x\sqrt{\frac{1}{a}} \right)} & \text{for}\: a > 0 \\ \operatorname{acosh}{\left(x\sqrt{-\frac{1}{a}} \right)} & \text{for}\: a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

3.505 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2,
-((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rubi [A] time = 0.0724693, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 364}

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```


```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2,
-((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```


Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{x^{1+n}}{\left(a+x^2\right)^2}\,dx, x, x-\sqrt{a+x^2}\right)\right)\\ &= -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0248509, size = 65, normalized size = 1.03

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+2}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)
/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^n \left(x^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.506 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
 -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rubi [A] time = 0.07218, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2122, 364}

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
 -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx &= -\left(16\,\text{Subst}\left(\int \frac{x^{3+n}}{\left(a+x^2\right)^4}\,dx,x,x-\sqrt{a+x^2}\right)\right)\\ &= -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n}\,{}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.0303316, size = 65, normalized size = 1.03

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}{}_2F_1\left(4,\frac{n+4}{2};\frac{n+4}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n
)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int\left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n}{x^6+3\,ax^4+3\,a^2x^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.507 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=365

$$\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5\left(d^2-af^2\right)^3\left(f\sqrt{a+\right.}}{16ef^4(}$$

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5
+ n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(
1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))
```

Rubi [A] time = 0.47376, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2121, 12, 270}

$$\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5\left(d^2-af^2\right)^3\left(f\sqrt{a+\right.}}{16ef^4(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5
+ n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(
1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5}{64e^6}dx, x, d+ex+f\sqrt{\ldots}\right.}{f^4}$$

$$= \frac{\text{Subst}\left(\int x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5 dx, x, d\ldots\right.}{32e^6f^4}$$

$$= \frac{\text{Subst}\left(\int\left(-e^5\left(d^2-af^2\right)^5x^{-6+n}+5e^5\left(d^2-af^2\right)^4x^{-4+\ldots}\right.\right.}{}$$

$$= \frac{\left(d^2-af^2\right)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)} - \frac{5\left(d^2\ldots\right.}{}$$

Mathematica [A] time = 2.98739, size = 280, normalized size = 0.77

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-5}\left(-\frac{5\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^8}{n+3}+\frac{10\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^6}{n+1}-\frac{10\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\ldots\right.}{n-1}\ldots\right.}{32ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n)*(-((d^2 - a*f^2)^5/
(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
 e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^10/
(5 + n)))/(32*e*f^4)
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \left(a+2\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+2\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2
))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2
))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2 \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 1.22951, size = 1384, normalized size = 3.79

$$-\frac{\left(5\,a^2df^4n^4 + 225\,a^2df^4 - 300\,ad^3f^2 + 5\left(e^5n^4 - 10\,e^5n^2 + 9\,e^5\right)x^5 + 120\,d^5 + 25\left(de^4n^4 - 10\,de^4n^2 + 9\,de^4\right)x^4 + 10\left(15\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 - 10*e^5*n^2
 + 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2 + 9*d*e^4)*x^4 + 10*
(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d^2*e^3)*n^4 - 2*(8*a*e^3*f^2 +
 17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*d*e^
2*f^2 + (3*a*d*e^2*f^2 + 2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)*n^2)
*x^2 + 5*(45*a^2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*e*f^4
+ 26*a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10*e^4*f*
n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*(d*e^3*f*n^5 -
10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3 + 2*d^2*e^2*f)*n^5 - 10*(
2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2*f^3 + 8*d^2*e^2*f)*n)*x^2 + (149*a
^2*f^5 - 260*a*d^2*f^3 + 120*d^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2*a*d*e*f^3
- d^3*e*f)*n^3 + (19*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^2 + a*f^2 +
2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^4
*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*
x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2 \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

3.508 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=239

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{2d}{f}}\right.}{8ef^2(n}$$

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3
+ n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d +
 e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)
) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*
(3 + n))
```

Rubi [A] time = 0.250062, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2121, 12, 270}

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{2d}{f}}\right.}{8ef^2(n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n,x]
```


```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3
+ n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d +
 e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)
) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*
(3 + n))
```


Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}{16e^4}dx,x,d+ex+f\sqrt{\ldots}\right.}{f^2}$$

$$=\frac{\text{Subst}\left(\int x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3 dx,x,d+\ldots\right.}{8e^4f^2}$$

$$=\frac{\text{Subst}\left(\int\left(-e^3\left(d^2-af^2\right)^3x^{-4+n}+3e^3\left(d^2-af^2\right)^2x^{-2+\ldots}\right.\right.}{\ldots}$$

$$=\frac{\left(d^2-af^2\right)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)}-\frac{3\left(d^2-\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.576077, size = 186, normalized size = 0.78

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-3}\left(-\frac{3\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}{n+1}+\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n-1}-\frac{\left(d^2-af^2\right)^3}{n-3}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\ldots\right.}{n+3}\right.}{8ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-((d^2 - a*f^2)^3/
(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n)))/
(8*e*f^2)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int\left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))
^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))
^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 1.15083, size = 482, normalized size = 2.02

$$-\frac{\left(3\,adf^2n^2 - 9\,adf^2 + 3\left(e^3n^2 - e^3\right)x^3 + 6\,d^3 + 9\left(de^2n^2 - de^2\right)x^2 - 3\left(3\,aef^2 - \left(aef^2 + 2\,d^2e\right)n^2\right)x - \left(af^3n^3 + \left(e^2fn^3 - e\right.\right.\right.}{ef^2n^4 - 10\,ef^2n^2 + 9\,ef^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(d*e^2*n^2
- d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*x - (a*f^3*n^3 + (e^
2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*n + 2*(d*e*f*n^3 - d*e*f*n)*x)
*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*
d*e*x)/f^2) + d)^n/(e*f^2*n^4 - 10*e*f^2*n^2 + 9*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**
2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

3.509 $\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rubi [A] time = 0.089253, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2116, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = 2\,\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}}\right.$$

$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\right.}{2e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e\left(d^2-af^2\right)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{2e^2}$$

$$= \frac{\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x}{f^2}}\right.}{2e(1+n)}$$

Mathematica [A] time = 0.367488, size = 89, normalized size = 0.83

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-1}\left(\frac{af^2-d^2}{n-1}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 1.09534, size = 163, normalized size = 1.52

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.510 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rubi [A] time = 0.292669, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2121, 364}

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = (2f^2)\,\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)\,(1+n)}$$

Mathematica [A] time = 0.14492, size = 112, normalized size = 0.92

$$-\frac{2f^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+1)\,(d^2-af^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{e^2x^2+af^2+2\,dex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^2
 + a*f^2 + 2*d*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

3.511 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)*Hyper
geometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Rubi [A] time = 0.267613, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2121, 12, 364}

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)*Hyper
geometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = \left(2f^4\right)\text{Subst}\left(\int \frac{4e^2x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\right.$$

$$= \left(8e^2f^4\right)\text{Subst}\left(\int \frac{x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}\right.$$

$$= -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n} {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right.}{e\left(d^2-af^2\right)^3(3+n)}$$

Mathematica [A] time = 0.193331, size = 112, normalized size = 0.92

$$-\frac{8f^4\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(3 + n)*Hypergeometri
c2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^2,x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex + f\sqrt{\frac{e^2x^2 + af^2 + 2\,dex}{f^2}} + d\right)^n f^4}{e^4x^4 + 4\,de^3x^3 + a^2f^4 + 4\,adef^2x + 2\left(ae^2f^2 + 2\,d^2e^2\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4/(e^4*x^4
 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^2, x)
```

3.512 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rubi [A] time = 0.132555, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2118, 2116, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 2118

```
Int[((g_.) + (h_.)*((u_) + (f_.)*Sqrt[v_])^(n_))^(p_.), x_Symbol] :> Int[(g
 + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f,
g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(LinearMatchQ[u, x]
&& QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x,
 2]*f^2, 0] && IntegerQ[p]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= 2\,\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}\,dx, x, d+ex+f\sqrt{a+\frac{2d}{f}}\right.$$

$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}}\right.}{2e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e\left(d^2-af^2\right)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{2e^2}$$

$$= \frac{\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e}{}}\right.}{2e(1+n)}$$

Mathematica [A] time = 0.0381455, size = 89, normalized size = 0.83

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-1}\left(\frac{af^2-d^2}{n-1}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(ex+2d)}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+f\sqrt{\frac{af^2+(ex+2d)ex}{f^2}}+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

Fricas [A] time = 1.10098, size = 163, normalized size = 1.52

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

3.513 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rubi [A] time = 0.498452, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2127, 2121, 364}

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2
+ (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$
$$=\left(2f^2\right)\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$
$$=-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

Mathematica [A] time = 0.0374039, size = 112, normalized size = 0.92

$$-\frac{2f^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(ex+2d)}{f^2}}\right)^n\left(a+2\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{e^2x^2+af^2+2\,dex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^2
 + a*f^2 + 2*d*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f**2
+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

3.514 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=297

$$-\frac{\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a}\right.}{8}$$

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-4
 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*(d
 + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2
- a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*
e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 +
 n)/(16*e*f^3*(4 + n))
```

Rubi [A] time = 0.419456, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a}\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-4
 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*(d
 + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2
- a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*
e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 +
 n)/(16*e*f^3*(4 + n))
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4}{32e^5}dx, x, d+ex+f\sqrt{a \ldots}\right)}{f^3}$$

$$= \frac{\text{Subst}\left(\int x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4 dx, x, d+\ldots\right)}{16e^5f^3}$$

$$= \frac{\text{Subst}\left(\int \left(e^4\left(d^2-af^2\right)^4x^{-5+n}-4e^4\left(d^2-af^2\right)^3x^{-3+n}+\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4+n}}{16ef^3(4-n)}+\frac{\left(d^2-\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.24453, size = 228, normalized size = 0.77

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^4}{(n-4)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}-\frac{4\left(d^2-af^2\right)^3}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}-\frac{4\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}+\frac{6\left(d^2-af^2\right)\ldots}{n}\ldots\right)}{16ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (
2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n + (
d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4) -
 (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(2
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n)))/(16*e*f^3
)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 1.16936, size = 778, normalized size = 2.62

$$\Big(a^2f^4n^4 + 24\,a^2f^4 - 48\,ad^2f^2 + \left(e^4n^4 - 4\,e^4n^2\right)x^4 + 24\,d^4 + 4\left(de^3n^4 - 4\,de^3n^2\right)x^3 - 4\left(4\,a^2f^4 - 3\,ad^2f^2\right)n^2 + 2\,\big((ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*f^4*n^4 + 24*a^2*f^4 - 48*a*d^2*f^2 + (e^4*n^4 - 4*e^4*n^2)*x^4 + 24*d
^4 + 4*(d*e^3*n^4 - 4*d*e^3*n^2)*x^3 - 4*(4*a^2*f^4 - 3*a*d^2*f^2)*n^2 + 2*
((a*e^2*f^2 + 2*d^2*e^2)*n^4 - 2*(5*a*e^2*f^2 + d^2*e^2)*n^2)*x^2 + 4*(a*d*
e*f^2*n^4 - 2*(5*a*d*e*f^2 - 3*d^3*e)*n^2)*x - 4*(a*d*f^3*n^3 + (e^3*f*n^3
- 4*e^3*f*n)*x^3 + 3*(d*e^2*f*n^3 - 4*d*e^2*f*n)*x^2 - 2*(5*a*d*f^3 - 3*d^3
*f)*n + ((a*e*f^3 + 2*d^2*e*f)*n^3 - 2*(5*a*e*f^3 + d^2*e*f)*n)*x)*sqrt((e^
2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^
2) + d)^n/(e*f^3*n^5 - 20*e*f^3*n^3 + 64*e*f^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

3.515 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=171

$$-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d \cdots}{4ef(n+2)}$$

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2
 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^
2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rubi [A] time = 0.324975, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d \cdots}{4ef(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2
 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^
2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}{8e^3}dx, x, d+ex+f\sqrt{a+\ldots}\right)}{f}$$

$$= \frac{\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2 dx, x, d+ex\ldots\right)}{4e^3f}$$

$$= \frac{\text{Subst}\left(\int \left(e^2\left(d^2-af^2\right)^2x^{-3+n}-2e^2\left(d^2-af^2\right)x^{-1+n}+e^2\ldots\right)\right)}{4e^3f}$$

$$= -\frac{\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)} - \frac{\left(d^2-af\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.355357, size = 135, normalized size = 0.79

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^2}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}+\frac{2\left(af^2-d^2\right)}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}\right)}{4ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d
^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) +
(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 1.10248, size = 261, normalized size = 1.53

$$\frac{\left(e^2n^2x^2 + af^2n^2 + 2\,den^2x - 2\,af^2 + 2\,d^2 - 2\left(efnx + dfn\right)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n}{efn^3 - 4\,efn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (e^2*n^2*x^2 + a*f^2*n^2 + 2*d*e*n^2*x - 2*a*f^2 + 2*d^2 - 2*(e*f*n*x + d*f
*n)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 +
 2*d*e*x)/f^2) + d)^n/(e*f*n^3 - 4*e*f*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

3.516 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}} dx$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rubi [A] time = 0.25443, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=(2f)\,\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$
$$=\frac{f\,\text{Subst}\left(\int x^{-1+n}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}$$
$$=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Mathematica [A] time = 0.0678211, size = 36, normalized size = 0.88

$$\frac{f\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \frac{1}{\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

Fricas [A] time = 1.01584, size = 85, normalized size = 2.07

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2 + af^2 + 2\,dex}{f^2}} + d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\, f + d\right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

3.517 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hyperg
eometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
(e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Rubi [A] time = 0.302176, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 364}

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hyperg
eometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
(e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \left(2f^3\right)\mathrm{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \left(4ef^3\right)\mathrm{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n} {}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^2(2+n)}$$

Mathematica [A] time = 0.149754, size = 112, normalized size = 0.92

$$\frac{4f^3\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+2} {}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(2 + n)*Hypergeometric
2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^
2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n f^4\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^4x^4 + 4\,de^3x^3 + a^2f^4 + 4\,adef^2x + 2\left(ae^2f^2 + 2\,d^2e^2\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e^
2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^4*x^4 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^
2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^(3/2), x)
```

3.518 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rubi [A] time = 0.444236, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2127, 2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(
2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$
$$=(2f)\,\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$
$$=\frac{f\,\text{Subst}\left(\int x^{-1+n}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}$$
$$=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Mathematica [A] time = 0.0409528, size = 36, normalized size = 0.88

$$\frac{f\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 +
 e*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(ex+2\,d)}{f^2}}\right)^n \frac{1}{\sqrt{\frac{af^2+ex(ex+2\,d)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^
2)^(1/2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^
2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 +
 (e*x + 2*d)*e*x)/f^2), x)
```

Fricas [A] time = 0.994076, size = 85, normalized size = 2.07

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(e
*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}} + d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 +
 (e*x + 2*d)*e*x)/f^2), x)
```

3.519 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \, dx$

Optimal. Leaf size=327

$$-\frac{\left(d^2-af^2\right)^2 \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{\left(d^2-af^2\right) \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

```
[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f*(2 - n)*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)/f
^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*
g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

Rubi [A] time = 0.616985, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2123, 2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^2 \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{\left(d^2-af^2\right) \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f*(2 - n)*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)/f
^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*
g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

Rule 2123

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
- 1/2)*Sqrt[g + h*x + i*x^2])/Sqrt[a + b*x + c*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && IGtQ[m + 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
```

```
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\ldots\right.}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$$= \frac{\left(2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\right)\text{Subst}\left(\int\frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)\ldots\right.}{8e^3}\ldots\right.}{f\sqrt{a+\frac{2dex}{f^2}+\ldots}}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\ldots\right.\right.\right.}{4e^3f\sqrt{a+\ldots}}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\text{Subst}\left(\int\left(e^2\left(d^2-af^2\right)^2x^{-3+n}\ldots\right.\right.}{4e^3\ldots}$$

$$= -\frac{\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\ldots}\right.}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Mathematica [A] time = 0.247199, size = 175, normalized size = 0.54

$$\frac{\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^2}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}+\frac{2\left(af^2-d^2\right)}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}\right)}{4ef\sqrt{a+\frac{ex(2d+ex)}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a
 + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x)
)/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sq
rt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^2/(2 + n)))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \sqrt{ag+2\,\frac{degx}{f^{2}}+\frac{e^{2}gx^{2}}{f^{2}}}\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^{2}}+\frac{e^{2}x^{2}}{f^{2}}}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x)
```

```
[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^{2}gx^{2}}{f^{2}}+ag+\frac{2\,degx}{f^{2}}}\left(ex+\sqrt{\frac{e^{2}x^{2}}{f^{2}}+a+\frac{2\,dex}{f^{2}}}f+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 1.17777, size = 486, normalized size = 1.49

$$-\frac{\left(2\,e^{3}nx^{3}+6\,de^{2}nx^{2}+2\,adf^{2}n+2\left(aef^{2}+2\,d^{2}e\right)nx-\left(e^{2}fn^{2}x^{2}+af^{3}n^{2}+2\,defn^{2}x-2\,af^{3}+2\,d^{2}f\right)\sqrt{\frac{e^{2}x^{2}+af^{2}+2\,dex}{f^{2}}}\right)\left(e\right.}{aef^{2}n^{3}-4\,aef^{2}n+\left(e^{3}n^{3}-4\,e^{3}n\right)x^{2}+2\left(de^{2}n^{3}-4\,de^{2}n\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(2*e^3*n*x^3 + 6*d*e^2*n*x^2 + 2*a*d*f^2*n + 2*(a*e*f^2 + 2*d^2*e)*n*x - (
e^2*f*n^2*x^2 + a*f^3*n^2 + 2*d*e*f*n^2*x - 2*a*f^3 + 2*d^2*f)*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) +
 d)^n*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(a*e*f^2*n^3 - 4*a*e*f^2*
n + (e^3*n^3 - 4*e^3*n)*x^2 + 2*(d*e^2*n^3 - 4*d*e^2*n)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/
f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.520 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rubi [A] time = 0.529009, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d
*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx &= \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\\
&= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\\
&= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\\
&= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0976617, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g
+ (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \frac{1}{\sqrt{ag+2\,\frac{degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

Fricas [A] time = 1.04327, size = 250, normalized size = 2.69

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2 + aef^2gn + 2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

3.521 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}} \, dx$

Optimal. Leaf size=177

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2
, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(
e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rubi [A] time = 0.586196, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2125, 2121, 12, 364}

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*g
*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2
, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(
e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
```

```
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\cdots}\right.}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(4ef^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\cdots}\right.}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n} {}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{(d+ex+\cdots}{\cdots}\right.}{e\left(d^2-af^2\right)^2 g(2+n)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A] time = 0.197995, size = 152, normalized size = 0.86

$$\frac{4f^3\left(a+\frac{ex(2d+ex)}{f^2}\right)^{3/2}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+2} {}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2\left(g\left(a+\frac{ex(2d+ex)}{f^2}\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2
*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(a + (e*x*(2*d + e*x))/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f
```

```
*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 +
 n)*(g*(a + (e*x*(2*d + e*x))/f^2))^(3/2))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \left(ag+2\frac{degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^
2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^4\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}}{e^4g^2x^4+4\,de^3g^2x^3+a^2f^4g^2+4\,adef^2g^2x+2\left(ae^2f^2+2\,d^2e^2\right)g^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e^
2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(e^4*g^2*x^4 + 4*d*e^3*g^2*x^3 + a^2*f^
4*g^2 + 4*a*d*e*f^2*g^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*g^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\left(\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^
2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

3.522 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rubi [A] time = 0.735419, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2127, 2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g
*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] && !GtQ[i/c, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
```

```
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx &= \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx \\
&= \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}} \\
&= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}} \\
&= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}} \\
&= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.0394468, size = 76, normalized size = 0.82

$$
\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g
 + e*g*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{\frac{af^2 + ex\,(ex + 2\,d)}{f^2}}\right)^n \frac{1}{\sqrt{\frac{af^2g + egx(ex+2\,d)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + f\sqrt{\frac{af^2 + (ex+2\,d)ex}{f^2}} + d\right)^n}{\sqrt{\frac{af^2g + (ex+2\,d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g
 + (e*x + 2*d)*e*g*x)/f^2), x)
```

Fricas [A] time = 1.06035, size = 250, normalized size = 2.69

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2 + af^2 + 2\,dex}{f^2}} + d\right)^n f^3\sqrt{\frac{e^2gx^2 + af^2g + 2\,degx}{f^2}}\sqrt{\frac{e^2x^2 + af^2 + 2\,dex}{f^2}}}{e^3gnx^2 + aef^2gn + 2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*
x*(e*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + f\sqrt{\frac{af^2+(ex+2d)ex}{f^2}} + d\right)^n}{\sqrt{\frac{af^2g+(ex+2d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g
 + (e*x + 2*d)*e*g*x)/f^2), x)
```

3.523 $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c + a^2*d]*Sqr
t[e + f*x^2])])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f])) + (Sqrt[-c]*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b^2*c)/(a^2*d)), ArcSin[(
Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])
```

Rubi [A] time = 0.512984, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2113, 538, 537, 571, 93, 208}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c + a^2*d]*Sqr
t[e + f*x^2])])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f])) + (Sqrt[-c]*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b^2*c)/(a^2*d)), ArcSin[(
Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = a\int \frac{1}{\left(a^2-b^2x^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx - b\int \frac{x}{\left(a^2-b^2x^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$= -\left(\frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{\left(a^2-b^2x\right)\sqrt{c+dx}\sqrt{e+fx}}\,dx, x, x^2\right)\right) + \frac{\left(a\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{}{\left(a^2-b^2x^2\right)}}{\sqrt{c+dx}}$$

$$= -\left(b\,\mathrm{Subst}\left(\int \frac{1}{b^2c+a^2d-\left(b^2e+a^2f\right)x^2}\,dx, x, \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\right)\right) + \frac{\left(a\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\right.}{\sqrt{c}}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{b^2e+a^2f}\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\sqrt{b^2e+a^2f}} + \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] time = 1.58234, size = 772, normalized size = 4.04

$$\frac{2\sqrt{d}\left(\sqrt{c}+i\sqrt{d}x\right)\left(\sqrt{e}+i\sqrt{f}x\right)\sqrt{\frac{\left(\sqrt{d}x+i\sqrt{c}\right)\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)}{\left(\sqrt{d}x-i\sqrt{c}\right)\left(\sqrt{c}\sqrt{f}+\sqrt{d}\sqrt{e}\right)}}\sqrt{\frac{\sqrt{c}\sqrt{d}\left(\sqrt{f}x+i\sqrt{e}\right)}{\left(\sqrt{d}x-i\sqrt{c}\right)\left(\sqrt{c}\sqrt{f}-\sqrt{d}\sqrt{e}\right)}}\left(\left(b\sqrt{c}+ia\sqrt{d}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)(}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)(}}\right.\right.\right.}{\sqrt{c+dx^2}\sqrt{e+fx^2}\left(b\sqrt{c}-ia\sqrt{d}\right)\left(b\sqrt{c}+ia\sqrt{d}\right)\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (2*Sqrt[d]*(Sqrt[c] + I*Sqrt[d]*x)*Sqrt[((Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f]
)*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[
c] + Sqrt[d]*x))]*(Sqrt[e] + I*Sqrt[f]*x)*Sqrt[(Sqrt[c]*Sqrt[d]*(I*Sqrt[e]
+ Sqrt[f]*x))/((-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[c] + Sqrt[
d]*x))]*((b*Sqrt[c] + I*a*Sqrt[d])*EllipticF[ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
- Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqr
t[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2/
(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2] - 2*b*Sqrt[c]*EllipticPi[((b*Sqrt[c]
 - I*a*Sqrt[d])*(Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f]))/((b*Sqrt[c] + I*a*Sqrt
```

```
[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])), ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
 - Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sq
rt[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2
/(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2]))/((b*Sqrt[c] - I*a*Sqrt[d])*(b*Sqr
t[c] + I*a*Sqrt[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*Sqrt[(Sqrt[c]*Sq
rt[d]*(Sqrt[e] + I*Sqrt[f]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*(Sqrt[c
] + I*Sqrt[d]*x))]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.064, size = 353, normalized size = 1.9

$$\frac{1}{2\,ab\left(dfx^4+cfx^2+ex^2d+ce\right)}\left(2\,b\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},-\frac{b^2c}{a^2d},\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2}{b^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*(2*b*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2
),-b^2*c/a^2/d,(-f/e)^(1/2)/(-1/c*d)^(1/2))*((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d
*e+b^4*c*e)/b^4)^(1/2)-arctanh(1/2*(2*a^2*d*f*x^2+b^2*c*f*x^2+b^2*d*e*x^2+a
^2*c*f+a^2*d*e+2*b^2*c*e)/b^2/((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d*e+b^4*c*e)/b^
4)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2))*(-1/c*d)^(1/2)*a*(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/a/((a^4*d*f+a^2*
b^2*c*f+a^2*b^2*d*e+b^4*c*e)/b^4)^(1/2)/(-1/c*d)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

3.524 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=81

$$\frac{\log\left(2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

```
[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + Log[e + 2*S
qrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rubi [A] time = 0.0583478, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1164, 628}

$$\frac{\log\left(2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + Log[e + 2*S
qrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx=\int \frac{e-2fx^2}{e^2+4(d+e)fx^2+4f^2x^4}\,dx$$

$$=-\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{-d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\sqrt{f}}-\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{-d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\sqrt{f}}$$

$$=-\frac{\log\left(e-2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}+\frac{\log\left(e+2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

Mathematica [B] time = 0.120562, size = 191, normalized size = 2.36

$$-\frac{\frac{\left(\sqrt{d}\sqrt{d+2e}-d-2e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-\sqrt{d}\sqrt{d+2e}+d+e}}\right)}{\sqrt{-\sqrt{d}\sqrt{d+2e}+d+e}}-\frac{\left(\sqrt{d}\sqrt{d+2e}+d+2e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{d}\sqrt{d+2e}+d+e}}\right)}{\sqrt{\sqrt{d}\sqrt{d+2e}+d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{d+2e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d +
e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d + 2
*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt[d]
*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*S
qrt[d + 2*e]*Sqrt[f])
```

Maple [B] time = 0.051, size = 394, normalized size = 4.9

$$-\frac{f\sqrt{2}d}{4}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{df+fe+\sqrt{f^2d(d+2e)}}}\right)\frac{1}{\sqrt{f^2d(d+2e)}}\frac{1}{\sqrt{df+fe+\sqrt{f^2d(d+2e)}}}-\frac{f\sqrt{2}e}{2}\arctan\left(fx\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] -1/4*f/(f^2*d*(d+2*e))^(1/2)*2^(1/2)/(d*f+f*e+(f^2*d*(d+2*e))^(1/2))^(1/2)*
arctan(f*x*2^(1/2)/(d*f+f*e+(f^2*d*(d+2*e))^(1/2))^(1/2))*d-1/2*f/(f^2*d*(d
+2*e))^(1/2)*2^(1/2)/(d*f+f*e+(f^2*d*(d+2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/
2)/(d*f+f*e+(f^2*d*(d+2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(d*f+f*e+(f^2*d*(d+
2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(d*f+f*e+(f^2*d*(d+2*e))^(1/2))^(1/2)
)-1/4*f/(f^2*d*(d+2*e))^(1/2)*2^(1/2)/(-d*f-f*e+(f^2*d*(d+2*e))^(1/2))^(1/2
)*arctanh(f*x*2^(1/2)/(-d*f-f*e+(f^2*d*(d+2*e))^(1/2))^(1/2))*d-1/2*f/(f^2*
d*(d+2*e))^(1/2)*2^(1/2)/(-d*f-f*e+(f^2*d*(d+2*e))^(1/2))^(1/2)*arctanh(f*x
*2^(1/2)/(-d*f-f*e+(f^2*d*(d+2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(-d*f-f*e+(f
^2*d*(d+2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(-d*f-f*e+(f^2*d*(d+2*e))^(1
/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,fx^2-e}{4\,f^2x^4+4\,dfx^2+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

Fricas [A] time = 1.41653, size = 325, normalized size = 4.01

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4\,f^2x^4-4\,(d-e)fx^2+e^2+4\,\left(2\,fx^3+ex\right)\sqrt{-df}}{4\,f^2x^4+4\,(d+e)fx^2+e^2}\right)}{4\,df},\,-\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2\,fx^3+(2\,d+e)x\right)\sqrt{df}}{de}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)
*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*ar
ctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^3 + (2*d + e)*x)*sqrt(d*f)/(d
*e)))/(d*f)]
```

Sympy [A] time = 0.617299, size = 70, normalized size = 0.86

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

Giac [C] time = 2.26747, size = 4845, normalized size = 59.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/
abs(f))))^2*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^
(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))) - 4^(3/4)*(
f^2)^(3/4)*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(
3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 9*4^(3/4
)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*c
osh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2
*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arcco
s(-d*f*e^(-1)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4)*cosh(1/2*imag_pa
rt(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(arcc
os(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos(-d*f*e^(-1)
/abs(f) - f/abs(f)))) + 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f
*e^(-1)/abs(f) - f/abs(f))))^2*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f)
 - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(
f))))*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3
/4)*(f^2)^(3/4)*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*
e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/
2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/
4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(
1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(ar
ccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3 + 4^(3/4)*(f^2)^(3/4)*e^(3/2)*sin(1
/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(a
rccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 2*4^(1/4)*(f^2)^(1/4)*f*cosh(1/2
*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_par
t(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))) + 2*4^(1/4)*(f^2)^(1/4)*f*e^(3/2)
*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_pa
rt(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))))*arctan(4*(1/4)^(3/4)*((1/4)^(1/
4)*(f^(-2))^(1/4)*cos(1/2*arccos(-(d*f + f*e)*e^(-1)/abs(f)))*e^(1/2) + x)*
(f^2)^(1/4)*e^(-1/2)/sin(1/2*arccos(-(d*f + f*e)*e^(-1)/abs(f))))/((d^2 + 2
*d*e)*f^2 - sqrt(d^2 + 2*d*e)*(d*f + f*e)*abs(f)) + 1/8*(3*4^(3/4)*(f^2)^(3
/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*cosh(1/2*im
ag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3/2)*sin(1/2*real_part
(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))) - 4^(3/4)*(f^2)^(3/4)*cosh(1/2*ima
g_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3/2)*sin(1/2*real_part(
arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*r
eal_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*cosh(1/2*imag_part(arcco
s(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(arccos(-d*f*
e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) -
f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4)*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/a
bs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) -
 f/abs(f))))^3*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))) +
 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(
f))))^2*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*
sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_par
t(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*cosh(1/
2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_pa
rt(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos(-d*f
*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(ar
ccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(arccos(-d
*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f)
 - f/abs(f))))^3 + 4^(3/4)*(f^2)^(3/4)*e^(3/2)*sin(1/2*real_part(arccos(-d*
f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f
) - f/abs(f))))^3 - 2*4^(1/4)*(f^2)^(1/4)*f*cosh(1/2*imag_part(arccos(-d*f*
e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/ab
s(f) - f/abs(f)))) + 2*4^(1/4)*(f^2)^(1/4)*f*e^(3/2)*sin(1/2*real_part(arcc
os(-d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/a
bs(f) - f/abs(f)))))*arctan(-4*(1/4)^(3/4)*((1/4)^(1/4)*(f^(-2))^(1/4)*cos(
1/2*arccos(-(d*f + f*e)*e^(-1)/abs(f)))*e^(1/2) - x)*(f^2)^(1/4)*e^(-1/2)/s
in(1/2*arccos(-(d*f + f*e)*e^(-1)/abs(f))))/((d^2 + 2*d*e)*f^2 - sqrt(d^2 +
```

2*d*e)*(d*f + f*e)*abs(f)) + 1/16*(4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(a
rccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(-d*f*e^(
-1)/abs(f) - f/abs(f))))^3*e^(3/2) - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_par
t(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(-d*f*e^
(-1)/abs(f) - f/abs(f))))^3*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/ab
s(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*
e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f)
- f/abs(f))))^2*e^(3/2)*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/ab
s(f)))) + 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f)
 - f/abs(f))))*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2
*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*sinh(1
/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4
)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag
_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sinh(1/2*imag_part(ar
ccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*rea
l_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(-d
*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)
/abs(f) - f/abs(f))))^2*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/ab
s(f))))^2 - 4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f)
 - f/abs(f))))^3*e^(3/2)*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/a
bs(f))))^3 + 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs
(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/a
bs(f))))^2*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 2
4^(1/4)(f^2)^(1/4)*f*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(
f))))*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2) +
2*4^(1/4)*(f^2)^(1/4)*f*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs
(f))))*e^(3/2)*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))))*
log(2*(1/4)^(1/4)*(f^(-2))^(1/4)*x*cos(1/2*arccos(-(d*f + f*e)*e^(-1)/abs(f
)))*e^(1/2) + x^2 + 1/2*sqrt(f^(-2))*e)/((d^2 + 2*d*e)*f^2 - sqrt(d^2 + 2*d
e)(d*f + f*e)*abs(f)) - 1/16*(4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arcco
s(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/
abs(f) - f/abs(f))))^3*e^(3/2) - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(ar
ccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(-d*f*e^(-1)
/abs(f) - f/abs(f))))^3*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f)
 - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-
1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/
abs(f))))^2*e^(3/2)*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)
))) + 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f
/abs(f))))*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(
3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^2*sinh(1/2*i
mag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4)*co
s(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_par
t(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sinh(1/2*imag_part(arccos
(-d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_pa
rt(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(-d*f*e
^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs
(f) - f/abs(f))))^2*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)
)))^2 - 4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f
/abs(f))))^3*e^(3/2)*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f
))))^3 + 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f)
- f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f
))))^2*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 2*4^(
1/4)*(f^2)^(1/4)*f*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))
)*cosh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2) + 2*4^
(1/4)*(f^2)^(1/4)*f*cos(1/2*real_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f))
))*e^(3/2)*sinh(1/2*imag_part(arccos(-d*f*e^(-1)/abs(f) - f/abs(f)))))*log(
-2*(1/4)^(1/4)*(f^(-2))^(1/4)*x*cos(1/2*arccos(-(d*f + f*e)*e^(-1)/abs(f)))
*e^(1/2) + x^2 + 1/2*sqrt(f^(-2))*e)/((d^2 + 2*d*e)*f^2 - sqrt(d^2 + 2*d*e)
*(d*f + f*e)*abs(f))

3.525 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=73

$$\frac{\log\left(2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

```
[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log[e + 2*Sqr
t[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0445108, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1164, 628}

$$\frac{\log\left(2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log[e + 2*Sqr
t[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \int \frac{e-2fx^2}{e^2+(-4d+4e)fx^2+4f^2x^4}\,dx$$
$$= -\frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\sqrt{f}}$$
$$= -\frac{\log\left(e-2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}} + \frac{\log\left(e+2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.138129, size = 233, normalized size = 3.19

$$\frac{-\frac{\left(\sqrt{d}\sqrt{2e-d}-id+2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}-\frac{\left(\sqrt{d}\sqrt{2e-d}+id-2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{2e-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d +
 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]
*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[
-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

Maple [B] time = 0.048, size = 394, normalized size = 5.4

$$\frac{f\sqrt{2}d}{4}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{-df+fe+\sqrt{f^2d(d-2e)}}}\right)\frac{1}{\sqrt{f^2d(d-2e)}}\frac{1}{\sqrt{-df+fe+\sqrt{f^2d(d-2e)}}}-\frac{f\sqrt{2}e}{2}\arctan\left(fx\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4*f/(f^2*d*(d-2*e))^(1/2)*2^(1/2)/(-d*f+f*e+(f^2*d*(d-2*e))^(1/2))^(1/2)*
arctan(f*x*2^(1/2)/(-d*f+f*e+(f^2*d*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(f^2*d*(
d-2*e))^(1/2)*2^(1/2)/(-d*f+f*e+(f^2*d*(d-2*e))^(1/2))^(1/2)*arctan(f*x*2^(
1/2)/(-d*f+f*e+(f^2*d*(d-2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(-d*f+f*e+(f^2*d
*(d-2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+f*e+(f^2*d*(d-2*e))^(1/2))^
(1/2))+1/4*f/(f^2*d*(d-2*e))^(1/2)*2^(1/2)/(d*f-f*e+(f^2*d*(d-2*e))^(1/2))^
(1/2)*arctanh(f*x*2^(1/2)/(d*f-f*e+(f^2*d*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(f
^2*d*(d-2*e))^(1/2)*2^(1/2)/(d*f-f*e+(f^2*d*(d-2*e))^(1/2))^(1/2)*arctanh(f
*x*2^(1/2)/(d*f-f*e+(f^2*d*(d-2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(d*f-f*e+(f
^2*d*(d-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(d*f-f*e+(f^2*d*(d-2*e))^(1/
2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,fx^2-e}{4\,f^2x^4-4\,dfx^2+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

Fricas [A] time = 1.45365, size = 327, normalized size = 4.48

$$\left[\frac{\sqrt{df}\log\left(\frac{4\,f^2x^4+4\,(d+e)fx^2+e^2+4\,\left(2\,fx^3+ex\right)\sqrt{df}}{4\,f^2x^4-4\,(d-e)fx^2+e^2}\right)}{4\,df},-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{d}\right)-\sqrt{-df}\arctan\left(\frac{\left(2\,fx^3-(2\,d-e)x\right)\sqrt{-df}}{de}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*s
qrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arct
an(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^3 - (2*d - e)*x)*sqrt(-d*f)/(
d*e)))/(d*f)]
```

Sympy [A] time = 0.635968, size = 63, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

Giac [C] time = 2.22849, size = 4694, normalized size = 64.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

[Out] 1/8*(3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/a
bs(f))))^2*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3
/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)))) - 4^(3/4)*(f^2
)^(3/4)*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3/2)
*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 9*4^(3/4)*(f^
2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*cosh(1/
2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_p
art(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arccos(d*f*e^
(-1)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4)*cosh(1/2*imag_part(arccos
(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(
-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/
abs(f)))) + 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f
) - f/abs(f))))^2*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))
*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*
imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*
cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*r
eal_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos
(d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_par
t(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(arccos
(d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f
) - f/abs(f))))^3 + 4^(3/4)*(f^2)^(3/4)*e^(3/2)*sin(1/2*real_part(arccos(d*
f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f)
 - f/abs(f))))^3 - 2*4^(1/4)*(f^2)^(1/4)*f*cosh(1/2*imag_part(arccos(d*f*e^
(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f
) - f/abs(f)))) + 2*4^(1/4)*(f^2)^(1/4)*f*e^(3/2)*sin(1/2*real_part(arccos(
d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f)
 - f/abs(f)))))*arctan(4*(1/4)^(3/4)*((1/4)^(1/4)*(f^(-2))^(1/4)*cos(1/2*ar
ccos((d*f - f*e)*e^(-1)/abs(f)))*e^(1/2) + x)*(f^2)^(1/4)*e^(-1/2)/sin(1/2*
arccos((d*f - f*e)*e^(-1)/abs(f))))/((d^2 - 2*d*e)*f^2 + sqrt(d^2 - 2*d*e)*
(d*f - f*e)*abs(f)) + 1/8*(3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d
*f*e^(-1)/abs(f) - f/abs(f))))^2*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f
) - f/abs(f))))^3*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/ab
s(f)))) - 4^(3/4)*(f^2)^(3/4)*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) -
 f/abs(f))))^3*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f
))))^3 - 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) -
 f/abs(f))))^2*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*
e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*i
mag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4)*cos
h(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*re
al_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(arccos(
d*f*e^(-1)/abs(f) - f/abs(f)))) + 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*cosh(1/2*imag_part(arccos(d*f*e^(-1
)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) -
 f/abs(f))))*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2 -
3*4^(3/4)*(f^2)^(3/4)*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f
))))*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*sin
h(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^
(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*si
n(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3 + 4^(3/4)*(f^2)^(3/4)*e^(3/2)*sin(1
/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*sinh(1/2*imag_part(ar
ccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 2*4^(1/4)*(f^2)^(1/4)*f*cosh(1/2*i
mag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f)))) + 2*4^(1/4)*(f^2)^(1/4)*f*e^(3/2)*sin
(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*sinh(1/2*imag_part(ar
ccos(d*f*e^(-1)/abs(f) - f/abs(f)))))*arctan(-4*(1/4)^(3/4)*((1/4)^(1/4)*(f
^(-2))^(1/4)*cos(1/2*arccos((d*f - f*e)*e^(-1)/abs(f)))*e^(1/2) - x)*(f^2)^
(1/4)*e^(-1/2)/sin(1/2*arccos((d*f - f*e)*e^(-1)/abs(f))))/((d^2 - 2*d*e)*f
^2 + sqrt(d^2 - 2*d*e)*(d*f - f*e)*abs(f)) + 1/16*(4^(3/4)*(f^2)^(3/4)*cos(

```
1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3/2) - 3*4^(3/4)*(f^2)^(3/4)*cos
(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(ar
ccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3/2)*sin(1/2*real_part(arccos(d*f
*e^(-1)/abs(f) - f/abs(f))))^2 - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(ar
ccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(d*f*e^(-1)
/abs(f) - f/abs(f))))^2*e^(3/2)*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f)
 - f/abs(f)))) + 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/
abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)
)))^2*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*si
nh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(
3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*im
ag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sinh(1/2*imag_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*rea
l_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(d*f
*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/ab
s(f) - f/abs(f))))^2*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)
)))^2 - 4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/
abs(f))))^3*e^(3/2)*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))
))^3 + 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f
/abs(f))))*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))
^2*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3 - 2*4^(1/4)*
(f^2)^(1/4)*f*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*cosh
(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2) + 2*4^(1/4)*(
f^2)^(1/4)*f*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/
2)*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)))))*log(2*(1/4)^(
1/4)*(f^(-2))^(1/4)*x*cos(1/2*arccos((d*f - f*e)*e^(-1)/abs(f)))*e^(1/2) +
x^2 + 1/2*sqrt(f^(-2))*e)/((d^2 - 2*d*e)*f^2 + sqrt(d^2 - 2*d*e)*(d*f - f*e
)*abs(f)) - 1/16*(4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/a
bs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f
))))^3*e^(3/2) - 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/
abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)
)))^3*e^(3/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2 -
3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)
)))^3*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*s
inh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f)))) + 9*4^(3/4)*(f^2)^
(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*ima
g_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*e^(3/2)*sin(1/2*real_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*sinh(1/2*imag_part(arccos(d*f*e^(-1
)/abs(f) - f/abs(f)))) + 3*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f
*e^(-1)/abs(f) - f/abs(f))))^3*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f)
- f/abs(f))))*e^(3/2)*sinh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f
))))^2 - 9*4^(3/4)*(f^2)^(3/4)*cos(1/2*real_part(arccos(d*f*e^(-1)/abs(f) -
 f/abs(f))))*cosh(1/2*imag_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3
/2)*sin(1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*sinh(1/2*ima
g_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2 - 4^(3/4)*(f^2)^(3/4)*cos(1
/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3*e^(3/2)*sinh(1/2*imag
_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^3 + 3*4^(3/4)*(f^2)^(3/4)*cos(
1/2*real_part(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sin(1/2*real_p
art(arccos(d*f*e^(-1)/abs(f) - f/abs(f))))^2*sinh(1/2*imag_part(arccos(d*f*
e^(-1)/abs(f) - f/abs(f))))^3 - 2*4^(1/4)*(f^2)^(1/4)*f*cos(1/2*real_part(a
rccos(d*f*e^(-1)/abs(f) - f/abs(f))))*cosh(1/2*imag_part(arccos(d*f*e^(-1)/
abs(f) - f/abs(f))))*e^(3/2) + 2*4^(1/4)*(f^2)^(1/4)*f*cos(1/2*real_part(ar
ccos(d*f*e^(-1)/abs(f) - f/abs(f))))*e^(3/2)*sinh(1/2*imag_part(arccos(d*f*
e^(-1)/abs(f) - f/abs(f)))))*log(-2*(1/4)^(1/4)*(f^(-2))^(1/4)*x*cos(1/2*ar
ccos((d*f - f*e)*e^(-1)/abs(f)))*e^(1/2) + x^2 + 1/2*sqrt(f^(-2))*e)/((d^2
- 2*d*e)*f^2 + sqrt(d^2 - 2*d*e)*(d*f - f*e)*abs(f))
```

3.526 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0627856, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.0606442, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^2df+4\#1^3ef+4\#1^6f^2+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{3\#1^2e+6\#1^5f+2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1]) +
 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/(4*f)
```

Maple [C] time = 0.008, size = 70, normalized size = 1.8

$$\frac{1}{4f}\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6+4fe\_Z^3+4df\_Z^2+e^2\right)}\frac{\left(-4\_R^3f+e\right)\ln\left(x-\_R\right)}{6f\_R^5+3e\_R^2+2d\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z^
6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{4fx^3-e}{4f^2x^6+4efx^3+4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

Fricas [B] time = 1.47591, size = 347, normalized size = 9.13

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6+4efx^3-4dfx^2+e^2+4\left(2fx^4+ex\right)\sqrt{-df}}{4f^2x^6+4efx^3+4dfx^2+e^2}\right)}{4df},\ -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x^2}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2fx^5+ex^2+2dx\right)\sqrt{df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2 + 4*(2*f*x^4
+ e*x)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2))/(d*f), -1/2*(
sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + e*x^2 + 2*d
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

Sympy [B] time = 0.795029, size = 70, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{4 f x^3 - e}{4 f^2 x^6 + 4 e f x^3 + 4 d f x^2 + e^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

3.527 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0615178, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.0603246, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[-4\#1^2df+4\#1^3ef+4\#1^6f^2+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{3\#1^2e+6\#1^5f-2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1]) +
 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/(4*f)
```

Maple [C] time = 0.009, size = 70, normalized size = 1.8

$$\frac{1}{4f}\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6+4fe\_Z^3-4df\_Z^2+e^2\right)}\frac{\left(-4\_R^3f+e\right)\ln\left(x-\_R\right)}{6f\_R^5+3e\_R^2-2d\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e-2*_R*d)*ln(x-_R),_R=RootOf(4*_Z^
6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{4fx^3-e}{4f^2x^6+4efx^3-4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

Fricas [B] time = 1.49488, size = 348, normalized size = 9.16

$$\left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4efx^3+4dfx^2+e^2+4\left(2fx^4+ex\right)\sqrt{df}}{4f^2x^6+4efx^3-4dfx^2+e^2}\right)}{4df},-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x^2}{d}\right)-\sqrt{-df}\arctan\left(\frac{\left(2fx^5+ex^2-2dx\right)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2 + 4*(2*f*x^4 +
e*x)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqr
t(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + e*x^2 - 2*d
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

Sympy [A] time = 0.830833, size = 63, normalized size = 1.66

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{4\,fx^3 - e}{4\,f^2x^6 + 4\,efx^3 - 4\,dfx^2 + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

3.528 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0955535, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+2f(-1+n)x^n}\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.274566, size = 0, normalized size = 0.

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)), x]
```

Maple [B] time = 0.049, size = 78, normalized size = 2.1

$$-\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(2dfx+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(-2dfx+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)+1/4/(
-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,f(n-1)x^{n}-e}{4\,dfx^{2}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

Fricas [A] time = 1.54707, size = 325, normalized size = 8.55

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^{2}-4\,f^{2}x^{2\,n}-4\,\sqrt{-df}ex-e^{2}-4\left(2\,\sqrt{-df}fx+ef\right)x^{n}}{4\,dfx^{2}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\right)}{4\,df},\ -\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}fx^{n}+\sqrt{df}e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*e*x - e^2 -
 4*(2*sqrt(-d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*
f*x))/(d*f)]
```

Sympy [A] time = 126.247, size = 139, normalized size = 3.66

$$\begin{cases} \frac{x}{e} & \text{for}\ f = 0 \wedge \left(d = 0 \vee f = 0\right) \\ \frac{x}{e+2fx^n} & \text{for}\ d = 0 \\ -\frac{i \log\left(-\frac{ie\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}}{2} - ifx^n\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}} + x\right)}{4df\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}} + \frac{i \log\left(\frac{ie\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}}{2} + ifx^n\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}} + x\right)}{4df\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Piecewise((x/e, Eq(f, 0) & (Eq(d, 0) | Eq(f, 0))), (x/(e + 2*f*x**n), Eq(d,
 0)), (-I*log(-I*e*sqrt(1/d)*sqrt(1/f)/2 - I*f*x**n*sqrt(1/d)*sqrt(1/f) + x
)/(4*d*f*sqrt(1/d)*sqrt(1/f)) + I*log(I*e*sqrt(1/d)*sqrt(1/f)/2 + I*f*x**n*
sqrt(1/d)*sqrt(1/f) + x)/(4*d*f*sqrt(1/d)*sqrt(1/f)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,f(n-1)x^n - e}{4\,dfx^2 + 4\,f^2x^{2\,n} + 4\,efx^n + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

3.529 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0950575, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx &= -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+2f(-1+n)x^n}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [F] time = 0.277112, size = 0, normalized size = 0.

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)), x]
```

Maple [B] time = 0.045, size = 72, normalized size = 1.9

$$\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(2dfx+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(-2dfx+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)-1/4/(d*f)
^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2f(n-1)x^{n}-e}{4dfx^{2}-4f^{2}x^{2n}-4efx^{n}-e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^
2), x)
```

Fricas [A] time = 1.56521, size = 324, normalized size = 8.53

$$\left[\frac{\sqrt{df}\log\left(-\frac{4dfx^{2}+4f^{2}x^{2n}+4\sqrt{df}ex+e^{2}+4\left(2\sqrt{df}fx+ef\right)x^{n}}{4dfx^{2}-4f^{2}x^{2n}-4efx^{n}-e^{2}}\right)}{4df},\ -\frac{\sqrt{-df}\arctan\left(\frac{2\sqrt{-df}fx^{n}+\sqrt{-df}e}{2dfx}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*e*x + e^2 + 4*
(2*sqrt(d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2))
/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f
*x))/(d*f)]
```

Sympy [A] time = 121.347, size = 129, normalized size = 3.39

$$\begin{cases} \frac{x}{e} & \text{for } f = 0 \wedge \left(d = 0 \vee f = 0\right) \\ \frac{x}{e+2fx^n} & \text{for } d = 0 \\ -\frac{\log\left(-\frac{e\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}}{2} - fx^n\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}} + x\right)}{4df\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}} + \frac{\log\left(\frac{e\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}}{2} + fx^n\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}} + x\right)}{4df\sqrt{\frac{1}{d}}\sqrt{\frac{1}{f}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Piecewise((x/e, Eq(f, 0) & (Eq(d, 0) | Eq(f, 0))), (x/(e + 2*f*x**n), Eq(d,
 0)), (-log(-e*sqrt(1/d)*sqrt(1/f)/2 - f*x**n*sqrt(1/d)*sqrt(1/f) + x)/(4*d
*f*sqrt(1/d)*sqrt(1/f)) + log(e*sqrt(1/d)*sqrt(1/f)/2 + f*x**n*sqrt(1/d)*sq
rt(1/f) + x)/(4*d*f*sqrt(1/d)*sqrt(1/f)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,f(n-1)x^n - e}{4\,dfx^2 - 4\,f^2x^{2\,n} - 4\,efx^n - e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^
2), x)
```

3.530 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4} dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rubi [A] time = 0.0661598, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \int \frac{x}{e^2+4efx^2+4\left(df+f^2\right)x^4}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx, x, x^2\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx, x, 4f\left(e+2(d+f)x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Mathematica [A] time = 0.0212751, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Maple [A] time = 0.003, size = 42, normalized size = 1.

$$\frac{1}{4e}\arctan\left(\frac{2\left(4df+4f^2\right)x^2+4fe}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/e/(d*f)^(1/2)*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*f*e)/e/(d*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43143, size = 340, normalized size = 8.1

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4\left(d^2f+2df^2+f^3\right)x^4-de^2+e^2f+4\left(def+ef^2\right)x^2-2\left(2\left(de+ef\right)x^2+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^4+4efx^2+e^2}\right)}{8def}, \frac{\sqrt{df}\arctan\left(\frac{\left(2\left(d+f\right)x^2+e\right)\sqrt{df}}{de}\right)}{4def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d*
e*f + e*f^2)*x^2 - 2*(2*(d*e + e*f)*x^2 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x
^4 + 4*e*f*x^2 + e^2))/(d*e*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)*sq
rt(d*f)/(d*e))/(d*e*f)]
```

Sympy [B] time = 0.526625, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8 + sqrt
(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e
```

Giac [A] time = 1.30321, size = 51, normalized size = 1.21

$$\frac{\arctan\left(\frac{\left(2\,dfx^2+2\,f^2x^2+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

3.531 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^2(d-f)\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rubi [A] time = 0.0674179, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^2(d-f)\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = \int \frac{x}{e^2+4efx^2+\left(-4df+4f^2\right)x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{e^2+4efx+\left(-4df+4f^2\right)x^2}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^2\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e+2(-d+f)x^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Mathematica [A] time = 0.0222899, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(-2dx^2+e+2fx^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f
])
```

Maple [A] time = 0.003, size = 42, normalized size = 1.

$$\frac{1}{4e}\text{Artanh}\left(\frac{2\left(4df-4f^2\right)x^2-4fe}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/e/(d*f)^(1/2)*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*f*e)/e/(d*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25194, size = 342, normalized size = 7.77

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\left(d^2f-2df^2+f^3\right)x^4+de^2+e^2f-4\left(def-ef^2\right)x^2+2\left(2\left(de-ef\right)x^2-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^4-4efx^2-e^2}\right)}{8def}, \frac{\sqrt{-df}\arctan\left(-\frac{\left(2\left(d-f\right)x^2-e\right)\sqrt{-df}}{de}\right)}{4def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*e
*f - e*f^2)*x^2 + 2*(2*(d*e - e*f)*x^2 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^4
 - 4*e*f*x^2 - e^2))/(d*e*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)*sq
rt(-d*f)/(d*e))/(d*e*f)]
```

Sympy [A] time = 0.560523, size = 75, normalized size = 1.7

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}-\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8 - sqrt(1
/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8)/e
```

Giac [A] time = 1.36528, size = 55, normalized size = 1.25

$$-\frac{\arctan\left(\frac{2dfx^2-2f^2x^2-fe}{\sqrt{-dfe^2}}\right)}{4\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)
```

3.532 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.129959, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx &= \left(3e^2\right)\text{Subst}\left(\int \frac{1}{e^2+36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.0507576, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[4\#1^6df+4\#1^2ef+4\#1^4f^2+e^2\&,\frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) & ]/(8*f)
```

Maple [C] time = 0.112, size = 74, normalized size = 1.9

$$\frac{1}{8f}\sum_{\_R=\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4fe\_Z^2+e^2\right)}\frac{\left(2\_R^4f+3\_R^2e\right)\ln\left(x-\_R\right)}{3d\_R^5+2f\_R^3+e\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(4
*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2fx^2+3e\right)x^2}{4dfx^6+4f^2x^4+4efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="maxima")
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)
```

Fricas [B] time = 1.26836, size = 462, normalized size = 11.55

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4dfx^6-4f^2x^4-4efx^2-e^2-4\left(2fx^5+ex^3\right)\sqrt{-df}}{4dfx^6+4f^2x^4+4efx^2+e^2}\right)}{4df},\ \frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{f}\right)-\sqrt{df}\arctan\left(\frac{2\left(2dfx^5-\left(de-2f^2\right)x^3+efx\right)\sqrt{df}}{de^2}\right)+\sqrt{d}}{2df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2 - 4*(2*f*x^5
+ e*x^3)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2))/(d*f), 1/2*
(sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(2*(2*d*f*x^5 - (d*e - 2
*f^2)*x^3 + e*f*x)*sqrt(d*f)/(d*e^2)) + sqrt(d*f)*arctan((2*d*f*x^3 - (d*e
- 2*f^2)*x)*sqrt(d*f)/(d*e*f)))/(d*f)]
```

Sympy [B] time = 1.08516, size = 90, normalized size = 2.25

$$-\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4 +
 sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="giac")
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)
```

3.533 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.127675, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx &= \left(3e^2\right)\text{Subst}\left(\int \frac{1}{e^2-36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.0518767, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[-4\#1^6df+4\#1^2ef+4\#1^4f^2+e^2\&,\frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{-3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(8*f)
```

Maple [C] time = 0.105, size = 77, normalized size = 1.9

$$-\frac{1}{8f}\sum_{\_R=\mathrm{RootOf}\left(4df\_Z^6-4f^2\_Z^4-4fe\_Z^2-e^2\right)}\frac{\left(2\_R^4f+3\_R^2e\right)\ln\left(x-\_R\right)}{3d\_R^5-2f\_R^3-e\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(x-_R),_R=RootOf(
4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2fx^2+3e\right)x^2}{4dfx^6-4f^2x^4-4efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="maxima")
```

```
[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

Fricas [B] time = 1.23696, size = 467, normalized size = 11.68

$$\left[\frac{\sqrt{df}\log\left(\frac{4dfx^6+4f^2x^4+4efx^2+e^2+4\left(2fx^5+ex^3\right)\sqrt{df}}{4dfx^6-4f^2x^4-4efx^2-e^2}\right)}{4df},-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{f}\right)-\sqrt{-df}\arctan\left(\frac{2\left(2dfx^5-\left(de+2f^2\right)x^3-efx\right)\sqrt{-df}}{de^2}\right)}{2df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2 + 4*(2*f*x^5 +
e*x^3)*sqrt(d*f))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(2*(2*d*f*x^5 - (d*e +
2*f^2)*x^3 - e*f*x)*sqrt(-d*f)/(d*e^2)) + sqrt(-d*f)*arctan((2*d*f*x^3 - (d
*e + 2*f^2)*x)*sqrt(-d*f)/(d*e*f)))/(d*f)]
```

Sympy [B] time = 1.0913, size = 80, normalized size = 2.

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + sq
rt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="giac")
```

```
[Out] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

3.534 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.219073, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x^
2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x, \frac{}{e(-1}\right.\right.$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.316644, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] time = 0.053, size = 78, normalized size = 1.9

$$-\frac{1}{4}\ln\left(x^{m}+\frac{2fx^{2}+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^{m}-\frac{2fx^{2}+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),
x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2f(m-1)x^{2}+e(m+1)\right)x^{m}}{4f^{2}x^{4}+4efx^{2}+4dfx^{2m+2}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 1.37995, size = 321, normalized size = 7.64

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4f^{2}x^{4}-4dfx^{2}x^{2m}+4efx^{2}+4\left(2fx^{3}+ex\right)\sqrt{-df}x^{m}+e^{2}}{4f^{2}x^{4}+4dfx^{2}x^{2m}+4efx^{2}+e^{2}}\right)}{4df},-\frac{\sqrt{df}\arctan\left(\frac{\left(2fx^{2}+e\right)\sqrt{df}}{2dfxx^{m}}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x
^3 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-1)x^2+e(m+1)\right)x^m}{4\,f^2x^4+4\,efx^2+4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

3.535 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.218034, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\left(1-m^2\right)x^{m+1}}{(1-m)(m+1)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x
^2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x, \frac{}{e(-1+n}\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\left(1-m^2\right)x^{1+m}}{(1-m)(1+m)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.0410601, size = 42, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] time = 0.053, size = 74, normalized size = 1.8

$$\frac{1}{4}\ln\left(x^m + \frac{2fx^2+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}} - \frac{1}{4}\ln\left(x^m - \frac{2fx^2+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),
x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-1)x^2 + e(m+1)\right)x^m}{4f^2x^4 + 4efx^2 - 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 1.21498, size = 320, normalized size = 7.62

$$\left[\frac{\sqrt{df}\log\left(-\frac{4f^2x^4+4dfx^2x^{2m}+4efx^2+4\left(2fx^3+ex\right)\sqrt{df}x^m+e^2}{4f^2x^4-4dfx^2x^{2m}+4efx^2+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\left(2fx^2+e\right)\sqrt{-df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x^3
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2 f(m-1)x^2 + e(m+1)\right)x^m}{4 f^2 x^4 + 4 e f x^2 - 4 d f x^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

3.536 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.089305, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2e-2fx^3\right)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx &= -\left(\left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.0502882, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[4\#1^4df+4\#1^3ef+4\#1^6f^2+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{4\#1^2d+6\#1^4f+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1]) +
 f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]/(2*f)
```

Maple [C] time = 0.009, size = 74, normalized size = 1.9

$$-\frac{1}{2f}\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4fe\_Z^3+e^2\right)}\frac{\left(\_R^4f-\_Re\right)\ln\left(x-\_R\right)}{6f\_R^5+4d\_R^3+3e\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(x-_R),_R=RootOf(4*
_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\left(fx^3-e\right)x}{4f^2x^6+4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

Fricas [B] time = 1.33453, size = 347, normalized size = 8.68

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6-4dfx^4+4efx^3+e^2+4\left(2fx^5+ex^2\right)\sqrt{-df}}{4f^2x^6+4dfx^4+4efx^3+e^2}\right)}{4df},-\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2fx^4+2dx^2+ex\right)\sqrt{df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5
+ e*x^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2
*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2 + e
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

Sympy [B] time = 1.15566, size = 73, normalized size = 1.82

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*
f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\left(fx^3-e\right)x}{4f^2x^6+4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

3.537 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rubi [A] time = 0.0891171, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2e-2fx^3\right)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx &= -\left(\left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.0481161, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[-4\#1^4df+4\#1^3ef+4\#1^6f^2+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{-4\#1^2d+6\#1^4f+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1]) +
 f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]/(2*f)
```

Maple [C] time = 0.009, size = 74, normalized size = 1.9

$$-\frac{1}{2f}\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4fe\_Z^3+e^2\right)}\frac{\left(\_R^4 f-\_R e\right)\ln\left(x-\_R\right)}{6f\_R^5-4d\_R^3+3e\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f-4*_R^3*d+3*_R^2*e)*ln(x-_R),_R=RootOf(4*
_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\left(fx^3-e\right)x}{4f^2x^6-4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

Fricas [B] time = 1.3109, size = 348, normalized size = 8.7

$$\left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4dfx^4+4efx^3+e^2+4\left(2fx^5+ex^2\right)\sqrt{df}}{4f^2x^6-4dfx^4+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{d}\right)-\sqrt{-df}\arctan\left(\frac{\left(2fx^4-2dx^2+ex\right)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 +
e*x^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^4 - 2*d*x^2 + e
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

Sympy [A] time = 1.16377, size = 66, normalized size = 1.65

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f)
)*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\left(fx^3-e\right)x}{4\,f^2x^6-4\,dfx^4+4\,efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

3.538 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rubi [A] time = 0.0589649, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx=\int \frac{x^2}{e^2+4efx^3+4\left(df+f^2\right)x^6}\,dx$$
$$=\frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx,x,x^3\right)$$
$$=-\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx,x,4f\left(e+2(d+f)x^3\right)\right)\right)$$
$$=\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^3\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Mathematica [A] time = 0.0195887, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Maple [A] time = 0.002, size = 42, normalized size = 1.

$$\frac{1}{6e}\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4fe}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/e/(d*f)^(1/2)*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*f*e)/e/(d*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24589, size = 342, normalized size = 8.14

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4\left(d^2f+2df^2+f^3\right)x^6+4\left(def+ef^2\right)x^3-de^2+e^2f-2\left(2\left(de+ef\right)x^3+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^6+4efx^3+e^2}\right)}{12def},\ \frac{\sqrt{df}\arctan\left(\frac{\left(2\left(d+f\right)x^3+e\right)\sqrt{df}}{de}\right)}{6def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x^
3 - d*e^2 + e^2*f - 2*(2*(d*e + e*f)*x^3 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*
x^6 + 4*e*f*x^3 + e^2))/(d*e*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e)*s
qrt(d*f)/(d*e))/(d*e*f)]
```

Sympy [B] time = 0.693828, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sqr
t(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e
```

Giac [A] time = 1.80116, size = 51, normalized size = 1.21

$$\frac{\arctan\left(\frac{\left(2\,dfx^3+2\,f^2x^3+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

```
[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

3.539 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^3(d-f)\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rubi [A] time = 0.0616221, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^3(d-f)\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = \int \frac{x^2}{e^2+4efx^3+\left(-4df+4f^2\right)x^6}\,dx$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{e^2+4efx+\left(-4df+4f^2\right)x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^3\right)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2(d-f)x^3\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Mathematica [A] time = 0.0218638, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(-2dx^3+e+2fx^3\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f
])
```

Maple [A] time = 0.003, size = 42, normalized size = 1.

$$\frac{1}{6e}\text{Artanh}\left(\frac{2\left(4df-4f^2\right)x^3-4fe}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/e/(d*f)^(1/2)*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*f*e)/e/(d*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28025, size = 343, normalized size = 7.8

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\left(d^2f-2\,df^2+f^3\right)x^6-4\left(def-ef^2\right)x^3+de^2+e^2f+2\left(2\left(de-ef\right)x^3-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^6-4\,efx^3-e^2}\right)}{12\,def}, \frac{\sqrt{-df}\arctan\left(-\frac{\left(2\left(d-f\right)x^3-e\right)\sqrt{-df}}{de}\right)}{6\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

```
[Out] [1/12*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^3
 + d*e^2 + e^2*f + 2*(2*(d*e - e*f)*x^3 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^
6 - 4*e*f*x^3 - e^2))/(d*e*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e)*s
qrt(-d*f)/(d*e))/(d*e*f)]
```

Sympy [A] time = 0.736859, size = 75, normalized size = 1.7

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12 - sqrt(
1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12)/e
```

Giac [A] time = 1.75315, size = 55, normalized size = 1.25

$$-\frac{\arctan\left(\frac{2\,dfx^3-2\,f^2x^3-fe}{\sqrt{-dfe^2}}\right)}{6\,\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

```
[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)
```

3.540 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.22301, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x, \frac{}{e(-2}\right.\right.$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.317145, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] time = 0.035, size = 78, normalized size = 1.9

$$-\frac{1}{4}\ln\left(x^{m}+\frac{2fx^{3}+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^{m}-\frac{2fx^{3}+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),
x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-2)x^{3}+e(m+1)\right)x^{m}}{4f^{2}x^{6}+4efx^{3}+4dfx^{2m+2}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 1.27622, size = 321, normalized size = 7.64

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4f^{2}x^{6}-4dfx^{2}x^{2m}+4efx^{3}+4\left(2fx^{4}+ex\right)\sqrt{-df}x^{m}+e^{2}}{4f^{2}x^{6}+4dfx^{2}x^{2m}+4efx^{3}+e^{2}}\right)}{4df},-\frac{\sqrt{df}\arctan\left(\frac{\left(2fx^{3}+e\right)\sqrt{df}}{2dfxx^{m}}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x
^4 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-2)x^3+e(m+1)\right)x^m}{4\,f^2x^6+4\,efx^3+4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

3.541 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.215992, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x, \frac{}{e(-2+}\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.0446058, size = 42, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] time = 0.033, size = 74, normalized size = 1.8

$$\frac{1}{4}\ln\left(x^m+\frac{2fx^3+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^m-\frac{2fx^3+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),
x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2f(m-2)x^3+e(m+1)\right)x^m}{4f^2x^6+4efx^3-4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 1.31763, size = 320, normalized size = 7.62

$$\left[\frac{\sqrt{df}\log\left(-\frac{4f^2x^6+4dfx^2x^{2m}+4efx^3+4\left(2fx^4+ex\right)\sqrt{df}x^m+e^2}{4f^2x^6-4dfx^2x^{2m}+4efx^3+e^2}\right)}{4df},-\frac{\sqrt{-df}\arctan\left(\frac{\left(2fx^3+e\right)\sqrt{-df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^4
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-2)x^3+e(m+1)\right)x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

3.542 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.249303, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(1+m)^2(1+m-n)^2x^2}\,dx, x,\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.480236, size = 0, normalized size = 0.

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

Maple [B] time = 0.066, size = 84, normalized size = 2.

$$-\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(2\,dfxx^{m}+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(-2\,dfxx^{m}+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)+1
/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-n+1)x^{n}+e(m+1)\right)x^{m}}{4\,dfx^{2\,m+2}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

Fricas [A] time = 1.28238, size = 363, normalized size = 8.64

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^{2}x^{2\,m}-4\,\sqrt{-df}exx^{m}-4\,f^{2}x^{2\,n}-e^{2}-4\,\left(2\,\sqrt{-df}fxx^{m}+ef\right)x^{n}}{4\,dfx^{2}x^{2\,m}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\right)}{4\,df},\,-\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}fx^{n}+\sqrt{df}e}{2\,dfxx^{m}}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*e*x*x^m - 4*f^2*x^(
2*n) - e^2 - 4*(2*sqrt(-d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f^2
*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*
f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-n+1)x^{n}+e(m+1)\right)x^{m}}{4\,dfx^{2\,m+2}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

3.543 $\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.242656, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(1+m)^2(1+m-n)^2x^2}\,dx, x, \frac{}{e(1}\right.$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.517193, size = 0, normalized size = 0.

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

Maple [B] time = 0.065, size = 78, normalized size = 1.9

$$\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(2dfxx^{m}+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(-2dfxx^{m}+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+e*(d*f)^(1/2))/(d*f)^(1/2)/f)-1/4/(
d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+e*(d*f)^(1/2))/(d*f)^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2f(m-n+1)x^{n}+e(m+1)\right)x^{m}}{4dfx^{2m+2}-4f^{2}x^{2n}-4efx^{n}-e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

Fricas [A] time = 1.32736, size = 362, normalized size = 8.62

$$\left[\frac{\sqrt{df}\log\left(-\frac{4dfx^{2}x^{2m}+4\sqrt{df}exx^{m}+4f^{2}x^{2n}+e^{2}+4\left(2\sqrt{df}fxx^{m}+ef\right)x^{n}}{4dfx^{2}x^{2m}-4f^{2}x^{2n}-4efx^{n}-e^{2}}\right)}{4df},-\frac{\sqrt{-df}\arctan\left(\frac{2\sqrt{-df}fx^{n}+\sqrt{-df}e}{2dfxx^{m}}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m) + 4*sqrt(d*f)*e*x*x^m + 4*f^2*x^(2*n
) + e^2 + 4*(2*sqrt(d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) - 4*f^2*x^(
2*n) - 4*e*f*x^n - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*
x^n + sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2 f(m-n+1)x^{n}+e(m+1)\right)x^{m}}{4 d f x^{2 m+2}-4 f^{2}x^{2 n}-4 e f x^{n}-e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

3.544 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=134

$$-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(a+bx^2\right)^2}{4b^3c}$$

```
[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/(b
^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + ((a
*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

Rubi [A] time = 0.372887, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(a+bx^2\right)^2}{4b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/(b
^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + ((a
*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\left(a-x^2\right)^2}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{\left(2ac^2-d^2\right)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{\left(ac^2-d^2\right)^2}{c^4(d+cx)}\right)dx,x,\sqrt{a+bx^2}\right)}{b^3}\\
&= -\frac{\left(2ac^2-d^2\right)x^2}{2b^2c^3}+\frac{d\left(2ac^2-d^2\right)\sqrt{a+bx^2}}{b^3c^4}-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(a+bx^2\right)^2}{4b^3c}+\frac{\left(ac^2-d^2\right)}{}
\end{aligned}$$

Mathematica [A] time = 0.218416, size = 126, normalized size = 0.94

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^2}-6bc^3x^2\right)+2bcdx^2\left(3d-2c\sqrt{a+bx^2}\right)-12d^3\sqrt{a+bx^2}+3b^2c^3x^4\right)+12\left(d^2-ac^2\right)^2\log\left(c\sqrt{a+bx^2}\cdots\right.}{12b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (c*(3*b^2*c^3*x^4 - 12*d^3*Sqrt[a + b*x^2] + 2*b*c*d*x^2*(3*d - 2*c*Sqrt[a
+ b*x^2]) + a*(-6*b*c^3*x^2 + 20*c^2*d*Sqrt[a + b*x^2])) + 12*(-(a*c^2) + d
^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(12*b^3*c^5)
```

Maple [B] time = 0.077, size = 4947, normalized size = 36.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^
2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/
2)*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2
)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+
1/c^2*d^2)^(1/2)*d^5-1/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((
-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2/b)+1/c^2*d^2)^(1/2)*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))
^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*((x+(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*d^5-1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-
c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c
^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-
2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2
*d^2)^(1/2))*a^2-1/2*d/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1/2)/b)^2*b
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-c^2
*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*c^2*((
x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*a^2+1/2*d/b^(5/2)/((-a*
b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d
^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b
*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+1/c^2*d^2)^(1/2))*a^2+1/2/b^(5/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2
-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^4*(-c^2*b*(a*
c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/
2))*d^5-1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)
*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^4*d^7/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^
```

```
2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d
^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*c^2*((x+(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*a^2+1/2/b^2/c^3*x^2*d^2+1/2/b^3/c^
5*d^4*ln(b*c^2*x^2+a*c^2-d^2)-1/2*a/c/b^2*x^2+1/2*a^2/c/b^3*ln(b*c^2*x^2+a*
c^2-d^2)-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2-1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a
*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(1/c^2*
d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/
b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b
)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a^2-1/2/b^(5/2)/(
(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c
^2-d^2))^(1/2))/c^4*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))*d^5-1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(
a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(1/c^2
*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/
b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a^2-1/2/b^2/((-a
*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-
d^2))^(1/2))/c^4*d^7/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b))-1/2*d/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/
((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-
d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*d^5/(1/c^2*d
^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b
)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a-1/2*d/b^(5/2)*c^
2*a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2
*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/
b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))-1
/b^(5/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-
c^2*b*(a*c^2-d^2))^(1/2))/c^2*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2
-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+
(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c
^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a+1
/b^(5/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-
c^2*b*(a*c^2-d^2))^(1/2))/c^2*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))*a*d^3+1/2*d/b^(5/2)*c^2*a^2/(
(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c
^2-d^2))^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)
+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/4/b/c*
x^4-a/c^3/b^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)
```

Maxima [A] time = 1.4181, size = 169, normalized size = 1.26

$$\frac{\frac{3\left(bx^2+a\right)^2c^3-4\left(bx^2+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^2+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^2+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^2+a}c+d\right)}{c^5}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3
```

Fricas [A] time = 1.48454, size = 498, normalized size = 3.72

$$\frac{3\,b^2c^4x^4-6\left(abc^4-bc^2d^2\right)x^2+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(bc^2x^2+ac^2-d^2\right)+3\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{}}{x^2}\right.}{12\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 3*(a^2*c^4 - 2*a
*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2)
 - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**5/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Giac [A] time = 1.13328, size = 209, normalized size = 1.56

$$\frac{\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{b^3c^5}+\frac{3\left(bx^2+a\right)^2b^9c^3-12\left(bx^2+a\right)ab^9c^3-4\left(bx^2+a\right)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^2+a}ab}{12\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) + 1
/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(3/
2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2 - 1
2*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)
```

3.545 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=69

$$-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}-\frac{d\sqrt{a+bx^2}}{b^2c^2}+\frac{x^2}{2bc}$$

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt
[a + b*x^2]])/(b^2*c^3)
```

Rubi [A] time = 0.214601, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}-\frac{d\sqrt{a+bx^2}}{b^2c^2}+\frac{x^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt
[a + b*x^2]])/(b^2*c^3)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b^2}\\ &= \frac{\operatorname{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)dx,x,\sqrt{a+bx^2}\right)}{b^2}\\ &= \frac{x^2}{2bc}-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}\end{aligned}$$

Mathematica [A] time = 0.0892205, size = 65, normalized size = 0.94

$$\frac{-\frac{\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^2}+d\right)}{c^3}-\frac{d\sqrt{a+bx^2}}{c^2}+\frac{bx^2}{2c}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*x^2)/(2*c) - (d*Sqrt[a + b*x^2])/c^2 - ((a*c^2 - d^2)*Log[d + c*Sqrt[a
+ b*x^2]])/c^3)/b^2
```

Maple [B] time = 0.019, size = 3410, normalized size = 49.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c
^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-a*b)^(1/2)*l
n((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/2*d/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2
-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b*((x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*c^2*a-1/2/((-a*b)^(1/2)*c^2+(-c^2
*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b*((x
-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*d^3+1/2*d/((-a*b)^(1/2)*
c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))/b^(3/2)*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b
*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+1/c^2*d^2)^(1/2))*a-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2
))*d^3-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2
+(-c^2*b*(a*c^2-d^2))^(1/2))/b*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)
^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b))*a+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2)
)/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b/c^2*d^5/(1/c^2*d^2)^(1/2
)*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d
^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d/((-a*b)^(1/2)*c^2+(-
c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b*
((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*
(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*c^2*a-1/2/((-a*b)^(1/
2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^
```

```
(1/2))/b*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*d^3-1/2*d/((
-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/b^(3/2)*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c
^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))*a+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*
c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^
2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1
/c^2*d^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-
a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^
2*d^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
+2*(1/c^2*d^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))
/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b/c^2*d^5/(1
/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)
/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*d*c^2*a/
((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a
*c^2-d^2))^(1/2))/b*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b
)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln((b*(x-(-a*b
)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b))^(1/2))-1/2*a/c/b^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*x^2/b/c+1/2
/b^2/c^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)
```

Maxima [A] time = 1.29795, size = 84, normalized size = 1.22

$$\frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}d}{c^2}-\frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^2+a}c+d\right)}{c^3}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^2 + a)*c + d)/c^3)/b^2
```

Fricas [B] time = 1.44253, size = 340, normalized size = 4.93

$$\frac{2\,bc^2x^2-4\sqrt{bx^2+a}cd-2\left(ac^2-d^2\right)\log\left(bc^2x^2+ac^2-d^2\right)-\left(ac^2-d^2\right)\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)+\left(ac^2-d^2\right)\log\left(-\right.}{4\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 +
a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*
```

```
d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c
*d + d^2)/x^2))/(b^2*c^3)
```

Sympy [A] time = 4.08136, size = 88, normalized size = 1.28

$$\begin{cases} \frac{\frac{a+bx^2}{2bc} - \frac{d\sqrt{a+bx^2}}{bc^2} - \frac{(ac^2-d^2)\left(\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c = 0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{bc^2}}{b} & \text{for } b \neq 0 \\ \frac{x^4}{2\left(2\sqrt{a}d+2ac\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((((a + b*x**2)/(2*b*c) - d*sqrt(a + b*x**2)/(b*c**2) - (a*c**2 -
d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(b*c**2))/b, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)), True))
```

Giac [A] time = 1.12227, size = 97, normalized size = 1.41

$$-\frac{\frac{2\left(ac^2-d^2\right)\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{bc^3} - \frac{\left(bx^2+a\right)bc-2\sqrt{bx^2+a}bd}{b^2c^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 + a
)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

3.546 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rubi [A] time = 0.085703, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2155, 31}

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}\end{aligned}$$

Mathematica [A] time = 0.0222406, size = 23, normalized size = 1.

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Maple [B] time = 0.015, size = 1931, normalized size = 84.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+
(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b))^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(
-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^
(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^
2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)-1/2*d/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2
-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2
-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))
/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c
^2+(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)
^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b))-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)+1/2*d/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2)
)^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2)
)^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)
/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/b^(
1/2)+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(
-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/
2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b))+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2)
)/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*
(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(
1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)+1/2/b/c*ln(b*c^2*x^2+a*c
^2-d^2)
```

Maxima [A] time = 1.57803, size = 28, normalized size = 1.22

$$\frac{\log\left(\sqrt{bx^2+ac}+d\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

Fricas [B] time = 1.2414, size = 227, normalized size = 9.87

$$\frac{2\,\log\left(bc^2x^2+ac^2-d^2\right)+\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right)-\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^
2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d +
d^2)/x^2))/(b*c)
```

Sympy [A] time = 2.38927, size = 29, normalized size = 1.26

$$\frac{\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, T
rue))/b
```

Giac [A] time = 1.1544, size = 30, normalized size = 1.3

$$\frac{\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

3.547 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=88

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/(
a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

Rubi [A] time = 0.247173, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/(
a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^2\right)\\ &= \text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx,x,\sqrt{a+bx^2}\right)\\ &= -\frac{c^2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{ac^2-d^2}+\frac{\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx,x,\sqrt{a+bx^2}\right)}{-ac^2+d^2}\\ &= -\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}+\frac{c\,\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx,x,\sqrt{a+bx^2}\right)}{ac^2-d^2}-\frac{d\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx,x\right.}{ac^2-d^2}\\ &= \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}-\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}\end{aligned}$$

Mathematica [A] time = 0.129352, size = 107, normalized size = 1.22

$$\frac{\left(\sqrt{a}c-d\right)\log\left(\sqrt{a}-\sqrt{a+bx^2}\right)+\left(\sqrt{a}c+d\right)\log\left(\sqrt{a+bx^2}+\sqrt{a}\right)-2\sqrt{a}c\log\left(c\sqrt{a+bx^2}+d\right)}{2\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^2]] + (Sqrt[a]*c + d)*Log[Sqrt[
a] + Sqrt[a + b*x^2]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^2]])/(2*Sqrt[a]*
(a*c^2 - d^2))
```

Maple [B] time = 0.027, size = 2175, normalized size = 24.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+c*ln(x)/(a*c^2-d^2)+1/2*
c/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*d*c^2*b/a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^
2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1/
2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*b^(1/2)/a/((-a
*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-
d^2))^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((
x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+d/a^(1/2)/(
```

```
a*c^2-d^2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-d/a/(a*c^2-d^2)*(b*x^2+a)^
(1/2)-1/2*d*c^4*b/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))
/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+1/c^2*d^2)^(1/2)-1/2*d*c^2*b^(1/2)/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(
-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-
c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+
2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2
*d^2)^(1/2))+1/2*c^2*b/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(1/c^2*d^2)^(1/2)*l
n((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)
/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)
^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*d*c^4*b/(a*c^2-d^2)/((-a*
b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d
^2))^(1/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)+1/2*d*c^2
*b^(1/2)/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^
(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+(
(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(
x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))+1/2*c^2*b/(a*c^2-d^2)
/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a
*c^2-d^2))^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b))+1/2*d*c^2*b/a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-
a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*b^(1/2)/a/((-a*b)^(1/2)*c^2+(-c^
2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b
)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^
2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

Fricas [A] time = 1.90406, size = 721, normalized size = 8.19

$$\left[-\frac{2\,ac\log\left(bc^2x^2 + ac^2 - d^2\right) - 4\,ac\log\left(x\right) + ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right) - ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right) + 2\sqrt{\cdots}}{4\left(a^2c^2 - ad^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*
x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c
^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + 2*sqrt(a)*d*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*log(b*c^2*x^2
 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2
 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d
 + d^2)/x^2) + 4*sqrt(-a)*d*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a^2*c^2 - a*
d^2)]
```

Sympy [A] time = 4.75033, size = 88, normalized size = 1.

$$-\frac{c^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{ac^2-d^2}-\frac{-\frac{c\log\left(-bx^2\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{ac^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(a*c**2 - d**2) - (-c*log(-b*x**2)/2 + d*atan(sqrt(a + b*x**2)/
sqrt(-a))/sqrt(-a))/(a*c**2 - d**2)
```

Giac [A] time = 1.16565, size = 127, normalized size = 1.44

$$-\frac{c^2\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{ac^3-cd^2}+\frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)}-\frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a*
c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))
```

3.548 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=151

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2
```

Rubi [A] time = 0.352349, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 741, 801, 635, 206, 260}

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)$$
$$= b\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx, x, \sqrt{a+bx^2}\right)$$
$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\,\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)(a-x^2)}\,dx, x, \sqrt{a+bx^2}\right)}{2a\left(ac^2-d^2\right)}$$
$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2ac^4}{(ac^2-d^2)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{(ac^2-d^2)(a-x^2)}\right)dx, x, \sqrt{a+bx^2}\right)}{2a\left(ac^2-d^2\right)}$$
$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,dx, x, \sqrt{a+\right.}{2a\left(ac^2-d^2\right)^2}$$
$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} + \frac{\left(bc^3\right)\text{Subst}\left(\int \frac{x}{a-x^2}\,dx, x, \sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2}$$
$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{bc^3\log\left(d+c\sqrt{\right.}{\left(ac^2-d^{\right.}$$

Mathematica [A] time = 1.05364, size = 291, normalized size = 1.93

$$\frac{\frac{\sqrt{a}\left(-a^2c^3\sqrt{a+bx^2}+a^2c^2d+abc^3x^2\sqrt{a+bx^2}\log\left(ac^2+bc^2x^2-d^2\right)+bdx^2\sqrt{\frac{bx^2}{a}+1}\left(ac^2-d^2\right)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+abc^2dx^2+2abc^3x^2\sqrt{a+bx^2}\tanh^{-1}\left(\frac{c\sqrt{a+bx^2}}{d}\right)-2abc^3x^2\ldots}{x^2\sqrt{a+bx^2}}}{2a^{3/2}\left(d^2-ac^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (2*b*d*(-2*a*c^2 + d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + (Sqrt[a]*(a^2*c^
2*d - a*d^3 + a*b*c^2*d*x^2 - b*d^3*x^2 - a^2*c^3*Sqrt[a + b*x^2] + a*c*d^2
*Sqrt[a + b*x^2] + 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*ArcTanh[(c*Sqrt[a + b*x^2]
)/d] + b*d*(a*c^2 - d^2)*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a
]] - 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*Log[x] + a*b*c^3*x^2*Sqrt[a + b*x^2]*Log
```

```
[a*c^2 - d^2 + b*c^2*x^2]))/(x^2*Sqrt[a + b*x^2]))/(2*a^(3/2)*(-(a*c^2) + d
^2)^2)
```

Maple [B] time = 0.047, size = 2459, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2*c/(a*c^2-d^2)/x^2
-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*ln(x)*d^2-1/2*b*c^3/(a*c
^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+b*c/a/(a*c^2-d^2)*ln(x)-1/2*d*c^2*b^2/a
^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*
(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b))^(1/2)+1/2*d*c^2*b^(3/2)/a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2
))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*
b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b))^(1/2))-2*d*b/a^(1/2)/(a*c^2-d^2)^2*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)*c^2+2*d*b/a/(a*c^2-d^2)^2*(b*x^2+a)^(1/2)*c^2+b/a^(3/2)/(a*
c^2-d^2)^2*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)*d^3-b/a^2/(a*c^2-d^2)^2*(b
*x^2+a)^(1/2)*d^3+1/2*d*c^6*b^2/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*
c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)+1/2*d*c^4*b^(3/2)/(a*c^2-d^2)^2/(
(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c
^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c
^2+b*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)+1/c^2*d^2)^(1/2))-1/2*c^4*b^2/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+
(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d
^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d*c
^6*b^2/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^
(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+
1/c^2*d^2)^(1/2)-1/2*d*c^4*b^(3/2)/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*
(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(
a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^
(1/2))-1/2*c^4*b^2/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln(
(2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(
1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*d*c^2*b^2/a^2/((-a*b)^(1/2)
*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/
2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*
c^2*b^(3/2)/a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)
*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*
b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
)^(1/2))+1/2*d/a^2/(a*c^2-d^2)/x^2*(b*x^2+a)^(3/2)+1/2*d/a^(3/2)/(a*c^2-d^2
)*b*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*d/a^2/(a*c^2-d^2)*b*(b*x^2+a)
```

```
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

Fricas [A] time = 5.32345, size = 1131, normalized size = 7.49

$$\left[\frac{2\,a^2bc^3x^2\log\left(bc^2x^2 + ac^2 - d^2\right) - 4\,a^2bc^3x^2\log\left(x\right) + a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right) - a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{b}}{x^2}\right.}{4\left(a^4c^4 - 2\,a^3c^2d^2 + a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x)
 + a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2
) - a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^
2) - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^2*log(-(b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^
2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x^2*log
(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x) + a^2*b*c^3*x^2*log(-(b*
c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a^2*b*c^3*x^2*log(-(b
*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 2*a^3*c^3 + 2*a^2*c*
d^2 + 2*(3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d
^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

Giac [A] time = 1.18803, size = 275, normalized size = 1.82

$$\frac{1}{2}\left(\frac{2\,c^4\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{c^3\log\left(bx^2\right)}{a^2c^4-2\,ac^2d^2+d^4}+\frac{\left(3\,ac^2d-d^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2c^3-acd^2-\left(ac^2d-d^3\right)\sqrt{bx^2}}{\left(ac^2-d^2\right)^2abx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(2*c^4*log(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4)
- c^3*log(b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + (3*a*c^2*d - d^3)*arctan(s
qrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(-a)) - (a^
2*c^3 - a*c*d^2 - (a*c^2*d - d^3)*sqrt(b*x^2 + a))/((a*c^2 - d^2)^2*a*b*x^2
))*b
```

3.549 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3
/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[
a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(
3/2)*c^2)
```

Rubi [A] time = 0.239423, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2156, 321, 205, 483, 217, 206, 377}

$$\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3
/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[
a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(
3/2)*c^2)
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = (ac)\int \frac{x^2}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{x^2}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$=\frac{x}{bc}-\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{bc^2}-\frac{\left(a\left(ac^2-d^2\right)\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{bc}+\frac{\left(ad\left(ac^2-d^2\right)\right)\int \frac{1}{\sqrt{a+bx^2}\left(a^2c^2\cdots\right.}}{bc^2}$$

$$=\frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{bc^2}+\frac{\left(ad\left(ac^2-d^2\right)\right)\text{S}\cdots}{\cdots}$$

$$=\frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+b\cdots}}\right.}{b^{3/2}c^2}$$

Mathematica [A] time = 0.257765, size = 157, normalized size = 1.07

$$\frac{\sqrt{ac^2-d^2}\left(\sqrt{b}cx-d\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)+\left(ac^2-d^2\right)\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)+\left(d^2-ac^2\right)\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((-(a*c^2) + d^2)*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + (a*c^2 - d^2)*A
rcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])] + Sqrt[a*c^2 - d^2
]*(Sqrt[b]*c*x - d*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(b^(3/2)*c^2*Sqrt[a
*c^2 - d^2])
```

Maple [B] time = 0.029, size = 3485, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out] 1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-
a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln((b*(x+(-a
*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b))^(1/2))/b^(1/2)+1/2*d*c^4/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^
2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^
2-d^2))^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*a-1/2*c
^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b
*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)+1/c^2*d^2)^(1/2)*d^3+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d
^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((-c^2*b*(a*c
^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/b^(1/2)*a-1/2/((-a*b)^(1/2)*
c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/
b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/b^(1/2)*
d^3-1/2*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^
2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(1/c^2*d^2)^(1
/2)*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+
2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2
*d^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a+1/2/((-a*b)^(1/2)*c^2+
(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/
(-c^2*b*(a*c^2-d^2))^(1/2)*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/
2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b))-1/2*d*c^4/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2)
)/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)
*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)*a+1/2*c^2/((-a*b)^(1
/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))
^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*
b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c
^2*d^2)^(1/2)*d^3+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(
-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/b^(1/2)*a-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*
(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)
+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/b^(1/2)*d^3+1/2*c^2
/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(
a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c
^2*d^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b
)+2*(1/c^2*d^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)
)/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*
c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*
c^2-d^2))^(1/2)*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x+(-c^
```

```
2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b))-1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-a*b)^(1/2)/b)^2*
b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-
c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln
((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-a/b/(b*(a*c^2-d^2))^(1/2)*arcta
n(x*c*b/(b*(a*c^2-d^2))^(1/2))+x/b/c+1/b/c^2*d^2/(b*(a*c^2-d^2))^(1/2)*arct
an(x*c*b/(b*(a*c^2-d^2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{bcx^2 + ac + \sqrt{bx^2 + ad}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Fricas [A] time = 2.30825, size = 2371, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b
^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2
+ 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^
3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*
c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*
log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 +
 a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^
2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*
a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*
c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 +
a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*
c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2
)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d
^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) - b*sqrt((a*c^2 -
 d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2
+ a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x
)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2),
1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b
)/(a*c^2 - d^2)) + 2*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - b*sqrt
((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sq
rt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d -
 a*d^3)*x)))/(b^2*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Giac [A] time = 1.18521, size = 244, normalized size = 1.66

$$\frac{x}{bc} - \frac{\left(ac^2 - d^2\right)\arctan\left(\frac{bcx}{\sqrt{abc^2 - bd^2}}\right)}{\sqrt{abc^2 - bd^2}\,bc^2} + \frac{d\log\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2\right)}{2\,b^{\frac{3}{2}}c^2} - \frac{\left(ac^2d - d^3\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 c^2 + ac^2 - 2\,d^2}{2\,\sqrt{ac^2 - d^2}d}\right)}{\sqrt{ac^2 - d^2}\,b^{\frac{3}{2}}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] x/(b*c) - (a*c^2 - d^2)*arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 -
 b*d^2)*b*c^2) + 1/2*d*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/(b^(3/2)*c^2) -
 (a*c^2*d - d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 -
2*d^2)/(sqrt(a*c^2 - d^2)*d))/(sqrt(a*c^2 - d^2)*b^(3/2)*c^2*d)
```

3.550 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTa
n[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 -
d^2])
```

Rubi [A] time = 0.0674658, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2156, 205, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTa
n[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 -
d^2])
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-(ad)\,\text{Subst}\left(\int \frac{1}{a^2c^2-ad^2-\left(-a^2bc^2+b\left(a^2c^2-ad^2\right)\right)x^2}\,dx, x, \frac{x}{\sqrt{a+b}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Mathematica [A] time = 0.100649, size = 83, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)-\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] (ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2
 - d^2]*Sqrt[a + b*x^2])])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

Maple [B] time = 0.019, size = 1995, normalized size = 19.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*c^2*b/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(
-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*b^(1/2)/((-a*b)^(1/2)*c^2+(-c^2
*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln((b
*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))-1/2*d*c^4*b/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c
^2-d^2))^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)-1/2*d*
c^2*b^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^
2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+
1/c^2*d^2)^(1/2))+1/2*c^2*b/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(
-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^
3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d*c^
4*b/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*
b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b)+1/c^2*d^2)^(1/2)-1/2*d*c^2*b^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*
```

```
c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((
x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))-1/2*c^2*b/((-a*b)^(1/2
)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^
2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d*c^2*b/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2
*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(
-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*b^(1/
2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(-(-a*b)^(1/2)*c^2+(-c^2*b
*(a*c^2-d^2))^(1/2))*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a
*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/(b*(a*c^2-d^2)
)^(1/2)*arctan(x*c*b/(b*(a*c^2-d^2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Fricas [B] time = 1.35453, size = 1017, normalized size = 9.87

$$\left[-\frac{\sqrt{-abc^2 + bd^2}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\left(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4\right)x^2-4\sqrt{-abc^2+bd^2}\left(\left(abc^2d-2\,bd^3\right)x^3+\left(a^2c^2d-a\right.\right.}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right.}{4\left(abc^2 - bd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2
*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^
2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3 +
(a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^
2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log((b*c
^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c^2 - d
^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(a*b*c^2
 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^2*c^2 -
a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2 + a)/((a*
b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2 - b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Giac [A] time = 1.10828, size = 144, normalized size = 1.4

$$\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}}+\frac{\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}d}\right)}{\sqrt{ac^2-d^2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(s
qrt(a*c^2 - d^2)*sqrt(b))
```

3.551 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=160

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]
*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b
]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d
^2)^(3/2)
```

Rubi [A] time = 0.240101, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2156, 325, 205, 480, 12, 377}

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]
*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b
]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d
^2)^(3/2)
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{ac^2-d^2}+\frac{d\int \frac{a^2bc^2}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2\right.}}{a\left(ac^2-d^2\right)}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\int \frac{1}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+a\right.}}{ac^2-d^2}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\text{Subst}\left(\int \frac{}{a^2c^2-ad^2-}\right.}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right.}{\left(ac^2-d^2\right)^{3/2}}$$

Mathematica [A] time = 0.276195, size = 139, normalized size = 0.87

$$\frac{\sqrt{ac^2-d^2}\left(d\sqrt{a+bx^2}-ac\right)+a\sqrt{b}c^2x\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)-a\sqrt{b}c^2x\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{ax\left(ac^2-d^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (Sqrt[a*c^2 - d^2]*(-(a*c) + d*Sqrt[a + b*x^2]) - a*Sqrt[b]*c^2*x*ArcTan[(S
qrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + a*Sqrt[b]*c^2*x*ArcTan[(Sqrt[b]*d*x)/(Sqrt
[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*(a*c^2 - d^2)^(3/2)*x)
```

Maple [B] time = 0.027, size = 2289, normalized size = 14.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] b*c^2/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*c*b/(b*(a*c^2-d^2))^(1/2))-a*c^4/(
a*c^2-d^2)*b/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*c*b/(b*(a*c^2-d^2))^(1/2))-
c/(a*c^2-d^2)/x-1/2*d*c^2*b^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-a*b)^(1
/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*b^(3/2)/a/((-
a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2
-d^2))^(1/2))*ln((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/
2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))-1/2*d*c^6*b^2/(a*c^2-d^
2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*
(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+1/c^2*d^2)^(1/2)-1/2*d*c^4*b^(3/2)/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(
-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln
(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+b*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^
(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))+1/2*c^4*b^2/(a
*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(
-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(1/c^2*d^2)^(1/2)
*ln((2/c^2*d^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)+2*(1/c^2*d^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(
-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^
2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))+d/(a*c^2-d^2)/a^2/x*(b*x^2+
a)^(3/2)-d/(a*c^2-d^2)/a^2*b*x*(b*x^2+a)^(1/2)-d/(a*c^2-d^2)/a*b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))+1/2*d*c^6*b^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2
*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*
b*(a*c^2-d^2))^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2)-
1/2*d*c^4*b^(3/2)/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))
/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2+b*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))/b^(1/2)+((x+(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))-1/2*c^4*b^2/(a*c^2-d^2)/((-a*b)^(1/2)*c
^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2)
)/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(1/c^2*d^2)^(
1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)
/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+1/c^2*d^2)^(1/2))/(x+(-c^2*b*(a*c
^2-d^2))^(1/2)/c^2/b))+1/2*d*c^2*b^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2
*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-
a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*b^(3/2
)/a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b
*(a*c^2-d^2))^(1/2))*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a
*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}\,d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

Fricas [A] time = 1.84793, size = 1149, normalized size = 7.18

$$\left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\left(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4\right)x^2+4\left(\left(a^2bc^4d-3\,abc^2d^3+2\,bd^5\right)x^3+\left(a^3c^4d-2\,a^2c^2d^{\cdots}\right.\right.}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right.}{4\left(a^2c^2-ad^2\right)x}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^
4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*
b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*x^3
 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 -
d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)
*x^2)) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3
 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) + 4*a*
c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqrt(b/(a*c
^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c^2 - d^2)
)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*s
qrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x^2 + a)*d)/
((a^2*c^2 - a*d^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

Giac [A] time = 1.16282, size = 285, normalized size = 1.78

$$-b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}d}\right)}{\left(abc^2-bd^2\right)\sqrt{ac^2-d^2}d}+\frac{2}{\left(abc^2-bd^2\right)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)}\right)-\frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}\left(ac^2-d^2\right)}-\frac{c}{\left(ac^2-d^{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2
*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/((
a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(b*c
*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2
 - d^2)*x)
```

3.552 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}$$

```
[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3])/
(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*c)
 + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

Rubi [A] time = 0.299107, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3])/
(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*c)
 + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{\left(a-x^2\right)^2}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3b^3}\\
&= \frac{2\,\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{\left(2ac^2-d^2\right)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{\left(ac^2-d^2\right)^2}{c^4(d+cx)}\right)dx,x,\sqrt{a+bx^3}\right)}{3b^3}\\
&= -\frac{\left(2ac^2-d^2\right)x^3}{3b^2c^3}+\frac{2d\left(2ac^2-d^2\right)\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}+\frac{2\left(ac^2-d^2\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.181143, size = 126, normalized size = 0.9

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^3}-6bc^3x^3\right)+2bcdx^3\left(3d-2c\sqrt{a+bx^3}\right)-12d^3\sqrt{a+bx^3}+3b^2c^3x^6\right)+12\left(d^2-ac^2\right)^2\log\left(c\sqrt{a+b\right.}{18b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(3*b^2*c^3*x^6 - 12*d^3*Sqrt[a + b*x^3] + 2*b*c*d*x^3*(3*d - 2*c*Sqrt[a
+ b*x^3]) + a*(-6*b*c^3*x^3 + 20*c^2*d*Sqrt[a + b*x^3])) + 12*(-(a*c^2) + d
^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(18*b^3*c^5)
```

Maple [C] time = 0.099, size = 1473, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+4/3*d/b^3/c^2*(b*x^3+a)^(1/2)*a-2/3/b^3/c^4*
d^3*(b*x^3+a)^(1/2)-1/3*I/b^5/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/
b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/
b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2
*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(
1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I
*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)
/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a^2+2/3*I*d/b^5/c
^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(
-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^
2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^
(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*
c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I/b^5/c^4*d^3*2^(1/2)*sum((-a*b^2)^(1/3)
*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3
))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(
-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(
1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2
/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-
1/3*a/c/b^2*x^3+1/3*a^2/c/b^3*ln(b*c^2*x^3+a*c^2-d^2)-2/3*a/c^3/b^3*d^2*ln(
b*c^2*x^3+a*c^2-d^2)+1/6/b/c*x^6+1/3/b^2/c^3*x^3*d^2+1/3/b^3/c^5*d^4*ln(b*c
```

```
^2*x^3+a*c^2-d^2)
```

Maxima [A] time = 1.31592, size = 169, normalized size = 1.21

$$\frac{\frac{3\left(bx^3+a\right)^2c^3-4\left(bx^3+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^3+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^3+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}c+d\right)}{c^5}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5)/b^3
```

Fricas [A] time = 1.28078, size = 409, normalized size = 2.92

$$\frac{3\,b^2c^4x^6-6\left(abc^4-bc^2d^2\right)x^3+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(bc^2x^3+ac^2-d^2\right)+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}c+\right.}{18\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 +
a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(x**8/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [A] time = 1.13094, size = 211, normalized size = 1.51

$$\frac{2\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,b^3c^5}+\frac{3\left(bx^3+a\right)^2b^9c^3-12\left(bx^3+a\right)ab^9c^3-4\left(bx^3+a\right)^{\frac{3}{2}}b^9c^2d+24\sqrt{bx^3+a}\,ac}{18\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5)
 + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 + a)
^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*d^2
 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)
```

3.553 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}+\frac{x^3}{3bc}$$

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d +
c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rubi [A] time = 0.201527, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}+\frac{x^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d +
c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3b^2}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)dx,x,\sqrt{a+bx^3}\right)}{3b^2}\\
&= \frac{x^3}{3bc}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2\left(ac^2-d^2\right)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}
\end{aligned}$$

Mathematica [A] time = 0.0692534, size = 63, normalized size = 0.86

$$\frac{\left(2d^2-2ac^2\right)\log\left(c\sqrt{a+bx^3}+d\right)+c\left(bcx^3-2d\sqrt{a+bx^3}\right)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqrt[a +
b*x^3]])/(3*b^2*c^3)
```

Maple [C] time = 0.015, size = 943, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/3*d*(b*x^3+a)^(1/2)/c^2/b^2+1/3*I/b^4/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*
I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/
2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^
(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2
)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_
alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_a
lpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*
I*d/b^4/c^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*
3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-
3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-
(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2*ln(b*c^2*x^3+a*c^2-d^2)+1
/3*x^3/b/c+1/3/b^2/c^3*d^2*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A] time = 1.17846, size = 84, normalized size = 1.15

$$\frac{\frac{(bx^3+a)c-2\sqrt{bx^3+a}d}{c^2}-\frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^3+a}c+d\right)}{c^3}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^3 + a)*c + d)/c^3)/b^2
```

Fricas [A] time = 1.24101, size = 246, normalized size = 3.37

$$\frac{bc^2x^3 - 2\sqrt{bx^3 + a}cd - \left(ac^2 - d^2\right)\log\left(bc^2x^3 + ac^2 - d^2\right) - \left(ac^2 - d^2\right)\log\left(\sqrt{bx^3 + a}c + d\right) + \left(ac^2 - d^2\right)\log\left(\sqrt{bx^3 + a}c - \right.}{3\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*x^3 + a*c^
2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (a*c^2 - d^2)*log(sqr
t(b*x^3 + a)*c - d))/(b^2*c^3)
```

Sympy [A] time = 4.27612, size = 95, normalized size = 1.3

$$\begin{cases} \frac{2\left(\frac{a+bx^3}{6bc} - \frac{d\sqrt{a+bx^3}}{3bc^2} - \frac{\left(ac^2-d^2\right)\left(\begin{cases} \frac{\sqrt{a+bx^3}}{d} & \text{for}\ c = 0 \\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{3bc^2}\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x^6}{2\left(3\sqrt{a}d+3ac\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Piecewise((2*((a + b*x**3)/(6*b*c) - d*sqrt(a + b*x**3)/(3*b*c**2) - (a*c**
2 - d**2)*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3)
 + d)/c, True))/(3*b*c**2))/b, Ne(b, 0)), (x**6/(2*(3*sqrt(a)*d + 3*a*c)),
True))
```

Giac [A] time = 1.11376, size = 97, normalized size = 1.33

$$-\frac{\frac{2\left(ac^2-d^2\right)\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{bc^3} - \frac{\left(bx^3+a\right)bc-2\sqrt{bx^3+a}bd}{b^2c^2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((b*x^3 + a
)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

3.554 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx$

Optimal. Leaf size=26

$$\frac{2 \log \left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rubi [A] time = 0.110529, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 31}

$$\frac{2 \log \left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}\end{aligned}$$

Mathematica [A] time = 0.0283146, size = 26, normalized size = 1.

$$\frac{2 \log \left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Maple [C] time = 0.013, size = 455, normalized size = 17.5

$$\frac{-\frac{i}{3}\sqrt{2}}{b^3 d} \sum_{\_alpha=RootOf\left(\_Z^3 bc^2+c^2a-d^2\right)} \sqrt[3]{-ab^2}\sqrt{\frac{i}{2}b\left(2x+\frac{1}{b}\left(\sqrt[3]{-ab^2}-i\sqrt{3}\sqrt[3]{-ab^2}\right)\right)\frac{1}{\sqrt[3]{-ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{-ab^2}\right)\left(-3\sqrt[3]{-ab^2}+i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I
*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(
-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*
b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2
-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3/b/c*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A] time = 1.19344, size = 30, normalized size = 1.15

$$\frac{2\log\left(\sqrt{bx^3+ac}+d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

Fricas [B] time = 1.26852, size = 135, normalized size = 5.19

$$\frac{\log\left(bc^2x^3+ac^2-d^2\right)+\log\left(\sqrt{bx^3+ac}+d\right)-\log\left(\sqrt{bx^3+ac}-d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(b
*x^3 + a)*c - d))/(b*c)
```

Sympy [A] time = 2.50954, size = 32, normalized size = 1.23

$$\frac{2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] 2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3) + d)/c,
 True))/(3*b)
```

Giac [A] time = 1.12645, size = 31, normalized size = 1.19

$$\frac{2\,\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

3.555 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=93

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x
])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))
```

Rubi [A] time = 0.222704, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x
])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)$$
$$= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx,x,\sqrt{a+bx^3}\right)$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}$$
$$= -\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} + \frac{(2c)\,\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} - \frac{(2d)\,\text{Subst}\left(\int \right.}{3}$$
$$= \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}$$

Mathematica [A] time = 0.126702, size = 107, normalized size = 1.15

$$\frac{\left(\sqrt{a}c-d\right)\log\left(\sqrt{a}-\sqrt{a+bx^3}\right)+\left(\sqrt{a}c+d\right)\log\left(\sqrt{a+bx^3}+\sqrt{a}\right)-2\sqrt{a}c\log\left(c\sqrt{a+bx^3}+d\right)}{3\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^3]] + (Sqrt[a]*c + d)*Log[Sqrt[
a] + Sqrt[a + b*x^3]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^3]])/(3*Sqrt[a]*
(a*c^2 - d^2))
```

Maple [C] time = 0.033, size = 636, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] c*ln(x)/(a*c^2-d^2)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3*c
/d^2*ln(b*c^2*x^3+a*c^2-d^2)-2/3*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)+2/3*d*arct
anh((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)-2/3/a/d*(b*x^3+a)^(1/2)+2/
3*c^2/(a*c^2-d^2)/d*(b*x^3+a)^(1/2)+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*sum
((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))
)/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1
```

```
/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a
*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/
2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b
-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a
*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*
b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c
^2+a*c^2-d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

Fricas [A] time = 1.55523, size = 536, normalized size = 5.76

$$\left[-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}c+d\right)-ac\log\left(\sqrt{bx^3+a}c-d\right)-3\,ac\log\left(x\right)-\sqrt{a}d\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)}{3\left(a^2c^2-ad^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*sq
rt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^2*x
^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3 + a
)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a))/
(a^2*c^2 - a*d^2)]
```

Sympy [A] time = 5.01097, size = 97, normalized size = 1.04

$$-\frac{2c^2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3\left(ac^2-d^2\right)}-\frac{2\left(-\frac{c\log\left(-bx^3\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] -2*c**2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3) +
 d)/c, True))/(3*(a*c**2 - d**2)) - 2*(-c*log(-b*x**3)/2 + d*atan(sqrt(a +
b*x**3)/sqrt(-a))/sqrt(-a))/(3*(a*c**2 - d**2))
```

Giac [A] time = 1.13327, size = 127, normalized size = 1.37

$$-\frac{2\,c^2\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\left(ac^3-cd^2\right)}+\frac{c\log\left(bx^3\right)}{3\left(ac^2-d^2\right)}-\frac{2\,d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3)
/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt
(-a))
```

3.556 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2)^
2)
```

Rubi [A] time = 0.298076, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 741, 801, 635, 206, 260}

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2)^
2)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx,x,\sqrt{a+bx^3}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\,\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)\left(a-x^2\right)}\,dx,x,\sqrt{a+bx^3}\right)}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2ac^4}{\left(ac^2-d^2\right)(d+cx)}+\frac{3ac^2d-d^3-2ac^3x}{\left(ac^2-d^2\right)\left(a-x^2\right)}\right)dx,x,\sqrt{a+bx^3}\right)}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,dx,x,\right.}{3a\left(ac^2-d^2\right)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} + \frac{\left(2bc^3\right)\text{Subst}\left(\int \frac{x}{a-x^2}\,dx,x,\sqrt{a\right.}{3\left(ac^2-d^2\right)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{2bc^3\log\left(d\right.}{3\left(ac\right.}$$

Mathematica [A] time = 0.684142, size = 307, normalized size = 1.99

$$\frac{\sqrt{a}\left(-a^2c^3\sqrt{a+bx^3}+a^2c^2d+abc^3x^3\sqrt{a+bx^3}\log\left(ac^2+bc^2x^3-d^2\right)+bdx^3\sqrt{\frac{bx^3}{a}+1}\left(ac^2-d^2\right)\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (-2*b*d*(2*a*c^2 - d^2)*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]
] + Sqrt[a]*(a^2*c^2*d - a*d^3 + a*b*c^2*d*x^3 - b*d^3*x^3 - a^2*c^3*Sqrt[a
 + b*x^3] + a*c*d^2*Sqrt[a + b*x^3] + 2*a*b*c^3*x^3*Sqrt[a + b*x^3]*ArcTanh
[(c*Sqrt[a + b*x^3])/d] + b*d*(a*c^2 - d^2)*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh
```

```
[Sqrt[1 + (b*x^3)/a]] - 3*a*b*c^3*x^3*Sqrt[a + b*x^3]*Log[x] + a*b*c^3*x^3*
Sqrt[a + b*x^3]*Log[a*c^2 - d^2 + b*c^2*x^3]))/(3*a^(3/2)*(-(a*c^2) + d^2)^
2*x^3*Sqrt[a + b*x^3])
```

Maple [C] time = 0.036, size = 863, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*c/(a*c^2-d^2)/x^3-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*ln
(x)*d^2+1/3*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^2-d^2)+b*c/a/(a*c^2-
d^2)*ln(x)-1/3*b*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+4/3*d*b/a/(a*c
^2-d^2)^2*(b*x^3+a)^(1/2)*c^2-2/3*b/a^2/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*d^3-4
/3*d*b/a^(1/2)/(a*c^2-d^2)^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))*c^2+2/3*b/a^(
3/2)/(a*c^2-d^2)^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))*d^3+2/3*b/a^2/d*(b*x^3+
a)^(1/2)-2/3*b*c^4/(a*c^2-d^2)^2/d*(b*x^3+a)^(1/2)-1/3*I/b*c^4/(a*c^2-d^2)^
2/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*
(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^
2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3
)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b
^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)
^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2
*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=
RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^3+1/3*d
/a^(3/2)/(a*c^2-d^2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

Fricas [A] time = 2.1742, size = 946, normalized size = 6.14

$$\left[\frac{2\,a^2bc^3x^3\log\left(bc^2x^3 + ac^2 - d^2\right) + 2\,a^2bc^3x^3\log\left(\sqrt{bx^3 + ac} + d\right) - 2\,a^2bc^3x^3\log\left(\sqrt{bx^3 + ac} - d\right) - 6\,a^2bc^3x^3\log\left(x\right) - 2}{6\left(a^4c^4 - 2\,a^3c^2d^2 + a^2d^4\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*log(sq
rt(b*x^3 + a)*c + d) - 2*a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 6*a^2*b
*c^3*x^3*log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^3*log((b*x^3
+ 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d - a*d^
3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^2*b*c
^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c +
 d) - a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 3*a^2*b*c^3*x^3*log(x) - a
^3*c^3 + (3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^
2*d^2 + a^2*d^4)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**4*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Giac [A] time = 1.11163, size = 275, normalized size = 1.79

$$\frac{1}{3}\left(\frac{2\,c^4\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{c^3\log\left(bx^3\right)}{a^2c^4-2\,ac^2d^2+d^4}+\frac{\left(3\,ac^2d-d^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2c^3-acd^2-\left(ac^2d-d^3\right)\sqrt{bx^3}}{\left(ac^2-d^2\right)^2abx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(2*c^4*log(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4)
 - c^3*log(b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + (3*a*c^2*d - d^3)*arctan(s
qrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(-a)) - (a^
2*c^3 - a*c*d^2 - (a*c^2*d - d^3)*sqrt(b*x^3 + a))/((a*c^2 - d^2)^2*a*b*x^3
))*b
```

3.557 $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=311

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-\sqrt[3]{ac^2-d^2}$$

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt[a + b*x^3]) + ((a*c
^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1
/3) + b^(1/3)*c^(2/3)*x])/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a
*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)
*x^2])/(6*b^(4/3)*c^(5/3))
```

Rubi [A] time = 0.503803, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2156, 321, 200, 31, 634, 617, 204, 628, 511, 510}

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-\sqrt[3]{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt[a + b*x^3]) + ((a*c
^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1
/3) + b^(1/3)*c^(2/3)*x])/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a
*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)
*x^2])/(6*b^(4/3)*c^(5/3))
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{x^3}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{x^3}{\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$=\frac{x}{bc}-\frac{\left(a\left(ac^2-d^2\right)\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{bc}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx}{\sqrt{a+bx^3}}$$

$$=\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{a}\sqrt[3]{ac^2-d^2}\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3bc}$$

$$=\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+$$

$$=\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+$$

$$=\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}}+\frac{\sqrt[3]{ac^2-d^2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}-\sqrt[3]{ac^2-d^2}\ldots$$

Mathematica [A] time = 0.561473, size = 296, normalized size = 0.95

$$\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)-2\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)-2\sqrt{3}\sqrt[3]{ac^2-d^2}\ldots}{6b^{4/3}c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))])/((4*a*c^2 - 4*d^2)*Sqrt[a + b*x^3])) + (6*b^(1/3)
*c^(2/3)*x - 2*Sqrt[3]*(a*c^2 - d^2)^(1/3)*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*
x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^
(1/3) + b^(1/3)*c^(2/3)*x] + (a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/3) -
b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(4/3)*c^
(5/3))
```

Maple [C] time = 0.045, size = 1544, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 2/3*I*d/b^2/c^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
```

```
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/3*I/d/b^4*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a
*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*
b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(
2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2
*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b
^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I*d/b^4/c^2*2^(1/
2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2
)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*
b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1
/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a
*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^
2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1
/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_a
lpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(
x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6*a/c/b^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x
^2-(1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))-1/3*a/c/b^2/(
1/c^2/b*(a*c^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^
2))^(1/3)*x-1))+x/b/c+1/3/b^2/c^3*d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x+(1/c
^2/b*(a*c^2-d^2))^(1/3))-1/6/b^2/c^3*d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x^2
-(1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3/b^2/c^3*d^2
/(1/c^2/b*(a*c^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-
d^2))^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{bcx^3 + ac + \sqrt{bx^3 + ad}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{bcx^3 + ac + \sqrt{bx^3 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.558 $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\cdots\right.}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2\cdots}}$$

```
[Out] -(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c
^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*Sqrt[a + b*x^3]) - ArcTan[(1 - (2
*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*c^(1/3)*
(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^(2
/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3
)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2 -
d^2)^(1/3))
```

Rubi [A] time = 0.299333, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2156, 292, 31, 634, 617, 204, 628, 511, 510}

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\cdots\right.}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c
^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*Sqrt[a + b*x^3]) - ArcTan[(1 - (2
*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*c^(1/3)*
(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^(2
/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3
)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2 -
d^2)^(1/3))
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{x}{\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$= -\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}+\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c}}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\left(a^{2/3}\sqrt[3]{c}\right)\int}{}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\log\left(\left(ac^2-\right.\right.}{}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\right.}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2}}$$

Mathematica [F] time = 0.165597, size = 0, normalized size = 0.

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]), x]
```

Maple [C] time = 0.046, size = 619, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_al
pha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-1/3/b/c/(1/c^2/b*(a*c^2-d^2
))^(1/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6/b/c/(1/c^2/b*(a*c^2-d^2))^(1
/3)*ln(x^2-(1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3/b
/c*3^(1/2)/(1/c^2/b*(a*c^2-d^2))^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^
2-d^2))^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(x/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.559 $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=300

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2}\right.}{3\sqrt[3]{b}\left(ac\right.}$$

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])) - (c^(1/3)*ArcTan[
(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*
(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x
])/(3*b^(1/3)*(a*c^2 - d^2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(
1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^
2 - d^2)^(2/3))
```

Rubi [A] time = 0.259067, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2156, 200, 31, 634, 617, 204, 628, 430, 429}

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2}\right.}{3\sqrt[3]{b}\left(ac\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])) - (c^(1/3)*ArcTan[
(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*
(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x
])/(3*b^(1/3)*(a*c^2 - d^2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(
1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^
2 - d^2)^(2/3))
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx - (ad)\int \frac{1}{\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$= \frac{\left(\sqrt[3]{a}c\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\left(ac^2-d^2\right)^{2/3}} + \frac{\left(\sqrt[3]{a}c\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{ac^2-d^2}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,d}{3\left(ac^2-d^2\right)^{2/3}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^3}} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}} - \frac{\sqrt[3]{c}\int \frac{}{a^{2/3}}}{}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^3}} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}} - \frac{\sqrt[3]{c}\log(}{}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d\right.}}{3\sqrt[3]{b}\left(ac^2-\right.}$$

Mathematica [F] time = 0.144361, size = 0, normalized size = 0.

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1), x]
```

Maple [C] time = 0.012, size = 619, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b
^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*
x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_
alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3/b/c/(1/c^2/b*(a*c^2-d
^2))^(2/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))-1/6/b/c/(1/c^2/b*(a*c^2-d^2))^
(2/3)*ln(x^2-(1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3
/b/c/(1/c^2/b*(a*c^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*
```

```
c^2-d^2))^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.560 $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=319

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{b}c^{5/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log(}{3(}$$

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/2, 1, 2/3,
 -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*x*Sqrt[a + b*x
^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/
3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3
)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)
*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```

Rubi [A] time = 0.407952, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2156, 325, 292, 31, 634, 617, 204, 628, 511, 510}

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{b}c^{5/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log(}{3(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```


```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/2, 1, 2/3,
 -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*x*Sqrt[a + b*x
^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/
3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3
)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)
*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```


Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=(ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,d$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\left(a^2c^2-ad^2+ab\right.}}{\sqrt{a+bx^3}}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\left(\sqrt[3]{a}b^{2/3}c^{7/3}\right)\int \frac{}{\sqrt[3]{a}\sqrt[3]{ac^2}}}{3\left(ac^2-d\right.}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}\right.}{3\left(ac^2-d^2\right)}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}\right.}{3\left(ac^2-d^2\right)}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^2}{\sqrt[3]{ac^2-}}}{\sqrt{3}}\right.}{\sqrt{3}\left(ac^2-d^2\right)^{4/3}}$$

Mathematica [A] time = 0.701374, size = 496, normalized size = 1.55

$$-6b^2c^2dx^6\sqrt{\frac{bx^3}{a}+1}\sqrt[3]{ac^2-d^2}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+15bdx^3\sqrt{\frac{bx^3}{a}+1}\sqrt[3]{ac^2-d^2}\left(ac^2+d^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (15*b*d*(a*c^2 - d^2)^(1/3)*(a*c^2 + d^2)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[
2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 6*b^2*c^2*d
*(a*c^2 - d^2)^(1/3)*x^6*Sqrt[1 + (b*x^3)/a]*AppellF1[5/3, 1/2, 1, 8/3, -((
b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 10*(a*c^2 - d^2)*(-6*a*d*(a*c^2
- d^2)^(1/3) - 6*b*d*(a*c^2 - d^2)^(1/3)*x^3 + 6*a*c*(a*c^2 - d^2)^(1/3)*Sq
rt[a + b*x^3] + 2*Sqrt[3]*a*b^(1/3)*c^(5/3)*x*Sqrt[a + b*x^3]*ArcTan[(-1 +
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(1/3)*c^(5/3)*x
*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(1/3)*c
^(5/3)*x*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 -
 d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(60*a*(a*c^2 - d^2)^(7/3)*x*Sqrt[a +
 b*x^3])
```

Maple [C] time = 0.034, size = 3560, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 1/3*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x+(1/c^2/b*(a*c^2-
d^2))^(1/3))-1/6*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x^2-(
1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))-1/3*a*c^3/(a*c^2-
d^2)/d^2*3^(1/2)/(1/c^2/b*(a*c^2-d^2))^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2/b
*(a*c^2-d^2))^(1/3)*x-1))-c/(a*c^2-d^2)/x-1/3*c/d^2/(1/c^2/b*(a*c^2-d^2))^(
1/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6*c/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3
)*ln(x^2-(1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3*c/d
^2*3^(1/2)/(1/c^2/b*(a*c^2-d^2))^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^
2-d^2))^(1/3)*x-1))+2/3*I/a/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))-1/a/d*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))/b-I/a/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/b
+I/b*c^2/(a*c^2-d^2)/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*c^2/(a*c^2-d^2)/d*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*
(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3)
)^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/
3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-
a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*(-a*b^2)^(1/3)*3^(1
/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/
3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-3
/2*I*d/a/(a*c^2-d^2)*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
```

```
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))/b+d/a/(a*c^2-d^2)/x*(b*x^3+a)^(1/2)-2/3*I/b*c^2/(a*c^2-d^2)/d*3^(1/
2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-3/2*d/a/(a*c^2-d^2
)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/b+I*d/a/(a*c^2-d^2)
*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + ad}\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

3.561 $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=324

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}\log}{3}$$

```
[Out] -c/(2*(a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3, 1/2, 1, 1/
3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*x^2*Sqrt[a
 + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^
2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[(a
*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3)*
c^(7/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b
^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```

Rubi [A] time = 0.413234, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2156, 325, 200, 31, 634, 617, 204, 628, 511, 510}

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}\log}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```


```
[Out] -c/(2*(a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3, 1/2, 1, 1/
3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*x^2*Sqrt[a
 + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^
2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[(a
*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3)*
c^(7/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b
^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```


Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}dx = (ac)\int \frac{1}{x^3\left(a^2c^2-ad^2+abc^2x^3\right)}dx-(ad)\int \frac{1}{x^3\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)} d$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}dx}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\left(a^2c^2-ad^2\right.}}{\sqrt{a+bx^3}}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{a}bc^3\right)\int \frac{}{\sqrt[3]{a}\sqrt[3]{ac^2-}}}{3\left(ac^2-\right.}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-}\right.}{3\left(ac^2-\right.}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-}\right.}{3\left(ac^2-\right.}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}+\frac{b^{2/3}c^{7/3}\tan^{-1}\left(\frac{1-\frac{2}{3}}{}\right.}{\sqrt{3}\left(ac^2-d^2\right)}$$

Mathematica [A] time = 6.3059, size = 604, normalized size = 1.86

$$\frac{b^2c^2dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{16a\sqrt{a+bx^3}\left(d^2-ac^2\right)^2}+\frac{2bdx\left(d^2-5ac\right.}{\sqrt{a+bx^3}\left(ac^2+bc^2x^3-d^2\right)\left(3bx^3\left(2ac^2F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (b^2*c^2*d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 -((b*c^2*x^3)/(a*c^2 - d^2))])/(16*a*(-(a*c^2) + d^2)^2*Sqrt[a + b*x^3]) +
 (2*b*d*(-5*a*c^2 + d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a*(-(
a*c^2) + d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2
 - d^2))] + 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^
3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (-3*a*c*(a*c^2 - d^2)^(2/3) + 3*d
*(a*c^2 - d^2)^(2/3)*Sqrt[a + b*x^3] - 2*Sqrt[3]*a*b^(2/3)*c^(7/3)*x^2*ArcT
an[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(2/3)*
c^(7/3)*x^2*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(2/3)*c^(7/3
)*x^2*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(
2/3)*c^(4/3)*x^2])/(6*a*(a*c^2 - d^2)^(5/3)*x^2)
```

Maple [C] time = 0.033, size = 1789, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 1/3*c/d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))-1/6
*c/d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x^2-(1/c^2/b*(a*c^2-d^2))^(1/3)*x+(1/
c^2/b*(a*c^2-d^2))^(2/3))+1/3*c/d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^2))^(1/3)*x-1))-1/3*a*c^3/(a*c^2-d^2)/
d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6*a*c^3
/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x^2-(1/c^2/b*(a*c^2-d^2))^(
1/3)*x+(1/c^2/b*(a*c^2-d^2))^(2/3))-1/3*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c
^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^2))^(1/3)*x-
1))-1/2*c/(a*c^2-d^2)/x^2+2/3*I/a/d*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))-2/3*I/(a*c^2-d^2)*c^2/d*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/3*I/(a*c^2-d^2)/b^2*c^2/d*2^(1/2)*sum(1
/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^
2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/
3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^
(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1
/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)
*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b
*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*b*c^2+a*c^2-d^2))+1/2*d/a/(a*c^2-d^2)/x^2*(b*x^3+a)^(1/2)+1/2*I*d/a/(
a*c^2-d^2)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + ad}\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

3.562 $\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=135

$$\frac{cx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a
), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^n])) + (c*x*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c
^2 - d^2)
```

Rubi [A] time = 0.0938964, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2156, 245, 430, 429}

$$\frac{cx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```


```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a
), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^n])) + (c*x*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c
^2 - d^2)
```


Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^n}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx$$

$$=\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{1}{\sqrt{1+\frac{bx^n}{a}}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx}{\sqrt{a+bx^n}}$$

$$=-\frac{dx\sqrt{1+\frac{bx^n}{a}}F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^n}}+\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

Mathematica [B] time = 0.633227, size = 320, normalized size = 2.37

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{2ad(n+1)x\left(ac^2-d^2\right)F_1\left(\right.}{\sqrt{a+bx^n}\left(ac^2+bc^2x^n-d^2\right)\left(\left(ac^2-d^2\right)\left(2a(n+1)F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)-bn\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] (-2*a*d*(a*c^2 - d^2)*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x
^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/(Sqrt[a + b*x^n]*(a*c^2 - d^2 + b*c^
2*x^n)*(-2*a*b*c^2*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)
/a), -((b*c^2*x^n)/(a*c^2 - d^2))] + (a*c^2 - d^2)*(-(b*n*x^n*AppellF1[1 +
n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) +
2*a*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n
)/(a*c^2 - d^2))]))) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c
^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$\int \left(ac+bcx^n+d\sqrt{a+bx^n}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^n+ac+\sqrt{bx^n+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bcx^n + ac - \sqrt{bx^n + a}d}{b^2c^2x^{2n} + a^2c^2 - ad^2 + \left(2\,abc^2 - bd^2\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^n + a*c - sqrt(b*x^n + a)*d)/(b^2*c^2*x^(2*n) + a^2*c^2 - a
*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.563 $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=167

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/
n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m)*Sqrt
[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m))
```

Rubi [A] time = 0.196985, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2156, 364, 511, 510}

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/
n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m)*Sqrt
[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m))
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = (ac)\int \frac{x^m}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{x^m}{\sqrt{a+bx^n}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx$$

$$= \frac{cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^m}{\sqrt{1+\frac{bx^n}{a}}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx}{\sqrt{a+bx^n}}$$

$$= -\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}F_1\left(\frac{1+m}{n};\frac{1}{2},1;\frac{1+m+n}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)\sqrt{a+bx^n}}+\frac{cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)}$$

Mathematica [A] time = 0.292834, size = 156, normalized size = 0.93

$$\frac{x^{m+1}\left(c\sqrt{a+bx^n}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)-d\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (x^(1 + m)*(-(d*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)
/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) + c*Sqrt[a + b*x^n]*Hyperg
eometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))]))/((
a*c^2 - d^2)*(1 + m)*Sqrt[a + b*x^n])
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int x^m\left(ac+bcx^n+d\sqrt{a+bx^n}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bcx^n+ac+\sqrt{bx^n+a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bcx^m x^n + acx^m - \sqrt{bx^n + a}dx^m}{b^2c^2x^{2n} + a^2c^2 - ad^2 + (2abc^2 - bd^2)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^m*x^n + a*c*x^m - sqrt(b*x^n + a)*d*x^m)/(b^2*c^2*x^(2*n) +
 a^2*c^2 - a*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Integral(x**m/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.564 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=27

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rubi [A] time = 0.10991, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2155, 31}

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^n}\right)}{bn}\\ &= \frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}\end{aligned}$$

Mathematica [A] time = 0.0609164, size = 27, normalized size = 1.

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int x^{-1+n} \left(ac + bcx^n + d\sqrt{a + bx^n}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [B] time = 1.32554, size = 82, normalized size = 3.04

$$-\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn} + \frac{2\log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}d}{d}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)/
(b*c*n)
```

Fricas [A] time = 1.86726, size = 51, normalized size = 1.89

$$\frac{2\log\left(\sqrt{bx^n + a}c + d\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

Sympy [A] time = 18.4952, size = 32, normalized size = 1.19

$$\frac{2\left(\begin{cases} \frac{\sqrt{a+bx^n}}{d} & \text{for}\ c = 0 \\ \frac{\log\left(c\sqrt{a+bx^n}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] 2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**n) + d)/c,
 True))/(b*n)
```

Giac [A] time = 1.12408, size = 55, normalized size = 2.04

$$\frac{2\log\left(\left|\sqrt{bx^n+a}c+d\right|\right)}{bcn}-\frac{2\log\left(|d|\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n) - 2*log(abs(d))/(b*c*n)
```

3.565 $\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(2\sqrt{x}\right)$$

```
[Out] ArcTan[2*Sqrt[x]]
```

Rubi [A] time = 0.0057815, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1593, 63, 203}

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx &= \int \frac{1}{\sqrt{x}(1+4x)}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sqrt{x}\right)\\
&= \tan^{-1}\left(2\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0024078, size = 8, normalized size = 1.

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^(3/2)+x^(1/2)),x)
```

```
[Out] arctan(2*x^(1/2))
```

Maxima [A] time = 1.84293, size = 8, normalized size = 1.

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(2*sqrt(x))
```

Fricas [A] time = 1.23929, size = 26, normalized size = 3.25

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan(2*sqrt(x))
```

Sympy [A] time = 0.202828, size = 7, normalized size = 0.88

$$\operatorname{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**(3/2)+x**(1/2)),x)
```

```
[Out] atan(2*sqrt(x))
```

Giac [A] time = 1.09106, size = 8, normalized size = 1.

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(2*sqrt(x))
```

3.566 $\int \frac{1}{\sqrt{x}-x^{5/2}} \, dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0084578, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - x^(5/2))^(-1),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx &= \int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0040201, size = 13, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A] time = 0.004, size = 10, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)+\text{Artanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(5/2)+x^(1/2)),x)
```

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [B] time = 2.49291, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] time = 1.3356, size = 85, normalized size = 6.54

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] time = 0.397158, size = 26, normalized size = 2.

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(5/2)+x**(1/2)),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Giac [B] time = 1.09752, size = 30, normalized size = 2.31

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.567 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} \, dx$

Optimal. Leaf size=27

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rubi [A] time = 0.0116985, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1593, 266, 43}

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} \, dx &= \int \frac{1}{\left(-1+\sqrt[4]{x}\right)\sqrt[4]{x}} \, dx \\
&= 4\,\text{Subst}\left(\int \frac{x^2}{-1+x} \, dx, x, \sqrt[4]{x}\right) \\
&= 4\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right) dx, x, \sqrt[4]{x}\right) \\
&= 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0100669, size = 27, normalized size = 1.

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Maple [A] time = 0.007, size = 20, normalized size = 0.7

$$2\sqrt{x}+4\sqrt[4]{x}+4\ln\left(\sqrt[4]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+4*x^(1/4)+4*ln(x^(1/4)-1)
```

Maxima [A] time = 1.31642, size = 26, normalized size = 0.96

$$2\sqrt{x}+4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Fricas [A] time = 1.17699, size = 59, normalized size = 2.19

$$2\sqrt{x}+4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Sympy [A] time = 0.212634, size = 22, normalized size = 0.81

$$4\sqrt[4]{x}+2\sqrt{x}+4\log\left(\sqrt[4]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/4)+x**(1/2)),x)
```

```
[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)
```

Giac [A] time = 1.13031, size = 27, normalized size = 1.

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(\left|x^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))
```

3.568 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi [A] time = 0.0142512, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]

[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\
&= 6\,\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx, x, \sqrt[6]{x}\right)\\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0119524, size = 32, normalized size = 1.

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Maple [B] time = 0.023, size = 92, normalized size = 2.9

$$2\ln\left(\sqrt[6]{x}-1\right)-\ln\left(\sqrt[3]{x}+\sqrt[6]{x}+1\right)+\ln\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)-2\ln\left(1+\sqrt[6]{x}\right)+2\sqrt{x}+\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+6\sqrt[6]{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*ln(x^(1/6)-1)-ln(x^(1/3)+x^(1/6)+1)+ln(1-x^(1/6)+x^(1/3))-2*ln(1+x^(1/6))
+2*x^(1/2)+ln(-1+x^(1/2))-ln(1+x^(1/2))+6*x^(1/6)-ln(x-1)-2*ln(x^(1/3)-1)+l
n(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

Maxima [A] time = 1.26138, size = 32, normalized size = 1.

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A] time = 1.24838, size = 76, normalized size = 2.38

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Giac [A] time = 1.12005, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.569 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rubi [A] time = 0.0112607, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx, x, \sqrt[4]{x}\right)\\
&= -4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0089011, size = 25, normalized size = 1.

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$-4\sqrt[4]{x}+4\ln\left(1+\sqrt[4]{x}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/2)),x)
```

```
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Maxima [A] time = 1.78604, size = 26, normalized size = 1.04

$$2\sqrt{x}-4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Fricas [A] time = 1.16614, size = 59, normalized size = 2.36

$$2\sqrt{x}-4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Sympy [A] time = 0.210391, size = 22, normalized size = 0.88

$$-4\sqrt[4]{x}+2\sqrt{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/2)),x)
```

```
[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

Giac [A] time = 1.09892, size = 26, normalized size = 1.04

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

3.570 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx$

Optimal. Leaf size=20

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rubi [A] time = 0.0091215, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1593, 266, 43}

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx &= \int \frac{1}{\left(-1+\sqrt[3]{x}\right)\sqrt[3]{x}} \, dx \\
&= 3\,\text{Subst}\left(\int \frac{x}{-1+x} \, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right) dx, x, \sqrt[3]{x}\right) \\
&= 3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.006239, size = 18, normalized size = 0.9

$$3\left(\sqrt[3]{x}+\log\left(1-\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*(x^(1/3) + Log[1 - x^(1/3)])
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$3\sqrt[3]{x}+3\ln\left(\sqrt[3]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/3)+x^(2/3)),x)
```

```
[Out] 3*x^(1/3)+3*ln(x^(1/3)-1)
```

Maxima [A] time = 1.27376, size = 19, normalized size = 0.95

$$3x^{\frac{1}{3}}+3\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Fricas [A] time = 1.28871, size = 43, normalized size = 2.15

$$3x^{\frac{1}{3}}+3\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Sympy [A] time = 0.137258, size = 15, normalized size = 0.75

$$3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/3)+x**(2/3)),x)
```

```
[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)
```

Giac [A] time = 1.10568, size = 20, normalized size = 1.

$$3x^{\frac{1}{3}} + 3\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="giac")
```

```
[Out] 3*x^(1/3) + 3*log(abs(x^(1/3) - 1))
```

3.571 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rubi [A] time = 0.0374304, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1593, 341, 321, 292, 31, 634, 618, 204, 628}

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx &= \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}-4\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right)-\frac{4}{3}\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)-2\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)+4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0065424, size = 24, normalized size = 0.39

$$-2\sqrt{x}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-x^{3/4}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -x^(3/4)])
```

Maple [A] time = 0.003, size = 46, normalized size = 0.7

$$2\sqrt{x}-\frac{2}{3}\ln\left(1-\sqrt[4]{x}+\sqrt{x}\right)-\frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[4]{x}-1\right)\right)+\frac{4}{3}\ln\left(1+\sqrt[4]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/3*(2*x^(1/4)-1)*3^
(1/2))+4/3*ln(1+x^(1/4))
```

Maxima [A] time = 1.77427, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Fricas [A] time = 1.37038, size = 167, normalized size = 2.69

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Sympy [A] time = 0.724254, size = 68, normalized size = 1.1

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.09093, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

3.572 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=73

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Rubi [A] time = 0.0282093, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[12]{x}\right)\sqrt[4]{x}}\,dx\\
&= 12\,\text{Subst}\left(\int \frac{x^8}{1+x}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\,\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7+\frac{1}{1+x}\right)dx, x, \sqrt[12]{x}\right)\\
&= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0240054, size = 73, normalized size = 1.

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Maple [B] time = 0.087, size = 173, normalized size = 2.4

$$-\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)+2\ln\left(\sqrt[3]{x}-1\right)+\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)-2\ln\left(1+\sqrt[6]{x}\right)+\ln\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)+2\ln\left(\sqrt[6]{x}-1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/3)),x)
```

```
[Out] -ln(x^(2/3)+x^(1/3)+1)+2*ln(x^(1/3)-1)+ln(-1+x^(1/2))-ln(1+x^(1/2))-2*ln(1+
x^(1/6))+ln(1-x^(1/6)+x^(1/3))+2*ln(x^(1/6)-1)-ln(x^(1/3)+x^(1/6)+1)-2*ln(x
^(1/4)-1)-12/7*x^(7/12)-12/5*x^(5/12)-12*x^(1/12)+2*ln(1+x^(1/4))+ln(x-1)+3
/2*x^(2/3)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)+2*x^(1/2)+4*ln(1+x^(1/12))-2*ln(1-
x^(1/12)+x^(1/6))-4*ln(x^(1/12)-1)+2*ln(x^(1/6)+x^(1/12)+1)
```

Maxima [A] time = 1.29995, size = 66, normalized size = 0.9

$$\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Fricas [A] time = 1.18504, size = 176, normalized size = 2.41

$$\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(1/(x**(1/4) + x**(1/3)), x)
```

Giac [A] time = 1.1225, size = 66, normalized size = 0.9

$$\frac{3}{2}\, x^{\frac{2}{3}} - \frac{12}{7}\, x^{\frac{7}{12}} + 2\, \sqrt{x} - \frac{12}{5}\, x^{\frac{5}{12}} + 3\, x^{\frac{1}{3}} - 4\, x^{\frac{1}{4}} + 6\, x^{\frac{1}{6}} - 12\, x^{\frac{1}{12}} + 12\, \log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.573 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} \, dx$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{}$$

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rubi [A] time = 0.045854, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx$$
$$=12\,\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx,x,\sqrt[12]{x}\right)$$
$$=12\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}+x^{12}-x^{13}+x^{14}\right)\right.$$
$$=12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12}{}$$

Mathematica [A] time = 0.0363599, size = 130, normalized size = 1.

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Maple [A] time = 0.003, size = 83, normalized size = 0.6

$$12\,x^{1/12}-6\,\sqrt[6]{x}+4\,\sqrt[4]{x}-3\,\sqrt[3]{x}+\frac{12}{5}x^{\frac{5}{12}}+\frac{12}{7}x^{\frac{7}{12}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{12}{11}x^{\frac{11}{12}}-x+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{4}{5}x^{\frac{5}{4}}-12\,\ln\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Maxima [A] time = 1.4032, size = 111, normalized size = 0.85

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}+4\,x^{\frac{1}{4}}-6\,x^{\frac{1}{6}}+12\,x^{\frac{1}{12}}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Fricas [A] time = 1.2763, size = 286, normalized size = 2.2

$$\frac{4}{5}(x+5)x^{\frac{1}{4}}-\frac{6}{7}(x+7)x^{\frac{1}{6}}+\frac{12}{13}(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}-12\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

Sympy [A] time = 1.98156, size = 121, normalized size = 0.93

$$\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

Giac [A] time = 1.21281, size = 111, normalized size = 0.85

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

3.574 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{}$$

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.395599, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {1593, 341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```


```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```


Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
egQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$$

$$= 6\,\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + 6\,\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} - \frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\,\text{S}$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) + \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{\right.}}{\sqrt{5}}$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\right.$$

$$= 2\sqrt{x} + 6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}}\left(5+\sqrt{5}\right)\left(1+\sqrt{5}+\right.\right.$$

Mathematica [C] time = 0.0068891, size = 22, normalized size = 0.11

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{5},1;\frac{8}{5};x^{5/6}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)])
```

Maple [A] time = 0.031, size = 175, normalized size = 0.9

$$2\sqrt{x} + \frac{6}{5}\ln\left(\sqrt[6]{x}-1\right) + \frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right) - \frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right) - \frac{12\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+6/5*ln(x^(1/6)-1)+3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^
(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-12/5/(10-2*5^(1/2))^(1/2
)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)-3/10*ln(2+x^(1
/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*
5^(1/2))+12/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/
2))^(1/2))*5^(1/2)
```

Maxima [B] time = 2.10946, size = 367, normalized size = 1.84

$$-\frac{6}{5}\,(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\,\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{2\,\sqrt{5}-10}}+\frac{6\,\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}-}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}-}\right.}{5\,\sqrt{-2\,\sqrt{5}-10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Fricas [B] time = 8.5966, size = 2130, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Giac [A] time = 1.46687, size = 188, normalized size = 0.94

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

3.575 $\int \frac{\sqrt{x}}{x+x^2} \, dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0047993, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {647, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{x+x^2} \, dx &= \int \frac{1}{\sqrt{x}(1+x)} \, dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{x}\right)\\
&= 2\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0018211, size = 8, normalized size = 1.

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+x),x)
```

```
[Out] 2*arctan(x^(1/2))
```

Maxima [A] time = 1.99945, size = 8, normalized size = 1.

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

Fricas [A] time = 1.223, size = 26, normalized size = 3.25

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [A] time = 0.292127, size = 7, normalized size = 0.88

$$2\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+x),x)
```

```
[Out] 2*atan(sqrt(x))
```

Giac [A] time = 1.11654, size = 8, normalized size = 1.

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.576 $\int \frac{x}{4\sqrt{x}+x} dx$

Optimal. Leaf size=19

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rubi [A] time = 0.0143723, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{4\sqrt{x}+x} dx &= \int \frac{\sqrt{x}}{4+\sqrt{x}} dx \\
&= 2\,\text{Subst}\left(\int \frac{x^2}{4+x} dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(-4+x+\frac{16}{4+x}\right) dx, x, \sqrt{x}\right) \\
&= -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.008548, size = 19, normalized size = 1.

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Maple [A] time = 0.001, size = 16, normalized size = 0.8

$$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+4*x^(1/2)),x)
```

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

Maxima [A] time = 1.29466, size = 20, normalized size = 1.05

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Fricas [A] time = 1.28512, size = 50, normalized size = 2.63

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Sympy [A] time = 0.143043, size = 17, normalized size = 0.89

$$-8\sqrt{x}+x+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x**(1/2)),x)
```

```
[Out] -8*sqrt(x) + x + 32*log(sqrt(x) + 4)
```

<u>Giac</u> [A] time = 1.08843, size = 20, normalized size = 1.05

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="giac")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

3.577 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$

Optimal. Leaf size=108

$$2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]
*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2]) + (
3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rubi [A] time = 0.0827125, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1584, 341, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]
*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2]) + (
3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = \int \frac{\sqrt[6]{x}}{1+x^{2/3}}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{x^{5/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 2\sqrt{x} - 3\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 2\sqrt{x} - 6\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)$$
$$= 2\sqrt{x} + 3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) - 3\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)$$
$$= 2\sqrt{x} - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{2}+}{-1-\sqrt{2}}\right.}{2\sqrt{}}$$
$$= 2\sqrt{x} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} + \frac{3\,\text{Sub}}{}$$
$$= 2\sqrt{x} + \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.006702, size = 24, normalized size = 0.22

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^{2/3}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^(2/3)])
```

Maple [A] time = 0.005, size = 71, normalized size = 0.7

$$2\sqrt{x} - \frac{3\sqrt{2}}{2}\arctan\left(1+\sqrt[6]{x}\sqrt{2}\right) - \frac{3\sqrt{2}}{2}\arctan\left(-1+\sqrt[6]{x}\sqrt{2}\right) - \frac{3\sqrt{2}}{4}\ln\left(\left(1+\sqrt[3]{x}-\sqrt[6]{x}\sqrt{2}\right)\left(1+\sqrt[3]{x}+\sqrt[6]{x}\sqrt{2}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3)+x),x)
```

```
[Out] 2*x^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(-1+x^(1/6)*2^(1/
2))*2^(1/2)-3/4*2^(1/2)*ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2
^(1/2)))
```

Maxima [A] time = 1.70134, size = 112, normalized size = 1.04

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right) - \frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

Fricas [A] time = 1.27114, size = 404, normalized size = 3.74

$$3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1}-\sqrt{2}x^{\frac{1}{6}}-1\right)+3\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4}-\sqrt{2}x^{\frac{1}{6}}+1\right)+\frac{3}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*x^(1/6) + x^(1/3) + 1) - sqrt(2)*x^(1
/6) - 1) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*x^(1/6) + 4*x^(1/3)
 + 4) - sqrt(2)*x^(1/6) + 1) + 3/4*sqrt(2)*log(4*sqrt(2)*x^(1/6) + 4*x^(1/3
) + 4) - 3/4*sqrt(2)*log(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) + 2*sqrt(x)
```

Sympy [A] time = 1.93226, size = 110, normalized size = 1.02

$$2\sqrt{x}-\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt[6]{x}+4\sqrt[3]{x}+4\right)}{4}+\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt[6]{x}+4\sqrt[3]{x}+4\right)}{4}-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[6]{x}-1\right)}{2}-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[6]{x}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/3)+x),x)
```

```
[Out] 2*sqrt(x) - 3*sqrt(2)*log(-4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 + 3*sqrt(
2)*log(4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 - 3*sqrt(2)*atan(sqrt(2)*x**(
1/6) - 1)/2 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6) + 1)/2
```

Giac [A] time = 1.10828, size = 112, normalized size = 1.04

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

3.578 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=76

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]
```

Rubi [A] time = 0.0344146, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1584, 341, 50, 58, 618, 204, 31}

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx &= \int \frac{\sqrt[12]{x}}{1+\sqrt[4]{x}}\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^{10/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= \frac{6x^{5/6}}{5} - 4\,\text{Subst}\left(\int \frac{x^{7/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= -\frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\,\text{Subst}\left(\int \frac{x^{4/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\,\text{Subst}\left(\int \frac{1}{x^{2/3}(1+x)}\,dx, x, \sqrt[4]{x}\right)\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 2\log\left(1+\sqrt[4]{x}\right) + 6\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[12]{x}\right) + 6\,\text{Subst}\Bigg(\int \frac{}{1}\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right) - 12\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0175064, size = 83, normalized size = 1.09

$$\frac{6x^{5/6}}{5} - \frac{12x^{7/12}}{7} + 3\sqrt[3]{x} - 12\sqrt[12]{x} + 4\log\left(\sqrt[12]{x}+1\right) - 2\log\left(\sqrt[6]{x} - \sqrt[12]{x}+1\right) + 4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[12]{x}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 + 4*Sqrt[3]*ArcT
an[(-1 + 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) +
x^(1/6)]
```

Maple [A] time = 0.003, size = 61, normalized size = 0.8

$$\frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3\sqrt[3]{x} - 12\,x^{1/12} - 2\,\ln\left(1 - x^{1/12} + \sqrt[6]{x}\right) + 4\,\sqrt{3}\arctan\left(1/3\,\left(2\,x^{1/12}-1\right)\sqrt{3}\right) + 4\,\ln\left(1 + x^{1/12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/4)+x^(1/2)),x)
```

```
[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)-2*ln(1-x^(1/12)+x^(1/6))+4*
3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))+4*ln(1+x^(1/12))
```

Maxima [A] time = 1.89103, size = 81, normalized size = 1.07

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

Fricas [A] time = 1.32073, size = 221, normalized size = 2.91

$$4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*x
^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x
^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

Giac [A] time = 1.12424, size = 81, normalized size = 1.07

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

3.579 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx$

Optimal. Leaf size=119

$$\frac{6x^{7/6}}{7} - \frac{12x^{13/12}}{13} - \frac{12x^{11/12}}{11} + \frac{6x^{5/6}}{5} - \frac{4x^{3/4}}{3} + \frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + x + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12\,\mathrm{lo}$$

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Rubi [A] time = 0.0493564, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1584, 266, 43}

$$\frac{6x^{7/6}}{7} - \frac{12x^{13/12}}{13} - \frac{12x^{11/12}}{11} + \frac{6x^{5/6}}{5} - \frac{4x^{3/4}}{3} + \frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + x + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12\,\mathrm{lo}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \int \frac{\sqrt[4]{x}}{1+\sqrt[12]{x}}\,dx$$
$$= 12\,\mathrm{Subst}\left(\int \frac{x^{14}}{1+x}\,dx, x, \sqrt[12]{x}\right)$$
$$= 12\,\mathrm{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7-x^8+x^9-x^{10}+x^{11}-x^{12}+x^{13}+\frac{1}{1+x}\right) dx, x$$
$$= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}-\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/12}}{11}+x-$$

Mathematica [A] time = 0.0329904, size = 119, normalized size = 1.

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+1$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Maple [A] time = 0.004, size = 76, normalized size = 0.6

$$-12x^{1/12}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12}{5}x^{\frac{5}{12}}-\frac{12}{7}x^{\frac{7}{12}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{11}x^{\frac{11}{12}}+x-\frac{12}{13}x^{\frac{13}{12}}+\frac{6}{7}x^{\frac{7}{6}}+12\ln\left(1+x^{1/12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/4)+x^(1/3)),x)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Maxima [A] time = 1.17092, size = 101, normalized size = 0.85

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{12}{13}x^{\frac{13}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Fricas [A] time = 1.23181, size = 273, normalized size = 2.29

$$\frac{6}{7}(x+7)x^{\frac{1}{6}} - \frac{12}{13}(x+13)x^{\frac{1}{12}} + x - \frac{12}{11}x^{\frac{11}{12}} + \frac{6}{5}x^{\frac{5}{6}} - \frac{4}{3}x^{\frac{3}{4}} + \frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 12\log\left(x^{\frac{1}{12}} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*x
^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/
12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

Giac [A] time = 1.11659, size = 101, normalized size = 0.85

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{12}{13}x^{\frac{13}{12}} + x - \frac{12}{11}x^{\frac{11}{12}} + \frac{6}{5}x^{\frac{5}{6}} - \frac{4}{3}x^{\frac{3}{4}} + \frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\log\left(x^{\frac{1}{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.580 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-$$

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.223926, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1584, 341, 302, 202, 634, 618, 204, 628, 31}

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```


```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
```

```
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx$$

$$= 6\,\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\,\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x}+x-\frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\text{Subst}$$

$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(5-\right.\right.$$

$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\right.$$

$$= 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+\right.\right.$$

Mathematica [C] time = 0.0057218, size = 29, normalized size = 0.14

$$-6\sqrt[6]{x}\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};x^{5/6}\right)+x+6\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - 6*x^(1/6)*Hypergeometric2F1[1/5, 1, 6/5, x^(5/6)]
```

Maple [A] time = 0.014, size = 242, normalized size = 1.2

$$x+6\sqrt[6]{x}+\frac{6}{5}\ln\left(\sqrt[6]{x}-1\right)-\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-6\frac{1}{\sqrt{10-2\sqrt{5}}}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)+6/5*ln(x^(1/6)-1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*
5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-6/(10-2*5^(1/2))^(1/2)
*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5/(10-2*5^(1/2))^(1/2
)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)+3/10*ln(2+x^(1
/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*
5^(1/2))-6/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))
^(1/2))-6/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2)
)^(1/2))*5^(1/2)
```

Maxima [B] time = 2.26769, size = 396, normalized size = 1.97

$$-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+\cdots}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+\cdots}\right)}{5\sqrt{-2\sqrt{5}-10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

Fricas [B] time = 8.51782, size = 1858, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

Sympy [B] time = 35.108, size = 561, normalized size = 2.79

$$-\frac{60\sqrt[6]{x}}{-10+10\sqrt{5}}+\frac{60\sqrt{5}\sqrt[6]{x}}{-10+10\sqrt{5}}-\frac{10x}{-10+10\sqrt{5}}+\frac{10\sqrt{5}x}{-10+10\sqrt{5}}-\frac{12\log\left(\sqrt[6]{x}-1\right)}{-10+10\sqrt{5}}+\frac{12\sqrt{5}\log\left(\sqrt[6]{x}-1\right)}{-10+10\sqrt{5}}-\frac{12\log\left(8\sqrt[6]{x}+8\ldots\right.}{-10\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] -60*x**(1/6)/(-10 + 10*sqrt(5)) + 60*sqrt(5)*x**(1/6)/(-10 + 10*sqrt(5)) -
10*x/(-10 + 10*sqrt(5)) + 10*sqrt(5)*x/(-10 + 10*sqrt(5)) - 12*log(x**(1/6)
 - 1)/(-10 + 10*sqrt(5)) + 12*sqrt(5)*log(x**(1/6) - 1)/(-10 + 10*sqrt(5))
- 12*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6) + 16*x**(1/3) + 16)/(-10 + 10*sqrt
(5)) - 6*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3) + 16)/(
-10 + 10*sqrt(5)) + 18*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3) +
 16)/(-10 + 10*sqrt(5)) - 6*sqrt(10)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1
/6)/sqrt(5 - sqrt(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5
- sqrt(5))))/(-10 + 10*sqrt(5)) + 6*sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2
)*x**(1/6)/sqrt(5 - sqrt(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*
sqrt(5 - sqrt(5))))/(-10 + 10*sqrt(5)) - 6*sqrt(10)*sqrt(sqrt(5) + 5)*atan(
2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2*sqrt(sqrt(5) + 5)) + sqr
t(2)/(2*sqrt(sqrt(5) + 5)))/(-10 + 10*sqrt(5)) + 6*sqrt(2)*sqrt(sqrt(5) + 5
)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2*sqrt(sqrt(5) + 5)
) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/(-10 + 10*sqrt(5))
```

Giac [A] time = 1.75064, size = 189, normalized size = 0.94

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

3.581 $\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=36

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rubi [A] time = 0.036967, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

Mathematica [A] time = 0.0212461, size = 35, normalized size = 0.97

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x}}}{\left(m+\frac{1}{2}\right)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

Maple [A] time = 0.003, size = 36, normalized size = 1.

$$2\,{\frac{{x}^{1+m}}{(1+2\,m)\sqrt{-bx+a}}\sqrt{-{\frac{-bx+a}{x}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x^(1+m)/(1+2*m)*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] time = 1.75715, size = 20, normalized size = 0.56

$$\frac{2\sqrt{x}x^m}{2i\,m+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*x^m/(2*I*m + I)
```

Fricas [A] time = 1.56237, size = 95, normalized size = 2.64

$$\frac{2\sqrt{-bx+a}\,xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] time = 1.2424, size = 212, normalized size = 5.89

$$\frac{2\sqrt{-ab}a|b|e^{\left(m\log\left(\frac{a}{b}\right)-\log\left(\frac{a}{b}\right)\right)}\operatorname{sgn}(x)}{2b^3m+b^3}-\frac{2\left(\frac{\sqrt{-ab}ae^{\left(m\log\left(\frac{a}{b}\right)-\log\left(\frac{a}{b}\right)\right)}}{2m+1}+\frac{(-(bx-a)b-ab)^{\frac{3}{2}}e^{\left(m\log\left(\frac{(bx-a)b+ab}{b^2}\right)-\log\left(\frac{(bx-a)b+ab}{b^2}\right)\right)}}{b(2m+1)}\right)|b|\operatorname{sgn}(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-a*b)*a*abs(b)*e^(m*log(a/b) - log(a/b))*sgn(x)/(2*b^3*m + b^3) - 2*
(sqrt(-a*b)*a*e^(m*log(a/b) - log(a/b))/(2*m + 1) + (-(b*x - a)*b - a*b)^(3
/2)*e^(m*log(((b*x - a)*b + a*b)/b^2) - log(((b*x - a)*b + a*b)/b^2))/(b*(2
*m + 1)))*abs(b)*sgn(x)/b^3
```

3.582 $\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rubi [A] time = 0.0342811, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int x^{3/2}\,dx}{\sqrt{a-bx}}\\ &= \frac{2\sqrt{b-\frac{a}{x}}x^3}{5\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] time = 0.015002, size = 29, normalized size = 1.

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$\frac{2x^3}{5}\sqrt{-\frac{-bx+a}{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] time = 1.44615, size = 7, normalized size = 0.24

$$-\frac{2}{5}i\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*x^(5/2)
```

Fricas [A] time = 1.40796, size = 72, normalized size = 2.48

$$-\frac{2\sqrt{-bx+a}x^3\sqrt{\frac{bx-a}{x}}}{5(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] time = 1.17096, size = 170, normalized size = 5.86

$$\frac{2\sqrt{-ab}a^2|b|\mathrm{sgn}(x)}{5b^4}-\frac{2\left(3\sqrt{-ab}a^2+\frac{5(-(bx-a)b-ab)^{\frac{3}{2}}a-\frac{5(-(bx-a)b-ab)^{\frac{3}{2}}ab+3((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b}}{b}\right)|b|\mathrm{sgn}(x)}{15b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/15*(3*sqrt(-a*b)*a^2 + (5*(-(b*x -
 a)*b - a*b)^(3/2)*a - (5*(-(b*x - a)*b - a*b)^(3/2)*a*b + 3*((b*x - a)*b +
 a*b)^2*sqrt(-(b*x - a)*b - a*b))/b)/b)*abs(b)*sgn(x)/b^4
```

3.583 $\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rubi [A] time = 0.023956, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {515, 23, 30}

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \sqrt{x}\,dx}{\sqrt{a-bx}}\\
&= \frac{2\sqrt{b-\frac{a}{x}}x^2}{3\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.0119389, size = 29, normalized size = 1.

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Maple [A] time = 0.002, size = 27, normalized size = 0.9

$$\frac{2x^2}{3}\sqrt{-\frac{-bx+a}{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] time = 1.18344, size = 7, normalized size = 0.24

$$-\frac{2}{3}i\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*x^(3/2)
```

Fricas [A] time = 1.47565, size = 72, normalized size = 2.48

$$-\frac{2\sqrt{-bx+a}\,x^2\sqrt{\frac{bx-a}{x}}}{3\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] time = 1.13623, size = 76, normalized size = 2.62

$$\frac{2\sqrt{-ab}a|b|\mathrm{sgn}\left(x\right)}{3\,b^3} - \frac{2\left(\sqrt{-ab}a + \frac{\left(-(bx-a)b-ab\right)^{\frac{3}{2}}}{b}\right)|b|\mathrm{sgn}\left(x\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x - a)*b - a*
b)^(3/2)/b)*abs(b)*sgn(x)/b^3
```

3.584 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Rubi [A] time = 0.014152, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {435, 23, 30}

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{\sqrt{x}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}}\,dx}{\sqrt{a-bx}}\\ &= \frac{2\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] time = 0.010256, size = 25, normalized size = 1.

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Maple [A] time = 0.003, size = 25, normalized size = 1.

$$2\frac{x}{\sqrt{-bx+a}}\sqrt{-\frac{-bx+a}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] time = 1.38986, size = 7, normalized size = 0.28

$$-2i\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(x)
```

Fricas [A] time = 1.46328, size = 66, normalized size = 2.64

$$-\frac{2\sqrt{-bx+a}x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] time = 1.11791, size = 69, normalized size = 2.76

$$\frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\mathrm{sgn}\left(x\right)}{b^2}+\frac{2\sqrt{-ab}|b|\mathrm{sgn}\left(x\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)*
abs(b)*sgn(x)/b^2
```

3.585 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rubi [A] time = 0.0343887, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{3/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{x^{3/2}}\,dx}{\sqrt{a-bx}}\\
&= -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.012466, size = 24, normalized size = 1.

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$-2\frac{1}{\sqrt{-bx+a}}\sqrt{-\frac{-bx+a}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x)
```

```
[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] time = 2.05264, size = 7, normalized size = 0.29

$$\frac{2i}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I/sqrt(x)
```

Fricas [A] time = 1.48782, size = 62, normalized size = 2.58

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

Giac [B] time = 1.12331, size = 57, normalized size = 2.38

$$\frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}}-\frac{b^3}{\sqrt{-ab}}\right)|b|\mathrm{sgn}\left(x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3
```

3.586 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rubi [A] time = 0.0330215, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{5/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{x^{5/2}}\,dx}{\sqrt{a-bx}}\\
&= -\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.0107928, size = 29, normalized size = 1.

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Maple [A] time = 0.002, size = 27, normalized size = 0.9

$$-\frac{2}{3x}\sqrt{-\frac{-bx+a}{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x)
```

```
[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Maxima [C] time = 1.61323, size = 7, normalized size = 0.24

$$\frac{2i}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*I/x^(3/2)
```

Fricas [A] time = 1.40859, size = 70, normalized size = 2.41

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\left(bx^2-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

Giac [B] time = 1.14102, size = 81, normalized size = 2.79

$$\frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}}-\frac{b^4}{\sqrt{-aba}}\right)|b|\mathrm{sgn}\left(x\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*a
))*abs(b)*sgn(x)/b^3
```

3.587 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$

Optimal. Leaf size=80

$$\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d*
x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rubi [A] time = 0.0591799, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {435, 135, 133}

$$\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d*
x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \left(\left(a+\frac{b}{x}\right)^m x^m (b+ax)^{-m}\right)\int x^{-m}(b+ax)^m(c+dx)^n\,dx$$
$$= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m}\right)\int x^{-m}\left(1+\frac{ax}{b}\right)^m (c+dx)^n\,dx$$
$$= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}\right)\int x^{-m}\left(1+\frac{ax}{b}\right)^m\left(1+\frac{dx}{c}\right)^n dx$$
$$= \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n} F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

Mathematica [F] time = 0.0647359, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c)^n,x)
```

```
[Out] int((a+b/x)^m*(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^n\left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^n\left(\frac{ax+b}{x}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^{m} (c + dx)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c)**n,x)
```

```
[Out] Integral((a + b/x)**m*(c + d*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{n} \left(a + \frac{b}{x}\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

3.588 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$

Optimal. Leaf size=138

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(m^2-3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{6a^4(m+1)}+\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd}{6a^2}$$

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a + b/x)^(1
 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) + b^2*d^2*(2 - 3*m + m
^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(6*
a^4*(1 + m))
```

Rubi [A] time = 0.116717, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 89, 78, 65}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(m^2-3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{6a^4(m+1)}+\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a + b/x)^(1
 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) + b^2*d^2*(2 - 3*m + m
^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(6*
a^4*(1 + m))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx &= \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right)^2 x^2\,dx\\ &= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)^2}{x^4}\,dx, x, \frac{1}{x}\right)\\ &= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}x^3}{3a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^m\left(d(6ac-bd(2-m))+3ac^2x\right)}{x^3}\,dx, x, \frac{1}{x}\right)}{3a}\\ &= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m}x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}x^3}{3a} - \frac{1}{6}\left(6c^2 - \frac{bd(6ac-bd(2-m))(1-m)}{a^2}\right.\\ &= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m}x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}x^3}{3a} - \frac{b\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(2-\right.\right.}{}\end{aligned}$$

Mathematica [A] time = 0.0750939, size = 112, normalized size = 0.81

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^m\left(a^2d(m+1)x^2(2a(3c+dx)+bd(m-2))-b\left(6a^2c^2+6abcd(m-1)+b^2d^2\left(m^2-3m+2\right)\right)\,{}_2F_1\left(2,m+\right.\right.}{6a^4(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2*(b*d*(-2 + m) + 2*a*(3*c + d*x))
- b*(6*a^2*c^2 + 6*a*b*c*d*(-1 + m) + b^2*d^2*(2 - 3*m + m^2))*Hypergeometr
ic2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(6*a^4*(1 + m)*x)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c)^2,x)
```

[Out] int((a+b/x)^m*(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^2*(a + b/x)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\left(\frac{ax+b}{x}\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="fricas")

[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)

Sympy [C] time = 5.36515, size = 121, normalized size = 0.88

$$\frac{b^m c^2 x x^{-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}+\frac{2b^m cdx^2x^{-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}+\frac{b^m d^2x^3x^{-m}\Gamma(3-m)\,{}_2F_1\left(\begin{matrix}-\end{matrix}\right.}{\Gamma(4-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)**m*(d*x+c)**2,x)

[Out] b**m*c**2*x*x**(-m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*pi)/b)/gamma(2 - m) + 2*b**m*c*d*x**2*x**(-m)*gamma(2 - m)*hyper((-m, 2 - m), (3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m) + b**m*d**2*x**3*x**(-m)*gamma(3 - m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/gamma(4 - m)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*(a + b/x)^m, x)

3.589 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

Optimal. Leaf size=79

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 + m
)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

Rubi [A] time = 0.0378231, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {434, 446, 78, 65}

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x),x]
```

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 + m
)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int\left(a+\frac{b}{x}\right)^m(c+dx)\,dx=\int\left(a+\frac{b}{x}\right)^m\left(d+\frac{c}{x}\right)x\,dx$$
$$=-\operatorname{Subst}\left(\int\frac{(a+bx)^m(d+cx)}{x^3}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{d\left(a+\frac{b}{x}\right)^{1+m}x^2}{2a}-\frac{(2ac+bd(-1+m))\operatorname{Subst}\left(\int\frac{(a+bx)^m}{x^2}\,dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{d\left(a+\frac{b}{x}\right)^{1+m}x^2}{2a}-\frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{2a^3(1+m)}$$

Mathematica [A] time = 0.0309613, size = 73, normalized size = 0.92

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^m\left(a^2d(m+1)x^2+b(-2ac-bd(m-1))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)\right)}{2a^3(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m*(c + d*x),x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2 + b*(-2*a*c - b*d*(-1 + m))*Hyper
geometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(2*a^3*(1 + m)*x)
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^m(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c),x)
```

```
[Out] int((a+b/x)^m*(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((dx+c)\left(\frac{ax+b}{x}\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*((a*x + b)/x)^m, x)
```

Sympy [C] time = 3.42879, size = 75, normalized size = 0.95

$$\frac{b^m cxx^{-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}+\frac{b^m dx^2x^{-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c),x)
```

```
[Out] b**m*c*x*x**(-m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*
pi)/b)/gamma(2 - m) + b**m*d*x**2*x**(-m)*gamma(2 - m)*hyper((-m, 2 - m), (
3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

3.590 $\int \left(a+\frac{b}{x}\right)^m \, dx$

Optimal. Leaf size=40

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a^
2*(1 + m)))
```

Rubi [A] time = 0.0101131, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {242, 65}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a^
2*(1 + m)))
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^m \, dx &= -\text{Subst}\left(\int \frac{(a+bx)^m}{x^2} \, dx, x, \frac{1}{x}\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{a^2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0173851, size = 50, normalized size = 1.25

$$-\frac{x\left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} {}_2F_1\left(1-m,-m;2-m;-\frac{ax}{b}\right)}{m-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m,x]
```

```
[Out] -(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])/((-1 + m)
*(1 + (a*x)/b)^m))
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m,x)
```

```
[Out] int((a+b/x)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\frac{ax+b}{x}\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m, x)
```

Sympy [C] time = 1.36163, size = 34, normalized size = 0.85

$$\frac{b^{m}xx^{-m}\Gamma(1-m)\,{}_{2}F_{1}\left(\begin{matrix} -m,1-m \\ 2-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m,x)
```

```
[Out] b**m*x*x**(-m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*pi
)/b)/gamma(2 - m)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m, x)
```

3.591 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx} \, dx$

Optimal. Leaf size=101

$$\frac{\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F1[
1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rubi [A] time = 0.0684855, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 86, 65, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F1[
1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)x}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{(a+bx)^m}{x(d+cx)}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(a+bx)^m}{x}\,dx, x, \frac{1}{x}\right)}{d} + \frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^m}{d+cx}\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(1,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)} + \frac{\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(1,1+m;2+m;1+\frac{b}{ax}\right)}{ad(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0444179, size = 97, normalized size = 0.96

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^m\left(ac\,{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(bd-ac)\,{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)\right)}{ad(m+1)x(bd-ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x),x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + m)*x)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c),x)
```

```
[Out] int((a+b/x)^m/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

3.592 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2} dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rubi [A] time = 0.0340559, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {434, 444, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{b\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}$$

Mathematica [A] time = 0.0244326, size = 57, normalized size = 1.02

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{c\left(a+\frac{b}{x}\right)}{bd-ac}\right)}{(m+1)(bd-ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + m)))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**2,x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

3.593 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

```
[Out] -(d*(a + b/x)^(1 + m))/(2*c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(1 +
 m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rubi [A] time = 0.0680983, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {434, 446, 78, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] -(d*(a + b/x)^(1 + m))/(2*c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(1 +
 m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^3 x^3}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{x(a+bx)^m}{(d+cx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{(2ac-bd(1+m))\operatorname{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{2c(ac-bd)}$$

$$= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}$$

Mathematica [A] time = 0.0676184, size = 99, normalized size = 0.88

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\left(\frac{b(bd(m+1)-2ac)\,{}_2F_1\left(2,m+1;m+2;\frac{bc+axc}{acx-bdx}\right)}{(m+1)(ac-bd)^2}-\frac{dx^2}{(c+dx)^2}\right)}{2c(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] ((a + b/x)^(1 + m)*(-((d*x^2)/(c + d*x)^2) + (b*(-2*a*c + b*d*(1 + m))*Hype
rgeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d*x)])/((a*c - b*d)
^2*(1 + m))))/(2*c*(a*c - b*d))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^3}\left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^3,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{\left(c + dx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**3,x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{\left(dx + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

3.594 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$

Optimal. Leaf size=185

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-$$

```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a*c - b*d*(
4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeo
metric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4
*(1 + m))
```

Rubi [A] time = 0.18285, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 89, 78, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^4,x]
```


```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a*c - b*d*(
4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeo
metric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4
*(1 + m))
```


Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^4 x^4}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{x^2(a+bx)^m}{(d+cx)^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^m(-d(3ac-bd(1+m))+3c(ac-bd)x)}{(d+cx)^3}\,dx, x, \frac{1}{x}\right)}{3c^2(ac-bd)}$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} - \frac{\left(6a^2c^2-6abcd(1+m)+b^2d^2\left(2+3m+m^2\right.\right.}{6c^2(ac-bd)^2}$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} - \frac{b\left(6a^2c^2-6abcd(1+m)+b^2d^2\left(2+3m+m\right.\right.}{6c^2(ac-b}$$

Mathematica [A] time = 0.144135, size = 155, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\left(-\frac{b\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{bc+axc}{acx-bdx}\right)}{(m+1)(ac-bd)^2}+\frac{2d^2x^3(ac-bd)}{(c+dx)^3}+\frac{dx^2(bd(m+4)-6ac)}{(c+dx)^2}\right)}{6c^2(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] ((a + b/x)^(1 + m)*((2*d^2*(a*c - b*d)*x^3)/(c + d*x)^3 + (d*(-6*a*c + b*d*
(4 + m))*x^2)/(c + d*x)^2 - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2
+ 3*m + m^2))*Hypergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d
*x)])/((a*c - b*d)^2*(1 + m))))/(6*c^2*(a*c - b*d)^2)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^4}\left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^4,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

3.595 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=33

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rubi [A] time = 0.0314056, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{x^{-1+m}\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int x^{-1+m}\,dx}{\sqrt{a-bx^2}}\\
&= \frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0174014, size = 33, normalized size = 1.

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Maple [A] time = 0.001, size = 35, normalized size = 1.1

$$\frac{x^{1+m}}{m}\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] x^(1+m)/m*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [C] time = 1.15638, size = 11, normalized size = 0.33

$$-\frac{i\,x^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*x^m/m
```

Fricas [A] time = 1.51179, size = 85, normalized size = 2.58

$$-\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

3.596 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=31

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rubi [A] time = 0.0351915, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{x\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int x\,dx}{\sqrt{a-bx^2}}\\ &= \frac{\sqrt{b-\frac{a}{x^2}}x^3}{2\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0095304, size = 31, normalized size = 1.

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Maple [A] time = 0.003, size = 31, normalized size = 1.

$$\frac{x^3}{2}\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [C] time = 1.18261, size = 7, normalized size = 0.23

$$-\frac{1}{2}i\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2
```

Fricas [A] time = 1.46176, size = 82, normalized size = 2.65

$$-\frac{\sqrt{-bx^2+a}\,x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [A] time = 1.10978, size = 53, normalized size = 1.71

$$-\frac{\left(bx^2-a\right)i\mathrm{sgn}\left(bx^2-a\right)\mathrm{sgn}\left(x\right)}{2\,b}+\frac{ai\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*x^2 - a)*i*sgn(b*x^2 - a)*sgn(x)/b + 1/2*a*i*sgn(a)*sgn(x)/b
```

3.597 $\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rubi [A] time = 0.0260666, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 8}

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int 1\,dx}{\sqrt{a-bx^2}}\\
&= \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0081602, size = 28, normalized size = 1.

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Maple [A] time = 0.01, size = 42, normalized size = 1.5

$$-\frac{x^2}{bx^2-a}\sqrt{\frac{bx^2-a}{x^2}}\sqrt{-bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -((b*x^2-a)/x^2)^(1/2)*x^2/(b*x^2-a)*(-b*x^2+a)^(1/2)
```

Maxima [C] time = 1.1588, size = 9, normalized size = 0.32

$$-i\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*sqrt(x^2)
```

Fricas [A] time = 1.44236, size = 77, normalized size = 2.75

$$-\frac{\sqrt{-bx^2+a}x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

3.598 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rubi [A] time = 0.0186658, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {435, 23, 29}

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{\sqrt{-a+bx^2}}{x\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int \frac{1}{x}\,dx}{\sqrt{a-bx^2}}\\
&= \frac{\sqrt{b-\frac{a}{x^2}}x\log(x)}{\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0082065, size = 28, normalized size = 1.

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Maple [A] time = 0.007, size = 42, normalized size = 1.5

$$-\frac{x\ln(x)}{bx^2-a}\sqrt{\frac{bx^2-a}{x^2}}\sqrt{-bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -((b*x^2-a)/x^2)^(1/2)*x/(b*x^2-a)*(-b*x^2+a)^(1/2)*ln(x)
```

Maxima [C] time = 1.06245, size = 5, normalized size = 0.18

$$-i\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*log(x)
```

Fricas [B] time = 1.61596, size = 115, normalized size = 4.11

$$-\arctan\left(\frac{\sqrt{-bx^2+a}\left(x^3+x\right)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*x
^2 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [A] time = 1.10214, size = 42, normalized size = 1.5

$$-\frac{1}{2}\,i\log\left(\left(bx^2-a\right)i+ai\right)\operatorname{sgn}\left(bx^2-a\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*i*log((b*x^2 - a)*i + a*i)*sgn(b*x^2 - a)*sgn(x)
```

3.599 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rubi [A] time = 0.0319128, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^2\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^2}\,dx}{\sqrt{a-bx^2}}\\
&= -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.0077534, size = 26, normalized size = 1.

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Maple [A] time = 0.003, size = 28, normalized size = 1.1

$$-\sqrt{-\frac{-bx^2+a}{x^2}}\,\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x)
```

```
[Out] -(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [C] time = 1.07519, size = 9, normalized size = 0.35

$$\frac{i}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] I/sqrt(x^2)
```

Fricas [A] time = 1.4819, size = 82, normalized size = 3.15

$$-\frac{\sqrt{-bx^2+a}(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{-bx^2 + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

3.600 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Rubi [A] time = 0.0344175, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]

[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^3\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^3}\,dx}{\sqrt{a-bx^2}}\\ &= -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] time = 0.0071627, size = 31, normalized size = 1.

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

```
[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Maple [A] time = 0.001, size = 31, normalized size = 1.

$$-\frac{1}{2x}\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Maxima [C] time = 1.16725, size = 7, normalized size = 0.23

$$\frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I/x^2
```

Fricas [A] time = 1.47842, size = 93, normalized size = 3.

$$-\frac{\sqrt{-bx^2+a}\left(x^2-1\right)\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^3-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

Giac [A] time = 1.07772, size = 27, normalized size = 0.87

$$-\frac{\operatorname{sgn}\left(bx^2-a\right)\operatorname{sgn}\left(x\right)}{2\,ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sgn(b*x^2 - a)*sgn(x)/(i*x^2)
```

3.601 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=406

$$-\frac{2\sqrt{b}c\sqrt{\frac{ax^2}{b}+1}\left(ac^2+bd^2\right)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}+\frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}\left(ac^2-3bd^2\right)E\left(\sin \dots\right.}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{\dots}}$$

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c + d*x)^(3/2
)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*c^2 - 3*b*d^2)*Sqrt[
c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]
]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3
/2)*d*Sqrt[a + b/x^2]*x*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (
2*Sqrt[b]*c*(a*c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*
Sqrt[1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]
], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqr
t[a + b/x^2]*x*Sqrt[c + d*x])
```

Rubi [A] time = 0.463681, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1450, 833, 844, 719, 424, 419}

$$-\frac{2\sqrt{b}c\sqrt{\frac{ax^2}{b}+1}\left(ac^2+bd^2\right)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}+\frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}\left(ac^2-3bd^2\right)E\left(\sin \dots\right.}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c + d*x)^(3/2
)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*c^2 - 3*b*d^2)*Sqrt[
c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]
]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3
/2)*d*Sqrt[a + b/x^2]*x*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (
2*Sqrt[b]*c*(a*c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*
Sqrt[1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]
], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqr
t[a + b/x^2]*x*Sqrt[c + d*x])
```

Rule 1450

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Sy
mbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2
*n))^FracPart[p], Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x], x] /
; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !I
ntegerQ[q] && PosQ[n]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{b+ax^2}\int \frac{x(c+dx)^{3/2}}{\sqrt{b+ax^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(2\sqrt{b+ax^2}\right)\int \frac{\left(-\frac{3bd}{2}+\frac{3acx}{2}\right)\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5a\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(4\sqrt{b+ax^2}\right)\int \frac{-3abcd+\frac{3}{4}a\left(ac^2-3bd^2\right)x}{\sqrt{c+dx}\sqrt{b+ax^2}}\,dx}{15a^2\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(\left(ac^2-3bd^2\right)\sqrt{b+ax^2}\right)\int \frac{\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5ad\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\left(c\left(ac^2+\right.\right.\cdots}{\cdots}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(2\sqrt{-a}\sqrt{b}\left(ac^2-3bd^2\right)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}\right)\text{Subst}\,\cdots}{5a^2d\sqrt{a+\frac{b}{x^2}}\,x\sqrt{\frac{a(c+\cdots}{ac-\sqrt{-\cdots}}}}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2\sqrt{b}\left(ac^2-3bd^2\right)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}\right.\right.\cdots}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}\,x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}\cdots}}}$$

<u>Mathematica</u> [C] time = 3.03285, size = 540, normalized size = 1.33

$$\frac{\sqrt{c+dx}\left(\frac{2\left(ax^2+b\right)(2c+dx)}{a} + \frac{2\left(d^2\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}\left(a^2c^2x^2+ab\left(c^2-3d^2x^2\right)-3b^2d^2\right)+\sqrt{a}(c+dx)^{3/2}\left(-ia^{3/2}c^3+a\sqrt{b}c^2d+3i\sqrt{a}bcd^2-3b^{3/2}d^3\right)\sqrt{\frac{d\left(x+\frac{i\sqrt{b}}{\sqrt{a}}\right)}{c+dx}}\sqrt{-\frac{-dx+\cdots}{c+dx}}\cdots\right.}{a^2\cdots}\right)}{5x\sqrt{a+\frac{b}{x^2}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c - (I*Sqrt[b
]*d)/Sqrt[a]]*(-3*b^2*d^2 + a^2*c^2*x^2 + a*b*(c^2 - 3*d^2*x^2)) + Sqrt[a]*
((-I)*a^(3/2)*c^3 + a*Sqrt[b]*c^2*d + (3*I)*Sqrt[a]*b*c*d^2 - 3*b^(3/2)*d^3
)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[
a] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt
[b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sq
rt[b]*d)] - Sqrt[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b*d^2)
*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a
] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[
b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqr
t[b]*d)]))/(a^2*d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a
 + b/x^2]*x)
```

Maple [B] time = 0.108, size = 1145, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x)
```

```
[Out] 2/5*((-a*b)^(1/2)*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/
2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c
))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*
d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*a*c^3*d+(-a*b)^(1/2)*(-(d*x+c)*a/((-a*b
)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((
a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*
b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*
b*c*d^3-3*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((
-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)
*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/(
(-a*b)^(1/2)*d+a*c))^(1/2))*a*b*c^2*d^2-3*b^2*(-(d*x+c)*a/((-a*b)^(1/2)*d-a
*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^
(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-
a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*d^4-(-(d*x+
c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c
))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticE((-(d*x
+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*
c))^(1/2))*a^2*c^4+2*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^
(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-
a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/
2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*a*b*c^2*d^2+3*b^2*(-(d*x+c)*a/((-a*b
)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((
a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*
b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*
d^4+x^4*a^2*d^4+3*x^3*a^2*c*d^3+2*x^2*a^2*c^2*d^2+x^2*a*b*d^4+3*x*a*b*c*d^3
+2*a*b*c^2*d^2)/(d*x+c)^(1/2)/d^2/a^2/x/((a*x^2+b)/x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^3+cx^2\right)\sqrt{dx+c}\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2)*sqrt(d*x + c)*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

3.602 $\int \frac{-1+x^3}{\left(-4x+x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

```
[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rubi [A] time = 0.0093661, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+x^3}{\left(-4x+x^4\right)^{2/3}}\,dx=\frac{3}{4}\sqrt[3]{-4x+x^4}$$

Mathematica [A] time = 0.0256064, size = 15, normalized size = 1.

$$\frac{3}{4}\sqrt[3]{x\left(x^3-4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(x*(-4 + x^3))^(1/3))/4
```

Maple [A] time = 0.006, size = 18, normalized size = 1.2

$$\frac{3\,x\left(x^3-4\right)}{4}\left(x^4-4\,x\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^4-4*x)^(2/3),x)
```

```
[Out] 3/4*x*(x^3-4)/(x^4-4*x)^(2/3)
```

Maxima [A] time = 1.10455, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^4 - 4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Fricas [A] time = 1.36459, size = 31, normalized size = 2.07

$$\frac{3}{4}\left(x^4 - 4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Sympy [A] time = 0.185364, size = 12, normalized size = 0.8

$$\frac{3\sqrt[3]{x^4 - 4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**4-4*x)**(2/3),x)
```

```
[Out] 3*(x**4 - 4*x)**(1/3)/4
```

Giac [A] time = 1.10566, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^4 - 4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

3.603 $\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx$

Optimal. Leaf size=17

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rubi [A] time = 0.0087056, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx=\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Mathematica [C] time = 0.045878, size = 72, normalized size = 4.24

$$-\frac{4\sqrt[4]{-x\left(x^2-6\right)}\left(5x^3\, _2F_1\left(-\frac{1}{4},\frac{13}{8};\frac{21}{8};\frac{x^2}{6}\right)-26x\, _2F_1\left(-\frac{1}{4},\frac{5}{8};\frac{13}{8};\frac{x^2}{6}\right)\right)}{65\sqrt[4]{1-\frac{x^2}{6}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/6
] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1/4
))
```

Maple [A] time = 0.004, size = 20, normalized size = 1.2

$$-\frac{4\,x\left(x^2-6\right)}{15}\sqrt[4]{-x^3+6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)*(-x^3+6*x)^(1/4),x)
```

```
[Out] -4/15*(-x^3+6*x)^(1/4)*x*(x^2-6)
```

Maxima [A] time = 1.09946, size = 18, normalized size = 1.06

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

Fricas [A] time = 1.42863, size = 51, normalized size = 3.

$$-\frac{4}{15}\left(x^3-6\,x\right)\left(-x^3+6\,x\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

Sympy [B] time = 0.251987, size = 31, normalized size = 1.82

$$-\frac{4x^3\sqrt[4]{-x^3+6x}}{15}+\frac{8x\sqrt[4]{-x^3+6x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
```

```
[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

Giac [A] time = 1.12287, size = 18, normalized size = 1.06

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")
```

```
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

3.604 $\int (1+x^4)\sqrt{5x+x^5}\,dx$

Optimal. Leaf size=15

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

Rubi [A] time = 0.008289, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(5*x + x^5)^(3/2))/15
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(1+x^4\right)\sqrt{5x+x^5}\,dx=\frac{2}{15}\left(5x+x^5\right)^{3/2}$$

Mathematica [A] time = 0.0278142, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x\left(x^4+5\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(x*(5 + x^4))^(3/2))/15
```

Maple [A] time = 0.004, size = 18, normalized size = 1.2

$$\frac{2x\left(x^4+5\right)}{15}\sqrt{x^5+5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^5+5*x)^(1/2),x)
```

```
[Out] 2/15*x*(x^4+5)*(x^5+5*x)^(1/2)
```

Maxima [A] time = 1.05358, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Fricas [A] time = 1.44797, size = 32, normalized size = 2.13

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Sympy [B] time = 0.246925, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+5x}}{15}+\frac{2x\sqrt{x^5+5x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

Giac [A] time = 1.10161, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

3.605 $\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx$

Optimal. Leaf size=15

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rubi [A] time = 0.0083743, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2}{3}\left(2x+x^5\right)^{3/2}$$

Mathematica [A] time = 0.0339676, size = 15, normalized size = 1.

$$\frac{2}{3}\left(x\left(x^4+2\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

```
[Out] (2*(x*(2 + x^4))^(3/2))/3
```

Maple [A] time = 0.003, size = 18, normalized size = 1.2

$$\frac{2x\left(x^4+2\right)}{3}\sqrt{x^5+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x)
```

```
[Out] 2/3*x*(x^4+2)*(x^5+2*x)^(1/2)
```

Maxima [A] time = 1.08896, size = 15, normalized size = 1.

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Fricas [A] time = 1.44811, size = 31, normalized size = 2.07

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Sympy [B] time = 0.243079, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+2x}}{3}+\frac{4x\sqrt{x^5+2x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

Giac [A] time = 1.14557, size = 15, normalized size = 1.

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

3.606 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} \, dx$

Optimal. Leaf size=13

$$\sqrt{2x^3+x^2}$$

```
[Out] Sqrt[x^2 + 2*x^3]
```

Rubi [A] time = 0.0097622, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\sqrt{2x^3+x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2 + 2*x^3]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} \, dx = \sqrt{x^2+2x^3}$$

Mathematica [A] time = 0.0092088, size = 13, normalized size = 1.

$$\sqrt{x^2(2x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2*(1 + 2*x)]
```

Maple [A] time = 0.002, size = 21, normalized size = 1.6

$$x^2(1+2x)\frac{1}{\sqrt{2x^3+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x)/(2*x^3+x^2)^(1/2),x)
```

```
[Out] x^2*(1+2*x)/(2*x^3+x^2)^(1/2)
```

Maxima [A] time = 1.089, size = 15, normalized size = 1.15

$$\sqrt{2\,x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2*x^3 + x^2)
```

Fricas [A] time = 1.40801, size = 26, normalized size = 2.

$$\sqrt{2\,x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*x^3 + x^2)
```

Sympy [A] time = 0.139449, size = 10, normalized size = 0.77

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
```

```
[Out] sqrt(2*x**3 + x**2)
```

Giac [A] time = 1.14004, size = 15, normalized size = 1.15

$$\sqrt{2\,x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2*x^3 + x^2)
```

3.607 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx$

Optimal. Leaf size=44

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Rubi [A] time = 0.0243852, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {431, 376, 77}

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx &= -\left(\frac{1}{5}\,\text{Subst}\left(\int \frac{2+\sqrt[3]{x}}{3+\sqrt[3]{x}}\,dx, x, 1-5x\right)\right)\\ &= -\left(\frac{3}{5}\,\text{Subst}\left(\int \frac{x^2(2+x)}{3+x}\,dx, x, \sqrt[3]{1-5x}\right)\right)\\ &= -\left(\frac{3}{5}\,\text{Subst}\left(\int \left(3-x+x^2-\frac{9}{3+x}\right)dx, x, \sqrt[3]{1-5x}\right)\right)\\ &= -\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)\end{aligned}$$

Mathematica [A] time = 0.0207475, size = 44, normalized size = 1.

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Maple [A] time = 0.003, size = 34, normalized size = 0.8

$$-\frac{1}{5}+x+\frac{3}{10}(1-5x)^{\frac{2}{3}}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\ln\left(3+\sqrt[3]{1-5x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x)
```

```
[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))
```

Maxima [A] time = 1.11109, size = 45, normalized size = 1.02

$$x+\frac{3}{10}(-5x+1)^{\frac{2}{3}}-\frac{9}{5}(-5x+1)^{\frac{1}{3}}+\frac{27}{5}\log\left((-5x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

Fricas [A] time = 1.59752, size = 112, normalized size = 2.55

$$x+\frac{3}{10}(-5x+1)^{\frac{2}{3}}-\frac{9}{5}(-5x+1)^{\frac{1}{3}}+\frac{27}{5}\log\left((-5x+1)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3)
```

Sympy [A] time = 0.175741, size = 39, normalized size = 0.89

$$x + \frac{3(1-5x)^{\frac{2}{3}}}{10} - \frac{9\sqrt[3]{1-5x}}{5} + \frac{27\log\left(\sqrt[3]{1-5x}+3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

```
[Out] x + 3*(1 - 5*x)**(2/3)/10 - 9*(1 - 5*x)**(1/3)/5 + 27*log((1 - 5*x)**(1/3)
+ 3)/5
```

Giac [A] time = 1.12737, size = 45, normalized size = 1.02

$$x + \frac{3}{10}(-5x+1)^{\frac{2}{3}} - \frac{9}{5}(-5x+1)^{\frac{1}{3}} + \frac{27}{5}\log\left((-5x+1)^{\frac{1}{3}}+3\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

3.608 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} dx$

Optimal. Leaf size=21

$$x + 4\sqrt{x} + 4\log\left(1-\sqrt{x}\right)$$

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rubi [A] time = 0.01262, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$x + 4\sqrt{x} + 4\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)dx, x, \sqrt{x}\right)\\
&= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0086964, size = 20, normalized size = 0.95

$$x + 4\left(\sqrt{x}+\log\left(1-\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] x + 4*(Sqrt[x] + Log[1 - Sqrt[x]])
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(-1+x^(1/2)),x)
```

```
[Out] x+4*x^(1/2)+4*ln(-1+x^(1/2))
```

Maxima [A] time = 1.0568, size = 20, normalized size = 0.95

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Fricas [A] time = 1.66223, size = 49, normalized size = 2.33

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A] time = 0.131752, size = 17, normalized size = 0.81

$$4\sqrt{x}+x+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

Giac [A] time = 1.13404, size = 22, normalized size = 1.05

$$x+4\sqrt{x}+4\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")

[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

3.609 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx$

Optimal. Leaf size=33

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rubi [A] time = 0.0216975, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {431, 376, 77}

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]

[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx,x,2+3x\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{(1-x)x}{1+x}\,dx,x,\sqrt{2+3x}\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \left(2-x-\frac{2}{1+x}\right)dx,x,\sqrt{2+3x}\right)\\ &= -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)\end{aligned}$$

Mathematica [A] time = 0.0144503, size = 33, normalized size = 1.

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{4}{3}\sqrt{2+3x}-x-\frac{2}{3}-\frac{4}{3}\ln\left(1+\sqrt{2+3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x)
```

```
[Out] 4/3*(2+3*x)^(1/2)-x-2/3-4/3*ln(1+(2+3*x)^(1/2))
```

Maxima [A] time = 1.0267, size = 35, normalized size = 1.06

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

Fricas [A] time = 1.58899, size = 72, normalized size = 2.18

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

Sympy [A] time = 0.144897, size = 27, normalized size = 0.82

$$-x+\frac{4\sqrt{3x+2}}{3}-\frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
```

```
[Out] -x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

Giac [A] time = 1.10943, size = 35, normalized size = 1.06

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log\left(\sqrt{3x+2}+1\right) - \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

3.610 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

Optimal. Leaf size=33

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rubi [A] time = 0.020707, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {431, 376, 77}

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+\sqrt{x}}{1+\sqrt{x}}\,dx, x, a+bx\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{(-1+x)x}{1+x}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-2+x+\frac{2}{1+x}\right)dx, x, \sqrt{a+bx}\right)}{b}$$
$$= x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(1+\sqrt{a+bx}\right)}{b}$$

Mathematica [A] time = 0.0149851, size = 33, normalized size = 1.

$$-\frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(\sqrt{a+bx}+1\right)}{b} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Maple [A] time = 0.001, size = 35, normalized size = 1.1

$$-4\,\frac{\sqrt{bx+a}}{b} + x + \frac{a}{b} + 4\,\frac{\ln\left(1+\sqrt{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x)
```

```
[Out] -4*(b*x+a)^(1/2)/b+x+a/b+4*ln(1+(b*x+a)^(1/2))/b
```

Maxima [A] time = 1.10034, size = 41, normalized size = 1.24

$$\frac{bx + a - 4\,\sqrt{bx+a} + 4\,\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Fricas [A] time = 1.68102, size = 73, normalized size = 2.21

$$\frac{bx - 4\,\sqrt{bx+a} + 4\,\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Sympy [A] time = 0.416818, size = 42, normalized size = 1.27

$$\begin{cases} x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(\sqrt{a+bx}+1\right)}{b} & \text{for } b \neq 0 \\ \frac{x\left(\sqrt{a}-1\right)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

```
[Out] Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)), (
x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

Giac [A] time = 1.14317, size = 51, normalized size = 1.55

$$\frac{4\log\left(\sqrt{bx+a}+1\right)}{b} + \frac{(bx+a)b - 4\sqrt{bx+a}b}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

3.611 $\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx$

Optimal. Leaf size=10

$$\log\left(ax+bx^n\right)$$

```
[Out] Log[a*x + b*x^n]
```

Rubi [A] time = 0.0523407, antiderivative size = 17, normalized size of antiderivative = 1.7, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 514, 446, 72}

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx &= \int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx \\
&= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)dx, x, x^{1-n}\right)}{1-n} \\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}$$

Mathematica [A] time = 0.0272881, size = 17, normalized size = 1.7

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Maple [A] time = 0.016, size = 13, normalized size = 1.3

$$\ln\left(ax+be^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(-1+n))/(a*x+b*x^n),x)
```

```
[Out] ln(a*x+b*exp(n*ln(x)))
```

Maxima [A] time = 1.10843, size = 14, normalized size = 1.4

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="maxima")
```

```
[Out] log(a*x + b*x^n)
```

Fricas [A] time = 1.83338, size = 24, normalized size = 2.4

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="fricas")
```

```
[Out] log(a*x + b*x^n)
```

Sympy [A] time = 8.93697, size = 32, normalized size = 3.2

$$\begin{cases} \log\left(x + \frac{bx^n}{a}\right) & \text{for } a \neq 0 \\ n\left(\frac{n^2\log(x)}{n^2-n} - \frac{n\log(x)}{n^2-n}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*(n**2*log(x)/(n**2 - n) - n*log
(x)/(n**2 - n)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnx^{n-1} + a}{ax + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

3.612 $\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx$

Optimal. Leaf size=17

$$\log \left(ax^{1-n}+b\right)+n \log (x)$$

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rubi [A] time = 0.0377438, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {514, 446, 72}

$$\log \left(ax^{1-n}+b\right)+n \log (x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx &= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx, x, x^{1-n}\right)}{1-n}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)dx, x, x^{1-n}\right)}{1-n}\\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}$$

Mathematica [A] time = 0.0158437, size = 17, normalized size = 1.

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Maple [A] time = 0.021, size = 13, normalized size = 0.8

$$\ln\left(ax+b\mathrm{e}^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x)
```

```
[Out] ln(a*x+b*exp(n*ln(x)))
```

Maxima [B] time = 1.11918, size = 116, normalized size = 6.82

$$bn\left(\frac{\log(x)}{b}-\frac{n\log(x)}{b(n-1)}+\frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)}\right)+a\left(\frac{n\log(x)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")
```

```
[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) +
a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))
```

Fricas [A] time = 1.59771, size = 24, normalized size = 1.41

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")
```

```
[Out] log(a*x + b*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnx^{n-1} + a}{\left(ax^{-n+1} + b\right)x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

3.613 $\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3\right.$

Optimal. Leaf size=37

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi [A] time = 0.0923031, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 176, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1590}

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a
*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+\right.$$

Mathematica [A] time = 0.363181, size = 34, normalized size = 0.92

$$x^2(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d
 + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*
e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b
*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] time = 0.023, size = 38, normalized size = 1.

$$x^{2} \left(c x^{2}+b x +a \right)^{1+m} \left(g \,x^{3}+f \,x^{2}+e x +d \right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d
)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*b*
f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b
*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x)
```

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [B] time = 1.56011, size = 131, normalized size = 3.54

$$\left(cgx^7 + \left(cf + bg\right)x^6 + \left(ce + bf + ag\right)x^5 + \left(cd + be + af\right)x^4 + adx^2 + (bd + ae)x^3\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e + a*f)*x^4
+ a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^
2 + b*x + a))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+
3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*g*
n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*
```

```
m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

3.614 $\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2a\right.$

Optimal. Leaf size=35

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi [A] time = 0.0972397, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 174, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1590}

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e +
 b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*
f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a
*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g
*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+bem+ben+\right.$$

Mathematica [A] time = 0.332371, size = 32, normalized size = 0.91

$$x(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2
*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n
+ 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
 + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4
 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] time = 0.022, size = 36, normalized size = 1.

$$x\left(cx^2+bx+a\right)^{1+m}\left(gx^3+fx^2+ex+d\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+
2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*g+5
*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x)
```

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [B] time = 1.48306, size = 128, normalized size = 3.66

$$\left(cgx^6+\left(cf+bg\right)x^5+\left(ce+bf+ag\right)x^4+\left(cd+be+af\right)x^3+adx+(bd+ae)x^2\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e + a*f)*x^3
+ a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2
+ b*x + a))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*
e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*
```

```
c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

3.615 $\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+\right.$

Optimal. Leaf size=34

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi [A] time = 0.11651, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 164, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1590}

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*
e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*
e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c
*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)
*x^4),x]
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3c\right.$$

Mathematica [A] time = 0.320869, size = 31, normalized size = 0.91

$$(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*
m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x +
 (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2
+ (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m
+ 3*n)*x^4),x]
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] time = 0.02, size = 35, normalized size = 1.

$$\left(cx^{2}+bx+a\right)^{1+m}\left(gx^{3}+fx^{2}+ex+d\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e
*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+
3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c*g*
(5+2*m+3*n)*x^4),x)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [B] time = 1.5879, size = 124, normalized size = 3.65

$$\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+(bd+ae)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c
*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c
```

```
*f)*x**3+c*g*(5+2*m+3*n)*x**4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.616 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}}{x}$$

```
[Out] ((a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n))/x
```

Rubi [F] time = 3.37432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2c$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e
*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e
+ 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f
 + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x
^5))/x^2,x]
```

```
[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int][(a + b*x + c*x
^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int][((a + b*x + c*x^2)^m
*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (b*d*m + a*e*n)*Defer[Int][((a + b*
x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x, x] + (c*e*(2 + 2*m + n) + b*f*
(2 + m + 2*n) + a*g*(2 + 3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x +
f*x^2 + g*x^3)^n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int]
[x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*m + 3
*n)*Defer[Int][x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2c$$

Mathematica [A] time = 0.950024, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m
 + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (
2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m +
```

```
3*n)*x^5))/x^2,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

Maple [A] time = 0.021, size = 38, normalized size = 1.

$$\frac{\left(cx^2+bx+a\right)^{1+m}\left(gx^3+fx^2+ex+d\right)^{1+n}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*
e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a
*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+
2*m+3*n)*x^5)/x^2,x)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

Maxima [B] time = 1.48582, size = 128, normalized size = 3.46

$$\frac{\left(cgx^5+\left(cf+bg\right)x^4+\left(ce+bf+ag\right)x^3+\left(cd+be+af\right)x^2+ad+(bd+ae)x\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e
*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f
)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cg(2\,m+3\,n+4)x^5+\left(2\,cfm+bgm+2\,cfn+3\,bgn+3\,cf+3\,bg\right)x^4+\left(2\,cem+bfm+cen+2\,bfn+3\,agn+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*g*(2*m + 3*n + 4)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n +
 3*c*f + 3*b*g)*x^4 + (2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n + 2*c*e
+ 2*b*f + 2*a*g)*x^3 + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n + c*d + b*e + a*f
)*x^2 - a*d + (b*d*m + a*e*n)*x)*(g*x^3 + f*x^2 + e*x + d)^n*(c*x^2 + b*x +
 a)^m/x^2, x)
```

3.617 $\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+be$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}}{x^2}$$

```
[Out] ((a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n))/x^2
```

Rubi [F] time = 2.99845, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf}{x^3}$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d) - a*
e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f
 + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g
+ 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3,
x]
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int][(a + b*x
 + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Defer[Int][((a + b*x +
c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^3, x] - (b*d*(1 - m) + a*e*(1 - n))
*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (2*
c*d*m + 2*a*f*n + b*e*(m + n))*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int]
[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m + 3*n
)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf}{x^3}$$

Mathematica [A] time = 1.39784, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d
) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e
 + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f +
2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5)
```

```
)/x^3,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

Maple [A] time = 0.023, size = 38, normalized size = 1.

$$\frac{\left(cx^2+bx+a\right)^{1+m}\left(gx^3+fx^2+ex+d\right)^{1+n}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3*n)
*x^5)/x^3,x)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Maxima [B] time = 1.61362, size = 128, normalized size = 3.46

$$\frac{\left(cgx^5+\left(cf+bg\right)x^4+\left(ce+bf+ag\right)x^3+\left(cd+be+af\right)x^2+ad+\left(bd+ae\right)x\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x^2
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-a
*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m
+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**4+c
*g*(3+2*m+3*n)*x**5)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cg(2\,m + 3\,n + 3)x^5 + \left(2\,cfm + bgm + 2\,cfn + 3\,bgn + 2\,cf + 2\,bg\right)x^4 + \left(2\,cem + bfm + cen + 2\,bfn + 3\,agn + ce + \right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*g*(2*m + 3*n + 3)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n +
 2*c*f + 2*b*g)*x^4 + (2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n + c*e +
b*f + a*g)*x^3 + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 - 2*a*d + (b*d*m +
 a*e*n - b*d - a*e)*x)*(g*x^3 + f*x^2 + e*x + d)^n*(c*x^2 + b*x + a)^m/x^3,
 x)
```

3.618 $\int x^3 \left(a + b\sqrt{c + dx}\right)^2 dx$

Optimal. Leaf size=185

$$\frac{c^2 (3a^2 - b^2 c) (c + dx)^2}{2d^4} + \frac{(a^2 - 3b^2 c) (c + dx)^4}{4d^4} - \frac{c (a^2 - b^2 c) (c + dx)^3}{d^4} - \frac{a^2 c^3 x}{d^3} + \frac{12abc^2 (c + dx)^{5/2}}{5d^4} - \frac{4abc^3 (c + dx)^3}{3d^4}$$

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^
2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2
- b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*
b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d
*x)^5)/(5*d^4)
```

Rubi [A] time = 0.25477, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{c^2 (3a^2 - b^2 c) (c + dx)^2}{2d^4} + \frac{(a^2 - 3b^2 c) (c + dx)^4}{4d^4} - \frac{c (a^2 - b^2 c) (c + dx)^3}{d^4} - \frac{a^2 c^3 x}{d^3} + \frac{12abc^2 (c + dx)^{5/2}}{5d^4} - \frac{4abc^3 (c + dx)^3}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^
2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2
- b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*
b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d
*x)^5)/(5*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int\left(-a^2c^3x-2abc^3x^2-c^2\left(-3a^2+b^2c\right)x^3+6abc^2x^4+3c\left(-a^2+b^2c\right)x^5-6abcx^6+\right.\right.}{d^4}$$
$$=-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{c\left(a^2-b^2c\right)(c+}{d^4}$$

Mathematica [A] time = 0.313332, size = 88, normalized size = 0.48

$$\frac{a^2x^4}{4}+\frac{4ab\sqrt{c+dx}\left(-6c^2d^2x^2+8c^3dx-16c^4+5cd^3x^3+35d^4x^4\right)}{315d^4}+\frac{1}{20}b^2x^4(5c+4dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*(5*c + 4*d*x))/20 + (4*a*b*Sqrt[c + d*x]*(-16*c^4 +
8*c^3*d*x - 6*c^2*d^2*x^2 + 5*c*d^3*x^3 + 35*d^4*x^4))/(315*d^4)
```

Maple [A] time = 0.003, size = 78, normalized size = 0.4

$$b^2\left(\frac{dx^5}{5}+\frac{cx^4}{4}\right)+4\,\frac{ab\left(1/9\,(dx+c)^{9/2}-3/7\,c\,(dx+c)^{7/2}+3/5\,c^2\,(dx+c)^{5/2}-1/3\,c^3\,(dx+c)^{3/2}\right)}{d^4}+\frac{a^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/5*d*x^5+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)+
3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4
```

Maxima [A] time = 1.10767, size = 204, normalized size = 1.1

$$\frac{252\,(dx+c)^5b^2+560\,(dx+c)^{\frac{9}{2}}ab-2160\,(dx+c)^{\frac{7}{2}}abc+3024\,(dx+c)^{\frac{5}{2}}abc^2-1680\,(dx+c)^{\frac{3}{2}}abc^3-1260\,(dx+c)a^2c^3-}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/2
)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 126
0*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 - a^2
*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4
```

Fricas [A] time = 1.98351, size = 217, normalized size = 1.17

$$\frac{252\, b^2 d^5 x^5 + 315\left(b^2 c + a^2\right) d^4 x^4 + 16\left(35\, abd^4 x^4 + 5\, abcd^3 x^3 - 6\, abc^2 d^2 x^2 + 8\, abc^3 dx - 16\, abc^4\right)\sqrt{dx + c}}{1260\, d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 +
5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*x
+ c))/d^4
```

Sympy [A] time = 5.39549, size = 139, normalized size = 0.75

$$\begin{cases} \frac{\frac{a^2 dx^4}{4} + \frac{4ab\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3} + \frac{3c^2(c+dx)^{\frac{5}{2}}}{5} - \frac{3c(c+dx)^{\frac{7}{2}}}{7} + \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3} + \frac{2b^2\left(-\frac{c^3(c+dx)^2}{4} + \frac{c^2(c+dx)^3}{2} - \frac{3c(c+dx)^4}{8} + \frac{(c+dx)^5}{10}\right)}{d^3}}{d} & \text{for}\ d \neq 0 \\ \frac{x^4\left(a+b\sqrt{c}\right)^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**4/4 + 4*a*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d
*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**2*(
-c**3*(c + d*x)**2/4 + c**2*(c + d*x)**3/2 - 3*c*(c + d*x)**4/8 + (c + d*x)
**5/10)/d**3)/d, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))
```

Giac [A] time = 1.24226, size = 173, normalized size = 0.94

$$\frac{315\left(dx^4 - \frac{c^4}{d^3}\right)a^2 + \frac{16\left(35\,(dx+c)^{\frac{9}{2}} - 135\,(dx+c)^{\frac{7}{2}}c + 189\,(dx+c)^{\frac{5}{2}}c^2 - 105\,(dx+c)^{\frac{3}{2}}c^3\right)ab}{d^3} + \frac{63\left(4\,(dx+c)^5 - 15\,(dx+c)^4 c + 20\,(dx+c)^3 c^2 - 10\,(dx+c)^2 c^3\right)b^2}{d^3}}{1260\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(315*(d*x^4 - c^4/d^3)*a^2 + 16*(35*(d*x + c)^(9/2) - 135*(d*x + c)^
(7/2)*c + 189*(d*x + c)^(5/2)*c^2 - 105*(d*x + c)^(3/2)*c^3)*a*b/d^3 + 63*(
4*(d*x + c)^5 - 15*(d*x + c)^4*c + 20*(d*x + c)^3*c^2 - 10*(d*x + c)^2*c^3)
*b^2/d^3)/d
```

3.619 $\int x^2 \left(a+b\sqrt{c+dx}\right)^2 dx$

Optimal. Leaf size=138

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+d}{4d^3}$$

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*
(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)
*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)
/(4*d^3)
```

Rubi [A] time = 0.16633, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+d}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*
(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)
*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)
/(4*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^2\,dx,x,c+dx\right)}{d^3}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}$$
$$=\frac{2\,\text{Subst}\left(\int\left(a^2c^2x+2abc^2x^2+c\left(-2a^2+b^2c\right)x^3-4abcx^4+\left(a^2-2b^2c\right)x^5+2abx^6+b^2x\right.\right.}{d^3}$$
$$=\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{\left(a^2-2b^2c\right)(c+d}{3d^3}$$

Mathematica [A] time = 0.189345, size = 77, normalized size = 0.56

$$\frac{a^2x^3}{3}+\frac{4ab\sqrt{c+dx}\left(-4c^2dx+8c^3+3cd^2x^2+15d^3x^3\right)}{105d^3}+\frac{1}{12}b^2x^3(4c+3dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*(4*c + 3*d*x))/12 + (4*a*b*Sqrt[c + d*x]*(8*c^3 - 4*
c^2*d*x + 3*c*d^2*x^2 + 15*d^3*x^3))/(105*d^3)
```

Maple [A] time = 0.003, size = 66, normalized size = 0.5

$$b^2\left(\frac{dx^4}{4}+\frac{cx^3}{3}\right)+4\,\frac{ab\left(1/7\,(dx+c)^{7/2}-2/5\,c\,(dx+c)^{5/2}+1/3\,c^2\,(dx+c)^{3/2}\right)}{d^3}+\frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/4*d*x^4+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*c*(d*x+c)^(5/2)+
1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

Maxima [A] time = 1.14879, size = 151, normalized size = 1.09

$$\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\left(2\,b^2c-a^2\right)(dx+}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)*
a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*c
- a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

Fricas [A] time = 1.92185, size = 188, normalized size = 1.36

$$\frac{105\,b^2d^4x^4+140\left(b^2c+a^2\right)d^3x^3+16\left(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3\right)\sqrt{dx+c}}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 + 3
*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

Sympy [A] time = 4.49434, size = 110, normalized size = 0.8

$$\begin{cases}\frac{\frac{a^2dx^3}{3}+\frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{2b^2\left(\frac{c^2(c+dx)^2}{4}-\frac{c(c+dx)^3}{3}+\frac{(c+dx)^4}{8}\right)}{d^2}}{d} & \text{for } d\neq 0\\ \frac{x^3\left(a+b\sqrt{c}\right)^2}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**3/3 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)*
*(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*(c**2*(c + d*x)**2/4 - c*(c +
d*x)**3/3 + (c + d*x)**4/8)/d**2)/d, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3,
 True))
```

Giac [A] time = 1.28828, size = 139, normalized size = 1.01

$$\frac{140\left(dx^3+\frac{c^3}{d^2}\right)a^2+\frac{16\left(15\,(dx+c)^{\frac{7}{2}}-42\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2\right)ab}{d^2}+\frac{35\left(3\,(dx+c)^4-8\,(dx+c)^3c+6\,(dx+c)^2c^2\right)b^2}{d^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/420*(140*(d*x^3 + c^3/d^2)*a^2 + 16*(15*(d*x + c)^(7/2) - 42*(d*x + c)^(5
/2)*c + 35*(d*x + c)^(3/2)*c^2)*a*b/d^2 + 35*(3*(d*x + c)^4 - 8*(d*x + c)^3
*c + 6*(d*x + c)^2*c^2)*b^2/d^2)/d
```

3.620 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=89

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*
x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)
```

Rubi [A] time = 0.090381, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*
x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\sqrt{c+dx}\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)\,dx,x,c+dx\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int\left(-a^2cx-2abcx^2+\left(a^2-b^2c\right)x^3+2abx^4+b^2x^5\right)dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= -\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}
\end{aligned}$$

Mathematica [A] time = 0.102714, size = 63, normalized size = 0.71

$$\frac{1}{30}\left(15a^2x^2+\frac{8ab\sqrt{c+dx}\left(-2c^2+cdx+3d^2x^2\right)}{d^2}+5b^2x^2(3c+2dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (15*a^2*x^2 + 5*b^2*x^2*(3*c + 2*d*x) + (8*a*b*Sqrt[c + d*x]*(-2*c^2 + c*d*
x + 3*d^2*x^2))/d^2)/30
```

Maple [A] time = 0.002, size = 54, normalized size = 0.6

$$b^2\left(\frac{dx^3}{3}+\frac{cx^2}{2}\right)+4\,\frac{ab\left(1/5\,(dx+c)^{5/2}-1/3\,c\,(dx+c)^{3/2}\right)}{d^2}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*c*(d*x+c)^(3/2))
+1/2*a^2*x^2
```

Maxima [A] time = 1.10944, size = 97, normalized size = 1.09

$$\frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\left(b^2c-a^2\right)(dx+c)^2}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b*
c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

Fricas [A] time = 1.91882, size = 151, normalized size = 1.7

$$\frac{10\,b^2d^3x^3+15\left(b^2c+a^2\right)d^2x^2+8\left(3\,abd^2x^2+abcdx-2\,abc^2\right)\sqrt{dx+c}}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c*
d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

Sympy [A] time = 3.60007, size = 94, normalized size = 1.06

$$\begin{cases} \frac{\frac{2a^2\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right)}{d}+\frac{4ab\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{2b^2\left(-\frac{c(c+dx)^2}{4}+\frac{(c+dx)^3}{6}\right)}{d}}{d} & \text{for } d \neq 0 \\ \frac{x^2\left(a+b\sqrt{c}\right)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((2*a**2*(-c*(c + d*x)/2 + (c + d*x)**2/4)/d + 4*a*b*(-c*(c + d*x
)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*b**2*(-c*(c + d*x)**2/4 + (c + d*x)*
*3/6)/d)/d, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

Giac [A] time = 1.21314, size = 115, normalized size = 1.29

$$\frac{\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d}+\frac{8\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)ab}{d}+\frac{5\left(2\,(dx+c)^3-3\,(dx+c)^2c\right)b^2}{d}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*((d*x + c)^2 - 2*(d*x + c)*c)*a^2/d + 8*(3*(d*x + c)^(5/2) - 5*(d*
x + c)^(3/2)*c)*a*b/d + 5*(2*(d*x + c)^3 - 3*(d*x + c)^2*c)*b^2/d)/d
```

3.621 $\int \left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=41

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rubi [A] time = 0.0303472, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {247, 190, 43}

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sqrt{c+dx}\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2\,dx,x,c+dx\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int x(a+bx)^2\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right)\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0270579, size = 40, normalized size = 0.98

$$\frac{6a^2dx+8ab(c+dx)^{3/2}+3b^2(c+dx)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (6*a^2*d*x + 8*a*b*(c + d*x)^(3/2) + 3*b^2*(c + d*x)^2)/(6*d)
```

Maple [A] time = 0.002, size = 35, normalized size = 0.9

$$b^2\left(\frac{dx^2}{2}+cx\right)+\frac{4\,ab}{3\,d}\,(dx+c)^{\frac{3}{2}}+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/2*d*x^2+c*x)+4/3*a*b*(d*x+c)^(3/2)/d+x*a^2
```

Maxima [A] time = 1.1046, size = 47, normalized size = 1.15

$$\frac{1}{2}\left(dx^2+2\,cx\right)b^2+a^2x+\frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

Fricas [A] time = 1.93548, size = 109, normalized size = 2.66

$$\frac{3\,b^2d^2x^2+6\left(b^2c+a^2\right)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x + c
))/d
```

Sympy [A] time = 0.190655, size = 68, normalized size = 1.66

$$\begin{cases} a^2x+\frac{4abc\sqrt{c+dx}}{3d}+\frac{4abx\sqrt{c+dx}}{3}+b^2cx+\frac{b^2dx^2}{2} & \text{for } d\neq 0 \\ x\left(a+b\sqrt{c}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3 +
 b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

Giac [A] time = 1.23808, size = 53, normalized size = 1.29

$$\frac{3\,(dx+c)^{2}b^{2}+8\,(dx+c)^{\frac{3}{2}}ab+6\,(dx+c)a^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)^2*b^2 + 8*(d*x + c)^(3/2)*a*b + 6*(d*x + c)*a^2)/d
```

3.622 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

Optimal. Leaf size=57

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]
] + (a^2 + b^2*c)*Log[x]
```

Rubi [A] time = 0.0655491, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 207, 260}

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]
] + (a^2 + b^2*c)*Log[x]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{-c+x}\,dx, x, c+dx\right)\\ &= 2\,\text{Subst}\left(\int \frac{x(a+bx)^2}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\ &= 2\,\text{Subst}\left(\int \left(2ab+b^2x+\frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\right)dx, x, \sqrt{c+dx}\right)\\ &= b^2dx+4ab\sqrt{c+dx}+2\,\text{Subst}\left(\int \frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\ &= b^2dx+4ab\sqrt{c+dx}+(4abc)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)+\left(2\left(a^2+b^2c\right)\right)\text{Subst}\left(\int \frac{x}{-c+x^2}\right.\\ &= b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.166515, size = 79, normalized size = 1.39

$$b\left(4a\sqrt{c+dx}+bdx\right)+\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)+\left(a+b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b*(b*d*x + 4*a*Sqrt[c + d*x]) + (a + b*Sqrt[c])^2*Log[Sqrt[c] - Sqrt[c + d*
x]] + (a - b*Sqrt[c])^2*Log[Sqrt[c] + Sqrt[c + d*x]]
```

Maple [A] time = 0.006, size = 51, normalized size = 0.9

$$\ln(x)\,b^2c+b^2dx-4\,ab\,\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}+4\,ab\sqrt{dx+c}+\ln(x)\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x,x)
```

```
[Out] ln(x)*b^2*c+b^2*d*x-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*b*(d*x
+c)^(1/2)+ln(x)*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98312, size = 301, normalized size = 5.28

$$\left[b^2dx + 2\,ab\sqrt{c}\log\left(\frac{dx - 2\,\sqrt{dx+c}\sqrt{c} + 2\,c}{x}\right) + 4\,\sqrt{dx+c}ab + \left(b^2c + a^2\right)\log\left(x\right), b^2dx + 4\,ab\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{\cdot}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*s
qrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(sq
rt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

Sympy [A] time = 18.6249, size = 65, normalized size = 1.14

$$a^2\log\left(x\right) - 2ab\left(-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - 2\sqrt{c+dx}\right) + b^2c\log\left(x\right) + b^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] a**2*log(x) - 2*a*b*(-2*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*sqrt(c
+ d*x)) + b**2*c*log(x) + b**2*d*x
```

Giac [A] time = 1.18772, size = 105, normalized size = 1.84

$$-b^2c\log\left(-c\right) + \frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx+c)b^2 - a^2\log\left(-c\right) + 4\,\sqrt{dx+c}ab + \left(b^2c + a^2\right)\log\left(dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] -b^2*c*log(-c) + 4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c
)*b^2 - a^2*log(-c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

3.623 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2} dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqr
t[c] + b^2*d*Log[x]
```

Rubi [A] time = 0.0659866, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 819, 635, 207, 260}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqr
t[c] + b^2*d*Log[x]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx &= d\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\,\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{d\,\text{Subst}\left(\int \frac{-2abc-2b^2cx}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{c}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} + (2abd)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right) + \left(2b^2d\right)\text{Subst}\left(\int \frac{x}{-c+x^2}\right.\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d\log(x)
\end{aligned}$$

Mathematica [B] time = 0.197767, size = 161, normalized size = 2.98

$$\frac{\sqrt{c}\left(2a^3b\sqrt{c+dx}+a^4-2ab^3c\sqrt{c+dx}-b^4c(c+2dx)\right)+bdx\left(a+b\sqrt{c}\right)\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)+bdx\left(b\sqrt{c}-\right.}{\sqrt{c}x\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] (Sqrt[c]*(a^4 + 2*a^3*b*Sqrt[c + d*x] - 2*a*b^3*c*Sqrt[c + d*x] - b^4*c*(c
+ 2*d*x)) + b*(-a + b*Sqrt[c])*(a + b*Sqrt[c])^2*d*x*Log[Sqrt[c] - Sqrt[c +
 d*x]] + b*(a - b*Sqrt[c])^2*(a + b*Sqrt[c])*d*x*Log[Sqrt[c] + Sqrt[c + d*x
]])/(Sqrt[c]*(-a^2 + b^2*c)*x)
```

Maple [A] time = 0.01, size = 60, normalized size = 1.1

$$b^2d\ln(x) - \frac{b^2c}{x} - 2\frac{ab\sqrt{dx+c}}{x} - 2\frac{abd}{\sqrt{c}}\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^2,x)
```

```
[Out] b^2*d*ln(x)-b^2*c/x-2*a*b/x*(d*x+c)^(1/2)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^(
1/2))/c^(1/2)-a^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86006, size = 347, normalized size = 6.43

$$\left[\frac{b^2cdx\log(x) + ab\sqrt{c}dx\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right) - b^2c^2 - 2\sqrt{dx+c}abc - a^2c}{cx}, \frac{b^2cdx\log(x) + 2ab\sqrt{-c}dx\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)}{cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] [(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2
*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x)
+ 2*a*b*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) - b^2*c^2 - 2*sqrt(d*
x + c)*a*b*c - a^2*c)/(c*x)]
```

Sympy [B] time = 41.0323, size = 139, normalized size = 2.57

$$-\frac{a^2}{x} - abcd\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + abcd\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + \frac{4abd\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2ab\sqrt{c+dx}}{x} - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - a*b*c*d*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c + d*x)) +
a*b*c*d*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) + 4*a*b*d*ata
n(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*a*b*sqrt(c + d*x)/x - b**2*c/x + b**
2*d*log(x)
```

Giac [B] time = 1.26706, size = 153, normalized size = 2.83

$$\frac{b^2d^2\log(dx) + \frac{2abd^2\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{b^2cd^2\log(-c)+b^2cd^2+a^2d^2}{c} - \frac{b^2cd^2+2\sqrt{dx+c}abd^2+a^2d^2}{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*log(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b^
2*c*d^2*log(-c) + b^2*c*d^2 + a^2*d^2)/c - (b^2*c*d^2 + 2*sqrt(d*x + c)*a*b
*d^2 + a^2*d^2)/(d*x))/d
```

3.624 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

```
[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2) +
(a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rubi [A] time = 0.073545, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 821, 12, 639, 207}

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2) +
(a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx &= d^2\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{d^2\,\text{Subst}\left(\int -\frac{2bc(a+bx)}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \left(bd^2\right)\text{Subst}\left(\int \frac{a+bx}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{\left(abd^2\right)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [B] time = 0.356943, size = 221, normalized size = 2.76

$$\frac{-\frac{2\sqrt{c}\left(a^4b^2\left(-c^2+2cdx+3d^2x^2\right)-a^2b^4c\left(c^2+4cdx+2d^2x^2\right)-2a^3b^3c\sqrt{c+dx}(2c+dx)+a^5b\sqrt{c+dx}(2c+dx)+a^6c+ab^5c^2\sqrt{c+dx}(2c+dx)+b^6c^2(c+dx)^2\right)}{x^2\left(a^2-b^2c\right)^2} - abd^2\log\left(\right.}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] ((-2*Sqrt[c]*(a^6*c + b^6*c^2*(c + d*x)^2 + a^5*b*Sqrt[c + d*x]*(2*c + d*x)
 - 2*a^3*b^3*c*Sqrt[c + d*x]*(2*c + d*x) + a*b^5*c^2*Sqrt[c + d*x]*(2*c + d
*x) - a^2*b^4*c*(c^2 + 4*c*d*x + 2*d^2*x^2) + a^4*b^2*(-c^2 + 2*c*d*x + 3*d
^2*x^2)))/((a^2 - b^2*c)^2*x^2) - a*b*d^2*Log[Sqrt[c] - Sqrt[c + d*x]] + a*
b*d^2*Log[Sqrt[c] + Sqrt[c + d*x]])/(4*c^(3/2))
```

Maple [A] time = 0.012, size = 81, normalized size = 1.

$$b^2\left(-\frac{d}{x}-\frac{c}{2\,x^2}\right)+4\,abd^2\left(\frac{1}{d^2x^2}\left(-1/8\,\frac{(dx+c)^{3/2}}{c}-1/8\,\sqrt{dx+c}\right)+1/8\,\frac{1}{c^{3/2}}\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^3,x)
```

```
[Out] b^2*(-d/x-1/2*c/x^2)+4*a*b*d^2*((-1/8/c*(d*x+c)^(3/2)-1/8*(d*x+c)^(1/2))/d^
2/x^2+1/8/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))-1/2*a^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72163, size = 427, normalized size = 5.34

$$\left[\frac{ab\sqrt{c}d^2x^2\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-4b^2c^2dx-2b^2c^3-2a^2c^2-2\left(abcdx+2abc^2\right)\sqrt{dx+c}}{4c^2x^2}, -\frac{ab\sqrt{-c}d^2x^2\arctan\left(\frac{\sqrt{dx+c}\sqrt{-}}{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4*
b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x +
c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c)
+ 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c)
)/(c^2*x^2)]
```

Sympy [B] time = 119.286, size = 292, normalized size = 3.65

$$-\frac{a^2}{2x^2}-\frac{20abc^2d^2\sqrt{c+dx}}{-8c^4-16c^3dx+8c^2(c+dx)^2}+\frac{12abcd^2(c+dx)^{\frac{3}{2}}}{-8c^4-16c^3dx+8c^2(c+dx)^2}+\frac{3abcd^2\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{c+dx}\right)}{4}-\frac{3abcd^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) - 20*a*b*c**2*d**2*sqrt(c + d*x)/(-8*c**4 - 16*c**3*d*x + 8*
c**2*(c + d*x)**2) + 12*a*b*c*d**2*(c + d*x)**(3/2)/(-8*c**4 - 16*c**3*d*x
+ 8*c**2*(c + d*x)**2) + 3*a*b*c*d**2*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5))
 + sqrt(c + d*x))/4 - 3*a*b*c*d**2*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + s
qrt(c + d*x))/4 - a*b*d**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c +
 d*x)) + a*b*d**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) - 2
*a*b*d*sqrt(c + d*x)/(c*x) - b**2*c/(2*x**2) - b**2*d/x
```

Giac [A] time = 1.16749, size = 170, normalized size = 2.12

$$-\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{b^2cd^3-a^2d^3}{c^2}+\frac{2(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}abcd^3+a^2cd^3}{cd^2x^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (b^2*c*d^3 - a^
2*d^3)/c^2 + (2*(d*x + c)*b^2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3
 + sqrt(d*x + c)*a*b*c*d^3 + a^2*c*d^3)/(c*d^2*x^2))/d
```

3.625 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=326

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}$$

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) + (4*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) - (12*a*(
7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) + (
4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d
^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) +
(12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a +
 b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/2))/(
17*b^8*d^4)
```

Rubi [A] time = 0.242677, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) + (4*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) - (12*a*(
7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) + (
4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d
^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) +
(12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a +
 b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/2))/(
17*b^8*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}(-c+x)^3\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\left(-c+x^2\right)^3\,dx, x, \sqrt{c+dx}\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^3\sqrt{a+bx}}{b^7} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{3/2}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^{5/2}}{b^7} + \frac{(35a}{}\right.\right.}{}$$
$$= -\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} - \frac{12a\left(7a^2\right.}{}$$

Mathematica [A] time = 0.420491, size = 232, normalized size = 0.71

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-48a^3b^4\left(616c^2-1080cdx+735d^2x^2\right)+24a^2b^5\sqrt{c+dx}\left(2960c^2-2716cdx+1617d^2x^2\right)+3840\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] - 48
*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]*(2
960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 57
54*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x
 + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

Maple [A] time = 0.004, size = 383, normalized size = 1.2

$$4\,\frac{1}{d^4b^8}\left(1/17\left(a+b\sqrt{dx+c}\right)^{17/2} - \frac{7\,a\left(a+b\sqrt{dx+c}\right)^{15/2}}{15} + 1/13\left(-3\,b^2c+21\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{13/2} + 1/11\left(-8\left(-b^2c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/17*(a+b*(d*x+c)^(1/2))^(17/2)-7/15*a*(a+b*(d*x+c)^(1/2))^(15/2
)+1/13*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(-8*(-b^2*c+a^2)*a
-2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(
(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c
+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-6*(-b^2*c+
a^2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*
a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b
*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Maxima [A] time = 1.17555, size = 362, normalized size = 1.11

$$\frac{4\left(45045\left(\sqrt{dx+c}b+a\right)^{\frac{17}{2}} - 357357\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}a - 176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}} + 348075\left(3ab^2c-7\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a)
^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*(3
*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a^2*
b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*
b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^
2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 -
 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*d^4)
```

Fricas [A] time = 2.37572, size = 697, normalized size = 2.14

$$4\left(45045\, b^{8} d^{4} x^{4} - 29568\, b^{8} c^{4} + 72960\, a^{2} b^{6} c^{3} - 96128\, a^{4} b^{4} c^{2} + 59904\, a^{6} b^{2} c - 14336\, a^{8} + 231\left(15\, b^{8} c - 14\, a^{2} b^{6}\right) d^{3} x^{3} - 28 \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^4
*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*x^
3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8*c^
3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^
3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 35
28*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*
a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3} \sqrt{a + b \sqrt{c + d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.34757, size = 1548, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/765765*(153153*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*b
^8*c^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 255255*sqrt((sqrt(d*x + c)*b +
a)*b^2)*(sqrt(d*x + c)*b + a)*a*b^8*c^3*sgn((sqrt(d*x + c)*b + a)*b - a*b)
```

- 255255*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^4*b^6*c^2*sg
n((sqrt(d*x + c)*b + a)*b - a*b) + 984555*sqrt((sqrt(d*x + c)*b + a)*b^2)*(
sqrt(d*x + c)*b + a)^3*a*b^6*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 13783
77*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^2*b^6*c^2*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) + 765765*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sq
rt(d*x + c)*b + a)*a^3*b^6*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 176715*
sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^6*b^4*c*sgn((sqrt(d*x
 + c)*b + a)*b - a*b) - 1044225*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x +
 c)*b + a)^5*a*b^4*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 2552550*sqrt((sqr
t(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^4*a^2*b^4*c*sgn((sqrt(d*x + c)
*b + a)*b - a*b) - 3281850*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b
 + a)^3*a^3*b^4*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 2297295*sqrt((sqrt(d
*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^4*b^4*c*sgn((sqrt(d*x + c)*b
+ a)*b - a*b) - 765765*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a
)*a^5*b^4*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 45045*sqrt((sqrt(d*x + c)*
b + a)*b^2)*(sqrt(d*x + c)*b + a)^8*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b)
+ 357357*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^7*a*b^2*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) - 1237005*sqrt((sqrt(d*x + c)*b + a)*b^2)*(s
qrt(d*x + c)*b + a)^6*a^2*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 2436525*
sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^5*a^3*b^2*sgn((sqrt(d
*x + c)*b + a)*b - a*b) - 2977975*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x
 + c)*b + a)^4*a^4*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 2297295*sqrt((s
qrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a^5*b^2*sgn((sqrt(d*x + c)
*b + a)*b - a*b) - 1072071*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b
 + a)^2*a^6*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 255255*sqrt((sqrt(d*x
+ c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^7*b^2*sgn((sqrt(d*x + c)*b + a)*b
- a*b))*abs(b)/(b^12*d^4)

3.626 $\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=224

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a(\cdots}{\cdots}$$

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4
- 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5
*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(
11/2))/(11*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rubi [A] time = 0.156336, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```


```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4
- 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5
*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(
11/2))/(11*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}(-c+x)^2\,dx, x, c+dx\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\left(-c+x^2\right)^2\,dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a(a^2-b^2c)^2\sqrt{a+bx}}{b^5}+\frac{(5a^4-6a^2b^2c+b^4c^2)(a+bx)^{3/2}}{b^5}-\frac{2(5a^3-3ab^2c)(a+bx)^{5/2}}{b^5}-\frac{2(-5a^2+b^2c)(a+bx}{b^5}\right.\right.}{d^3}$$
$$= -\frac{4a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{8a\left(5a^2-\right.}{}$$

Mathematica [A] time = 0.198368, size = 147, normalized size = 0.66

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(32a^3b^2(68c-75dx)+16a^2b^3\sqrt{c+dx}(175dx-254c)+1920a^4b\sqrt{c+dx}-1280a^5-6ab^4\left(96c^2-\right.\right.}{45045b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 19
20*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77*b
^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 380*c
*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

Maple [A] time = 0.003, size = 183, normalized size = 0.8

$$4\,\frac{1}{d^3b^6}\left(1/13\,\left(a+b\sqrt{dx+c}\right)^{13/2}-\frac{5\,a\left(a+b\sqrt{dx+c}\right)^{11/2}}{11}+1/9\,\left(-2\,b^2c+10\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{9/2}+1/7\,\left(-4\,\left(-b^2c+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/13*(a+b*(d*x+c)^(1/2))^(13/2)-5/11*a*(a+b*(d*x+c)^(1/2))^(11/2
)+1/9*(-2*b^2*c+10*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-4*(-b^2*c+a^2)*a-a*
(-2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^2+4*a^2*(-b^2
*c+a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))
^(3/2))
```

Maxima [A] time = 1.09632, size = 225, normalized size = 1.

$$\frac{4\left(3465\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}a-10010\left(b^2c-5\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}+12870\left(3\,ab^2c-5\,a^3\right)\left(\sqrt{\right.\right.}{45045\,b^6\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(1
1/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b^2
*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

Fricas [A] time = 2.34869, size = 441, normalized size = 1.97

$$\frac{4\left(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\left(11\,b^6c-10\,a^2b^4\right)d^2x^2-8\left(77\,b^6c^2-127\,a^2b^4c+\right.\right.}{45045\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2*
c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a^2
*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^3*b
^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(
d*x + c)*b + a)/(b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.21627, size = 923, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/45045*(9009*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*b^6*c
^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 15015*sqrt((sqrt(d*x + c)*b + a)*b^
2)*(sqrt(d*x + c)*b + a)*a*b^6*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 100
10*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^4*b^4*c*sgn((sqrt(
d*x + c)*b + a)*b - a*b) + 38610*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x
+ c)*b + a)^3*a*b^4*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 54054*sqrt((sqrt
(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^2*b^4*c*sgn((sqrt(d*x + c)*
b + a)*b - a*b) + 30030*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b +
a)*a^3*b^4*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 3465*sqrt((sqrt(d*x + c)*
b + a)*b^2)*(sqrt(d*x + c)*b + a)^6*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b)
- 20475*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^5*a*b^2*sgn((
sqrt(d*x + c)*b + a)*b - a*b) + 50050*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt
```

```
(d*x + c)*b + a)^4*a^2*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 64350*sqrt(
(sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a^3*b^2*sgn((sqrt(d*x +
c)*b + a)*b - a*b) + 45045*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b
 + a)^2*a^4*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 15015*sqrt((sqrt(d*x +
 c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^5*b^2*sgn((sqrt(d*x + c)*b + a)*b -
 a*b))*abs(b)/(b^10*d^3)
```

3.627 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=133

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x
])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)
```

Rubi [A] time = 0.0958616, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x
])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}(-c+x)\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{\left(-a^3+ab^2c\right)\sqrt{a+bx}}{b^3}+\frac{\left(3a^2-b^2c\right)(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

Mathematica [A] time = 0.0871912, size = 84, normalized size = 0.63

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(24a^2b\sqrt{c+dx}-16a^3+6ab^2(2c-5dx)+7b^3\sqrt{c+dx}(5dx-4c)\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)
```

Maple [A] time = 0.002, size = 94, normalized size = 0.7

$$4\,\frac{1/9\,\left(a+b\sqrt{dx+c}\right)^{9/2}-3/7\,\left(a+b\sqrt{dx+c}\right)^{7/2}a+1/5\,\left(-b^2c+3\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{5/2}-1/3\,\left(-b^2c+a^2\right)a\left(a+b\sqrt{dx+c}\right)^{3/2}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/9*(a+b*(d*x+c)^(1/2))^(9/2)-3/7*(a+b*(d*x+c)^(1/2))^(7/2)*a+1/
5*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)*a*(a+b*(d*x+c)^
(1/2))^(3/2))
```

Maxima [A] time = 1.10106, size = 126, normalized size = 0.95

$$\frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}a-63\left(b^2c-3\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}+105\left(ab^2c-a^3\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}\right)}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a -
 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sqrt
(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

Fricas [A] time = 2.39281, size = 240, normalized size = 1.8

$$\frac{4\left(35\,b^4d^2x^2-28\,b^4c^2+36\,a^2b^2c-16\,a^4+\left(7\,b^4c-6\,a^2b^2\right)dx+\left(5\,ab^3dx-16\,ab^3c+8\,a^3b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b+a}}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*a
^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.16013, size = 460, normalized size = 3.46

$$-\frac{4\left(63\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)^2b^4c\,\mathrm{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-105\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)ab^4c\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/315*(63*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*b^4*c*sg
n((sqrt(d*x + c)*b + a)*b - a*b) - 105*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqr
t(d*x + c)*b + a)*a*b^4*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 35*sqrt((sqr
t(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^4*b^2*sgn((sqrt(d*x + c)*b + a
)*b - a*b) + 135*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a*
b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 189*sqrt((sqrt(d*x + c)*b + a)*b^2
)*(sqrt(d*x + c)*b + a)^2*a^2*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 105*
sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^3*b^2*sgn((sqrt(d*x
 + c)*b + a)*b - a*b))*abs(b)/(b^8*d^2)
```

3.628 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/
2))/(5*b^2*d)
```

Rubi [A] time = 0.0327106, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/
2))/(5*b^2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,dx,x,\sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,\sqrt{c+dx}\right)}{d}$$
$$= -\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

Mathematica [A] time = 0.0241736, size = 43, normalized size = 0.77

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(3b\sqrt{c+dx}-2a\right)}{15b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-2*a + 3*b*Sqrt[c + d*x]))/(15*b^2*d)
```

Maple [A] time = 0.004, size = 41, normalized size = 0.7

$$4\,\frac{1/5\,\left(a+b\sqrt{dx+c}\right)^{5/2}-1/3\,a\left(a+b\sqrt{dx+c}\right)^{3/2}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d/b^2*(1/5*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Maxima [A] time = 1.10962, size = 58, normalized size = 1.04

$$\frac{4\left(\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/b
^2)/d
```

Fricas [A] time = 2.38948, size = 122, normalized size = 2.18

$$\frac{4\left(3\,b^2dx+3\,b^2c+\sqrt{dx+c}ab-2\,a^2\right)\sqrt{\sqrt{dx+c}b+a}}{15\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*b
 + a)/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.11971, size = 159, normalized size = 2.84

$$\frac{4\left(3\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)^2b^2\mathrm{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-5\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)ab^2\mathrm{sgn}\left(\right.\right.}{15\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(3*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*b^2*sgn((sq
rt(d*x + c)*b + a)*b - a*b) - 5*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x +
 c)*b + a)*a*b^2*sgn((sqrt(d*x + c)*b + a)*b - a*b))*abs(b)/(b^6*d)
```

3.629 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

Optimal. Leaf size=116

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rubi [A] time = 0.155936, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 825, 827, 1166, 207}

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
```

```
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{-c+x}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + 2\,\text{Subst}\left(\int \frac{bc+ax}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + 4\,\text{Subst}\left(\int \frac{-a^2+b^2c+ax^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + \left(2\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right) + \left(2\left(a+\right.\right.\\
&= 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+}}}{\sqrt{a+b\sqrt{c}}}\right.
\end{aligned}$$

Mathematica [A] time = 0.135855, size = 116, normalized size = 1.

$$4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Maple [B] time = 0.035, size = 221, normalized size = 1.9

$$4\sqrt{a+b\sqrt{dx+c}} + 2\frac{b^2c}{\sqrt{b^2c}\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right) + 2\frac{a}{\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right) - 2\,\frac{}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x,x)
```

```
[Out] 4*(a+b*(d*x+c)^(1/2))^(1/2)+2/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan
((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*b^2*c+2/(-(b^2*c)^(1/2
)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*a-2/(
b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2
*c)^(1/2)-a)^(1/2))*b^2*c+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/
2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{dx+c}b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

Fricas [B] time = 2.51916, size = 489, normalized size = 4.22

$$-\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a+\sqrt{b^2c}}\right)+\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a+\sqrt{b^2c}}\right)-\sqrt{a-\sqrt{b^2c}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b^
2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a +
 sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*
sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b +
 a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.630 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqr
t[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a +
b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])
```

Rubi [A] time = 0.168879, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 1398, 821, 12, 708, 1093, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqr
t[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a +
b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx &= d\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} - \frac{d\,\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{1}{2}(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \left(b^2d\right)\text{Subst}\left(\int \frac{1}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\sqrt{c}} - \frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+\cdots}\right.}{2\cdots}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\sqrt{c}} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.189986, size = 181, normalized size = 1.32

$$
\frac{\left(a-b\sqrt{c}\right)\left(2\sqrt{c}\left(a+b\sqrt{c}\right)\sqrt{a+b\sqrt{c+dx}}+bdx\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)\right)-bdx\sqrt{a-b\sqrt{c}}\left(a+b\sqrt{c}\right)\tanh^{-1}\left(\cdots\right.}{2\sqrt{c}x\left(b^2c-a^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (-(b*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*
x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c])*(2*(a + b*Sqrt[c])*Sqrt[c]*Sqrt
[a + b*Sqrt[c + d*x]] + b*Sqrt[a + b*Sqrt[c]]*d*x*ArcTanh[Sqrt[a + b*Sqrt[c
 + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[c]*(-a^2 + b^2*c)*x)
```

Maple [A] time = 0.017, size = 151, normalized size = 1.1

$$-\frac{b^2 d}{b^2 (dx + c) - b^2 c}\sqrt{a + b\sqrt{dx + c}} + \frac{b^2 d}{2}\arctan\left(\sqrt{a + b\sqrt{dx + c}}\frac{1}{\sqrt{-\sqrt{b^2 c} - a}}\right)\frac{1}{\sqrt{b^2 c}}\frac{1}{\sqrt{-\sqrt{b^2 c} - a}} - \frac{b^2 d}{2}\arctan\left(\sqrt{a + b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -b^2*d*(a+b*(d*x+c)^(1/2))^(1/2)/(b^2*(d*x+c)-b^2*c)+1/2*b^2*d/(b^2*c)^(1/2
)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)
-a)^(1/2))-1/2*b^2*d/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x
+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{dx + c}b + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

Fricas [B] time = 2.57014, size = 1871, normalized size = 13.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2
- a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)
)*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^
4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*
log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3
- 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*
d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)
)) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(
```

```
b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
+ (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 -
 a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4
/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*l
og(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d
^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))
) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.631 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

Optimal. Leaf size=224

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}-\sqrt{\ }$$

```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a +
b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan
h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2)
*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rubi [A] time = 0.428875, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {371, 1398, 821, 12, 741, 827, 1166, 207}

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}-\sqrt{\ }$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```


```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a +
b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan
h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2)
*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
```

```
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = d^2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} - \frac{d^2\,\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-2a^2+3b^2c\right)-\frac{abx}{2}}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x,\right.}{8c\left(a^2-b^2c\right)}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{\frac{a^2b}{2}+\frac{1}{2}b\left(-2a^2+3b^2c\right)-\frac{1}{2}abx^2}{a^2-b^2c-2ax^2+x^4}\right.}{4c\left(a^2-b^2c\right)}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(b\left(2a-3b\sqrt{c}\right)d^2\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x}\right.}{16\left(a-b\sqrt{c}\right)c^3}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3/2}}$$

Mathematica [A] time = 0.437401, size = 258, normalized size = 1.15

$$\frac{-\left(a-b\sqrt{c}\right)\left(2\sqrt{c}\left(a+b\sqrt{c}\right)\sqrt{a+b\sqrt{c+dx}}\left(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx)\right)-bd^2x^2\sqrt{a+b\sqrt{c}}\left(2a^2+ab\sqrt{c}-3b^2c\right)\right.}{16c^{3/2}x^2\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] (-(b*(2*a - 3*b*Sqrt[c])*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])^2*d^2*x^2*ArcT
anh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) - (a - b*Sqrt[c])*(2*(a
 + b*Sqrt[c])*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c +
 d*x] - b^2*c*(4*c + d*x)) - b*Sqrt[a + b*Sqrt[c]]*(2*a^2 + a*b*Sqrt[c] - 3
*b^2*c)*d^2*x^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(1
6*c^(3/2)*(a^2 - b^2*c)^2*x^2)
```

Maple [B] time = 0.029, size = 784, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x)
```

```
[Out] -1/8*b^2*d^2/(b^2*(d*x+c)-b^2*c)^2*a/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/
2)+1/8*b^4*d^2/(b^2*(d*x+c)-b^2*c)^2/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)
+3/8*b^2*d^2/(b^2*(d*x+c)-b^2*c)^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)
*a^2-1/8*b^4*d^2/(b^2*(d*x+c)-b^2*c)^2*a/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(
3/2)-3/8*b^2*d^2/(b^2*(d*x+c)-b^2*c)^2*a^3/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2
))^(3/2)-3/8*b^4*d^2/(b^2*(d*x+c)-b^2*c)^2*(a+b*(d*x+c)^(1/2))^(1/2)+1/8*b^
2*d^2/(b^2*(d*x+c)-b^2*c)^2/c*(a+b*(d*x+c)^(1/2))^(1/2)*a^2+3/16*b^4*d^2/(-
b^2*c+a^2)/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2)
)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/(-(b^2*c)^(1/
2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*a-1/
8*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b
*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*a^2-3/16*b^4*d^2/(-b^2*c+a^
2)/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(
(b^2*c)^(1/2)-a)^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/((b^2*c)^(1/2)-a)^(1/2)
*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a+1/8*b^2*d^2/c/
(-b^2*c+a^2)/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2
))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

Fricas [B] time = 3.44089, size = 5759, normalized size = 25.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2
)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c
^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b
^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2
*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2
*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*
c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b
^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*
a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^
4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*
sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10
*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 +
a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^
2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*
```

```
c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b
^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a
^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a
^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*
a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b
^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^1
2*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 -
 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4
*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(
b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) + (b^2*c^2 - a^2*c)*x^
2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b
^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^
4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 +
15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*
a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(
sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*
d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*
c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2
*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c
^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2
*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20
*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a
^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*
a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^
4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(
b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^
5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 -
 a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)
*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*
b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81
*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 1
5*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3
)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*
a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6
+ 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 +
3*a^4*b^2*c^4 - a^6*c^3))) - 4*(b^2*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c
^2 - 4*a^2*c)*sqrt(sqrt(d*x + c)*b + a))/((b^2*c^2 - a^2*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.632 $\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=230

$$\frac{2\left(-3a^2b^2c+a^4+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}-\frac{ax\left(-3a^2b^2c+a^4+3b^4c^2\right)}{b^6d^3}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+$$

```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqr
t[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))
/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^
2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d
*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*
d^4)
```

Rubi [A] time = 0.258577, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-3a^2b^2c+a^4+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}-\frac{ax\left(-3a^2b^2c+a^4+3b^4c^2\right)}{b^6d^3}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x]),x]
```


```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqr
t[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))
/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^
2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d
*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*
d^4)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)^3}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{(-a^2+b^2c)^3}{b^7} - \frac{a(a^4-3a^2b^2c+3b^4c^2)x}{b^6} + \frac{(a^4-3a^2b^2c+3b^4c^2)x^2}{b^5} - \frac{a(a^2-3b^2c)x^3}{b^4} - \frac{(-a^2+3b^2c)x^4}{b^3} - \frac{ax^5}{b^2} \cdots\right.\right.}{d^4}$$

$$= -\frac{a\left(a^4-3a^2b^2c+3b^4c^2\right)x}{b^6d^3} + \frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4} + \frac{2\left(a^4-3a^2b^2c+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4} - \frac{a\,(\cdots}{\cdots}$$

Mathematica [A] time = 0.204044, size = 213, normalized size = 0.93

$$\frac{b\left(84a^2b^4\sqrt{c+dx}\left(11c^2-3cdx+d^2x^2\right)-140a^4b^2(8c-dx)\sqrt{c+dx}-105a^3b^3dx(dx-4c)-210a^5bdx+420a^6\sqrt{c+dx}\cdots\right.}{21\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-210*a^5*b*d*x - 105*a^3*b^3*d*x*(-4*c + d*x) + 420*a^6*Sqrt[c + d*x] -
 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] + 84*a^2*b^4*Sqrt[c + d*x]*(11*c^2 -
 3*c*d*x + d^2*x^2) - 35*a*b^5*d*x*(6*c^2 - 3*c*d*x + 2*d^2*x^2) + 12*b^6*S
qrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^2 + 5*d^3*x^3)) - 420*a*(a^2
- b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210*b^8*d^4)
```

Maple [A] time = 0.006, size = 394, normalized size = 1.7

$$-\frac{a^5c}{d^4b^6} + \frac{5\,a^3c^2}{2\,d^4b^4} - \frac{11\,ac^3}{6\,d^4b^2} - \frac{6\,c}{5\,bd^4}(dx+c)^{\frac{5}{2}} + \frac{2\,a^2}{5\,d^4b^3}(dx+c)^{\frac{5}{2}} + 2\,\frac{c^2(dx+c)^{3/2}}{bd^4} - 2\,\frac{c^3\sqrt{dx+c}}{bd^4} + \frac{2\,a^4}{3\,d^4b^5}(dx+c)^{\frac{3}{2}} + 2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -1/d^4/b^6*a^5*c+5/2/d^4/b^4*a^3*c^2-11/6/d^4/b^2*a*c^3-6/5/d^4/b*(d*x+c)^(
5/2)*c+2/5/d^4/b^3*(d*x+c)^(5/2)*a^2+2/d^4/b*(d*x+c)^(3/2)*c^2-2/d^4/b*c^3*
(d*x+c)^(1/2)+2/3/d^4/b^5*(d*x+c)^(3/2)*a^4+2/d^3/b^4*x*a^3*c-1/d^3/b^2*x*a
*c^2+1/2/d^2/b^2*x^2*a*c-1/d^3/b^6*x*a^5-1/2/d^2/b^4*x^2*a^3-6/d^4/b^5*a^4*
c*(d*x+c)^(1/2)-1/3/d/b^2*x^3*a-2/d^4/b^3*(d*x+c)^(3/2)*a^2*c+6/d^4/b^3*a^2
*c^2*(d*x+c)^(1/2)+2/7*(d*x+c)^(7/2)/b/d^4+2/d^4/b^7*a^6*(d*x+c)^(1/2)+2/d^
4*a/b^2*ln(a+b*(d*x+c)^(1/2))*c^3-6/d^4*a^3/b^4*ln(a+b*(d*x+c)^(1/2))*c^2+6
/d^4*a^5/b^6*ln(a+b*(d*x+c)^(1/2))*c-2/d^4*a^7/b^8*ln(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 1.39428, size = 328, normalized size = 1.43

$$\frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\left(3\,b^6c-a^2b^4\right)(dx+c)^{\frac{5}{2}}+105\left(3\,ab^5c-a^3b^3\right)(dx+c)^2+140\left(3\,b^6c^2-3\,a^2b^4c+a^4b^2\right)(dx+c)^{\frac{3}{2}}-210\left(3\,ab^5c^2-3\,a^3b^3c+a^5b\right)(dx+c)-42\cdots}{b^7}}{210\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*b
^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*c^
2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^3*c
 + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqr
t(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(s
qrt(d*x + c)*b + a)/b^8)/d^4
```

Fricas [A] time = 1.8614, size = 501, normalized size = 2.18

$$-\frac{70\,ab^6d^3x^3 - 105\left(ab^6c - a^3b^4\right)d^2x^2 + 210\left(ab^6c^2 - 2\,a^3b^4c + a^5b^2\right)dx - 420\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)\log\left(\sqrt{dx + \cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^2
 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*
c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231*a^
2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (
24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\sqrt{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x)), x)
```

Giac [A] time = 1.37316, size = 533, normalized size = 2.32

$$\frac{2\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)\log\left(\left|\sqrt{dx + c}b + a\right|\right)}{b^8d^4} - \frac{2\left(ab^6c^3\log\left(|a|\right) - 3\,a^3b^4c^2\log\left(|a|\right) + 3\,a^5b^2c\log\left(|a|\right) - a^7\log\left(\cdots\right.\right.}{b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b +
 a))/(b^8*d^4) - 2*(a*b^6*c^3*log(abs(a)) - 3*a^3*b^4*c^2*log(abs(a)) + 3*a
^5*b^2*c*log(abs(a)) - a^7*log(abs(a)))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/
2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*
d^24 - 420*sqrt(d*x + c)*b^6*c^3*d^24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*
```

```
x + c)^2*a*b^5*c*d^24 - 630*(d*x + c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a
^2*b^4*d^24 - 420*(d*x + c)^(3/2)*a^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b
^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 +
 140*(d*x + c)^(3/2)*a^4*b^2*d^24 - 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210
*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*x + c)*a^6*d^24)/(b^7*d^28)
```

3.633 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=151

$$\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{a(c+dx)^2}{2b^2d^3}+2($$

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5
*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(
2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b
*Sqrt[c + d*x]])/(b^6*d^3)
```

Rubi [A] time = 0.157475, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{a(c+dx)^2}{2b^2d^3}+2($$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5
*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(
2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b
*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)^2}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-a^2+b^2c)^2}{b^5} - \frac{a(a^2-2b^2c)x}{b^4} - \frac{(-a^2+2b^2c)x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^2-b^2c)^2}{b^5(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{a(a^2-2b^2c)x}{b^4d^2} + \frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3} + \frac{2(a^2-2b^2c)(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3} + \frac{2(c+dx)^{5/}}{5bd^3}$$

Mathematica [A] time = 0.134633, size = 138, normalized size = 0.91

$$\frac{b\left(-20a^2b^2(5c-dx)\sqrt{c+dx} - 30a^3bdx + 60a^4\sqrt{c+dx} - 15ab^3dx(dx-2c) + 4b^4\sqrt{c+dx}\left(8c^2-4cdx+3d^2x^2\right)\right) - 60a}{30b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-30*a^3*b*d*x - 15*a*b^3*d*x*(-2*c + d*x) + 60*a^4*Sqrt[c + d*x] - 20*a
^2*b^2*(5*c - d*x)*Sqrt[c + d*x] + 4*b^4*Sqrt[c + d*x]*(8*c^2 - 4*c*d*x + 3
*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(30*b^6*d^3)
```

Maple [A] time = 0.005, size = 235, normalized size = 1.6

$$\frac{2}{5\,bd^3}(dx+c)^{\frac{5}{2}} - \frac{ax^2}{2\,b^2d} + \frac{axc}{b^2d^2} + \frac{3\,c^2a}{2\,d^3b^2} - \frac{4\,c}{3\,bd^3}(dx+c)^{\frac{3}{2}} + \frac{2\,a^2}{3\,b^3d^3}(dx+c)^{\frac{3}{2}} + 2\,\frac{c^2\sqrt{dx+c}}{bd^3} - \frac{a^3x}{b^4d^2} - \frac{a^3c}{b^4d^3} - 4\,\frac{a^2c\sqrt{}}{b^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b/d^3-1/2/d/b^2*x^2*a+1/d^2/b^2*x*a*c+3/2/d^3/b^2*a*c^2-4
/3/d^3/b*(d*x+c)^(3/2)*c+2/3/d^3/b^3*(d*x+c)^(3/2)*a^2+2/d^3/b*c^2*(d*x+c)^
(1/2)-1/d^2/b^4*x*a^3-1/d^3/b^4*a^3*c-4/d^3/b^3*a^2*c*(d*x+c)^(1/2)+2/d^3/b
^5*a^4*(d*x+c)^(1/2)-2/d^3*a/b^2*ln(a+b*(d*x+c)^(1/2))*c^2+4/d^3*a^3/b^4*ln
(a+b*(d*x+c)^(1/2))*c-2/d^3*a^5/b^6*ln(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 1.2224, size = 200, normalized size = 1.32

$$\frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4 - 15\,(dx+c)^2ab^3 - 20\left(2\,b^4c - a^2b^2\right)(dx+c)^{\frac{3}{2}} + 30\left(2\,ab^3c - a^3b\right)(dx+c) + 60\left(b^4c^2 - 2\,a^2b^2c + a^4\right)\sqrt{dx+c}}{b^5} - \frac{60\left(ab^4c^2 - 2\,a^3b^2c + a^5\right)\log\left(\sqrt{dx+c}b + a\right)}{b^6}}{30\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b^
2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a^2
*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(s
qrt(d*x + c)*b + a)/b^6)/d^3
```

Fricas [A] time = 1.74785, size = 306, normalized size = 2.03

$$-\frac{15\,ab^4d^2x^2 - 30\left(ab^4c - a^3b^2\right)dx + 60\left(ab^4c^2 - 2\,a^3b^2c + a^5\right)\log\left(\sqrt{dx+c}b + a\right) - 4\left(3\,b^5d^2x^2 + 8\,b^5c^2 - 25\,a^2b^3c + 15\,a\right.}{30\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a^
3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 - 25
*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\sqrt{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x)), x)
```

Giac [A] time = 1.2403, size = 320, normalized size = 2.12

$$-\frac{2\left(ab^4c^2 - 2\,a^3b^2c + a^5\right)\log\left(\left|\sqrt{dx+c}b + a\right|\right)}{b^6d^3} + \frac{2\left(ab^4c^2\log\left(|a|\right) - 2\,a^3b^2c\log\left(|a|\right) + a^5\log\left(|a|\right)\right)}{b^6d^3} + \frac{12\,(dx+c)^{\frac{5}{2}}b^4d^{12} - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
+ 2*(a*b^4*c^2*log(abs(a)) - 2*a^3*b^2*c*log(abs(a)) + a^5*log(abs(a)))/(b^
6*d^3) + 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12
+ 60*sqrt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*
a*b^3*c*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*
c*d^12 - 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

3.634 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=90

$$\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)
^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rubi [A] time = 0.0803427, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)
^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{a^2-b^2c}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} + \frac{-a^3+ab^2c}{b^3(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{ax}{b^2d} + \frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2} + \frac{2(c+dx)^{3/2}}{3bd^2} - \frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Mathematica [A] time = 0.0651472, size = 82, normalized size = 0.91

$$\frac{b\left(6a^2\sqrt{c+dx} - 3abdx + 2b^2(dx-2c)\sqrt{c+dx}\right) - 6\left(a^3-ab^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-3*a*b*d*x + 6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x]) -
6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

Maple [A] time = 0.004, size = 116, normalized size = 1.3

$$\frac{2}{3\,bd^2}(dx+c)^{\frac{3}{2}} - \frac{ax}{b^2d} - \frac{ac}{b^2d^2} - 2\,\frac{c\sqrt{dx+c}}{bd^2} + 2\,\frac{\sqrt{dx+c}a^2}{b^3d^2} + 2\,\frac{a\ln\left(a+b\sqrt{dx+c}\right)c}{b^2d^2} - 2\,\frac{a^3\ln\left(a+b\sqrt{dx+c}\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b/d^2-a*x/b^2/d-1/d^2/b^2*a*c-2/d^2/b*c*(d*x+c)^(1/2)+2/d
^2/b^3*(d*x+c)^(1/2)*a^2+2/d^2*a/b^2*ln(a+b*(d*x+c)^(1/2))*c-2/d^2*a^3/b^4*
ln(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 1.11058, size = 109, normalized size = 1.21

$$\frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2 - 3\,(dx+c)ab - 6\left(b^2c-a^2\right)\sqrt{dx+c}}{b^3} + \frac{6\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Fricas [A] time = 1.64355, size = 166, normalized size = 1.84

$$-\frac{3\,ab^2dx-6\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}b+a\right)-2\left(b^3dx-2\,b^3c+3\,a^2b\right)\sqrt{dx+c}}{3\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*x
 - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

Sympy [A] time = 3.07034, size = 109, normalized size = 1.21

$$\begin{cases} \frac{2\left(-\frac{a(c+dx)}{2b^2d}-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3d}+\frac{(c+dx)^{\frac{3}{2}}}{3bd}+\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d}\right)}{d} & \text{for } d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)/(2*b**2*d) - a*(a**2 - b**2*c)*Piecewise((sqrt(c
 + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d) + (c + d
*x)**(3/2)/(3*b*d) + (a**2 - b**2*c)*sqrt(c + d*x)/(b**3*d))/d, Ne(d, 0)),
(x**2/(2*(a + b*sqrt(c))), True))
```

Giac [A] time = 1.19539, size = 177, normalized size = 1.97

$$\frac{\frac{6\left(ab^2c-a^3\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}-\frac{6\left(ab^2c\log(|a|)-a^3\log(|a|)\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}a^2d^2}{b^3d^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - 6*(a*b^2*c*l
og(abs(a)) - a^3*log(abs(a)))/(b^4*d) + (2*(d*x + c)^(3/2)*b^2*d^2 - 6*sqrt
(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt(d*x + c)*a^2*d^2)/(b^3*d
^3))/d
```

3.635 $\int \frac{1}{a+b\sqrt{c+dx}} dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rubi [A] time = 0.0241772, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {247, 190, 43}

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sqrt{c+dx}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{x}{a+bx}\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\operatorname{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] time = 0.0188489, size = 39, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{c+dx}}{b}-\frac{a\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*(Sqrt[c + d*x]/b - (a*Log[a + b*Sqrt[c + d*x]])/b^2))/d
```

Maple [B] time = 0.007, size = 87, normalized size = 2.1

$$2\,\frac{\sqrt{dx+c}}{bd}+\frac{a}{b^2d}\ln\left(-a+b\sqrt{dx+c}\right)-\frac{a}{b^2d}\ln\left(a+b\sqrt{dx+c}\right)-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2*(d*x+c)^(1/2)/b/d+1/b^2/d*a*ln(-a+b*(d*x+c)^(1/2))-a*ln(a+b*(d*x+c)^(1/2)
)/b^2/d-a*ln(b^2*d*x+b^2*c-a^2)/b^2/d
```

Maxima [A] time = 0.976455, size = 47, normalized size = 1.15

$$-\frac{2\left(\frac{a\log\left(\sqrt{dx+c}b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

Fricas [A] time = 1.73875, size = 80, normalized size = 1.95

$$-\frac{2\left(a\log\left(\sqrt{dx+c}b+a\right)-\sqrt{dx+c}b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

Sympy [A] time = 0.496423, size = 49, normalized size = 1.2

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(d
, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), Tru
e))
```

Giac [A] time = 1.17932, size = 68, normalized size = 1.66

$$-\frac{2\,a\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d}+\frac{2\,a\log\left(|a|\right)}{b^2d}+\frac{2\,\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a*log(abs(a))/(b^2*d) + 2*sq
rt(d*x + c)/(b*d)
```

3.636 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*Log[x])/(a^
2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rubi [A] time = 0.0791696, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 206, 260}

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*Log[x])/(a^
2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)}\,dx,x,c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)}\,dx,x,\sqrt{c+dx}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)}+\frac{bc-ax}{\left(a^2-b^2c\right)\left(c-x^2\right)}\right)dx,x,\sqrt{c+dx}\right)\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2\,\text{Subst}\left(\int \frac{bc-ax}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{a^2-b^2c}\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{(2a)\,\text{Subst}\left(\int \frac{x}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{a^2-b^2c}\\
&= \frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}
\end{aligned}$$

Mathematica [A] time = 0.081121, size = 61, normalized size = 0.74

$$\frac{-2a\log\left(a+b\sqrt{c+dx}\right)+a\log(dx)+2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[d*x] - 2*a*Log[a + b*Sq
rt[c + d*x]])/(a^2 - b^2*c)
```

Maple [A] time = 0.007, size = 77, normalized size = 0.9

$$-2\,\frac{a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}+\frac{a\ln(dx)}{-b^2c+a^2}+2\,\frac{b\sqrt{c}}{-b^2c+a^2}\text{Artanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+1/(-b^2*c+a^2)*a*ln(d*x)+2*b*arctan
h((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89238, size = 301, normalized size = 3.67

$$\left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}b+a\right)-a\log\left(x\right)}{b^2c-a^2}, \frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2a\log\left(\sqrt{dx+c}b+a\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*x
 + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

Sympy [A] time = 8.24437, size = 85, normalized size = 1.04

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{a^2-b^2c}-\frac{2\left(-\frac{a\log\left(-dx\right)}{2}+\frac{bc\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^2-b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b,
True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c + d*x)/sqrt(-c
))/sqrt(-c))/(a**2 - b**2*c)
```

Giac [A] time = 1.20587, size = 155, normalized size = 1.89

$$\frac{2ab\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^3c-a^2b}+\frac{2bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^2c-a^2\right)\sqrt{-c}}-\frac{a\log\left(dx\right)}{b^2c-a^2}+\frac{a\log\left(-c\right)-2a\log\left(|a|\right)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*x
 + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2) + (a*lo
g(-c) - 2*a*log(abs(a)))/(b^2*c - a^2)
```

3.637 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=130

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^
2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rubi [A] time = 0.178304, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```


```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^
2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx &= d\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^2}\,dx,x,c+dx\right)\\
&= (2d)\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^2}\,dx,x,\sqrt{c+dx}\right)\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}+\frac{d\,\text{Subst}\left(\int \frac{-abc+b^2cx}{(a+bx)\left(-c+x^2\right)}\,dx,x,\sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}+\frac{d\,\text{Subst}\left(\int \left(-\frac{2ab^3c}{\left(a^2-b^2c\right)(a+bx)}-\frac{bc\left(a^2+b^2c-2abx\right)}{\left(-a^2+b^2c\right)\left(c-x^2\right)}\right)dx,x,\sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{a^2+b^2c-2abx}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}-\frac{\left(2ab^2d\right)\text{Subst}\left(\int \frac{x}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{(b
\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}+\frac{b\left(a^2+b^2c\right)d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.194544, size = 144, normalized size = 1.11

$$
\frac{\sqrt{c}\left(-\left(a^2-b^2c\right)\left(a-b\sqrt{c+dx}\right)-ab^2dx\log\left(a^2-b^2(c+dx)\right)+ab^2dx\log(x)\right)+bdx\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-2ab^2}{\sqrt{c}x\left(a^2-b^2c\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (-2*a*b^2*Sqrt[c]*d*x*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^2 + b^2*c)*d*x*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^2*c)*(a - b*Sqrt[c + d*x
])) + a*b^2*d*x*Log[x] - a*b^2*d*x*Log[a^2 - b^2*(c + d*x)]))/(Sqrt[c]*(a^2
 - b^2*c)^2*x)
```

Maple [A] time = 0.017, size = 216, normalized size = 1.7

$$-2\frac{ab^2d\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^2}-\frac{b^3c}{\left(-b^2c+a^2\right)^2x}\sqrt{dx+c}+\frac{a^2b}{\left(-b^2c+a^2\right)^2x}\sqrt{dx+c}+\frac{ab^2c}{\left(-b^2c+a^2\right)^2x}-\frac{a^3}{\left(-b^2c+a^2\right)^2x}+\frac{ab^2d}{\left(-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -2*a*b^2*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2-1/(-b^2*c+a^2)^2/x*(d*x+c)^
(1/2)*b^3*c+1/(-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*a^2*b+1/(-b^2*c+a^2)^2/x*a*b^2
*c-1/(-b^2*c+a^2)^2/x*a^3+d/(-b^2*c+a^2)^2*a*b^2*ln(d*x)+d/(-b^2*c+a^2)^2*c
^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*b^3+d/(-b^2*c+a^2)^2*b/c^(1/2)*arctan
h((d*x+c)^(1/2)/c^(1/2))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.31543, size = 636, normalized size = 4.89

$$\left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-2\,ab^2cdx\log\left(x\right)-2\,ab^2c^2-\left(b^3c+a^2b\right)\sqrt{c}dx\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,a^3c+2\left(b^3c^2-a^2\right.}{2\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a*
b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c*d*x
*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqr
t(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2
*c^2 + a^4*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

Giac [B] time = 1.23524, size = 342, normalized size = 2.63

$$-\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}+\frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3cd+a^2bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}-\frac{ab^2cd\log\left(c\right)-2\,ab^2cd\log\left(|a|\right)}{b^4c^3-2\,a^2b^2c^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arcta
n(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) - (a*b^2
*c*d*log(c) - 2*a*b^2*c*d*log(abs(a)) - a*b^2*c*d + a^3*d)/(b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c) + (a*b^2*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))
/((b^2*c - a^2)^2*d*x)
```

3.638 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{bd^2\left(-6a^2b^2c+a^4-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\right.}{4c}$$

```
[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b^2
*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b^4
*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (a*
b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^3
```

Rubi [A] time = 0.277508, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{bd^2\left(-6a^2b^2c+a^4-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b^2
*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b^4
*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (a*
b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^3
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\,\text{Subst}\left(\int \frac{-abc+3b^2cx}{(a+bx)\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\text{Subst}\left(\int \frac{abc\left(a^2-5b^2c\right)+b^2c\left(a^2+3b^2c\right)x}{(a+bx)\left(-c+x^2\right)}\right.}{4c^2\left(a^2-b^2c\right)^2}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\text{Subst}\left(\int\left(-\frac{8ab^5c^2}{\left(a^2-b^2c\right)(a+bx)} - \frac{bc\left(-a^4\right.}{}\right.\right.}{4c^2\left(a^2\right.}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3} + \frac{\left(bd^2\right)}{}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3} - \frac{\left(2ab^4\right.}{}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2\tanh^{-1}\left(\right.}{4c^{3/2}\left(a^2-b^2c\right)^3}$$

Mathematica [A] time = 0.433237, size = 228, normalized size = 1.12

$$\frac{bd^2x^2\left(-6a^2b^2c+a^4-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(4ab^4cd^2x^2\log\left(a^2-b^2(c+dx)\right)+\left(a^2-b^2c\right)\left(-a^2b\sqrt{c+dx}(2c+dx)\right.\right.}{4c^{3/2}x^2\left(b^2c-a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (8*a*b^4*c^(3/2)*d^2*x^2*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^4 - 6*a^2*b^2*
c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*((a^2 - b^2
*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3*d*x)*Sqrt[c + d*x] -
a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2*Log[x] + 4*a*b^4*c*d^2
*x^2*Log[a^2 - b^2*(c + d*x)]))/(4*c^(3/2)*(-a^2 + b^2*c)^3*x^2)
```

Maple [B] time = 0.014, size = 459, normalized size = 2.3

$$-2\,\frac{ab^4d^2\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^3}-\frac{3\,b^5c}{4\,\left(-b^2c+a^2\right)^3x^2}(dx+c)^{\frac{3}{2}}+\frac{a^2b^3}{2\,\left(-b^2c+a^2\right)^3x^2}(dx+c)^{\frac{3}{2}}+\frac{a^4b}{4\,\left(-b^2c+a^2\right)^3x^2c}(dx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -2*a*b^4*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^3-3/4/(-b^2*c+a^2)^3/x^2*b^
5*c*(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^3/x^2*b^3*(d*x+c)^(3/2)*a^2+1/4/(-b^2*c+
a^2)^3/x^2*b/c*(d*x+c)^(3/2)*a^4+d/(-b^2*c+a^2)^3/x*a*b^4*c-1/2/(-b^2*c+a^2
)^3/x^2*a*b^4*c^2-d/(-b^2*c+a^2)^3/x*a^3*b^2+1/(-b^2*c+a^2)^3/x^2*a^3*b^2*c
-3/2/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*a^2*b^3*c+1/4/(-b^2*c+a^2)^3/x^2*(d*x
+c)^(1/2)*a^4*b+5/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*b^5*c^2-1/2/(-b^2*c+a^
2)^3/x^2*a^5+d^2/(-b^2*c+a^2)^3*b^4*a*ln(d*x)+3/4*d^2/(-b^2*c+a^2)^3*c^(1/2
)*b^5*arctanh((d*x+c)^(1/2)/c^(1/2))+3/2*d^2/(-b^2*c+a^2)^3/c^(1/2)*b^3*arc
tanh((d*x+c)^(1/2)/c^(1/2))*a^2-1/4*d^2/(-b^2*c+a^2)^3/c^(3/2)*b*arctanh((d
*x+c)^(1/2)/c^(1/2))*a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.62273, size = 1112, normalized size = 5.45

$$\left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}b+a\right)-8\,ab^4c^2d^2x^2\log\left(x\right)+4\,ab^4c^4-8\,a^3b^2c^3+4\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\sqrt{c}d^2x^2\log}{8\left(b^6c^5-3\,a^2b^4c^4+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*l
og(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c
- a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 8*(
a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 -
 (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^
2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*log(sqr
t(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3*b^2*c^
3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*arctan(s
qrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*c^4 - 4
*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqr
t(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))), x)
```

Giac [B] time = 1.23144, size = 649, normalized size = 3.18

$$\frac{2\,ab^{5}d^{2}\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^{7}c^{3}-3\,a^{2}b^{5}c^{2}+3\,a^{4}b^{3}c-a^{6}b}-\frac{ab^{4}d^{2}\log\left(dx\right)}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}}+\frac{\left(3\,b^{5}c^{2}d^{2}+6\,a^{2}b^{3}cd^{2}-a^{4}bd^{2}\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\left(b^{6}c^{4}-3\,a^{2}b^{4}c^{3}+3\,a^{4}b^{2}c^{2}-a^{6}c\right)\sqrt{-c}}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*
b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c
- a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d*x
+ c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c))
 + 1/2*(2*a*b^4*c^2*d^2*log(-c) - 4*a*b^4*c^2*d^2*log(abs(a)) - 3*a*b^4*c^2
*d^2 + 4*a^3*b^2*c*d^2 - a^5*d^2)/(b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3
- a^6*c^2) + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^
5*c^2*d^2 - 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2
 - a^3*b^2*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*
d^2)*sqrt(d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)
```

3.639 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=240

$$\frac{x\left(-9a^2b^2c+5a^4+3b^4c^2\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}+\frac{3\left(a^2-b^2c\right)(c+dx)^2}{2b^4d^4}-\frac{4a\left(2a^2-3b^2c\right)(}{3b^5d^4}$$

```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqr
t[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4)
 + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3
*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqr
t[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(b^8*d^4)
```

Rubi [A] time = 0.279523, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{x\left(-9a^2b^2c+5a^4+3b^4c^2\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}+\frac{3\left(a^2-b^2c\right)(c+dx)^2}{2b^4d^4}-\frac{4a\left(2a^2-3b^2c\right)(}{3b^5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```


```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqr
t[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4)
 + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3
*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqr
t[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(b^8*d^4)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{6a\left(a^2-b^2c\right)^2}{b^7} + \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6} - \frac{2a\left(2a^2-3b^2c\right)x^2}{b^5} - \frac{3\left(-a^2+b^2c\right)x^3}{b^4} - \frac{2ax^4}{b^3} + \frac{x^5}{b^2} - \frac{a\left(a^2-\right.}{b^7(a+}\right.\right.}{d^4}$$

$$= \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6d^3} - \frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5d^4} + \frac{3\left(a^2-\right.}{}$$

Mathematica [A] time = 0.281365, size = 273, normalized size = 1.14

$$\frac{a^3b^4\left(856c^2+380cdx-35d^2x^2\right) - 3a^2b^5\sqrt{c+dx}\left(76c^2+36cdx-7d^2x^2\right) - 6a^5b^2(102c+35dx) + 2a^4b^3\sqrt{c+dx}(284c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (96*a^7 - 324*a^6*b*Sqrt[c + d*x] - 6*a^5*b^2*(102*c + 35*d*x) + 2*a^4*b^3*
Sqrt[c + d*x]*(284*c + 35*d*x) + a^3*b^4*(856*c^2 + 380*c*d*x - 35*d^2*x^2)
 - 3*a^2*b^5*Sqrt[c + d*x]*(76*c^2 + 36*c*d*x - 7*d^2*x^2) + 5*b^7*d*x*Sqrt
[c + d*x]*(6*c^2 - 3*c*d*x + 2*d^2*x^2) - a*b^6*(324*c^3 + 162*c^2*d*x - 33
*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c
 + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqrt[c + d*x]))
```

Maple [A] time = 0.008, size = 416, normalized size = 1.7

$$\frac{x^3}{3\,b^2d} - \frac{cx^2}{2\,b^2d^2} + \frac{c^2x}{d^3b^2} + \frac{11\,c^3}{6\,d^4b^2} - \frac{4\,a}{5\,b^3d^4}(dx+c)^{\frac{5}{2}} + \frac{3\,a^2x^2}{2\,b^4d^2} - 6\,\frac{xa^2c}{b^4d^3} - \frac{15\,a^2c^2}{2\,d^4b^4} + 4\,\frac{(dx+c)^{3/2}ac}{b^3d^4} - \frac{8\,a^3}{3\,d^4b^5}(dx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 1/3/d/b^2*x^3-1/2/d^2/b^2*x^2*c+1/d^3/b^2*x*c^2+11/6/d^4/b^2*c^3-4/5*a*(d*x
+c)^(5/2)/b^3/d^4+3/2/d^2/b^4*x^2*a^2-6/d^3/b^4*x*a^2*c-15/2/d^4/b^4*a^2*c^
2+4/d^4/b^3*(d*x+c)^(3/2)*a*c-8/3/d^4/b^5*(d*x+c)^(3/2)*a^3-12/d^4/b^3*a*c^
2*(d*x+c)^(1/2)+5/d^3/b^6*x*a^4+5/d^4/b^6*a^4*c+24/d^4/b^5*a^3*c*(d*x+c)^(1
/2)-12/d^4/b^7*a^5*(d*x+c)^(1/2)-2/d^4*a/b^2/(a+b*(d*x+c)^(1/2))*c^3+6/d^4*
a^3/b^4/(a+b*(d*x+c)^(1/2))*c^2-6/d^4*a^5/b^6/(a+b*(d*x+c)^(1/2))*c+2/d^4*a
^7/b^8/(a+b*(d*x+c)^(1/2))-2/d^4/b^2*ln(a+b*(d*x+c)^(1/2))*c^3+18/d^4/b^4*l
n(a+b*(d*x+c)^(1/2))*a^2*c^2-30/d^4/b^6*ln(a+b*(d*x+c)^(1/2))*a^4*c+14/d^4/
b^8*ln(a+b*(d*x+c)^(1/2))*a^6
```

Maxima [A] time = 1.06348, size = 339, normalized size = 1.41

$$-\frac{\frac{60\left(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7\right)}{\sqrt{dx+c}b^9+ab^8}-\frac{10\,(dx+c)^3b^5-24\,(dx+c)^{\frac{5}{2}}ab^4-45\left(b^5c-a^2b^3\right)(dx+c)^2+40\left(3\,ab^4c-2\,a^3b^2\right)(dx+c)^{\frac{3}{2}}+30\left(3\,b^5c^2-9\,a^2b^3c+5\,a^4b\right)(dx+c)-360\,(a}{b^7}}{30\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b^
9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c - a
^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3*b^
5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a
^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6
)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

Fricas [A] time = 1.61897, size = 838, normalized size = 3.49

$$\frac{10\,b^8d^4x^4+55\,b^8c^4-220\,a^2b^6c^3+195\,a^4b^4c^2+30\,a^6b^2c-60\,a^8-5\left(b^8c-7\,a^2b^6\right)d^3x^3+15\left(b^8c^2-8\,a^2b^6c+7\,a^4b^4\right)d^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30*
a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*b^
6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c -
30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^6*b^
2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x)*log
(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b^5*c^2
 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2*(24*a*
b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*x + (b^1
0*c - a^2*b^8)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.640 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal. Leaf size=166

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}-\frac{4a(c+}{3b^3}$$

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^
3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^
2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b
^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rubi [A] time = 0.172017, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}-\frac{4a(c+}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```


```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^
3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^
2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b
^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{4a\left(a^2-b^2c\right)}{b^5} - \frac{\left(-3a^2+2b^2c\right)x}{b^4} - \frac{2ax^2}{b^3} + \frac{x^3}{b^2} - \frac{a\left(a^2-b^2c\right)^2}{b^5(a+bx)^2} + \frac{5a^4-6a^2b^2c+b^4c^2}{b^5(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{\left(3a^2-2b^2c\right)x}{b^4d^2} - \frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}$$

Mathematica [A] time = 0.172758, size = 185, normalized size = 1.11

$$\frac{12\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right) - 2a^3b^2(38c+15dx) + 2a^2b^3\sqrt{c+dx}(18c+5dx) - 44a^4}{6b^6d^3\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (16*a^5 - 44*a^4*b*Sqrt[c + d*x] + 3*b^5*d*x*(-2*c + d*x)*Sqrt[c + d*x] + 2
*a^2*b^3*Sqrt[c + d*x]*(18*c + 5*d*x) - 2*a^3*b^2*(38*c + 15*d*x) + a*b^4*(
52*c^2 + 26*c*d*x - 5*d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*
Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sqrt[c + d*x]))
```

Maple [A] time = 0.009, size = 253, normalized size = 1.5

$$\frac{x^2}{2\,b^2d} - \frac{cx}{b^2d^2} - \frac{3\,c^2}{2\,d^3b^2} - \frac{4\,a}{3\,b^3d^3}(dx+c)^{\frac{3}{2}} + 3\,\frac{xa^2}{b^4d^2} + 3\,\frac{a^2c}{b^4d^3} + 8\,\frac{ac\sqrt{dx+c}}{b^3d^3} - 8\,\frac{a^3\sqrt{dx+c}}{d^3b^5} + 2\,\frac{c^2a}{d^3b^2\left(a+b\sqrt{dx+c}\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 1/2/d/b^2*x^2-1/d^2/b^2*x*c-3/2/d^3/b^2*c^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+3/d
^2/b^4*x*a^2+3/d^3/b^4*a^2*c+8/d^3/b^3*a*c*(d*x+c)^(1/2)-8/d^3/b^5*a^3*(d*x
+c)^(1/2)+2/d^3*a/b^2/(a+b*(d*x+c)^(1/2))*c^2-4/d^3*a^3/b^4/(a+b*(d*x+c)^(1
/2))*c+2/d^3*a^5/b^6/(a+b*(d*x+c)^(1/2))+2/d^3/b^2*ln(a+b*(d*x+c)^(1/2))*c^
2-12/d^3/b^4*ln(a+b*(d*x+c)^(1/2))*a^2*c+10/d^3/b^6*ln(a+b*(d*x+c)^(1/2))*a
^4
```

Maxima [A] time = 1.03901, size = 213, normalized size = 1.28

$$\frac{\frac{12\left(ab^4c^2-2\,a^3b^2c+a^5\right)}{\sqrt{dx+c}b^7+ab^6} + \frac{3\,(dx+c)^2b^3-8\,(dx+c)^{\frac{3}{2}}ab^2-6\left(2\,b^3c-3\,a^2b\right)(dx+c)+48\left(ab^2c-a^3\right)\sqrt{dx+c}}{b^5} + \frac{12\left(b^4c^2-6\,a^2b^2c+5\,a^4\right)\log\left(\sqrt{dx+c}b+a\right)}{b^6}}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(d
*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c) +
 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)
*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Fricas [A] time = 1.74116, size = 564, normalized size = 3.4

$$3\,b^6d^3x^3 - 9\,b^6c^3 + 15\,a^2b^4c^2 + 6\,a^4b^2c - 12\,a^6 - 3\left(b^6c - 5\,a^2b^4\right)d^2x^2 - 3\left(5\,b^6c^2 - 14\,a^2b^4c + 6\,a^4b^2\right)dx + 12\left(b^6c^3 - 7\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3*
(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*x
+ 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*b^4
*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13*a*b
^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt(d*x
+ c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a + b\sqrt{c + dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.641 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a
 + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^
2)
```

Rubi [A] time = 0.0898834, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a
 + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^
2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}+\frac{-a^3+ab^2c}{b^3(a+bx)^2}+\frac{3a^2-b^2c}{b^3(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{x}{b^2d} - \frac{4a\sqrt{c+dx}}{b^3d^2} + \frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)} + \frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Mathematica [A] time = 0.088006, size = 112, normalized size = 1.18

$$\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)-4a^2b\sqrt{c+dx}+2a^3-3ab^2(2c+dx)+b^3dx\sqrt{c+dx}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (2*a^3 - 4*a^2*b*Sqrt[c + d*x] + b^3*d*x*Sqrt[c + d*x] - 3*a*b^2*(2*c + d*x
) + 2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(b^4*
d^2*(a + b*Sqrt[c + d*x]))
```

Maple [A] time = 0.007, size = 125, normalized size = 1.3

$$\frac{x}{b^2d}+\frac{c}{b^2d^2}-4\frac{a\sqrt{dx+c}}{b^3d^2}-2\frac{ac}{b^2d^2\left(a+b\sqrt{dx+c}\right)}+2\frac{a^3}{b^4d^2\left(a+b\sqrt{dx+c}\right)}-2\frac{\ln\left(a+b\sqrt{dx+c}\right)c}{b^2d^2}+6\frac{\ln\left(a+b\sqrt{dx+\right.}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] x/b^2/d+1/d^2/b^2*c-4*a*(d*x+c)^(1/2)/b^3/d^2-2/d^2*a/b^2/(a+b*(d*x+c)^(1/2
))*c+2/d^2*a^3/b^4/(a+b*(d*x+c)^(1/2))-2/d^2/b^2*ln(a+b*(d*x+c)^(1/2))*c+6/
d^2/b^4*ln(a+b*(d*x+c)^(1/2))*a^2
```

Maxima [A] time = 1.11361, size = 122, normalized size = 1.28

$$-\frac{\frac{2\left(ab^2c-a^3\right)}{\sqrt{dx+c}b^5+ab^4}-\frac{(dx+c)b-4\sqrt{dx+c}a}{b^3}+\frac{2\left(b^2c-3a^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*x
 + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Fricas [A] time = 1.73069, size = 335, normalized size = 3.53

$$\frac{b^4d^2x^2 + b^4c^2 + a^2b^2c - 2a^4 + \left(2b^4c - a^2b^2\right)dx - 2\left(b^4c^2 - 4a^2b^2c + 3a^4 + \left(b^4c - 3a^2b^2\right)dx\right)\log\left(\sqrt{dx+c}\,b + a\right) - 2(}{b^6d^3x + \left(b^6c - a^2b^4\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(b
^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)*b
 + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x + (
b^6*c - a^2*b^4)*d^2)
```

Sympy [A] time = 21.9147, size = 131, normalized size = 1.38

$$\begin{cases} \frac{2\left(-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^3d} - \frac{2a\sqrt{c+dx}}{b^3d} + \frac{c+dx}{2b^2d} + \frac{\left(3a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3d}\right)}{d} & \text{for } d \neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
(-1/(b*(a + b*sqrt(c + d*x))), True))/(b**3*d) - 2*a*sqrt(c + d*x)/(b**3*d)
 + (c + d*x)/(2*b**2*d) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(
b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d))/d, Ne(d, 0)), (x**2/(
2*(a + b*sqrt(c))**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.642 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal. Leaf size=47

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rubi [A] time = 0.0327996, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {247, 190, 43}

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)} + \frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Mathematica [A] time = 0.0335396, size = 40, normalized size = 0.85

$$\frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[a + b*Sqrt[c + d*x]]))/(b^2*d)
```

Maple [B] time = 0.016, size = 142, normalized size = 3.

$$-2\,\frac{a^2}{\left(b^2dx+b^2c-a^2\right)b^2d}+\frac{\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}+\frac{a}{b^2d}\left(-a+b\sqrt{dx+c}\right)^{-1}-\frac{1}{b^2d}\ln\left(-a+b\sqrt{dx+c}\right)+\frac{a}{b^2d}\left(a+b\sqrt{d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] -2*a^2/(b^2*d*x+b^2*c-a^2)/b^2/d+ln(b^2*d*x+b^2*c-a^2)/b^2/d+a/b^2/d/(-a+b*
(d*x+c)^(1/2))-1/b^2/d*ln(-a+b*(d*x+c)^(1/2))+a/b^2/d/(a+b*(d*x+c)^(1/2))+l
n(a+b*(d*x+c)^(1/2))/b^2/d
```

Maxima [A] time = 1.01556, size = 58, normalized size = 1.23

$$\frac{2\left(\frac{a}{\sqrt{dx+c}b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}b+a\right)}{b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

Fricas [A] time = 1.71093, size = 154, normalized size = 3.28

$$\frac{2\left(\sqrt{dx+c}ab-a^2+\left(b^2dx+b^2c-a^2\right)\log\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2x+\left(b^4c-a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b +
a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)
```

Sympy [A] time = 0.918834, size = 124, normalized size = 2.64

$$\begin{cases} \frac{x}{a^2} & \text{for}\ b=0\wedge(b=0\vee d=0) \\ \frac{x}{\left(a+b\sqrt{c}\right)^2} & \text{for}\ d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2a}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**2
, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)
) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b + sqr
t(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.643 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2}$$

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2
- (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rubi [A] time = 0.119484, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 206, 260}

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2
- (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)}\,dx, x, c+dx\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)^2}-\frac{b\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2(a+bx)}+\frac{2abc-\left(a^2+b^2c\right)x}{\left(a^2-b^2c\right)^2\left(c-x^2\right)}\right)dx, x, \sqrt{c+\right.$$
$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{2\,\text{Subst}\left(\int\frac{2abc-\left(a^2+b^2c\right)x}{c-x^2}\,dx, x,\right.}{\left(a^2-b^2c\right)^2}$$
$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{(4abc)\,\text{Subst}\left(\int\frac{1}{c-x^2}\,dx, x, \sqrt{c\right.}{\left(a^2-b^2c\right)^2}$$
$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\left(a^2+b^2c\right)\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2\left(a^2+b^2c\right)\log\left(a\right.}{\left(a^2-b^{2}\right.}$$

Mathematica [A] time = 0.282711, size = 164, normalized size = 1.27

$$\frac{-2\left(a^2+b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)+2a^3-2ab^2c+\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)\left(a+b\sqrt{c+dx}\right)+\left(a+b\right.}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a^3 - 2*a*b^2*c + (a - b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] -
Sqrt[c + d*x]] + (a + b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] + Sqrt
[c + d*x]] - 2*(a^2 + b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]]
)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x]))
```

Maple [A] time = 0.008, size = 161, normalized size = 1.3

$$2\,\frac{a}{\left(-b^2c+a^2\right)\left(a+b\sqrt{dx+c}\right)}-2\,\frac{\ln\left(a+b\sqrt{dx+c}\right)b^2c}{\left(-b^2c+a^2\right)^2}-2\,\frac{\ln\left(a+b\sqrt{dx+c}\right)a^2}{\left(-b^2c+a^2\right)^2}+\frac{\ln(dx)\,b^2c}{\left(-b^2c+a^2\right)^2}+\frac{\ln(dx)\,a^2}{\left(-b^2c+a^2\right)^2}+4\,\frac{}{\left(-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 2*a/(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))-2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))
*b^2*c-2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))*a^2+1/(-b^2*c+a^2)^2*ln(d*x)*
b^2*c+1/(-b^2*c+a^2)^2*ln(d*x)*a^2+4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(
1/2)/(-b^2*c+a^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89787, size = 936, normalized size = 7.26

$$\left[\frac{2\,a^2b^2c - 2\,a^4 + 2\left(ab^3dx + ab^3c - a^3b\right)\sqrt{c}\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right) - 2\left(b^4c^2 - a^4 + \left(b^4c + a^2b^2\right)dx\right)\log\left(\sqrt{dx+c}b + a\right)}{b^6c^3 - 3\,a^2b^4c^2 + 3\,a^4b^2c - a^6 + \left(b^6c^2 - 2\,a^2b^4c + a^4b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*
x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x
) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2
*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^4 - 4
*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) -
2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c
^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c
))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c +
a^4*b^2)*d*x)]
```

Sympy [A] time = 26.9701, size = 153, normalized size = 1.19

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{a^2-b^2c} - \frac{2b\left(a^2+b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{\left(a^2-b^2c\right)^2} - \frac{2\left(\frac{2abc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \left(-\frac{a^2}{2} - \frac{b^2c}{2}\right)\right.}{\left(a^2-b^2c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x
))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((sqrt(c + d*x)/
```

```
a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**2*c)**2 - 2*(2
*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b**2*c/2)*log(-d*
x))/(a**2 - b**2*c)**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.644 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+\right.}{\left(a^2-b^2c\right)^3}$$

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c)^3
```

Rubi [A] time = 0.245105, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+\right.}{\left(a^2-b^2c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c)^3
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = d\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{d\,\text{Subst}\left(\int \frac{-2abc+2b^2cx}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{d\,\text{Subst}\left(\int \left(-\frac{4ab^3c}{\left(a^2-b^2c\right)(a+bx)^2} - \frac{2b^3c\left(3a^2+b^2c\right)}{\left(-a^2+b^2c\right)^2(a+bx)} + \frac{2bc\left(a\left(a^2+3b^2c\right)-\right.}{\left(a^2-b^2c\right)^2}\right.\right.}{c\left(a^2-b^2c\right)}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2\left(3a^2+b^2c\right)d\log\left(a+b\sqrt{c+\right.}}{\left(a^2-b^2c\right)^3}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2\left(3a^2+b^2c\right)d\log\left(a+b\sqrt{c+\right.}}{\left(a^2-b^2c\right)^3}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab\left(a^2+3b^2c\right)d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right.}{\sqrt{c}\left(a^2-b^2c\right)^3}$$

Mathematica [A] time = 0.799447, size = 230, normalized size = 1.14

$$\frac{\frac{\sqrt{c}\left(\frac{\sqrt{c}(a^2-b^2c)\left(-a^2b\sqrt{c+dx}+a^3-ab^2(c+4dx)+b^3c\sqrt{c+dx}\right)}{x\left(a+b\sqrt{c+dx}\right)}+2b^2\sqrt{c}d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)-bd\left(a+b\sqrt{c}\right)^3\log\left(\sqrt{c+dx}+\sqrt{c}\right)\right)}{\left(a^2-b^2c\right)^2}+\frac{\left(ab\sqrt{c}d-b^2cd\right)\log\left(\sqrt{c}-\sqrt{c+dx}\right)}{\left(a+b\sqrt{c}\right)^2}}{c\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (((a*b*Sqrt[c]*d - b^2*c*d)*Log[Sqrt[c] - Sqrt[c + d*x]])/(a + b*Sqrt[c])^2
 + (Sqrt[c]*((Sqrt[c]*(a^2 - b^2*c)*(a^3 - a^2*b*Sqrt[c + d*x] + b^3*c*Sqrt
[c + d*x] - a*b^2*(c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) - b*(a + b*Sqrt[c
])^3*d*Log[Sqrt[c] + Sqrt[c + d*x]] + 2*b^2*Sqrt[c]*(3*a^2 + b^2*c)*d*Log[a
 + b*Sqrt[c + d*x]]))/(a^2 - b^2*c)^2)/(c*(-a^2 + b^2*c))
```

Maple [A] time = 0.016, size = 312, normalized size = 1.5

$$2\,\frac{ab^2d}{\left(-b^2c+a^2\right)^2\left(a+b\sqrt{dx+c}\right)}-2\,\frac{b^4d\ln\left(a+b\sqrt{dx+c}\right)c}{\left(-b^2c+a^2\right)^3}-6\,\frac{b^2d\ln\left(a+b\sqrt{dx+c}\right)a^2}{\left(-b^2c+a^2\right)^3}-2\,\frac{a\sqrt{dx+c}b^3c}{\left(-b^2c+a^2\right)^3x}+2\,\frac{\sqrt{dx}}{(-b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 2*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-2*d*b^4/(-b^2*c+a^2)^3*ln(a+b*
(d*x+c)^(1/2))*c-6*d*b^2/(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))*a^2-2/(-b^2*c
+a^2)^3/x*(d*x+c)^(1/2)*a*b^3*c+2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*a^3*b+1/(-
b^2*c+a^2)^3/x*c^2*b^4-1/(-b^2*c+a^2)^3/x*a^4+d/(-b^2*c+a^2)^3*ln(d*x)*b^4*
c+3*d/(-b^2*c+a^2)^3*ln(d*x)*a^2*b^2+6*d/(-b^2*c+a^2)^3*c^(1/2)*arctanh((d*
x+c)^(1/2)/c^(1/2))*a*b^3+2*d/(-b^2*c+a^2)^3*b/c^(1/2)*arctanh((d*x+c)^(1/2
)/c^(1/2))*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.91541, size = 1724, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 -
 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b
^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2
*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)
*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2
 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^
2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6
*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2
*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((3*a*b^5*c
+ a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(-c)*arct
an(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2
+ 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x
) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)
*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^
2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.645 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=306

$$-\frac{abd^2\left(-10a^2b^2c+a^4-15b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}+\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2\left(5\right.}{}$$

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) -
(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3
*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt
[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x]
)/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^4
```

Rubi [A] time = 0.402982, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{abd^2\left(-10a^2b^2c+a^4-15b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}+\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) -
(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3
*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt
[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x]
)/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^4
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```

```
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int \frac{-2abc+4b^2cx}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int \frac{2abc\left(a^2\right.}{}\right.}{}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int\left(-\frac{2ab^3}{\left(a^2-\right.}\right.\right.}{}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\right.}{2c\left(a^2-b^2c\right)^2x\left(a+b\right.}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\right.}{2c\left(a^2-b^2c\right)^2x\left(a+b\right.}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\right.}{2c\left(a^2-b^2c\right)^2x\left(a+b\right.}$$

Mathematica [A] time = 0.909832, size = 401, normalized size = 1.31

$$\frac{d^2\left(\frac{\frac{2b\sqrt{c}\left(a^2+2b^2c\right)\left(\left(b\sqrt{c}-a\right)\log\left(\sqrt{c}-\sqrt{c+dx}\right)+\left(a+b\sqrt{c}\right)\log\left(\sqrt{c+dx}+\sqrt{c}\right)-2b\sqrt{c}\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2c-a^2}-abc\left(a^2+11b^2c\right)\left(\frac{2b\left(\frac{b^2c-a^2}{a+b\sqrt{c+dx}}+2a\log\left(a+b\sqrt{c+dx}\right)\right)}{\left(a^2-b^2c\right)^2}+\frac{\log\left(\sqrt{c}-\sqrt{c+dx}\right)}{\sqrt{c}\left(a+b\sqrt{c}\right)^2}-\frac{\log\left(\sqrt{c+\right.}{\sqrt{c}\left(a-\right.}\right.}{2c\left(a^2-b^2c\right)}\right.}{2c\left(a^2-b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (-((c*(a - b*Sqrt[c + d*x]))/(x^2*(a + b*Sqrt[c + d*x]))) + (b*d*(-3*a*b*c
+ a^2*Sqrt[c + d*x] + 2*b^2*c*Sqrt[c + d*x]))/((a^2 - b^2*c)*x*(a + b*Sqrt[
c + d*x])) + (d^2*((2*b*Sqrt[c]*(a^2 + 2*b^2*c)*((-a + b*Sqrt[c])*Log[Sqrt[
c] - Sqrt[c + d*x]] + (a + b*Sqrt[c])*Log[Sqrt[c] + Sqrt[c + d*x]] - 2*b*Sq
rt[c]*Log[a + b*Sqrt[c + d*x]]))/(-a^2 + b^2*c) - a*b*c*(a^2 + 11*b^2*c)*(L
og[Sqrt[c] - Sqrt[c + d*x]]/((a + b*Sqrt[c])^2*Sqrt[c]) - Log[Sqrt[c] + Sqr
t[c + d*x]]/((a - b*Sqrt[c])^2*Sqrt[c]) + (2*b*((-a^2 + b^2*c)/(a + b*Sqrt[
c + d*x]) + 2*a*Log[a + b*Sqrt[c + d*x]]))/(a^2 - b^2*c)^2)))/(2*c*(a^2 - b
^2*c)))/(2*c*(a^2 - b^2*c))
```

Maple [B] time = 0.018, size = 610, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 2*d^2*b^4/(-b^2*c+a^2)^3*a/(a+b*(d*x+c)^(1/2))-2*d^2*b^6/(-b^2*c+a^2)^4*ln(
a+b*(d*x+c)^(1/2))*c-10*d^2*b^4/(-b^2*c+a^2)^4*ln(a+b*(d*x+c)^(1/2))*a^2-7/
2/(-b^2*c+a^2)^4/x^2*a*b^5*c*(d*x+c)^(3/2)+3/(-b^2*c+a^2)^4/x^2*a^3*b^3*(d*
x+c)^(3/2)+1/2/(-b^2*c+a^2)^4/x^2*a^5*b/c*(d*x+c)^(3/2)+d/(-b^2*c+a^2)^4/x*
b^6*c^2-1/2/(-b^2*c+a^2)^4/x^2*c^3*b^6+2*d/(-b^2*c+a^2)^4/x*a^2*b^4*c+1/2/(
-b^2*c+a^2)^4/x^2*a^2*b^4*c^2-3*d/(-b^2*c+a^2)^4/x*a^4*b^2+1/2/(-b^2*c+a^2)
^4/x^2*a^4*b^2*c+9/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*a*b^5*c^2-5/(-b^2*c+a
^2)^4/x^2*(d*x+c)^(1/2)*a^3*b^3*c+1/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*a^5*
b-1/2/(-b^2*c+a^2)^4/x^2*a^6+d^2/(-b^2*c+a^2)^4*c*b^6*ln(d*x)+5*d^2/(-b^2*c
+a^2)^4*b^4*ln(d*x)*a^2+15/2*d^2/(-b^2*c+a^2)^4*c^(1/2)*b^5*arctanh((d*x+c)
^(1/2)/c^(1/2))*a+5*d^2/(-b^2*c+a^2)^4/c^(1/2)*b^3*arctanh((d*x+c)^(1/2)/c^
(1/2))*a^3-1/2*d^2/(-b^2*c+a^2)^4/c^(3/2)*b*arctanh((d*x+c)^(1/2)/c^(1/2))*
a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.6694, size = 2507, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^
4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d
^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqr
t(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^2*b^6
*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt
(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*
b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b^5*c^4 +
 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*
d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*sqrt
(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8
*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c
^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6*c^5 + 2*a^6*
b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - (
b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x + ((15*a*b^7*c
^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a
^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 4*((
b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2
```

```
)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3
+ (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - (2*a*b^7*c^5
- 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*
c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a
^7*b*c)*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*
a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c
^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] undef
```

3.646 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=324

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}$$

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^
2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4)
- (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7
*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqr
t[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8
*d^4)
```

Rubi [A] time = 0.229957, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^
2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4)
- (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7
*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqr
t[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8
*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^3}{b^7\sqrt{a+bx}} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2\sqrt{a+bx}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^{3/2}}{b^7} + \frac{\left(35a^4-30a^2b^2c+\right.}{b}\right.\right.}{d}$$

$$= -\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} - \frac{12a\left(7a^2-3\right.}{}$$

Mathematica [A] time = 0.355469, size = 232, normalized size = 0.72

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-16a^3b^4\left(2936c^2-680cdx+245d^2x^2\right)+24a^2b^5\sqrt{c+dx}\left(784c^2-356cdx+147d^2x^2\right)+768a^5b^2(58\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 716
8*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2*b
^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*c^2
 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d^2*x
^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*d^2*x
^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

Maple [A] time = 0.003, size = 383, normalized size = 1.2

$$4\,\frac{1}{d^4b^8}\left(1/15\,\left(a+b\sqrt{dx+c}\right)^{15/2}-\frac{7\,a\left(a+b\sqrt{dx+c}\right)^{13/2}}{13}+1/11\,\left(-3\,b^2c+21\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{11/2}+1/9\,\left(-8\,\left(-b^2c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/15*(a+b*(d*x+c)^(1/2))^(15/2)-7/13*a*(a+b*(d*x+c)^(1/2))^(13/2
)+1/11*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(-8*(-b^2*c+a^2)*a-
2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*((-
b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c+a
^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-6*(-b^2*c+a^
2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*a)
*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b*(
d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Maxima [A] time = 1.02297, size = 362, normalized size = 1.12

$$4\left(3003\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}+25025\left(3ab^2c-7a^3\right)\left(\sqrt{dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(1
3/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*b^
2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*c +
 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c +
7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^
4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4
*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

Fricas [A] time = 2.27983, size = 568, normalized size = 1.75

$$-\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\left(141\,ab^6c-140\,a^3b^4\right)d^2x^2+64\left(87\,ab^6c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544*
a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a*b
^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*c^3
 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^
2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d
*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.33864, size = 1449, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/45045*(15015*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*b^6*c
^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 45045*sqrt((sqrt(d*x + c)*b + a)*b^
2)*a*b^6*c^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 19305*sqrt((sqrt(d*x + c)
*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*b^4*c^2*sgn((sqrt(d*x + c)*b + a)*b -
a*b) + 81081*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a*b^4*
c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 135135*sqrt((sqrt(d*x + c)*b + a)*
b^2)*(sqrt(d*x + c)*b + a)*a^2*b^4*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) +
 135135*sqrt((sqrt(d*x + c)*b + a)*b^2)*a^3*b^4*c^2*sgn((sqrt(d*x + c)*b +
a)*b - a*b) + 12285*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^5
*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 75075*sqrt((sqrt(d*x + c)*b + a
)*b^2)*(sqrt(d*x + c)*b + a)^4*a*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) +
 193050*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a^2*b^2*c*s
gn((sqrt(d*x + c)*b + a)*b - a*b) - 270270*sqrt((sqrt(d*x + c)*b + a)*b^2)*
(sqrt(d*x + c)*b + a)^2*a^3*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 2252
25*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^4*b^2*c*sgn((sqr
t(d*x + c)*b + a)*b - a*b) - 135135*sqrt((sqrt(d*x + c)*b + a)*b^2)*a^5*b^2
*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3003*sqrt((sqrt(d*x + c)*b + a)*b^2
)*(sqrt(d*x + c)*b + a)^7*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 24255*sqrt((
sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^6*a*sgn((sqrt(d*x + c)*b +
a)*b - a*b) - 85995*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^5
*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 175175*sqrt((sqrt(d*x + c)*b + a)
*b^2)*(sqrt(d*x + c)*b + a)^4*a^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 2252
25*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a^4*sgn((sqrt(d*
x + c)*b + a)*b - a*b) + 189189*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x +
 c)*b + a)^2*a^5*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 105105*sqrt((sqrt(d*x
 + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^6*sgn((sqrt(d*x + c)*b + a)*b - a
*b) + 45045*sqrt((sqrt(d*x + c)*b + a)*b^2)*a^7*sgn((sqrt(d*x + c)*b + a)*b
 - a*b))/(b^8*d^4*abs(b))
```

3.647 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=222

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a\,(}{}$$

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(5*a^4 - 6*
a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2
 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2)
)/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rubi [A] time = 0.158127, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```



```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(5*a^4 - 6*
a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2
 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2)
)/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^2}{b^5\sqrt{a+bx}} + \frac{\left(5a^4-6a^2b^2c+b^4c^2\right)\sqrt{a+bx}}{b^5} - \frac{2\left(5a^3-3ab^2c\right)(a+bx)^{3/2}}{b^5} - \frac{2\left(-5a^2+b^2c\right)(a+bx)^{5/2}}{b^5} - \frac{5a(a}{}\right.\right.}{d^3}$$

$$= -\frac{4a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} - \frac{8a\left(5a^2-3b^2\right.}{}$$

Mathematica [A] time = 0.173945, size = 147, normalized size = 0.66

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(96a^3b^2(28c-5dx)-16a^2b^3(74c-25dx)\sqrt{c+dx}+640a^4b\sqrt{c+dx}-1280a^5-2ab^4\left(736c^2-244cd\right.\right.}{3465b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*a
^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sqrt
[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*x +
 175*d^2*x^2)))/(3465*b^6*d^3)
```

Maple [A] time = 0.003, size = 183, normalized size = 0.8

$$4\,\frac{1/11\,\left(a+b\sqrt{dx+c}\right)^{11/2}-5/9\,a\left(a+b\sqrt{dx+c}\right)^{9/2}+1/7\left(-2\,b^2c+10\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{7/2}+1/5\left(-4\left(-b^2c+a^2\right)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/11*(a+b*(d*x+c)^(1/2))^(11/2)-5/9*a*(a+b*(d*x+c)^(1/2))^(9/2)+
1/7*(-2*b^2*c+10*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-4*(-b^2*c+a^2)*a-a*(-
2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*((-b^2*c+a^2)^2+4*a^2*(-b^2*c
+a^2))*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(1/2)
)
```

Maxima [A] time = 1.07706, size = 225, normalized size = 1.01

$$\frac{4\left(315\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}a-990\left(b^2c-5\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+1386\left(3\,ab^2c-5\,a^3\right)\left(\sqrt{dx+cb}\right.\right.}{3465\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)
*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c - 5*
a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sq
rt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d
*x + c)*b + a))/(b^6*d^3)
```

Fricas [A] time = 2.24425, size = 342, normalized size = 1.54

$$-\frac{4\left(350\,ab^4d^2x^2+1472\,ab^4c^2-2688\,a^3b^2c+1280\,a^5-8\left(61\,ab^4c-60\,a^3b^2\right)dx-\left(315\,b^5d^2x^2+480\,b^5c^2-1184\,a^2b^3c+\right.\right.}{3465\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 - 8
*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^2*
b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.29198, size = 849, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*b^4*c^2*
sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3465*sqrt((sqrt(d*x + c)*b + a)*b^2)*a
*b^4*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 990*sqrt((sqrt(d*x + c)*b + a
)*b^2)*(sqrt(d*x + c)*b + a)^3*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 4
158*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a*b^2*c*sgn((sq
rt(d*x + c)*b + a)*b - a*b) - 6930*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*
x + c)*b + a)*a^2*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 6930*sqrt((sqr
t(d*x + c)*b + a)*b^2)*a^3*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 315*s
qrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^5*sgn((sqrt(d*x + c)*b
```

```
 + a)*b - a*b) - 1925*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)
^4*a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 4950*sqrt((sqrt(d*x + c)*b + a)*b
^2)*(sqrt(d*x + c)*b + a)^3*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 6930*s
qrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^3*sgn((sqrt(d*x +
c)*b + a)*b - a*b) + 5775*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b
+ a)*a^4*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3465*sqrt((sqrt(d*x + c)*b +
a)*b^2)*a^5*sgn((sqrt(d*x + c)*b + a)*b - a*b))/(b^6*d^3*abs(b))
```

3.648 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=131

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(
5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)
```

Rubi [A] time = 0.0939594, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(
5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{-a^3+ab^2c}{b^3\sqrt{a+bx}} + \frac{(3a^2-b^2c)\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2} +$$

Mathematica [A] time = 0.100955, size = 84, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(24a^2b\sqrt{c+dx} - 48a^3 + 2ab^2(26c-9dx) + 5b^3\sqrt{c+dx}(3dx-4c)\right)}{105b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*S
qrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)
```

Maple [A] time = 0.002, size = 94, normalized size = 0.7

$$4\,\frac{1/7\left(a+b\sqrt{dx+c}\right)^{7/2} - 3/5\,a\left(a+b\sqrt{dx+c}\right)^{5/2} + 1/3\left(-b^2c+3\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{3/2} - \left(-b^2c+a^2\right)a\sqrt{a+b\sqrt{dx+}}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/7*(a+b*(d*x+c)^(1/2))^(7/2)-3/5*a*(a+b*(d*x+c)^(1/2))^(5/2)+1/
3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)*a*(a+b*(d*x+c)^(1/2
))^(1/2))
```

Maxima [A] time = 1.10432, size = 126, normalized size = 0.96

$$\frac{4\left(15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}} - 63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a - 35\left(b^2c-3\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}} + 105\left(ab^2c-a^3\right)\sqrt{\sqrt{dx+c}b+a}\right)}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt(s
```

```
qrt(d*x + c)*b + a))/(b^4*d^2)
```

Fricas [A] time = 2.34615, size = 178, normalized size = 1.36

$$-\frac{4\left(18\,ab^2dx-52\,ab^2c+48\,a^3-\left(15\,b^3dx-20\,b^3c+24\,a^2b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b+a}}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a^
2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.22184, size = 412, normalized size = 3.15

$$-\frac{4\left(35\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)b^2c\,\mathrm{csgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-105\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}ab^2c\,\mathrm{sgn}\left(\left(\sqrt{dx+c}b+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/105*(35*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*b^2*c*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) - 105*sqrt((sqrt(d*x + c)*b + a)*b^2)*a*b^2*
c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 15*sqrt((sqrt(d*x + c)*b + a)*b^2)*(
sqrt(d*x + c)*b + a)^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 63*sqrt((sqrt(d
*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a*sgn((sqrt(d*x + c)*b + a)*b -
 a*b) - 105*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^2*sgn((
sqrt(d*x + c)*b + a)*b - a*b) + 105*sqrt((sqrt(d*x + c)*b + a)*b^2)*a^3*sgn
((sqrt(d*x + c)*b + a)*b - a*b))/(b^4*d^2*abs(b))
```

3.649 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=54

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/
(3*b^2*d)
```

Rubi [A] time = 0.0314209, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/
(3*b^2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx,x,\sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx,x,\sqrt{c+dx}\right)}{d}$$
$$= -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Mathematica [A] time = 0.0227483, size = 42, normalized size = 0.78

$$\frac{4\left(b\sqrt{c+dx}-2a\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

Maple [A] time = 0.005, size = 41, normalized size = 0.8

$$4\,\frac{1/3\,\left(a+b\sqrt{dx+c}\right)^{3/2}-a\sqrt{a+b\sqrt{dx+c}}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d/b^2*(1/3*(a+b*(d*x+c)^(1/2))^(3/2)-a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Maxima [A] time = 0.997243, size = 57, normalized size = 1.06

$$\frac{4\left(\frac{\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{\sqrt{dx+c}b+a}a}{b^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/d
```

Fricas [A] time = 2.29351, size = 85, normalized size = 1.57

$$\frac{4\sqrt{\sqrt{dx+c}b+a}\left(\sqrt{dx+c}b-2a\right)}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] time = 1.22179, size = 135, normalized size = 2.5

$$\frac{4\Big(\sqrt{(\sqrt{dx+c}b+a)b^2}(\sqrt{dx+c}b+a)\operatorname{sgn}\big((\sqrt{dx+c}b+a)b-ab\big)-3\sqrt{(\sqrt{dx+c}b+a)b^2}a\operatorname{sgn}\big((\sqrt{dx+c}b+a)b-ab}{3b^2d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x +
c)*b + a)*b - a*b) - 3*sqrt((sqrt(d*x + c)*b + a)*b^2)*a*sgn((sqrt(d*x + c)
*b + a)*b - a*b))/(b^2*d*abs(b))
```

3.650 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}} \, dx$

Optimal. Leaf size=97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Rubi [A] time = 0.0850731, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {371, 1398, 827, 1166, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\,\text{Subst}\left(\int \frac{-a+x^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)+2\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a-\right.\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}
\end{aligned}$$

Mathematica [A] time = 0.11024, size = 97, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Maple [A] time = 0.012, size = 92, normalized size = 1.

$$2\,\frac{1}{\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)+2\,\frac{1}{\sqrt{\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 2/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)
-a)^(1/2))+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2
*c)^(1/2)-a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{dx+c}b+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

Fricas [B] time = 2.44426, size = 1438, normalized size = 14.82

$$\sqrt{-\frac{\left(b^2c-a^2\right)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(4\left(\left(b^2c-a^2\right)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{\left(b^2c-a^2\right)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}+4\sqrt{\sqrt{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
- a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)
*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(
b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqr
t(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c
/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*
b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(
b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a
^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x
 + c)*b + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.651 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c
])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[
c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])
```

Rubi [A] time = 0.203434, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c
])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[
c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx &= d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x}+\frac{d\,\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{1}{2}b^2cx}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x}+\frac{(2d)\,\text{Subst}\left(\int \frac{-ab^2c+\frac{1}{2}b^2cx^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{c\left(a^2-b^2c\right)}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x}+\frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\left(a-b\sqrt{c}\right)\sqrt{c}}-\frac{(bd)}{}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.257769, size = 216, normalized size = 1.33

$$\frac{\sqrt{a-b\sqrt{c}}\left(bdx\left(a-b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-2\sqrt{c}\sqrt{a+b\sqrt{c}}\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}\right)-bdx\left(a+b\sqrt{c}\right)^{3/2}\text{ta}}{2\sqrt{c}x\sqrt{a-b\sqrt{c}}\sqrt{a+b\sqrt{c}}\left(a^2-b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-(b*(a + b*Sqrt[c])^(3/2)*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b
*Sqrt[c]]]) + Sqrt[a - b*Sqrt[c]]*(-2*Sqrt[a + b*Sqrt[c]]*Sqrt[c]*(a - b*Sq
rt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]] + b*(a - b*Sqrt[c])*d*x*ArcTanh[Sqrt
[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[a
+ b*Sqrt[c]]*Sqrt[c]*(a^2 - b^2*c)*x)
```

Maple [B] time = 0.022, size = 265, normalized size = 1.6

$$-2\,{\frac {d\sqrt {{b}^{2}c}\sqrt {a+b\sqrt {dx+c}}}{c \left(4\,\sqrt {{b}^{2}c}-4\,a \right) \left(b\sqrt {dx+c}+\sqrt {{b}^{2}c} \right) }}-2\,{\frac {d\sqrt {{b}^{2}c}}{c \left(4\,\sqrt {{b}^{2}c}-4\,a \right) \sqrt {\sqrt {{b}^{2}c}-a}}\arctan \left({\frac {\sqrt {a+b\sqrt {dx+c}}}{\sqrt {\sqrt {{b}^{2}c}-a}}} \right) }-2\,{\frac {d\sqrt {{b}^{2}c}\sqrt {a}}{c \left(-4\,\sqrt {{b}^{2}c}-4\,a \right) (}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] -2*d*(b^2*c)^(1/2)/c*(a+b*(d*x+c)^(1/2))^(1/2)/(4*(b^2*c)^(1/2)-4*a)/(b*(d*
x+c)^(1/2)+(b^2*c)^(1/2))-2*d*(b^2*c)^(1/2)/c/(4*(b^2*c)^(1/2)-4*a)/((b^2*c
)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))-
2*d*(b^2*c)^(1/2)/c*(a+b*(d*x+c)^(1/2))^(1/2)/(-4*(b^2*c)^(1/2)-4*a)/(-b*(d
*x+c)^(1/2)+(b^2*c)^(1/2))+2*d*(b^2*c)^(1/2)/c/(-4*(b^2*c)^(1/2)-4*a)/(-(b^
2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{dx + c}b + a}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

Fricas [B] time = 2.9717, size = 4913, normalized size = 30.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^
4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^
4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4
*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2
- a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6
```

```
*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)
*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 +
 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c
)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2
*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a
^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2
*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2
*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3
*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c
^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*
a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))
*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*
a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^
10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b
^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(
-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^
6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c
^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^
12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)
*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*
c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^
2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*
c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6
*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c
)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 +
6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 -
20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^
4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt
((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a
^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-
((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6
*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^
6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^1
2*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^
2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (
b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8
*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^
6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3
*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^
2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^
5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 -
 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt(sqrt(d*x + c)*b + a)*(sq
rt(d*x + c)*b - a))/((b^2*c - a^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + b\sqrt{c + dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.652 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}}$$

```
[Out] -((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(2*(a^2 - b^2*c)*x^2) -
(b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*Sqrt[c + d*x]))/(
8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*(2
*a + 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]
])/(16*(a + b*Sqrt[c])^(5/2)*c^(3/2))
```

Rubi [A] time = 0.481436, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(2*(a^2 - b^2*c)*x^2) -
(b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*Sqrt[c + d*x]))/(
8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*(2
*a + 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]
])/(16*(a + b*Sqrt[c])^(5/2)*c^(3/2))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```

```
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = d^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{a+bx}\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{5}{2}b^2cx}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\mathrm{Sub}}{}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\mathrm{Sub}}{}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} - \frac{\left(b\left(2a\right.\right.}{}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} + \frac{b\left(2a-\right.}{}$$

Mathematica [A] time = 0.69639, size = 281, normalized size = 1.08

$$-\frac{\frac{8\left(a^2-b^2c\right)\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{x^2}-\frac{2bd\sqrt{a+b\sqrt{c+dx}}\left(a^2\sqrt{c+dx}-6abc+5b^2c\sqrt{c+dx}\right)}{cx}+\frac{bd^2\left(\left(a-b\sqrt{c}\right)^{5/2}\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-\left(2a-5b\sqrt{c}\right)\left(a+\cdots\right)}{c^{3/2}\sqrt{a-b\sqrt{c}}\sqrt{a+b\sqrt{c}}}}{16\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -((8*(a^2 - b^2*c)*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/x^2 - (
2*b*d*Sqrt[a + b*Sqrt[c + d*x]]*(-6*a*b*c + a^2*Sqrt[c + d*x] + 5*b^2*c*Sqr
t[c + d*x]))/(c*x) + (b*d^2*(-((2*a - 5*b*Sqrt[c])*(a + b*Sqrt[c])^(5/2)*Ar
cTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c])^(5/
2)*(2*a + 5*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]
]]))/(Sqrt[a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c]]*c^(3/2)))/(16*(a^2 - b^2*c)^2
)
```

Maple [B] time = 0.069, size = 840, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 5/16*b^2*d^2/c/(b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a
^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b*(d*x+c)^(1/2)-
(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)*a-
7/16*b^2*d^2/c/(b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2/((b^2*c)^(1/2)+a)*(a+b*(d*
x+c)^(1/2))^(1/2)-1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b*(d*x+c)^(1/2)-(b^2*c)^(1/2
))^2/((b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2)*a+5/16*b^2*d^2/c/(b^2*c+2*
a*(b^2*c)^(1/2)+a^2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1
/2)/(-(b^2*c)^(1/2)-a)^(1/2))+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b^2*c+2*a*(b^2*c
)^(1/2)+a^2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b
^2*c)^(1/2)-a)^(1/2))*a+5/16*b^2*d^2/c/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(b
^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-1/8*b^2*d^2/c/(b^2*c)
^(1/2)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(b^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+b
*(d*x+c)^(1/2))^(3/2)*a-7/16*b^2*d^2/c/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(-
(b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2)+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b*(
d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(-(b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2)*
a-5/16*b^2*d^2/c/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/((b^2*c)^(1/2)-a)^(1/2)*arc
tan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))+1/8*b^2*d^2/c/(b^2*c
)^(1/2)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*
(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{dx+c}b+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

Fricas [B] time = 6.74201, size = 9651, normalized size = 36.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3
*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a
^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4
+ 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10
)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10
+ 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^
6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c
^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*
b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x
+ c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 +
35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*
a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c
^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6
*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^
11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^
8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*
sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b
^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 -
 a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10
780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*
b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*
a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*
c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*
b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a
*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a
^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sq
rt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*
c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 1
20*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7
- 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*
c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^1
0*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^
6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 6
09*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a
^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b
^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^1
4*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12
 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*
c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c
^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*
a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5
+ 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*
a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^
18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^1
0*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^1
```

```
8*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6
*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*
x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4
- (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c
^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2
- 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*
a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 +
210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a
^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*
a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b
^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977
*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a
^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^
4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^
3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 -
 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9
 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^
5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 -
35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6
- 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*
b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20
*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b
^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16
*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4
*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c
+ 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*
c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21
966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^
2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252
*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10
*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10
*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10
*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*
a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5
*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6
*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 +
 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)
*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 +
 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6
 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 +
70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7
+ 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^1
8*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^
8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14
*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*
b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2
*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) +
4*(6*a*b^2*c*d*x - 4*a*b^2*c^2 + 4*a^3*c + (4*b^3*c^2 - 4*a^2*b*c - (5*b^3*
c + a^2*b)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a))/((b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c + d*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.653 $\int x^3 \left(a + b\sqrt{c + dx}\right)^p \, dx$

Optimal. Leaf size=350

$$\frac{2\left(-30a^2b^2c + 35a^4 + 3b^4c^2\right)\left(a + b\sqrt{c + dx}\right)^{p+4}}{b^8d^4(p+4)} - \frac{2a\left(a^2 - b^2c\right)^3\left(a + b\sqrt{c + dx}\right)^{p+1}}{b^8d^4(p+1)} + \frac{2\left(a^2 - b^2c\right)^2\left(7a^2 - b^2c\right)\left(a + b\right.}{b^8d^4(p+2)}$$

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2
*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2
 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p)
)/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c
+ d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c
+ d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x
])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d
^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))
```

Rubi [A] time = 0.278758, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-30a^2b^2c + 35a^4 + 3b^4c^2\right)\left(a + b\sqrt{c + dx}\right)^{p+4}}{b^8d^4(p+4)} - \frac{2a\left(a^2 - b^2c\right)^3\left(a + b\sqrt{c + dx}\right)^{p+1}}{b^8d^4(p+1)} + \frac{2\left(a^2 - b^2c\right)^2\left(7a^2 - b^2c\right)\left(a + b\right.}{b^8d^4(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2
*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2
 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p)
)/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c
+ d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c
+ d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x
])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d
^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p(-c+x)^3 dx, x, c+dx\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^3 dx, x, \sqrt{c+dx}\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^3(a+bx)^p}{b^7}-\frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{1+p}}{b^7}-\frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^{2+p}}{b^7}+\frac{(35a}{}\right.\right.}{}$$
$$= -\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)}-\frac{6a\left(7a^2\right.}{}$$

Mathematica [A] time = 0.870672, size = 555, normalized size = 1.59

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(6a^3b^4\left(8c^2\left(p^4-14p^3-139p^2-124p+315\right)+40cd\left(p^4+4p^3-16p^2-61p-42\right)x+35d^2\left(p^4+1\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c + d
*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*b^
3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x) +
 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61
*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x
^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3)
+ 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 71*p^2
 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*Sqrt[c +
d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 + d^3*(48
+ 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p^2 + 38*p^
3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x +
 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 +
 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/
(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(8 + p))
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int x^3\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [B] time = 1.11213, size = 983, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^
4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b
^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4 + 1
0*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x +
 c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(
p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2
 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/((p^6 + 2
1*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7 + 28*p^6
+ 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b
^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x
+ c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d
*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5
/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^4*b^4 + 840*(p
^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*(d*x + c)*a^6*b^
2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*b + a)^p/((p^8 +
36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p
 + 40320)*b^8))/d^4
```

Fricas [B] time = 2.92685, size = 3067, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c
+ 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^7
+ 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3 +
13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 +
7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6
)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^2*b^6)
*d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959*a^2*b^
6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81*a^2*b^6
*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5 + (b^8*c
^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^4)*d^2*p^4
 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8*c^2 - 806*
a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c + a^4*b^4)*d^
2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 - 15*a^6*b^2*c
)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*a^2*b^6*c^2 -
5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b^4*c)*d*p^3 + (
176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2)*d*p^2 + 105*(b
^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (192*(a*b^7*c^3 +
a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a^5*b^3*c)*p^3 - (a
*b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 735*a*b^7*d^3*p^4 + 1
624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d^3*p)*x^3 + 192*(56*a*b
^7*c^3 - 49*a^3*b^5*c^2 + 15*a^5*b^3*c)*p^2 + 6*(2*a*b^7*c*d^2*p^6 + (33*a*
b^7*c - 7*a^3*b^5)*d^2*p^5 + 10*(20*a*b^7*c - 7*a^3*b^5)*d^2*p^4 + 5*(111*a
*b^7*c - 49*a^3*b^5)*d^2*p^3 + 2*(349*a*b^7*c - 175*a^3*b^5)*d^2*p^2 + 24*(
13*a*b^7*c - 7*a^3*b^5)*d^2*p)*x^2 + 48*(279*a*b^7*c^3 - 511*a^3*b^5*c^2 +
385*a^5*b^3*c - 105*a^7*b)*p - 24*((3*a*b^7*c^2 + a^3*b^5*c)*d*p^5 + 2*(21*
```

```
a*b^7*c^2 - 5*a^3*b^5*c)*d*p^4 + (192*a*b^7*c^2 - 135*a^3*b^5*c + 35*a^5*b^
3)*d*p^3 + (327*a*b^7*c^2 - 320*a^3*b^5*c + 105*a^5*b^3)*d*p^2 + 2*(87*a*b^
7*c^2 - 98*a^3*b^5*c + 35*a^5*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b
+ a)^p/(b^8*d^4*p^8 + 36*b^8*d^4*p^7 + 546*b^8*d^4*p^6 + 4536*b^8*d^4*p^5 +
 22449*b^8*d^4*p^4 + 67284*b^8*d^4*p^3 + 118124*b^8*d^4*p^2 + 109584*b^8*d^
4*p + 40320*b^8*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.654 $\int x^2 \left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=242

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}$$

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2
*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2
 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3
+ p)) + (4*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p))
 - (10*a*(a + b*Sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[
c + d*x])^(6 + p))/(b^6*d^3*(6 + p))
```

Rubi [A] time = 0.18244, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2
*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2
 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3
+ p)) + (4*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p))
 - (10*a*(a + b*Sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[
c + d*x])^(6 + p))/(b^6*d^3*(6 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p(-c+x)^2\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^2\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^2(a+bx)^p}{b^5}+\frac{\left(5a^4-6a^2b^2c+b^4c^2\right)(a+bx)^{1+p}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)(a+bx)^{2+p}}{b^5}-\frac{2\left(-5a^2+b^2c\right)(a+bx}{b^5}\right.\right.}{d^3}$$

$$= -\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)}-\frac{4a\left(5a^2-\right.}{}$$

Mathematica [A] time = 0.388854, size = 284, normalized size = 1.17

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(12a^3b^2\left(4c\left(p^2+p-5\right)+5d\left(p^2+3p+2\right)x\right)-4a^2b^3(p+1)\sqrt{c+dx}\left(2c\left(p^2-4p-30\right)+5d\left(p^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*x
] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(1
+ p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5*(1
5 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6
*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 -
 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)
*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int x^2\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 1.145, size = 543, normalized size = 2.24

$$2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^pc^2}{\left(p^2+3p+2\right)b^2}-\frac{2\left(\left(p^3+6p^2+11p+6\right)(dx+c)^2b^4+\left(p^3+3p^2+2p\right)(dx+c)^{\frac{3}{2}}ab^3-3\left(p^2+p\right)(dx+c)a^2b^2+6\sqrt{dx+c}a^3bp-6a^4\right)\left(\sqrt{\right.}{\left(p^4+10p^3+35p^2+50p+24\right)b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p
^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p +
 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(
5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 +
 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 12
0*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 + 21*p^5 +
 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

Fricas [B] time = 2.45443, size = 1492, normalized size = 6.17

$$\frac{2\left(120\,b^6c^3 - 360\,a^2b^4c^2 + 360\,a^4b^2c - 120\,a^6 + 8\left(b^6c^3 + 3\,a^2b^4c^2\right)p^3 + \left(b^6d^3p^5 + 15\,b^6d^3p^4 + 85\,b^6d^3p^3 + 225\,b^6d^3p^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 + 3
*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^
6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4*c^
2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (4
1*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c
 - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p -
4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6*
c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^
2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3*a^3*b^3*
c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^
2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p
- 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3
*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x +
c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735*b^6*d^3*p
^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.655 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=145

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 + p)) + (2*(
3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*(2 + p)) - (6*a*(a +
 b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^(4
+ p))/(b^4*d^2*(4 + p))
```

Rubi [A] time = 0.108389, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^p,x]
```


```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 + p)) + (2*(
3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*(2 + p)) - (6*a*(a +
 b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^(4
+ p))/(b^4*d^2*(4 + p))
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\sqrt{c+dx}\right)^p dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p(-c+x)\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-a^3+ab^2c)(a+bx)^p}{b^3} + \frac{(3a^2-b^2c)(a+bx)^{1+p}}{b^3} - \frac{3a(a+bx)^{2+p}}{b^3} + \frac{(a+bx)^{3+p}}{b^3}\right) dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)} + \frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)} - \frac{6a\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^4d^2(3+p)}$$

Mathematica [A] time = 0.168909, size = 128, normalized size = 0.88

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(-6a^2b(p+1)\sqrt{c+dx}+6a^3+ab^2\left(2c\left(p^2+p-3\right)+3d\left(p^2+3p+2\right)x\right)-b^3\left(p^2+4p+3\right)\sqrt{c+d}\right.}{b^4d^2(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 + p
 + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 + p)
)
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int x\left(a+b\sqrt{dx+c}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 1.14836, size = 252, normalized size = 1.74

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p c}{\left(p^2+3p+2\right)b^2} - \frac{\left(\left(p^3+6p^2+11p+6\right)(dx+c)^2b^4+\left(p^3+3p^2+2p\right)(dx+c)^{\frac{3}{2}}ab^3-3\left(p^2+p\right)(dx+c)a^2b^2+6\sqrt{dx+c}a^3bp-6a^4\right)\left(\sqrt{d}\right.}{\left(p^4+10p^3+35p^2+50p+24\right)b^4}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 +
(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 +
```

```
 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 +
35*p^2 + 50*p + 24)*b^4))/d^2
```

Fricas [B] time = 2.09734, size = 599, normalized size = 4.13

$$-\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\left(b^4c^2+a^2b^2c\right)p^2-\left(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2\right)x^2+4\left(2\,b^4c^2-a^2b^2c\right)p-\right.}{b^4d^2p^4+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*d
^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a^2
*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b^2
)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*
b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^4*d^2
*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x*(a + b*sqrt(c + d*x))**p, x)
```

Giac [B] time = 2.57629, size = 5399, normalized size = 37.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*
b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b^2*c*p^3*sgn((sqr
t(d*x + c)*b + a)*b - a*b) - ((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b +
a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2*b^2*c*p^3*sgn
((sqrt(d*x + c)*b + a)*b - a*b) + (sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b
 + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b -
a*b) + a)^p*b^2*c*p^3 - 2*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^
p*a*b^2*c*p^3 + ((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) -
 a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2*b^2*c*p^3 + 7*(sqrt(d*x +
c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn
((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b^2*c*p^2*sgn((sqrt(d*x + c)*b + a
)*b - a*b) - 7*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) -
a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2*b^2*c*p^2*sgn((sqrt(d*x + c
```

```
)*b + a)*b - a*b) - (sqrt(d*x + c)*b + a)^3*((sqrt(d*x + c)*b + a)*sgn((sqr
t(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*
p^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 3*(sqrt(d*x + c)*b + a)^2*((sqrt(d
*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b
 + a)*b - a*b) + a)^p*a^2*p^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3*(sqrt(d
*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) -
a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^3*p^3*sgn((sqrt(d*x + c)*b +
a)*b - a*b) + ((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a
*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^4*p^3*sgn((sqrt(d*x + c)*b + a
)*b - a*b) + 8*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x
 + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*b^2*c*p
^2 - 16*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b +
 a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b^2*c*p^2 + 7*
((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x
 + c)*b + a)*b - a*b) + a)^p*a^2*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*((sqrt
(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*
b + a)*b - a*b) + a)^p*p^3 + 4*(sqrt(d*x + c)*b + a)^3*((sqrt(d*x + c)*b +
a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b
) + a)^p*a*p^3 - 6*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt
(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2
*p^3 + 4*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b
 + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^3*p^3 - ((sqr
t(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)
*b + a)*b - a*b) + a)^p*a^4*p^3 + 12*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*
b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b -
 a*b) + a)^p*a*b^2*c*p*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 12*((sqrt(d*x +
 c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)
*b - a*b) + a)^p*a^2*b^2*c*p*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3*(sqrt(d
*x + c)*b + a)^3*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*p^2*sgn((sqrt(d*x + c)*b +
a)*b - a*b) + 9*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*
x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2*p^
2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 9*(sqrt(d*x + c)*b + a)*((sqrt(d*x +
 c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)
*b - a*b) + a)^p*a^3*p^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 3*((sqrt(d*x
+ c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a
)*b - a*b) + a)^p*a^4*p^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 19*(sqrt(d*x
 + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) -
a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*b^2*c*p - 38*(sqrt(d*x + c)*b +
 a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt
(d*x + c)*b + a)*b - a*b) + a)^p*a*b^2*c*p + 12*((sqrt(d*x + c)*b + a)*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^
p*a^2*b^2*c*p - 6*(sqrt(d*x + c)*b + a)^4*((sqrt(d*x + c)*b + a)*sgn((sqrt(
d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*p^2
+ 24*(sqrt(d*x + c)*b + a)^3*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b +
a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*p^2 - 33*(sqrt(
d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2*p^2 + 18*(sqrt(d*x + c)*
b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((s
qrt(d*x + c)*b + a)*b - a*b) + a)^p*a^3*p^2 - 3*((sqrt(d*x + c)*b + a)*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^
p*a^4*p^2 - 2*(sqrt(d*x + c)*b + a)^3*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x
+ c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*p*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) + 6*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*
b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b -
 a*b) + a)^p*a^2*p*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 12*(sqrt(d*x + c)*b
 + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sq
rt(d*x + c)*b + a)*b - a*b) + a)^p*a^3*p*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 + 8*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqr
```

```
t(d*x + c)*b + a)*b - a*b) + a)^p*a^4*p*sgn((sqrt(d*x + c)*b + a)*b - a*b)
+ 12*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b +
a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*b^2*c - 24*(sqrt(
d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) -
 a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b^2*c - 11*(sqrt(d*x + c)*b
+ a)^4*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((s
qrt(d*x + c)*b + a)*b - a*b) + a)^p*p + 44*(sqrt(d*x + c)*b + a)^3*((sqrt(d
*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b
+ a)*b - a*b) + a)^p*a*p - 63*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a
)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 + a)^p*a^2*p + 38*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d
*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^3*p
 - 8*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqr
t(d*x + c)*b + a)*b - a*b) + a)^p*a^4*p - 6*(sqrt(d*x + c)*b + a)^4*((sqrt(
d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b
 + a)*b - a*b) + a)^p + 24*(sqrt(d*x + c)*b + a)^3*((sqrt(d*x + c)*b + a)*s
gn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) +
a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x +
c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2 + 24*(
sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a
*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^3)/((b*p^4*sgn((sqrt(d*
x + c)*b + a)*b - a*b) + 10*b*p^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 35*b
*p^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 50*b*p*sgn((sqrt(d*x + c)*b + a)*
b - a*b) + 24*b*sgn((sqrt(d*x + c)*b + a)*b - a*b))*b^3*d^2)
```

3.656 $\int \left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=62

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d
*x])^(2 + p))/(b^2*d*(2 + p))
```

Rubi [A] time = 0.0401228, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {247, 190, 43}

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d
*x])^(2 + p))/(b^2*d*(2 + p))
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^p\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

Mathematica [A] time = 0.0358123, size = 53, normalized size = 0.85

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(b(p+1)\sqrt{c+dx}-a\right)}{b^2d(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
+ p)*(2 + p))
```

Maple [F] time = 0.003, size = 0, normalized size = 0.

$$\int \left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int((a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 1.13776, size = 81, normalized size = 1.31

$$\frac{2\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p}{\left(p^2+3p+2\right)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)
^p/((p^2 + 3*p + 2)*b^2*d)
```

Fricas [A] time = 2.01997, size = 174, normalized size = 2.81

$$\frac{2\left(b^2cp+\sqrt{dx+c}abp+b^2c-a^2+\left(b^2dp+b^2d\right)x\right)\left(\sqrt{dx+c}b+a\right)^p}{b^2dp^2+3\,b^2dp+2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqrt
(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p, x)
```

Giac [B] time = 1.4505, size = 817, normalized size = 13.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b
 - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b*p*sgn((sqrt(d*x +
 c)*b + a)*b - a*b) - ((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b -
a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a^2*b*p*sgn((sqrt(d*x +
c)*b + a)*b - a*b) + (sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a)*sgn((sq
rt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*b
*p - 2*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b +
a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b*p + ((sqrt(d*
x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b +
 a)*b - a*b) + a)^p*a^2*b*p + (sqrt(d*x + c)*b + a)^2*((sqrt(d*x + c)*b + a
)*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 + a)^p*b - 2*(sqrt(d*x + c)*b + a)*((sqrt(d*x + c)*b + a)*sgn((sqrt(d*x +
c)*b + a)*b - a*b) - a*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a)^p*a*b)/((p^2
*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 3*p*sgn((sqrt(d*x + c)*b + a)*b - a*b
) + 2*sgn((sqrt(d*x + c)*b + a)*b - a*b))*b^3*d)
```

3.657 $\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$

Optimal. Leaf size=139

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*
Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 + p))) - ((a + b*Sqrt[
c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/
(a + b*Sqrt[c])])/((a + b*Sqrt[c])*(1 + p))
```

Rubi [A] time = 0.13054, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {371, 1398, 831, 68}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*
Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 + p))) - ((a + b*Sqrt[
c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/
(a + b*Sqrt[c])])/((a + b*Sqrt[c])*(1 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^p}{-c+x}\,dx,x,c+dx\right)\\ &= 2\,\text{Subst}\left(\int \frac{x(a+bx)^p}{-c+x^2}\,dx,x,\sqrt{c+dx}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{(a+bx)^p}{2\left(\sqrt{c}-x\right)}+\frac{(a+bx)^p}{2\left(\sqrt{c}+x\right)}\right)dx,x,\sqrt{c+dx}\right)\\ &= -\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}-x}\,dx,x,\sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}+x}\,dx,x,\sqrt{c+dx}\right)\\ &= -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)}-\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0995351, size = 136, normalized size = 0.98

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\left(\left(a+b\sqrt{c}\right)\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)\right)}{(p+1)\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hypergeo
metric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])]))/((a - b
*Sqrt[c])*(a + b*Sqrt[c])*(1 + p)))
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^p/x,x)
```

```
[Out] int((a+b*(d*x+c)^(1/2))^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\sqrt{dx+c}b+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((sqrt(d*x + c)*b + a)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p/x,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

3.658 $\int \frac{(a+b(cx)^n)^{5/2}}{x} \, dx$

Optimal. Leaf size=93

$$\frac{2a^2\sqrt{a+b(cx)^n}}{n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a\left(a+b(cx)^n\right)^{3/2}}{3n} + \frac{2\left(a+b(cx)^n\right)^{5/2}}{5n}$$

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a +
 b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])
/n
```

Rubi [A] time = 0.0752527, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2a^2\sqrt{a+b(cx)^n}}{n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a\left(a+b(cx)^n\right)^{3/2}}{3n} + \frac{2\left(a+b(cx)^n\right)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a +
 b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])
/n
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b
\right.}{bn}\\
&= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.0780542, size = 77, normalized size = 0.83

$$\frac{2\sqrt{a+b(cx)^n}\left(23a^2+11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a
^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A] time = 0.006, size = 70, normalized size = 0.8

$$\frac{1}{n}\left(\frac{2}{5}\left(a+b\,(cx)^n\right)^{\frac{5}{2}}+\frac{2\,a}{3}\left(a+b\,(cx)^n\right)^{\frac{3}{2}}+2\sqrt{a+b\,(cx)^n}a^2-2\,a^{5/2}\text{Artanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/2
)*a^2-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b + a\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Fricas [A] time = 1.84996, size = 393, normalized size = 4.23

$$\left[\frac{15\, a^{\frac{5}{2}} \log\left(\frac{(cx)^n b - 2\, \sqrt{(cx)^n b + a}\sqrt{a} + 2\, a}{(cx)^n}\right) + 2\left(11\, (cx)^n ab + 3\, (cx)^{2\, n} b^2 + 23\, a^2\right)\sqrt{(cx)^n\, b + a}}{15\, n}, \frac{2\left(15\, \sqrt{-a}a^2 \arctan\left(\frac{\sqrt{(cx)^n b + a}\sqrt{-a}}{a}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*
x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a)
)/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*(c*
x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] time = 73.9411, size = 124, normalized size = 1.33

$$-\begin{cases} -\left(a^2\sqrt{a+b} + 2ab\sqrt{a+b} + b^2\sqrt{a+b}\right)\log(cx) & \text{for}\ n = 0 \\ -\frac{\frac{2a^3 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2a^2\sqrt{a+b(cx)^n} + \frac{2a\left(a+b(cx)^n\right)^{\frac{3}{2}}}{3} + \frac{2\left(a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] -Piecewise((-(a**2*sqrt(a + b) + 2*a*b*sqrt(a + b) + b**2*sqrt(a + b))*log(
c*x), Eq(n, 0)), (-(2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) + 2
*a**2*sqrt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*x)
**n)**(5/2)/5)/n, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b + a\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

3.659 $\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx$

Optimal. Leaf size=70

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\left(a+b(cx)^n\right)^{3/2}}{3n}$$

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*
ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi [A] time = 0.0572577, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\left(a+b(cx)^n\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*
ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.0433458, size = 61, normalized size = 0.87

$$\frac{2\sqrt{a+b(cx)^n}\,(4a+b(cx)^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c*
x)^n]/Sqrt[a]])/(3*n)
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{1}{n}\left(\frac{2}{3}\left(a+b\,(cx)^n\right)^{\frac{3}{2}} + 2\,a\sqrt{a+b\,(cx)^n} - 2\,a^{3/2}\text{Artanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] 1/n*(2/3*(a+b*(c*x)^n)^(3/2)+2*a*(a+b*(c*x)^n)^(1/2)-2*a^(3/2)*arctanh((a+b
*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b + a\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Fricas [A] time = 1.84232, size = 309, normalized size = 4.41

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a}\sqrt{a} + 2\,a}{(cx)^n}\right) + 2\left((cx)^n b + 4\,a\right)\sqrt{(cx)^n b + a}}{3\,n}, \frac{2\left(3\sqrt{-a}a\arctan\left(\frac{\sqrt{(cx)^n b + a}\sqrt{-a}}{a}\right) + \left((cx)^n b + 4\,a\right)\sqrt{(cx)}\right.}{3\,n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)
^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arctan(
sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] time = 49.312, size = 102, normalized size = 1.46

$$\begin{cases} \frac{-a\left(-\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 2\sqrt{a+b(cx)^n}\right) - b\left(\begin{cases} -\sqrt{a}\,(cx)^n & \text{for}\ b = 0 \\ -\frac{2\left(a+b(cx)^n\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)}{n} & \text{for}\ n \neq 0 \\ \left(a\sqrt{a+b} + b\sqrt{a+b}\right)\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((-a*(-2*a*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) - 2*sqrt(
a + b*(c*x)**n)) - b*Piecewise((-sqrt(a)*(c*x)**n, Eq(b, 0)), (-2*(a + b*(c
*x)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b)
)*log(x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b + a\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

3.660 $\int \frac{\sqrt{a+b(cx)^n}}{x} dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

Rubi [A] time = 0.0415735, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{a+bx^n}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.0214182, size = 47, normalized size = 0.96

$$\frac{2\sqrt{a+b(cx)^n} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Maple [A] time = 0.003, size = 40, normalized size = 0.8

$$\frac{1}{n}\left(2\sqrt{a+b(cx)^n} - 2\sqrt{a}\,\text{Artanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] 1/n*(2*(a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(cx)^n b + a}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

Fricas [A] time = 1.7595, size = 244, normalized size = 4.98

$$\left[\frac{\sqrt{a}\log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a}\sqrt{a} + 2a}{(cx)^n}\right) + 2\sqrt{(cx)^n b + a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b + a}\sqrt{-a}}{a}\right) + \sqrt{(cx)^n b + a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a
) + sqrt((c*x)^n*b + a))/n]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\,(cx)^n}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x)**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(cx)^n b + a}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

3.661 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] time = 0.0315944, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Mathematica [A] time = 0.0194779, size = 30, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$-2\,\frac{1}{n\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^n\,b+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Fricas [A] time = 1.83479, size = 185, normalized size = 6.17

$$\left[\frac{\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n)
, 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\,(cx)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^n\,b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

3.662 $\int \frac{1}{x(a+b(cx)^n)^{3/2}} dx$

Optimal. Leaf size=52

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Rubi [A] time = 0.046182, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{3/2}}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{n} \\
&= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{an} \\
&= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{abn} \\
&= \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}
$$

Mathematica [C] time = 0.0286098, size = 41, normalized size = 0.79

$$
\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(cx)^n}{a}+1\right)}{an\sqrt{a+b(cx)^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*(c*x)^n)/a])/(a*n*Sqrt[a + b*(c*x
)^n])
```

Maple [A] time = 0.005, size = 43, normalized size = 0.8

$$
\frac{1}{n}\left(-2\,\frac{1}{a^{3/2}}\text{Artanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)+2\,\frac{1}{a\sqrt{a+b\,(cx)^n}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^n)^(3/2),x)
```

```
[Out] 1/n*(-2/a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))+2/a/(a+b*(c*x)^n)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

Fricas [A] time = 1.78772, size = 378, normalized size = 7.27

$$\left[\frac{\left((cx)^n\sqrt{a}b+a^{\frac{3}{2}}\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}a}{(cx)^n a^2bn+a^3n}, \frac{2\left(\left((cx)^n\sqrt{-a}b+\sqrt{-a}a\right)\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right)+\sqrt{(cx)}\right.}{(cx)^n a^2bn+a^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt
(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n), 2
*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)
+ sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

Sympy [A] time = 6.01044, size = 48, normalized size = 0.92

$$\frac{2}{an\sqrt{a+b\,(cx)^n}}+\frac{2\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] 2/(a*n*sqrt(a + b*(c*x)**n)) + 2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a*n*s
qrt(-a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

3.663 $\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{3an\,(a+b(cx)^n)^{3/2}}$$

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan
h[Sqrt[a + b*(c*x)^n/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] time = 0.0642267, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{3an\,(a+b(cx)^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{5/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{a^2n}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{a^2bn}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}$$

Mathematica [C] time = 0.0339142, size = 43, normalized size = 0.57

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(cx)^n}{a}+1\right)}{3an\left(a+b(cx)^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*(c*x)^n)/a])/(3*a*n*(a + b*(c*x)
^n)^(3/2))
```

Maple [A] time = 0.007, size = 59, normalized size = 0.8

$$\frac{1}{n}\left(2\frac{1}{\sqrt{a+b\,(cx)^n}\,a^2}+\frac{2}{3\,a}\left(a+b\,(cx)^n\right)^{-\frac{3}{2}}-2\,\frac{1}{a^{5/2}}\text{Artanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^n)^(5/2),x)
```

```
[Out] 1/n*(2/a^2/(a+b*(c*x)^n)^(1/2)+2/3/a/(a+b*(c*x)^n)^(3/2)-2/a^(5/2)*arctanh(
(a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b + a\right)^{\frac{5}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Fricas [B] time = 1.97957, size = 603, normalized size = 8.04

$$\left[\frac{3\left(2\,(cx)^n a^{\frac{3}{2}} b + (cx)^{2n}\sqrt{a}b^2 + a^{\frac{5}{2}}\right)\log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a}\sqrt{a} + 2a}{(cx)^n}\right) + 2\left(3\,(cx)^n ab + 4a^2\right)\sqrt{(cx)^n b + a}}{3\left(2\,(cx)^n a^4 bn + (cx)^{2n} a^3 b^2 n + a^5 n\right)}, \frac{2\left(3\left(2\,(cx)^n \sqrt{-a}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*x
)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b +
4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^
5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(-a)*
a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*sqrt(
(c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

Sympy [A] time = 11.8503, size = 70, normalized size = 0.93

$$\frac{2}{3an\left(a + b\,(cx)^n\right)^{\frac{3}{2}}} + \frac{2}{a^2 n\sqrt{a + b\,(cx)^n}} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{a^2 n\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] 2/(3*a*n*(a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(a + b*(c*x)**n)) + 2*ata
n(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b + a\right)^{\frac{5}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

3.664 $\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx$

Optimal. Leaf size=101

$$\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\left(b(cx)^n-a\right)^{3/2}}{3n}+\frac{2\left(b(cx)^n-a\right)^{5/2}}{5n}$$

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-
a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a
]])/n
```

Rubi [A] time = 0.0730843, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\left(b(cx)^n-a\right)^{3/2}}{3n}+\frac{2\left(b(cx)^n-a\right)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-
a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a
]])/n
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a\,\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a\cdots}\right)}{bn}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.0802183, size = 81, normalized size = 0.8

$$\frac{2\sqrt{b(cx)^n-a}\left(23a^2-11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A] time = 0.009, size = 86, normalized size = 0.9

$$-\frac{2a}{3n}\left(-a+b(cx)^n\right)^{\frac{3}{2}}+\frac{2}{5n}\left(-a+b(cx)^n\right)^{\frac{5}{2}}-2\frac{a^{5/2}}{n}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)+2\frac{a^2\sqrt{-a+b(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((
-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b - a\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Fricas [A] time = 1.98703, size = 392, normalized size = 3.88

$$\left[\frac{15\sqrt{-a}a^2\log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b - a}\sqrt{-a} - 2a}{(cx)^n}\right) - 2\left(11\,(cx)^n ab - 3\,(cx)^{2n} b^2 - 23\,a^2\right)\sqrt{(cx)^n b - a}}{15n}, -\frac{2\left(15\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{(cx)^n b -}}{\sqrt{a}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*
a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*
b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(c*x
)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

Sympy [A] time = 69.661, size = 117, normalized size = 1.16

$$-\begin{cases} -\left(a^2\sqrt{-a+b} - 2ab\sqrt{-a+b} + b^2\sqrt{-a+b}\right)\log(cx) & \text{for } n = 0 \\ -\frac{-2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right) + 2a^2\sqrt{-a+b(cx)^n} - \frac{2a\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3} + \frac{2\left(-a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] -Piecewise((-(a**2*sqrt(-a + b) - 2*a*b*sqrt(-a + b) + b**2*sqrt(-a + b))*l
og(c*x), Eq(n, 0)), (-(-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*
a**2*sqrt(-a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c*
x)**n)**(5/2)/5)/n, True))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b - a\right)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

3.665 $\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/
2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi [A] time = 0.0570967, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/
2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{3/2}}{x}\,dx,x,cx\right)}{c}\\ &= \text{Subst}\left(\int \frac{(-a+bx^n)^{3/2}}{x}\,dx,x,cx\right)\\ &= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx,x,(cx)^n\right)}{n}\\ &= \frac{2(-a+b(cx)^n)^{3/2}}{3n}-\frac{a\,\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx,x,(cx)^n\right)}{n}\\ &= -\frac{2a\sqrt{-a+b(cx)^n}}{n}+\frac{2(-a+b(cx)^n)^{3/2}}{3n}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx,x,(cx)^n\right)}{n}\\ &= -\frac{2a\sqrt{-a+b(cx)^n}}{n}+\frac{2(-a+b(cx)^n)^{3/2}}{3n}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+b(cx)^n}\right)}{bn}\\ &= -\frac{2a\sqrt{-a+b(cx)^n}}{n}+\frac{2(-a+b(cx)^n)^{3/2}}{3n}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0482343, size = 66, normalized size = 0.87

$$\frac{6a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)-2(4a-b(cx)^n)\sqrt{b(cx)^n-a}}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*(
c*x)^n]/Sqrt[a]])/(3*n)
```

Maple [A] time = 0.004, size = 65, normalized size = 0.9

$$\frac{2}{3n}\left(-a+b(cx)^n\right)^{\frac{3}{2}}+2\frac{a^{3/2}}{n}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)-2\frac{a\sqrt{-a+b(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] 2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n
-2*a*(-a+b*(c*x)^n)^(1/2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b - a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Fricas [A] time = 1.57029, size = 306, normalized size = 4.03

$$\left[\frac{3\sqrt{-a}a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\left((cx)^n b-4a\right)}{3n}, \frac{2\left(3a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n b-a}\left((cx)^n b\right.\right.}{3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(
c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arctan
(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n]
```

Sympy [A] time = 44.4081, size = 97, normalized size = 1.28

$$-\begin{cases}\left(a\sqrt{-a+b}-b\sqrt{-a+b}\right)\log(x) & \text{for } n=0\\ \frac{-a\left(2\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)-2\sqrt{-a+b(cx)^n}\right)+b\left(\begin{cases}-\sqrt{-a}\,(cx)^n & \text{for } b=0\\ -\frac{2\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] -Piecewise(((a*sqrt(-a + b) - b*sqrt(-a + b))*log(x), Eq(n, 0)), ((-a*(2*sq
rt(a)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*sqrt(-a + b*(c*x)**n)) + b*Pi
ecewise((-sqrt(-a)*(c*x)**n, Eq(b, 0)), (-2*(-a + b*(c*x)**n)**(3/2)/(3*b),
 True)))/n, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b - a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

3.666 $\int \frac{\sqrt{-a+b(cx)^n}}{x} dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{b(cx)^n - a}}{n} - \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

Rubi [A] time = 0.0414853, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2\sqrt{b(cx)^n - a}}{n} - \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)}{c}\\ &= \text{Subst}\left(\int \frac{\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\ &= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\ &= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\ &= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0223945, size = 51, normalized size = 0.96

$$\frac{2\sqrt{b(cx)^n-a}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Maple [A] time = 0.003, size = 46, normalized size = 0.9

$$-2\,\frac{\sqrt{a}}{n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)+2\,\frac{\sqrt{-a+b\,(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(-a+b*(c*x)^n)^(1/2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(cx)^n b - a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

Fricas [A] time = 1.60105, size = 243, normalized size = 4.58

$$\left[\frac{\sqrt{-a}\log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b - a}\sqrt{-a} - 2a}{(cx)^n}\right) + 2\sqrt{(cx)^n b - a}}{n}, -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right) - \sqrt{(cx)^n b - a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
+ 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
 - sqrt((c*x)^n*b - a))/n]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a + b (cx)^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(cx)^n b - a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

3.667 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] time = 0.031478, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {367, 12, 266, 63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Mathematica [A] time = 0.0202475, size = 32, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$2\,\frac{1}{n\sqrt{a}}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b*(c*x)^n)^(1/2),x)
```

```
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^n\,b-a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Fricas [A] time = 1.58977, size = 184, normalized size = 5.75

$$\left[-\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)}{an}, \frac{2\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)}{\sqrt{a}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)/
(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^n b-a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

3.668 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Rubi [A] time = 0.0442907, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 51, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{3/2}}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{3/2}}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx,x,(cx)^n\right)}{n}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx,x,(cx)^n\right)}{an}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+b(cx)^n}\right)}{abn}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [C] time = 0.0308177, size = 44, normalized size = 0.79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{b(cx)^n}{a}\right)}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*(c*x)^n)/a])/(a*n*Sqrt[-a + b*(c
*x)^n])
```

Maple [A] time = 0.005, size = 49, normalized size = 0.9

$$-2\,\frac{1}{a^{3/2}n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)-2\,\frac{1}{an\sqrt{-a+b\,(cx)^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b*(c*x)^n)^(3/2),x)
```

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(-a+b*(c*x)^n)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((cx)^n b - a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

Fricas [A] time = 1.5589, size = 378, normalized size = 6.75

$$\left[-\frac{\left((cx)^n \sqrt{-a}b - \sqrt{-a}a\right) \log\left(\frac{(cx)^n b + 2\sqrt{(cx)^n b - a}\sqrt{-a} - 2a}{(cx)^n}\right) + 2\sqrt{(cx)^n b - a}a}{(cx)^n a^2 bn - a^3 n}, -\frac{2\left(\left((cx)^n \sqrt{a}b - a^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right) + \sqrt{(\right.}{(cx)^n a^2 bn - a^3 n} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)
*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3
*n), -2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
+ sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

Sympy [A] time = 7.75545, size = 44, normalized size = 0.79

$$-\frac{2}{an\sqrt{-a + b(cx)^n}} - \frac{2 \operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

```
[Out] -2/(a*n*sqrt(-a + b*(c*x)**n)) - 2*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**
(3/2)*n)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((cx)^n b - a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

3.669 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2}{a^2n\sqrt{b(cx)^n-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}-\frac{2}{3an\left(b(cx)^n-a\right)^{3/2}}$$

```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*Arc
Tan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] time = 0.0609469, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 51, 63, 205}

$$\frac{2}{a^2n\sqrt{b(cx)^n-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}-\frac{2}{3an\left(b(cx)^n-a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*Arc
Tan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{5/2}}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{5/2}}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{5/2}}\,dx,x,(cx)^n\right)}{n}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx,x,(cx)^n\right)}{an}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{-a+b(cx)^n}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx,x,(cx)^n\right)}{a^2n}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{-a+b(cx)^n}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+b(cx)^n}\right)}{a^2bn}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{-a+b(cx)^n}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}$$

Mathematica [C] time = 0.0382383, size = 46, normalized size = 0.57

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b(cx)^n}{a}\right)}{3an\left(b(cx)^n-a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*(c*x)^n)/a])/(3*a*n*(-a + b*(c*
x)^n)^(3/2))
```

Maple [A] time = 0.006, size = 70, normalized size = 0.9

$$-\frac{2}{3\,an}\left(-a+b\,(cx)^n\right)^{-\frac{3}{2}}+2\,\frac{1}{a^{5/2}n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)+2\,\frac{1}{a^2n\sqrt{-a+b\,(cx)^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b*(c*x)^n)^(5/2),x)
```

```
[Out] -2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2
)/n+2/a^2/n/(-a+b*(c*x)^n)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

Fricas [A] time = 1.55115, size = 602, normalized size = 7.43

$$\left[-\frac{3\left(2\,(cx)^n\,\sqrt{-a}ab-(cx)^{2n}\,\sqrt{-a}b^2-\sqrt{-a}a^2\right)\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b-a}\sqrt{-a}-2\,a}{(cx)^n}\right)+2\left(3\,(cx)^n\,ab-4\,a^2\right)\sqrt{(cx)^n\,b-a}}{3\left(2\,(cx)^n\,a^4bn-(cx)^{2n}\,a^3b^2n-a^5n\right)},\ \frac{2\left(3\left(2\,(c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2)
*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(c*
x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3
*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2 - a^
(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*sqrt((
c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

Sympy [A] time = 10.965, size = 63, normalized size = 0.78

$$-\frac{2}{3an\left(-a+b\,(cx)^n\right)^{\frac{3}{2}}}+\frac{2}{a^2n\sqrt{-a+b\,(cx)^n}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(-a + b*(c*x)**n)) + 2*
atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

3.670 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0071878, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0048364, size = 23, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$-2\frac{1}{\sqrt{a}}\operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52874, size = 142, normalized size = 6.17

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A] time = 1.02018, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A] time = 1.15868, size = 28, normalized size = 1.22

$$\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.671 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rubi [A] time = 0.031459, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^m}}\,dx,x,cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^m}}\,dx,x,cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^m\right)}{m}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^m}\right)}{bm}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

Mathematica [A] time = 0.0319271, size = 30, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$-2\,\frac{1}{m\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\,(cx)^m}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^m)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^m\,b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Fricas [A] time = 1.50127, size = 185, normalized size = 6.17

$$\left[\frac{\log\left(\frac{(cx)^m b-2\sqrt{(cx)^m b+a}\sqrt{a}+2a}{(cx)^m}\right)}{\sqrt{a}m}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^m b+a}\sqrt{-a}}{a}\right)}{am}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m)
, 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^m b+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

3.672 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rubi [A] time = 0.166927, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx, x, (dx)^m\right)}{m}$$
$$= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{cm}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{m}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c(dx)^m\right)^n\right)}{mn}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c(dx)^m\right)^n}\right)}{bmn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

Mathematica [A] time = 0.0658406, size = 37, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Maple [A] time = 0.006, size = 32, normalized size = 0.9

$$-2\,\frac{1}{nm\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\left(c\,(dx)^m\right)^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left((dx)^m\,c\right)^n b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)

Fricas [A] time = 1.58939, size = 300, normalized size = 8.11

$$\left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))} - 2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\sqrt{a} + 2a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}mn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{be^{(mn\log(dx)+n\log(c))}+a}}{a}\right.}{amn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")

[Out] [log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + b\left(c\left(dx\right)^{m}\right)^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left(dx\right)^{m}c\right)^{n}b + a}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)

3.673 $\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rubi [A] time = 0.372721, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^n)^p}}\,dx,x,(ex)^m\right)}{m}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^p}}\,dx,x,\left(d(ex)^m\right)^n\right)}{mn}\\
&= \frac{\operatorname{Subst}\left(\int \frac{c}{x\sqrt{a+bx^p}}\,dx,x,c\left(d(ex)^m\right)^n\right)}{cmn}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx^p}}\,dx,x,c\left(d(ex)^m\right)^n\right)}{mn}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\left(c\left(d(ex)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}\right)}{bmnp}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}
\end{aligned}
$$

Mathematica [A] time = 0.146035, size = 44, normalized size = 1.

$$
-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Maple [A] time = 0.015, size = 39, normalized size = 0.9

$$
-2\,\frac{1}{pnm\sqrt{a}}\operatorname{Artanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(ex\right)^{m}\right)^{n}\right)^{p}}}{\sqrt{a}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

Fricas [A] time = 1.56798, size = 386, normalized size = 8.77

$$\left[\frac{\log\left(\left(be^{(mnp\log(ex)+np\log(d)+p\log(c))} - 2\sqrt{be^{(mnp\log(ex)+np\log(d)+p\log(c))} + a}\sqrt{a} + 2a\right)e^{(-mnp\log(ex)-np\log(d)-p\log(c))}\right)}{\sqrt{a}mnp}, \frac{2\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*p*
log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + b\left(c\left(d\,(ex)^m\right)^n\right)^p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

3.674 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Rubi [A] time = 0.655875, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\left(c(d(ex)^n)^p\right)^q}}\,dx, x, (fx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^p)^q}}\,dx, x, \left(e(fx)^m\right)^n\right)}{mn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^q}}\,dx, x, \left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{cmnp}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q\right)}{mnpq}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}\right)}{bmnpq}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}
\end{aligned}$$

Mathematica [A] time = 0.265099, size = 51, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Maple [A] time = 0.02, size = 46, normalized size = 0.9

$$-2\,\frac{1}{mnpq\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.54825, size = 483, normalized size = 9.47

$$\left[\frac{\log\left(\left(\left(be^{\left(mnpq\log\left(fx\right)+npq\log(e)+pq\log(d)+q\log(c)\right)}-2\sqrt{be^{\left(mnpq\log\left(fx\right)+npq\log(e)+pq\log(d)+q\log(c)\right)}+a}\sqrt{a}+2\,a\right)e^{\left(-mnpq\log\left(fx\right)-npq\log\right.}}{\sqrt{a}mnpq}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) - 2*sqr
t(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt(a
) + 2*a)*e^(-m*n*p*q*log(f*x) - n*p*q*log(e) - p*q*log(d) - q*log(c)))/(sqr
t(a)*m*n*p*q), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e)
+ p*q*log(d) + q*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p*q)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left(\left((fx)^m e\right)^n d\right)^p c\right)^q b + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

3.675 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-
2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-
2)]])/16
```

Rubi [A] time = 0.026315, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-
2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-
2)]])/16
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx &= -\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6+\frac{7}{12}\operatorname{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6+\frac{35}{48}\operatorname{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6+\frac{35}{32}\operatorname{Subst}\left(\int \frac{\sqrt{-1+}}{x}\right.\\
&= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}}-\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6-\frac{35}{32}\operatorname{Sub}\\
&= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}}-\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6-\frac{35}{16}\operatorname{Sub}\\
&= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}}-\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6-\frac{35}{16}\tan
\end{aligned}
$$

Mathematica [C] time = 0.0104486, size = 34, normalized size = 0.45

$$\frac{\sqrt{\frac{1}{x^2}-1}\,{}_2F_1\left(-\frac{7}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-7/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

Maple [A] time = 0.011, size = 83, normalized size = 1.1

$$\frac{1}{48}\sqrt{-\frac{x^2-1}{x^2}}\left(-8\,x^4\left(-x^2+1\right)^{3/2}+30\,x^2\left(-x^2+1\right)^{3/2}+48\left(-x^2+1\right)^{3/2}+105\,x^2\sqrt{-x^2+1}+105\,\arcsin(x)\,x\right)\frac{1}{\sqrt{-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/48*(-(x^2-1)/x^2)^(1/2)*(-8*x^4*(-x^2+1)^(3/2)+30*x^2*(-x^2+1)^(3/2)+48*(
-x^2+1)^(3/2)+105*x^2*(-x^2+1)^(1/2)+105*arcsin(x)*x)/(-x^2+1)^(1/2)
```

Maxima [B] time = 2.13629, size = 162, normalized size = 2.13

$$\frac{3}{2}x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3+3\left(\frac{1}{x^2}-1\right)^2+\frac{3}{x^2}-2\right)}+\frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}-\frac{35}{16}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*(
1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 + 3/
x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2
 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

Fricas [A] time = 1.48702, size = 140, normalized size = 1.84

$$\frac{1}{48}\left(8x^6-38x^4+87x^2+48\right)\sqrt{-\frac{x^2-1}{x^2}}-\frac{35}{8}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*s
qrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1925, size = 104, normalized size = 1.37

$$\frac{1}{48}\left(2\left(4x^2\mathrm{sgn}(x)-19\,\mathrm{sgn}(x)\right)x^2+87\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x+\frac{35}{16}\arcsin(x)\,\mathrm{sgn}(x)-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/1
6*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1
) - 1)*sgn(x)/x
```

3.676 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx$

Optimal. Leaf size=60

$$\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4+\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^
(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rubi [A] time = 0.0199571, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {25, 266, 47, 50, 63, 203}

$$\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4+\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^
(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx &= \int \left(-1+\frac{1}{x^2}\right)^{5/2} x^3\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{5}{8}\operatorname{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4-\frac{15}{16}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4+\frac{15}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,\right.\\
&= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4+\frac{15}{8}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.\\
&= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4+\frac{15}{8}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0071944, size = 35, normalized size = 0.58

$$
-\frac{\sqrt{\frac{1}{x^2}-1}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] -((Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-5/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
)
```

Maple [A] time = 0.007, size = 69, normalized size = 1.2

$$
-\frac{1}{8}\sqrt{-\frac{x^2-1}{x^2}}\left(2x^2\left(-x^2+1\right)^{3/2}+8\left(-x^2+1\right)^{3/2}+15x^2\sqrt{-x^2+1}+15\arcsin(x)\,x\right)\frac{1}{\sqrt{-x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] -1/8*(-(x^2-1)/x^2)^(1/2)*(2*x^2*(-x^2+1)^(3/2)+8*(-x^2+1)^(3/2)+15*x^2*(-x
^2+1)^(1/2)+15*arcsin(x)*x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 2.02796, size = 90, normalized size = 1.5

$$-x^2\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}-1}-\frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}+\frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^
2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

Fricas [A] time = 1.54223, size = 124, normalized size = 2.07

$$\frac{1}{8}\left(2x^4-9x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
1)/x^2) - 1)/x)
```

Sympy [A] time = 73.4257, size = 60, normalized size = 1.

$$\frac{x^4\sqrt{-1+\frac{1}{x^2}}\left(2-\frac{1}{x^2}\right)}{8}-x^2\sqrt{-1+\frac{1}{x^2}}-\sqrt{-1+\frac{1}{x^2}}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
+ x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

Giac [A] time = 1.11504, size = 90, normalized size = 1.5

$$\frac{1}{8}\left(2\,x^2\mathrm{sgn}\left(x\right)-9\,\mathrm{sgn}\left(x\right)\right)\sqrt{-x^2+1}x-\frac{15}{8}\,\arcsin\left(x\right)\mathrm{sgn}\left(x\right)+\frac{x\mathrm{sgn}\left(x\right)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1/
2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.677 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)}{x}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 +
 x^(-2)]])/2
```

Rubi [A] time = 0.0138863, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 +
 x^(-2)]])/2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)}{x}\,dx &= -\int \left(-1+\frac{1}{x^2}\right)^{3/2} x\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{3}{4}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0073557, size = 34, normalized size = 0.77

$$\frac{\sqrt{\frac{1}{x^2}-1}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

Maple [A] time = 0.006, size = 55, normalized size = 1.3

$$\frac{1}{2}\sqrt{-\frac{x^2-1}{x^2}}\left(2\left(-x^2+1\right)^{3/2}+3\,x^2\sqrt{-x^2+1}+3\,\arcsin\left(x\right)x\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)+3*x^2*(-x^2+1)^(1/2)+3*arcsin(x)
*x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 2.05112, size = 41, normalized size = 0.93

$$\frac{1}{2}x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))
```

Fricas [A] time = 1.44859, size = 107, normalized size = 2.43

$$\frac{1}{2}\left(x^2+2\right)\sqrt{-\frac{x^2-1}{x^2}}-3\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/
x)
```

Sympy [A] time = 27.1288, size = 39, normalized size = 0.89

$$\frac{x^2\sqrt{-1+\frac{1}{x^2}}}{2}+\sqrt{-1+\frac{1}{x^2}}-\frac{3\operatorname{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2)))
/2
```

Giac [A] time = 1.16428, size = 77, normalized size = 1.75

$$\frac{1}{2}\sqrt{-x^2+1}x\operatorname{sgn}(x)+\frac{3}{2}\arcsin(x)\operatorname{sgn}(x)-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^
2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.678 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx$

Optimal. Leaf size=9

$$\sqrt{\frac{1}{x^2}-1}$$

```
[Out] Sqrt[-1 + x^(-2)]
```

Rubi [A] time = 0.0040838, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {25, 261}

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx &= -\int \frac{1}{\sqrt{-1+\frac{1}{x^2}}\,x^3}\,dx\\ &= \sqrt{-1+\frac{1}{x^2}}\end{aligned}$$

Mathematica [A] time = 0.0025487, size = 9, normalized size = 1.

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[-1 + x^(-2)]
```

Maple [A] time = 0.004, size = 13, normalized size = 1.4

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1),x)
```

```
[Out] (-(x^2-1)/x^2)^(1/2)
```

Maxima [B] time = 1.50257, size = 22, normalized size = 2.44

$$\frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1)/x
```

Fricas [A] time = 1.4282, size = 30, normalized size = 3.33

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
```

```
[Out] sqrt(-(x^2 - 1)/x^2)
```

Sympy [A] time = 1.88333, size = 8, normalized size = 0.89

$$\sqrt{-1+\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

```
[Out] sqrt(-1 + x**(-2))
```

Giac [B] time = 1.16072, size = 50, normalized size = 5.56

$$-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.679 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

```
[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rubi [A] time = 0.0107285, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {25, 266, 43}

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx=\int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{3/2}x^5}\,dx$$
$$=-\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(-1+x)^{3/2}}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$=-\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{\sqrt{-1+x}}\right)dx,x,\frac{1}{x^2}\right)\right)$$
$$=\frac{1}{\sqrt{-1+\frac{1}{x^2}}}-\sqrt{-1+\frac{1}{x^2}}$$

Mathematica [A] time = 0.0057347, size = 24, normalized size = 1.14

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(1-2x^2\right)}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

Maple [A] time = 0.003, size = 29, normalized size = 1.4

$$-\frac{2x^2-1}{x^2-1}\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x)
```

```
[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

Maxima [A] time = 1.04155, size = 41, normalized size = 1.95

$$-\frac{\left(2x^2-1\right)\sqrt{x+1}\sqrt{-x+1}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

Fricas [A] time = 1.45863, size = 61, normalized size = 2.9

$$-\frac{\left(2x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

Sympy [A] time = 3.16238, size = 20, normalized size = 0.95

$$-\sqrt{-1+\frac{1}{x^2}}+\frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
```

```
[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

Giac [B] time = 1.17584, size = 78, normalized size = 3.71

$$-\frac{\sqrt{-x^2+1}x\operatorname{sgn}(x)}{x^2-1}+\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/
2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.680 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

Optimal. Leaf size=34

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

```
[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]
```

Rubi [A] time = 0.0142253, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {25, 266, 43}

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = -\int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{5/2}x^7}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(-1+x)^{5/2}}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{5/2}} + \frac{2}{(-1+x)^{3/2}} + \frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} + \sqrt{-1+\frac{1}{x^2}}$$

Mathematica [A] time = 0.0072378, size = 32, normalized size = 0.94

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(8x^4-12x^2+3\right)}{3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

Maple [A] time = 0.003, size = 34, normalized size = 1.

$$\frac{8\,x^4-12\,x^2+3}{3\,\left(x^2-1\right)^2}\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x)
```

```
[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2
```

Maxima [A] time = 1.02952, size = 51, normalized size = 1.5

$$\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{x+1}\sqrt{-x+1}}{3\,\left(x^5-2\,x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)
```

Fricas [A] time = 1.41155, size = 88, normalized size = 2.59

$$\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

Sympy [A] time = 4.48906, size = 34, normalized size = 1.

$$\sqrt{-1+\frac{1}{x^2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

```
[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

Giac [B] time = 1.15893, size = 92, normalized size = 2.71

$$-\frac{x\mathrm{sgn}\left(x\right)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}\left(x\right)}{2\,x}-\frac{\left(5\,x^2\mathrm{sgn}\left(x\right)-6\,\mathrm{sgn}\left(x\right)\right)x}{3\left(x^2-1\right)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1/
3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))
```

3.681 $\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rubi [A] time = 0.0038201, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2}x^3}\,dx\\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

Mathematica [B] time = 0.0062454, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [B] time = 0.003, size = 23, normalized size = 2.6

$$\frac{x^2}{x^2+1}\sqrt{\frac{x^2+1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

Maxima [A] time = 1.54989, size = 15, normalized size = 1.67

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/sqrt((x^2 + 1)/x^2)
```

Fricas [B] time = 1.44177, size = 63, normalized size = 7.

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] time = 2.40479, size = 8, normalized size = 0.89

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

```
[Out] x/sqrt(x**2 + 1)
```

Giac [A] time = 1.12942, size = 15, normalized size = 1.67

$$\frac{x\operatorname{sgn}(x)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x*sgn(x)/sqrt(x^2 + 1)
```

3.682 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)} dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rubi [A] time = 0.003854, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]

[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)} dx &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2} x^3} dx \\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}} \end{aligned}$$

Mathematica [B] time = 0.0040753, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [A] time = 0.002, size = 12, normalized size = 1.3

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x)
```

```
[Out] 1/((x^2+1)/x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

Fricas [B] time = 1.58161, size = 63, normalized size = 7.

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] time = 2.04231, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
```

```
[Out] 1/sqrt(1 + x**(-2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

3.683 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rubi [A] time = 0.0717561, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0413454, size = 18, normalized size = 1.

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Maple [B] time = 0.039, size = 1059, normalized size = 58.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*((x+(-
b*(a-1))^(1/2)/b)^2*b-2*(-b*(a-1))^(1/2)*(x+(-b*(a-1))^(1/2)/b)+1)^(1/2)-1/
2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*(-b*(a-1
))^(1/2)*ln((b*(x+(-b*(a-1))^(1/2)/b)-(-b*(a-1))^(1/2))/b^(1/2)+((x+(-b*(a-
1))^(1/2)/b)^2*b-2*(-b*(a-1))^(1/2)*(x+(-b*(a-1))^(1/2)/b)+1)^(1/2))/b^(1/2
)-1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*arct
anh(1/2*(2-2*(-b*(a-1))^(1/2)*(x+(-b*(a-1))^(1/2)/b))/((x+(-b*(a-1))^(1/2)/
b)^2*b-2*(-b*(a-1))^(1/2)*(x+(-b*(a-1))^(1/2)/b)+1)^(1/2))-1/2/((-b*(a-1))^
(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*((x+(-a*b)^(1/2)/b)^2*
b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/
2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)/b)-
(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b))^(1/2))/b^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)
+(-a*b)^(1/2))*((x-(-b*(a-1))^(1/2)/b)^2*b+2*(-b*(a-1))^(1/2)*(x-(-b*(a-1))
^(1/2)/b)+1)^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(
-a*b)^(1/2))*(-b*(a-1))^(1/2)*ln((b*(x-(-b*(a-1))^(1/2)/b)+(-b*(a-1))^(1/2)
)/b^(1/2)+((x-(-b*(a-1))^(1/2)/b)^2*b+2*(-b*(a-1))^(1/2)*(x-(-b*(a-1))^(1/2
)/b)+1)^(1/2))/b^(1/2)-1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/
2)+(-a*b)^(1/2))*arctanh(1/2*(2+2*(-b*(a-1))^(1/2)*(x-(-b*(a-1))^(1/2)/b))/
((x-(-b*(a-1))^(1/2)/b)^2*b+2*(-b*(a-1))^(1/2)*(x-(-b*(a-1))^(1/2)/b)+1)^(1
/2))-1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*(
(x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2/((-b*(a
-1))^(1/2)+(-a*b)^(1/2))/(-(-b*(a-1))^(1/2)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln((
b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)+1/2/b*ln(b*x^2+a-1)
```

Maxima [A] time = 1.12493, size = 22, normalized size = 1.22

$$\frac{\log\left(\sqrt{bx^2 + a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

Fricas [B] time = 1.78275, size = 167, normalized size = 9.28

$$\frac{2\,\log\left(bx^2+a-1\right)+\log\left(\frac{bx^2+a+2\,\sqrt{bx^2+a}+1}{x^2}\right)-\log\left(\frac{bx^2+a-2\,\sqrt{bx^2+a}+1}{x^2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

Sympy [A] time = 1.97963, size = 14, normalized size = 0.78

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
```

```
[Out] log(sqrt(a + b*x**2) + 1)/b
```

Giac [A] time = 1.15959, size = 22, normalized size = 1.22

$$\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

3.684 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rubi [A] time = 0.061193, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6715, 1593, 260}

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\sqrt[3]{x}+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}}\,dx,x,x^2\right)\\ &= \frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)\end{aligned}$$

Mathematica [A] time = 0.0267944, size = 16, normalized size = 1.

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Maple [B] time = 0.033, size = 70, normalized size = 4.4

$$\frac{\ln\left(x^2-1\right)}{4}+\frac{\ln\left(x^2+1\right)}{4}+\frac{1}{2}\ln\left(\sqrt[3]{x^2}-1\right)-\frac{1}{4}\ln\left(\left(x^2\right)^{\frac{2}{3}}+\sqrt[3]{x^2}+1\right)-\frac{1}{4}\ln\left(\left(x^2\right)^{\frac{2}{3}}-\sqrt[3]{x^2}+1\right)+\frac{1}{2}\ln\left(\sqrt[3]{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-(x^2)^(1/3)),x)
```

```
[Out] 1/4*ln(x^2-1)+1/4*ln(x^2+1)+1/2*ln((x^2)^(1/3)-1)-1/4*ln((x^2)^(2/3)+(x^2)^
(1/3)+1)-1/4*ln((x^2)^(2/3)-(x^2)^(1/3)+1)+1/2*ln((x^2)^(1/3)+1)
```

Maxima [A] time = 1.12492, size = 28, normalized size = 1.75

$$\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}+1\right)+\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

Fricas [B] time = 1.64885, size = 80, normalized size = 5.

$$-3\log\left(\frac{\left(x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{3}{4}\log\left(-\frac{x^2-\left(x^2\right)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

Sympy [A] time = 0.204456, size = 19, normalized size = 1.19

$$-\frac{\log(x)}{2}+\frac{3\log\left(x^2-\sqrt[3]{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-(x**2)**(1/3)),x)
```

```
[Out] -log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4
```

Giac [A] time = 1.14409, size = 22, normalized size = 1.38

$$\frac{3}{4} \log \left(\left| (x\mathrm{sgn}\,(x))^{\frac{1}{3}} x\mathrm{sgn}\,(x) - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

3.685 $\int x\left(1+x^2\right)^3 \sqrt{2+2x^2+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

```
[Out] -(2 + 2*x^2 + x^4)^(3/2)/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/10
```

Rubi [A] time = 0.030516, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1247, 692, 629}

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] -(2 + 2*x^2 + x^4)^(3/2)/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/10
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int (1+x)^3\sqrt{2+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}-\frac{1}{5}\,\text{Subst}\left(\int (1+x)\sqrt{2+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{15}\left(2+2x^2+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0140994, size = 30, normalized size = 0.68

$$\frac{1}{30}\left(x^4+2x^2+2\right)^{3/2}\left(3x^4+6x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] ((2 + 2*x^2 + x^4)^(3/2)*(1 + 6*x^2 + 3*x^4))/30
```

Maple [A] time = 0.006, size = 27, normalized size = 0.6

$$\frac{3x^4+6x^2+1}{30}\left(x^4+2x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)
```

Maxima [A] time = 1.69889, size = 66, normalized size = 1.5

$$\frac{1}{10}\left(x^4+2x^2+2\right)^{\frac{3}{2}}x^4+\frac{1}{5}\left(x^4+2x^2+2\right)^{\frac{3}{2}}x^2+\frac{1}{30}\left(x^4+2x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*(
x^4 + 2*x^2 + 2)^(3/2)
```

Fricas [A] time = 1.62559, size = 90, normalized size = 2.05

$$\frac{1}{30}\left(3x^8+12x^6+19x^4+14x^2+2\right)\sqrt{x^4+2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

Sympy [B] time = 0.634387, size = 94, normalized size = 2.14

$$\frac{x^8\sqrt{x^4+2x^2+2}}{10}+\frac{2x^6\sqrt{x^4+2x^2+2}}{5}+\frac{19x^4\sqrt{x^4+2x^2+2}}{30}+\frac{7x^2\sqrt{x^4+2x^2+2}}{15}+\frac{\sqrt{x^4+2x^2+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
```

```
[Out] x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x**
4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x**
4 + 2*x**2 + 2)/15
```

Giac [A] time = 1.16469, size = 51, normalized size = 1.16

$$\frac{1}{30}\sqrt{x^4+2x^2+2}\left(\left(\left(3\left(x^2+4\right)x^2+19\right)x^2+14\right)x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(x^4 + 2*x^2 + 2)*(((3*(x^2 + 4)*x^2 + 19)*x^2 + 14)*x^2 + 2)
```

3.686 $\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$

Optimal. Leaf size=121

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}$$

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 +
 (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))
/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51
```

Rubi [A] time = 0.111314, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1821, 1620}

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 +
 (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))
/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{1-x}x\left(1+x^3\right)^2\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(4\sqrt{1-x}-16(1-x)^{3/2}+33(1-x)^{5/2}-43(1-x)^{7/2}+37(1-x)^{9/2}-21(1-x)^{1}\right.\right.\\ &= -\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{14}{13}\left(1-\right.\end{aligned}$$

Mathematica [A] time = 0.0427483, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3}\left(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^9+126561x^6-86507x^3-173014\right)}{2297295}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12
 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

Maple [A] time = 0.012, size = 58, normalized size = 0.5

$$\frac{\left(90090\,x^{21}+84084\,x^{18}+77616\,x^{15}+349020\,x^{12}+310240\,x^{9}+265920\,x^{6}+519042\,x^{3}+346028\right)(x-1)\left(x^{2}+x+1\right.}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x)
```

```
[Out] 2/2297295*(-x^3+1)^(1/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+1551
20*x^9+132960*x^6+259521*x^3+173014)*(x-1)*(x^2+x+1)
```

Maxima [A] time = 1.85608, size = 120, normalized size = 0.99

$$\frac{2}{51}\left(-x^{3}+1\right)^{\frac{17}{2}}-\frac{14}{45}\left(-x^{3}+1\right)^{\frac{15}{2}}+\frac{14}{13}\left(-x^{3}+1\right)^{\frac{13}{2}}-\frac{74}{33}\left(-x^{3}+1\right)^{\frac{11}{2}}+\frac{86}{27}\left(-x^{3}+1\right)^{\frac{9}{2}}-\frac{22}{7}\left(-x^{3}+1\right)^{\frac{7}{2}}+\frac{32}{15}\left(-x^{3}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
- 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2)
+ 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

Fricas [A] time = 1.8591, size = 184, normalized size = 1.52

$$\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}-173014\right)\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

Sympy [A] time = 14.7357, size = 133, normalized size = 1.1

$$\frac{2x^{24}\sqrt{1-x^{3}}}{51}-\frac{2x^{21}\sqrt{1-x^{3}}}{765}-\frac{28x^{18}\sqrt{1-x^{3}}}{9945}+\frac{1436x^{15}\sqrt{1-x^{3}}}{12155}-\frac{1108x^{12}\sqrt{1-x^{3}}}{65637}-\frac{8864x^{9}\sqrt{1-x^{3}}}{459459}+\frac{84374x^{6}\sqrt{1-}}{765765}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/65
637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765 -
173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

Giac [A] time = 1.14174, size = 186, normalized size = 1.54

$$\frac{2}{51}\left(x^3-1\right)^8\sqrt{-x^3+1}+\frac{14}{45}\left(x^3-1\right)^7\sqrt{-x^3+1}+\frac{14}{13}\left(x^3-1\right)^6\sqrt{-x^3+1}+\frac{74}{33}\left(x^3-1\right)^5\sqrt{-x^3+1}+\frac{86}{27}\left(x^3-1\right)^4\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13*
(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^3
- 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1)^2
*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

3.687 $\int \left(\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right)dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.0433835, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right)dx=\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx+\int \frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx$$
$$=-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$=-\frac{1}{b\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}$$
$$=-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.0306064, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]
)/(b*(-a + b)*Sqrt[a + b*x^2])
```

Maple [A] time = 0.018, size = 42, normalized size = 0.8

$$-\frac{1}{b}\frac{1}{\sqrt{bx^2+a}}+\arctan\left(\sqrt{bx^2+a}\frac{1}{\sqrt{b-a}}\right)\frac{1}{\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)+1/(b-a)^(1/2)*arctan((b*x^2+a)^(1/2)/(b-a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79609, size = 571, normalized size = 11.42

$$\left[\frac{\left(b^2x^2+ab\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\left(4\,ab-3\,b^2\right)x^2-4\left(bx^2+2\,a-b\right)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,\sqrt{bx^2+a}(a-b)}{4\left(a^2b-ab^2+\left(ab^2-b^3\right)x^2\right)},-\frac{\left(b^2x^2+ab\right)\sqrt{-a+\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [A] time = 2.49693, size = 49, normalized size = 0.98

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b)
```

Giac [A] time = 1.15177, size = 55, normalized size = 1.1

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

3.688 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.0644341, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6, 571, 78, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(1+a+x^2+bx^2\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx &= \int \frac{x\left(1+a+(1+b)x^2\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+a+(1+b)x}{(1+x)(a+bx)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= -\frac{1}{b\sqrt{a+bx^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= -\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\end{aligned}$$

Mathematica [A] time = 0.0133638, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]
)/(b*(-a + b)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.013, size = 133, normalized size = 2.7

$$-\frac{1}{\sqrt{bx^2+a}}-\frac{1}{b}\frac{1}{\sqrt{bx^2+a}}-\frac{b}{-b+a}\arctan\left(\sqrt{bx^2+a}\frac{1}{\sqrt{b-a}}\right)\frac{1}{\sqrt{b-a}}-\frac{b}{-b+a}\frac{1}{\sqrt{bx^2+a}}+\frac{a}{-b+a}\arctan\left(\sqrt{bx^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/(b*x^2+a)^(1/2)-1/b/(b*x^2+a)^(1/2)-b/(-b+a)/(b-a)^(1/2)*arctan((b*x^2+a
)^(1/2)/(b-a)^(1/2))-b/(-b+a)/(b*x^2+a)^(1/2)+a/(-b+a)/(b-a)^(1/2)*arctan((
b*x^2+a)^(1/2)/(b-a)^(1/2))+a/(-b+a)/(b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.61191, size = 571, normalized size = 11.42

$$\left[\frac{\left(b^2x^2+ab\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\left(4\,ab-3\,b^2\right)x^2-4\left(bx^2+2\,a-b\right)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\sqrt{bx^2+a}(a-b)}{4\left(a^2b-ab^2+\left(ab^2-b^3\right)x^2\right)}, -\frac{\left(b^2x^2+ab\right)\sqrt{-a+b}\,\mathrm{ar}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [A] time = 38.9967, size = 37, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

```
[Out] atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b) - 1/(b*sqrt(a + b*x**2))
```

Giac [A] time = 1.18557, size = 55, normalized size = 1.1

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

3.689 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.0417059, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2
]),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(\frac{x}{\left(a+bx^2\right)^{5/2}}+\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right)dx = \int \frac{x}{\left(a+bx^2\right)^{5/2}}\,dx + \int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx + \int \frac{x}{\left(1+x^2\right)\sqrt{a+b}}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\right.$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{b}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.190112, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a +
 b*x^2]),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a - b]]/Sqrt[a - b]
```

Maple [A] time = 0.013, size = 56, normalized size = 0.8

$$-\frac{1}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}} - \frac{1}{b}\frac{1}{\sqrt{bx^2+a}} + \arctan\left(\sqrt{bx^2+a}\frac{1}{\sqrt{b-a}}\right)\frac{1}{\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)+1/(b-a)^(1/2)*arctan((b*x^2+a)^(
1/2)/(b-a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64651, size = 799, normalized size = 11.75

$$\left[\frac{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\left(4\,ab-3\,b^2\right)x^2-4\left(bx^2+2\,a-b\right)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\left(3\left(ab-b^2\right)x^2+3\,a^2-(3\right.}{12\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [A] time = 3.15144, size = 97, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1
/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(5/2)), True)) + atan(sqrt(a + b*x**2)/sqrt(-a + b))/sq
rt(-a + b)
```

Giac [A] time = 1.16293, size = 74, normalized size = 1.09

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b) -
 1/3/((b*x^2 + a)^(3/2)*b)
```

3.690 $\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.509217, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {6, 6715, 897, 1261, 207}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/(
(1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{x\left(1+a+a^2+(1+a)x^2+bx^2+2abx^2+bx^4+b^2x\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+2abx^2+(1+a+b)x^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+(1+a+b+2ab)x^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx$$

$$= \int \frac{x\left(1+a+a^2+(1+a+b+2ab)x^2+\left(b+b^2\right)x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+a+a^2+(1+a+b+2ab)x+\left(b+b^2\right)}{(1+x)(a+bx)^{5/2}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\frac{\left(1+a+a^2\right)b^2-ab(1+a+b+2ab)+a^2\left(b+b^2\right)}{b^2}-\frac{\left(-b(1+a+b+2ab)+2a\left(b+b^2\right)\right)}{b^2}}{x^4\left(\frac{-a+b}{b}+\frac{x^2}{b}\right)}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{1}{x^4}+\frac{1}{x^2}+\frac{b}{-a+b+x^2}\right)dx,x,\sqrt{a+bx^2}\right)}{b}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\text{Subst}\left(\int \frac{1}{-a+b+x}\right)$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.0734214, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x
^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a - b]]/Sqrt[a - b]
```

Maple [B] time = 0.02, size = 314, normalized size = 4.6

$$-bx^2\left(bx^2+a\right)^{-\frac{3}{2}}-x^2\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{4a}{3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{a}{b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{b}{3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{1}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{a^2}{(-b+a)^2}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(
5/2),x)
```

```
[Out] -x^2*b/(b*x^2+a)^(3/2)-x^2/(b*x^2+a)^(3/2)-4/3*a/(b*x^2+a)^(3/2)-a/b/(b*x^2
+a)^(3/2)+1/3*b/(b*x^2+a)^(3/2)-1/3/b/(b*x^2+a)^(3/2)+1/(-b+a)^2/(b-a)^(1/2
)*arctan((b*x^2+a)^(1/2)/(b-a)^(1/2))*a^2-2/(-b+a)^2/(b-a)^(1/2)*arctan((b*
x^2+a)^(1/2)/(b-a)^(1/2))*a*b+1/(-b+a)^2/(b-a)^(1/2)*arctan((b*x^2+a)^(1/2)
/(b-a)^(1/2))*b^2+1/(-b+a)^2/(b*x^2+a)^(1/2)*a^2-2/(-b+a)^2/(b*x^2+a)^(1/2)
*a*b+1/(-b+a)^2/(b*x^2+a)^(1/2)*b^2+1/3/(-b+a)/(b*x^2+a)^(3/2)*a^2-2/3/(-b+
a)/(b*x^2+a)^(3/2)*a*b+1/3/(-b+a)/(b*x^2+a)^(3/2)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60535, size = 799, normalized size = 11.75

$$\left[\frac{3\left(b^3x^4+2ab^2x^2+a^2b\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\left(4ab-3b^2\right)x^2-4\left(bx^2+2a-b\right)\sqrt{bx^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{x^4+2x^2+1}\right)-4\left(3\left(ab-b^2\right)x^2+3a^2-(3a+\right.}{12\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2
+1)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18646, size = 70, normalized size = 1.03

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{3\,bx^2+3\,a+1}{3\,\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1)
/((b*x^2 + a)^(3/2)*b)
```

3.691 $\int \frac{1}{\sqrt{\sqrt{x}+x}} \, dx$

Optimal. Leaf size=34

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rubi [A] time = 0.0314615, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2010, 2013, 620, 206}

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[x] + x],x]

[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx &= 2\sqrt{\sqrt{x}+x}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+x}}\,dx\\ &= 2\sqrt{\sqrt{x}+x}-\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{\sqrt{x}+x}-2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\\ &= 2\sqrt{\sqrt{x}+x}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.033284, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}}\left(1-\frac{\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sqrt[x] + x],x]
```

```
[Out] 2*Sqrt[Sqrt[x] + x]*(1 - ArcSinh[x^(1/4)]/(Sqrt[1 + Sqrt[x]]*x^(1/4)))
```

Maple [A] time = 0.009, size = 45, normalized size = 1.3

$$\sqrt{x+\sqrt{x}}\left(2\sqrt{x+\sqrt{x}}-\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)\right)\frac{1}{\sqrt{\sqrt{x}\left(1+\sqrt{x}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(1/2))^(1/2),x)
```

```
[Out] (x+x^(1/2))^(1/2)/(x^(1/2)*(1+x^(1/2)))^(1/2)*(2*(x+x^(1/2))^(1/2)-ln(x^(1/
2)+1/2+(x+x^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x)), x)
```

Fricas [A] time = 3.88872, size = 122, normalized size = 3.59

$$2\sqrt{x+\sqrt{x}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8
*sqrt(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\sqrt{x}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(x) + x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.692 $\int \sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=74

$$\frac{2}{3}\sqrt{x+\sqrt{x}x}+\frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x] +
x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rubi [A] time = 0.0451278, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2004, 2018, 670, 640, 620, 206}

$$\frac{2}{3}\sqrt{x+\sqrt{x}x}+\frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[x] + x],x]
```

```
[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x] +
x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{6}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\,dx\\
&= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}-\frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0434416, size = 51, normalized size = 0.69

$$\frac{1}{12}\sqrt{x+\sqrt{x}}\left(8x+2\sqrt{x}+\frac{3\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x + (3*ArcSinh[x^(1/4)])/(Sqrt[1 + S
qrt[x]]*x^(1/4))))/12
```

Maple [A] time = 0.003, size = 42, normalized size = 0.6

$$\frac{2}{3}\left(x+\sqrt{x}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}+\frac{1}{8}\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)+1/8*ln(x^(1/2)+1/
2+(x+x^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x)), x)
```

Fricas [A] time = 4.05134, size = 157, normalized size = 2.12

$$\frac{1}{12}\left(8x+2\sqrt{x}-3\right)\sqrt{x+\sqrt{x}}+\frac{1}{16}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)+8x+8\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x))
*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.693 $\int \sqrt{-x} \left(\sqrt{-x}+x\right) \, dx$

Optimal. Leaf size=19

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rubi [A] time = 0.0047248, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{-x} \left(\sqrt{-x}+x\right) \, dx &= \int \left(-(-x)^{3/2}-x\right) \, dx \\ &= \frac{2}{5}(-x)^{5/2}-\frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0077638, size = 19, normalized size = 1.

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{2}{5}\left(-x\right)^{\frac{5}{2}}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x+(-x)^(1/2)),x)
```

```
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Maxima [A] time = 1.08456, size = 18, normalized size = 0.95

$$\frac{2}{5}(-x)^{\frac{5}{2}}-\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*(-x)^(5/2) - 1/2*x^2
```

Fricas [A] time = 1.39263, size = 38, normalized size = 2.

$$\frac{2}{5}\sqrt{-x}x^2-\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

Sympy [C] time = 0.18352, size = 14, normalized size = 0.74

$$\frac{2ix^{\frac{5}{2}}}{5}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
```

```
[Out] 2*I*x**(5/2)/5 - x**2/2
```

Giac [A] time = 1.10963, size = 22, normalized size = 1.16

$$\frac{2}{5}\sqrt{-x}x^2-\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

3.694 $\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx$

Optimal. Leaf size=54

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[6 - x]
```

Rubi [A] time = 0.0831094, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1831, 260, 321, 212, 206, 203}

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[6 - x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx &= 4\,\text{Subst}\left(\int \frac{x^3(5+x)}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(\frac{5x^3}{-6+x^4}+\frac{x^4}{-6+x^4}\right)dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^4}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)+20\,\text{Subst}\left(\int \frac{x^3}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)+24\,\text{Subst}\left(\int \frac{1}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}-x^2}\,dx, x, \sqrt[4]{x}\right)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)
\end{aligned}$$

Mathematica [C] time = 0.0773881, size = 107, normalized size = 1.98

$$4\sqrt[4]{x}+\left(5+\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}-\sqrt[4]{x}\right)+\left(5-i\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}-i\sqrt[4]{x}\right)+\left(5+i\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}+i\sqrt[4]{x}\right)-\left(\sqrt[4]{6}-5\right)\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) + (5 + 6^(1/4))*Log[6^(1/4) - x^(1/4)] + (5 - I*6^(1/4))*Log[6^(1
/4) - I*x^(1/4)] + (5 + I*6^(1/4))*Log[6^(1/4) + I*x^(1/4)] - (-5 + 6^(1/4)
)*Log[6^(1/4) + x^(1/4)]
```

Maple [A] time = 0.004, size = 52, normalized size = 1.

$$4\sqrt[4]{x}-2\sqrt[4]{6}\arctan\left(1/6\sqrt[4]{x}6^{3/4}\right)-\sqrt[4]{6}\ln\left(\left(\sqrt[4]{x}+\sqrt[4]{6}\right)\left(\sqrt[4]{x}-\sqrt[4]{6}\right)^{-1}\right)+5\ln(-6+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+x^(1/4))/(-6+x),x)
```

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-6^(1/4)*ln((x^(1/4)+6^(1/4)
)/(x^(1/4)-6^(1/4)))+5*ln(-6+x)
```

Maxima [A] time = 1.48588, size = 90, normalized size = 1.67

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(-\frac{6^{\frac{1}{4}}-x^{\frac{1}{4}}}{6^{\frac{1}{4}}+x^{\frac{1}{4}}}\right)+4x^{\frac{1}{4}}+5\log\left(\sqrt{6}+\sqrt{x}\right)+5\log\left(-\sqrt{6}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) - x^(1/4))/(
6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(x)) + 5*log(-sqrt(6)
 + sqrt(x))
```

Fricas [B] time = 1.5883, size = 281, normalized size = 5.2

$$-\left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+\left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+4\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}\sqrt{\sqrt{6}+\sqrt{x}}-\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+4\,x^{\frac{1}{4}}+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="fricas")
```

```
[Out] -(6^(1/4) - 5)*log(2*6^(1/4) + 2*x^(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) +
2*x^(1/4)) + 4*6^(1/4)*arctan(1/6*6^(3/4)*sqrt(sqrt(6) + sqrt(x)) - 1/6*6^(
3/4)*x^(1/4)) + 4*x^(1/4) + 5*log(4*sqrt(6) + 4*sqrt(x))
```

Sympy [A] time = 2.489, size = 100, normalized size = 1.85

$$4\sqrt[4]{x}+\sqrt[4]{6}\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right)+5\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right)-\sqrt[4]{6}\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)+5\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)+5\log\left(\sqrt{x}+\sqrt{6}\right)-2\sqrt[4]{6}\operatorname{atan}\left(\frac{6^{\frac{3}{4}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 4*x**(1/4) + 6**(1/4)*log(x**(1/4) - 6**(1/4)) + 5*log(x**(1/4) - 6**(1/4))
 - 6**(1/4)*log(x**(1/4) + 6**(1/4)) + 5*log(x**(1/4) + 6**(1/4)) + 5*log(s
qrt(x) + sqrt(6)) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
```

Giac [A] time = 1.17942, size = 74, normalized size = 1.37

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)-6^{\frac{1}{4}}\log\left(6^{\frac{1}{4}}+x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(\left|-6^{\frac{1}{4}}+x^{\frac{1}{4}}\right|\right)+4\,x^{\frac{1}{4}}+5\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*log(6^(1/4) + x^(1/4)) + 6
^(1/4)*log(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(abs(x - 6))
```

3.695 $\int \frac{1}{4+\sqrt{4-x}-x} \, dx$

Optimal. Leaf size=14

$$-2\log\left(\sqrt{4-x}+1\right)$$

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rubi [A] time = 0.0185971, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {31}

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+\sqrt{4-x}-x} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{4-x}\right)\right)\\ &= -2\log\left(1+\sqrt{4-x}\right)\end{aligned}$$

Mathematica [A] time = 0.0052837, size = 14, normalized size = 1.

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Maple [A] time = 0.01, size = 18, normalized size = 1.3

$$-\ln\left(-3+x\right)-2\,\text{Artanh}\left(\sqrt{4-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x+(4-x)^(1/2)),x)
```

```
[Out] -ln(-3+x)-2*arctanh((4-x)^(1/2))
```

Maxima [A] time = 0.950734, size = 16, normalized size = 1.14

$$-2 \log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

Fricas [A] time = 1.40136, size = 35, normalized size = 2.5

$$-2 \log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

Sympy [B] time = 2.10782, size = 32, normalized size = 2.29

$$\log\left(2\sqrt{4-x}\right) - \log\left(2\sqrt{4-x}+2\right) - \log\left(x-\sqrt{4-x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)**(1/2)),x)
```

```
[Out] log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(x - sqrt(4 - x) - 4)
```

Giac [A] time = 1.14667, size = 16, normalized size = 1.14

$$-2 \log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

3.696 $\int \frac{1}{1+x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rubi [A] time = 0.0435459, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x-\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx, x, \sqrt{2+x}\right)\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x\right.\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0431099, size = 58, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Maple [A] time = 0.009, size = 91, normalized size = 1.5

$$-\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)+\frac{\ln\left(x^2+x-1\right)}{2}-\frac{1}{2}\ln\left(x+1+\sqrt{2+x}\right)-\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{\sqrt{5}}{5}\left(2\sqrt{2+x}+1\right)\right)+\frac{1}{2}\ln\left(1+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x-(2+x)^(1/2)),x)
```

```
[Out] -1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))+1/2*ln(x^2+x-1)-1/2*ln(x+1+(2+x)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(2+x)^(1/2)+1)*5^(1/2))+1/2*ln(1+x-(2+x)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(2+x)^(1/2)-1)*5^(1/2))
```

Maxima [A] time = 1.43907, size = 62, normalized size = 1.02

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) - 1
)) + log(x - sqrt(x + 2) + 1)
```

Fricas [A] time = 1.47596, size = 180, normalized size = 2.95

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+3)-\left(\sqrt{5}(2x+1)-5\right)\sqrt{x+2}+7x+3}{x^2+x-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x +
 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)
```

Sympy [A] time = 1.255, size = 94, normalized size = 1.54

$$4\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+2}-\frac{1}{2}\right)^2>\frac{5}{4}\\-\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+2}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (sqrt(x +
2) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (
sqrt(x + 2) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 2) + 1)
```

Giac [A] time = 1.14996, size = 68, normalized size = 1.11

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+2}-1\right|}{\left|\sqrt{5}+2\sqrt{x+2}-1\right|}\right)+\log\left(\left|x-\sqrt{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x +
2) - 1)) + log(abs(x - sqrt(x + 2) + 1))
```

3.697 $\int \frac{1}{4+x+\sqrt{1+x}} \, dx$

Optimal. Leaf size=37

$$\log \left(x+\sqrt{x+1}+4\right)-\frac{2 \tan ^{-1}\left(\frac{2 \sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rubi [A] time = 0.0377686, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\log \left(x+\sqrt{x+1}+4\right)-\frac{2 \tan ^{-1}\left(\frac{2 \sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= \log\left(4+x+\sqrt{1+x}\right) + 2\,\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

Mathematica [A] time = 0.0147414, size = 37, normalized size = 1.

$$\log\left(x+\sqrt{x+1}+4\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Maple [B] time = 0.01, size = 93, normalized size = 2.5

$$\frac{1}{2}\ln\left(4+x+\sqrt{1+x}\right) - \frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(1+2\sqrt{1+x}\right)\right) - \frac{1}{2}\ln\left(x+4-\sqrt{1+x}\right) - \frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(2\sqrt{1+x}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x+(1+x)^(1/2)),x)
```

```
[Out] 1/2*ln(4+x+(1+x)^(1/2))-1/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/
2)-1/2*ln(x+4-(1+x)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(1+x)^(1/2)-1)*11^(
1/2))+1/11*11^(1/2)*arctan(1/11*(2*x+7)*11^(1/2))+1/2*ln(x^2+7*x+15)
```

Maxima [A] time = 1.44144, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right) + \log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Fricas [A] time = 1.52984, size = 126, normalized size = 3.41

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

Sympy [A] time = 1.1981, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

Giac [A] time = 1.13631, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

3.698 $\int \frac{1}{x-\sqrt{1+x}} dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rubi [A] time = 0.0311208, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+x}} dx &= 2\,\text{Subst}\left(\int \frac{x}{-1-x+x^2} dx, x, \sqrt{1+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x} dx, x, \sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x} dx, x, \sqrt{1\right.\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0248369, size = 58, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Maple [A] time = 0.006, size = 91, normalized size = 1.5

$$\frac{\ln\left(x^2-x-1\right)}{2}-\frac{\sqrt{5}}{5}\mathrm{Artanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)-\frac{1}{2}\ln\left(x+\sqrt{1+x}\right)-\frac{\sqrt{5}}{5}\mathrm{Artanh}\left(\frac{\sqrt{5}}{5}\left(1+2\sqrt{1+x}\right)\right)+\frac{1}{2}\ln\left(x-\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(1+x)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2-x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/2*ln(x+(1+x)^(1/
2))-1/5*5^(1/2)*arctanh(1/5*(1+2*(1+x)^(1/2))*5^(1/2))+1/2*ln(x-(1+x)^(1/2)
)-1/5*5^(1/2)*arctanh(1/5*(2*(1+x)^(1/2)-1)*5^(1/2))
```

Maxima [A] time = 1.46453, size = 61, normalized size = 1.

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) - 1
)) + log(x - sqrt(x + 1))
```

Fricas [A] time = 1.48591, size = 174, normalized size = 2.85

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+2)-\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}+3x-2}{x^2-x-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x +
 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))
```

Sympy [A] time = 1.09701, size = 92, normalized size = 1.51

$$4\left(\begin{cases}-\frac{\sqrt{5}\,\mathrm{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (sqrt(x +
1) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (
sqrt(x + 1) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 1))
```

Giac [A] time = 1.15241, size = 66, normalized size = 1.08

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+1}-1\right|}{\left|\sqrt{5}+2\sqrt{x+1}-1\right|}\right)+\log\left(\left|x-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x +
1) - 1)) + log(abs(x - sqrt(x + 1)))
```

3.699 $\int \frac{1}{x-\sqrt{2+x}} \, dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rubi [A] time = 0.0211831, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{2+x}} \, dx &= 2\,\text{Subst}\left(\int \frac{x}{-2-x+x^2} \, dx, x, \sqrt{2+x}\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{2+x}\right)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-2+x} \, dx, x, \sqrt{2+x}\right)\\ &= \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0107035, size = 31, normalized size = 1.

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [B] time = 0.013, size = 54, normalized size = 1.7

$$\frac{\ln(1+x)}{3}+\frac{2\ln(-2+x)}{3}-\frac{1}{3}\ln\left(\sqrt{2+x}-1\right)-\frac{2}{3}\ln\left(\sqrt{2+x}+2\right)+\frac{1}{3}\ln\left(1+\sqrt{2+x}\right)+\frac{2}{3}\ln\left(-2+\sqrt{2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(2+x)^(1/2)),x)
```

```
[Out] 1/3*ln(1+x)+2/3*ln(-2+x)-1/3*ln((2+x)^(1/2)-1)-2/3*ln((2+x)^(1/2)+2)+1/3*ln
(1+(2+x)^(1/2))+2/3*ln(-2+(2+x)^(1/2))
```

Maxima [A] time = 0.960797, size = 28, normalized size = 0.9

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Fricas [A] time = 1.42538, size = 72, normalized size = 2.32

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [A] time = 1.28863, size = 36, normalized size = 1.16

$$\log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

Giac [A] time = 1.1582, size = 30, normalized size = 0.97

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

3.700 $\int \frac{1}{-\sqrt{1-x}+x} \, dx$

Optimal. Leaf size=65

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rubi [A] time = 0.0372839, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{1-x}+x} \, dx &= 2 \, \text{Subst}\left(\int \frac{x}{-1+x+x^2} \, dx, x, \sqrt{1-x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right) \text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x} \, dx, x, \sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right) \text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x} \, dx, x, \sqrt{1-x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)\end{aligned}$$

Mathematica [A] time = 0.0274428, size = 62, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Maple [B] time = 0.004, size = 101, normalized size = 1.6

$$\frac{\ln\left(x^2+x-1\right)}{2}+\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)+\frac{1}{2}\ln\left(-x+\sqrt{1-x}\right)+\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{\sqrt{5}}{5}\left(2\sqrt{1-x}+1\right)\right)-\frac{1}{2}\ln\left(-x-\sqrt{1-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(1-x)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))+1/2*ln(-x+(1-x)^(1
/2))+1/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)+1)*5^(1/2))-1/2*ln(-x-(1-x)^(1/
2))+1/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)-1)*5^(1/2))
```

Maxima [A] time = 1.47626, size = 69, normalized size = 1.06

$$-\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1)
+ 1)) + log(-x + sqrt(-x + 1))
```

Fricas [A] time = 1.43954, size = 178, normalized size = 2.74

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(x-2)-\left(\sqrt{5}(2x+1)+5\right)\sqrt{-x+1}-3x-2}{x^2+x-1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x
+ 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))
```

Sympy [A] time = 1.15198, size = 92, normalized size = 1.42

$$-4\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{1-x}+\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{1-x}+\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{1-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)**(1/2)),x)
```

```
[Out] -4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10, (sqrt(1 -
 x) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10,
(sqrt(1 - x) + 1/2)**2 < 5/4)) + log(x - sqrt(1 - x))
```

Giac [A] time = 1.13885, size = 73, normalized size = 1.12

$$-\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{-x+1}+1\right|}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(\left|-x+\sqrt{-x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x +
1) + 1)) + log(abs(-x + sqrt(-x + 1)))
```

3.701 $\int \sqrt{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)^(3/2))/3
- (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rubi [A] time = 0.0253673, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1341, 640, 612, 619, 215}

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)^(3/2))/3
- (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2\sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0186094, size = 49, normalized size = 0.79

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x}+1}\left(8x+2\sqrt{x}+5\right)-9\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x) - 9*ArcSinh[(1 + 2*Sqrt[x])/
Sqrt[3]])/24
```

Maple [A] time = 0.006, size = 42, normalized size = 0.7

$$\frac{2}{3}\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+x+\sqrt{x}}-\frac{3}{8}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(\sqrt{x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(1+x+x^(1/2))^(1/2)-3/8*arcsinh(2
/3*3^(1/2)*(x^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x) + 1), x)
```

Fricas [A] time = 4.14163, size = 167, normalized size = 2.69

$$\frac{1}{12}\left(8x+2\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{16}\log\left(4\sqrt{x+\sqrt{x}+1}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqrt
(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{x}+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x + 1), x)
```

Giac [A] time = 1.14488, size = 61, normalized size = 0.98

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{8}\log\left(2\sqrt{x+\sqrt{x}+1}-2\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*log(2*sqrt
(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

3.702 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[
1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4
```

Rubi [A] time = 0.0440683, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1980, 640, 612, 620, 206}

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[
1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{x(1+x)}\,dx, x, \sqrt{1+x}\right)\\ &= 2\,\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)\\ &= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)\end{aligned}$$

Mathematica [A] time = 0.0565714, size = 62, normalized size = 0.83

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}+1}\left(8x+2\sqrt{x+1}+\frac{3\sinh^{-1}\left(\sqrt[4]{x+1}\right)}{\sqrt[4]{x+1}\sqrt{\sqrt{x+1}+1}}+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x] + (3*ArcSinh[(1 + x)^(1
/4)])/((1 + x)^(1/4)*Sqrt[1 + Sqrt[1 + x]])))/12
```

Maple [A] time = 0.004, size = 55, normalized size = 0.7

$$\frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{\frac{3}{2}} - \frac{1}{4}\left(1+2\sqrt{1+x}\right)\sqrt{1+x+\sqrt{1+x}} + \frac{1}{8}\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{1+x+\sqrt{1+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+(1+x)^(1/2))^(3/2)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2))^(1/2)+1
/8*ln(1/2+(1+x)^(1/2)+(1+x+(1+x)^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

Fricas [A] time = 4.14999, size = 196, normalized size = 2.61

$$\frac{1}{12}\left(8x+2\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{16}\log\left(-4\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqrt
(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x+\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1) + 1), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.703 $\int \sqrt{\sqrt{-1+x}+x}\,dx$

Optimal. Leaf size=68

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 + x] + x)^(3
/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rubi [A] time = 0.0421857, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {640, 612, 619, 215}

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 + x] + x)^(3
/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, \sqrt{-\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.0249856, size = 54, normalized size = 0.79

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x-1}}\left(8x+2\sqrt{x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] (2*Sqrt[Sqrt[-1 + x] + x]*(-3 + 2*Sqrt[-1 + x] + 8*x) - 9*ArcSinh[(1 + 2*Sq
rt[-1 + x])/Sqrt[3]])/24
```

Maple [A] time = 0.006, size = 48, normalized size = 0.7

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(\sqrt{x-1}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x-1)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+(x-1)^(1/2))^(3/2)-1/4*(1+2*(x-1)^(1/2))*(x+(x-1)^(1/2))^(1/2)-3/8*a
rcsinh(2/3*3^(1/2)*((x-1)^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x - 1)), x)
```

Fricas [A] time = 4.19606, size = 185, normalized size = 2.72

$$\frac{1}{12}\left(8x+2\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{16}\log\left(-4\sqrt{x+\sqrt{x-1}}\left(2\sqrt{x-1}+1\right)+8x+8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x +
 sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x - 1)), x)
```

Giac [A] time = 1.17886, size = 72, normalized size = 1.06

$$\frac{1}{12}\left(2\sqrt{x-1}\left(4\sqrt{x-1}+1\right)+5\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{8}\log\left(2\sqrt{x+\sqrt{x-1}}-2\sqrt{x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)) + 3/8*lo
g(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

3.704 $\int \sqrt{2x+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1
 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16
```

Rubi [A] time = 0.0393168, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {640, 612, 619, 215}

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1
 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x\right.$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{1}{16}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x\right.$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\sinh^{-1}\left(\frac{1+2\sqrt{-1+2x}}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.0298329, size = 62, normalized size = 0.78

$$\frac{1}{48}\left(2\sqrt{2x+\sqrt{2x-1}}\left(16x+2\sqrt{2x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) - 9*ArcSinh[(1
 + 2*Sqrt[-1 + 2*x]])/Sqrt[3]])/48
```

Maple [A] time = 0.006, size = 60, normalized size = 0.8

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}} - \frac{1}{8}\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}} - \frac{3}{16}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] 1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1)^(1/2))^(
1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x+\sqrt{2x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + sqrt(2*x - 1)), x)
```

Fricas [A] time = 4.11253, size = 207, normalized size = 2.59

$$\frac{1}{24}\left(16x + 2\sqrt{2x-1} - 3\right)\sqrt{2x + \sqrt{2x-1}} + \frac{3}{32}\log\left(-4\sqrt{2x + \sqrt{2x-1}}\left(2\sqrt{2x-1} + 1\right) + 16x + 8\sqrt{2x-1} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*s
qrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) - 3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x + \sqrt{2x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2*x + sqrt(2*x - 1)), x)
```

Giac [A] time = 1.23049, size = 92, normalized size = 1.15

$$\frac{1}{24}\left(2\sqrt{2x-1}\left(4\sqrt{2x-1} + 1\right) + 5\right)\sqrt{2x + \sqrt{2x-1}} + \frac{3}{16}\log\left(2\sqrt{2x + \sqrt{2x-1}} - 2\sqrt{2x-1} - 1\right) - \frac{5}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(2*x - 1))
+ 3/16*log(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1) - 1) - 5/24
```

3.705 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

Optimal. Leaf size=109

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

```
[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/(36*Sqr
t[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcS
inh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Rubi [A] time = 0.06955, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {640, 612, 619, 215}

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/(36*Sqr
t[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcS
inh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int x\sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{3}\operatorname{Subst}\left(\int \sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{47}{144}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{9}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{47}{}$$

Mathematica [A] time = 0.0519043, size = 65, normalized size = 0.6

$$\frac{1}{216}\left(12\sqrt{3x+\sqrt{8x-7}}\left(12x+\sqrt{8x-7}-4\right)-47\sqrt{6}\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] (12*Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]) - 47*Sqrt[6]*Ar
cSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/216
```

Maple [A] time = 0.007, size = 67, normalized size = 0.6

$$\frac{1}{288}\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}-\frac{1}{288}\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}-\frac{47\sqrt{6}}{216}\operatorname{Arcsinh}\left(\frac{3\sqrt{47}}{47}\left(\sqrt{-7+8x}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+(-7+8*x)^(1/2))^(1/2),x)
```

```
[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16*
(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2)+
4/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

Fricas [A] time = 8.64316, size = 320, normalized size = 2.94

$$\frac{1}{18}\left(12\,x+\sqrt{8\,x-7}-4\right)\sqrt{3\,x+\sqrt{8\,x-7}}+\frac{47}{864}\,\sqrt{6}\log\left(-41472\,x^2-192\,(144\,x-47)\sqrt{8\,x-7}+8\left(3\,\sqrt{6}(144\,x+17)\sqrt{8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)*
log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 17)
*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 9792*
x + 30047)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(3*x + sqrt(8*x - 7)), x)
```

Giac [A] time = 1.19073, size = 174, normalized size = 1.6

$$\frac{1}{72}\,\sqrt{2}\left(\left(3\,\sqrt{2}\sqrt{8\,x-7}+2\,\sqrt{2}\right)\sqrt{8\,x-7}+13\,\sqrt{2}\right)\sqrt{3\,x+\sqrt{8\,x-7}}+\frac{47}{216}\,\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}\left(\sqrt{3}\sqrt{8\,x-7}-2\,\sqrt{2}\sqrt{3\,x+\sqrt{8}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(2)*((3*sqrt(2)*sqrt(8*x - 7) + 2*sqrt(2))*sqrt(8*x - 7) + 13*sqrt
(2))*sqrt(3*x + sqrt(8*x - 7)) + 47/216*sqrt(3)*sqrt(2)*log(-sqrt(3)*(sqrt(
3)*sqrt(8*x - 7) - 2*sqrt(2)*sqrt(3*x + sqrt(8*x - 7))) - 4) - 1/432*sqrt(3
)*(13*sqrt(21)*sqrt(3)*sqrt(2) + 94*sqrt(2)*log(sqrt(21)*sqrt(3) - 4))
```

3.706 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=47

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]
```

Rubi [A] time = 0.0295322, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {640, 621, 206}

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - \text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - 2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - \tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A] time = 0.012928, size = 47, normalized size = 1.

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + 
x]])]
```

Maple [A] time = 0.009, size = 32, normalized size = 0.7

$$2\sqrt{x+\sqrt{1+x}} - \ln\left(\sqrt{1+x} + \frac{1}{2} + \sqrt{x+\sqrt{1+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 2*(x+(1+x)^(1/2))^(1/2)-ln((1+x)^(1/2)+1/2+(x+(1+x)^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)
```

Fricas [A] time = 3.67419, size = 143, normalized size = 3.04

$$2\sqrt{x+\sqrt{x+1}}+\frac{1}{2}\,\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8\,x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) +
1) - 8*x - 8*sqrt(x + 1) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.707 $\int \frac{1+x}{4+x+\sqrt{-9+6x}} \, dx$

Optimal. Leaf size=67

$$x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqr
t[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rubi [A] time = 0.129946, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1628, 634, 618, 204, 628}

$$x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqr
t[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(15+x^2)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-6+x+\frac{18(11+x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+6\,\text{Subst}\left(\int \frac{11+x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+3\,\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)+48\,\text{Subst}\left(\int \frac{1}{33+6x+\right.$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)-96\,\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx, x, 6+2\sqrt{-\right.$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+4\sqrt{6}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

Mathematica [A] time = 0.0714985, size = 56, normalized size = 0.84

$$x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] +
 3*Log[4 + x + Sqrt[-9 + 6*x]]
```

Maple [A] time = 0.003, size = 52, normalized size = 0.8

$$-2\sqrt{-9+6x}-\frac{3}{2}+x+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\arctan\left(1/24\left(2\sqrt{-9+6x}+6\right)\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(4+x+(-9+6*x)^(1/2)),x)
```

```
[Out] -2*(-9+6*x)^(1/2)-3/2+x+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/24
*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

Maxima [A] time = 1.48491, size = 66, normalized size = 0.99

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

Fricas [A] time = 1.4617, size = 153, normalized size = 2.28

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{6x-9}+\frac{1}{4}\sqrt{6}\right)+x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*x
 - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

Sympy [A] time = 24.8384, size = 58, normalized size = 0.87

$$x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(sq
rt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2
```

Giac [A] time = 1.14779, size = 113, normalized size = 1.69

$$-\frac{1}{2}\sqrt{3}\sqrt{2}\left(\sqrt{3}\sqrt{2}\log\left(33\right)+8\arctan\left(\frac{1}{4}\sqrt{3}\sqrt{2}\right)\right)+4\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*sqrt(2)*(sqrt(3)*sqrt(2)*log(33) + 8*arctan(1/4*sqrt(3)*sqrt(2
))) + 4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*(sqrt(6*x - 9) + 3)) +
x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

3.708 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=71

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2
*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rubi [A] time = 0.109763, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1628, 634, 618, 204, 628}

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2
*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x\left(-63+x^2\right)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\right)$$
$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(-6+x+\frac{6(33-10x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)\right)$$
$$= -x+2\sqrt{3}\sqrt{-3+2x}-2\,\text{Subst}\left(\int \frac{33-10x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)$$
$$= -x+2\sqrt{3}\sqrt{-3+2x}+10\,\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)-126\,\text{Subst}\left(\int \frac{1}{33+6x}\right.$$
$$= -x+2\sqrt{3}\sqrt{-3+2x}+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)+252\,\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx, x, 6+2\sqrt{}\right.$$
$$= -x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

Mathematica [A] time = 0.0383051, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[-9 + 6*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6]
)] + 10*Log[4 + x + Sqrt[-9 + 6*x]]
```

Maple [A] time = 0.004, size = 54, normalized size = 0.8

$$2\sqrt{-9+6x}+\frac{3}{2}-x+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}}{2}\arctan\left(\frac{\sqrt{6}}{24}\left(2\sqrt{-9+6x}+6\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12-x)/(4+x+(-9+6*x)^(1/2)),x)
```

```
[Out] 2*(-9+6*x)^(1/2)+3/2-x+10*ln(24+6*x+6*(-9+6*x)^(1/2))-21/2*6^(1/2)*arctan(1
/24*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

Maxima [A] time = 1.48008, size = 69, normalized size = 0.97

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

Fricas [A] time = 1.46101, size = 192, normalized size = 2.7

$$-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\sqrt{6x-9}+\frac{1}{4}\sqrt{3}\sqrt{2}\right)-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt(
3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)
```

Sympy [A] time = 33.6422, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{21\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)}{2}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] -x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) - 21*sqrt(6)*atan
(sqrt(6)*(sqrt(6*x - 9) + 3)/12)/2 + 3/2
```

Giac [A] time = 1.17464, size = 117, normalized size = 1.65

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\left(10\sqrt{3}\sqrt{2}\log(33)-63\arctan\left(\frac{1}{4}\sqrt{3}\sqrt{2}\right)\right)-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*(10*sqrt(3)*sqrt(2)*log(33) - 63*arctan(1/4*sqrt(3)*sq
rt(2))) - 21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*(sqrt(6*x - 9) +
 3)) - x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

3.709 $\int \frac{-1+x^3}{\sqrt{x}(1+x^2)}\,dx$

Optimal. Leaf size=52

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Rubi [A] time = 0.0532846, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1802, 827, 1162, 617, 204}

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{\sqrt{x}\left(1+x^2\right)}\,dx &= \int \left(\sqrt{x}-\frac{1+x}{\sqrt{x}\left(1+x^2\right)}\right)dx\\ &= \frac{2x^{3/2}}{3}-\int \frac{1+x}{\sqrt{x}\left(1+x^2\right)}\,dx\\ &= \frac{2x^{3/2}}{3}-2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\ &= \frac{2x^{3/2}}{3}-\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)\\ &= \frac{2x^{3/2}}{3}-\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)+\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{x}\right)\\ &= \frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.023999, size = 52, normalized size = 1.

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Maple [B] time = 0.008, size = 97, normalized size = 1.9

$$\frac{2}{3}x^{\frac{3}{2}}-\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}-\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}-\frac{\sqrt{2}}{4}\ln\left(\left(x+\sqrt{2}\sqrt{x}+1\right)\left(x-\sqrt{2}\sqrt{x}+1\right)^{-1}\right)-\frac{\sqrt{2}}{4}\ln\left(\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+1)/x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)-arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(-1+2^(1/2)*x^(1/2))*2^
(1/2)-1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-1/4*2^(1/
2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))
```

Maxima [A] time = 1.50326, size = 62, normalized size = 1.19

$$\frac{2}{3}x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

Fricas [A] time = 1.47922, size = 80, normalized size = 1.54

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

Sympy [A] time = 0.896422, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right) - \sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(sqrt(2)*sqr
t(x) + 1)
```

Giac [A] time = 1.19831, size = 62, normalized size = 1.19

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

3.710 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rubi [A] time = 0.104925, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {12, 619, 215}

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}\\
&= -\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0382423, size = 18, normalized size = 0.9

$$\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

Maple [A] time = 0.006, size = 14, normalized size = 0.7

$$\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(\sqrt{x-1}-\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)
```

```
[Out] arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\int\frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Fricas [B] time = 3.68223, size = 108, normalized size = 5.4

$$\frac{1}{2}\log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) -
 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)/2
```

Giac [A] time = 1.13189, size = 34, normalized size = 1.7

$$-\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

3.711 $\int \frac{1+x^{7/2}}{1-x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}-\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log(x+1)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]] + Log[1 + x
]/2
```

Rubi [A] time = 0.068065, antiderivative size = 31, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1833, 275, 206, 302, 212, 203}

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]] + ArcTanh[x
]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^{7/2}}{1-x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(1+x^7\right)}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{x}{1-x^4}+\frac{x^8}{1-x^4}\right)dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-x^4}\,dx,x,\sqrt{x}\right)+2\,\text{Subst}\left(\int \frac{x^8}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-1-x^4+\frac{1}{1-x^4}\right)dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,x\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [C] time = 0.0291391, size = 67, normalized size = 1.56

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}+\left(\frac{1}{2}-\frac{i}{2}\right)\log\left(-\sqrt{x}+i\right)-\log\left(1-\sqrt{x}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\log\left(\sqrt{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + (1/2 - I/2)*Log[I - Sqrt[x]] - Log[1 - Sqrt[x]
] + (1/2 + I/2)*Log[I + Sqrt[x]]
```

Maple [A] time = 0.006, size = 34, normalized size = 0.8

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}-\frac{1}{2}\ln\left(-1+\sqrt{x}\right)+\frac{1}{2}\ln\left(1+\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)+\text{Artanh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(7/2))/(-x^2+1),x)
```

```
[Out] -2/5*x^(5/2)-2*x^(1/2)-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))+arctan(x^(1/2))
+arctanh(x)
```

Maxima [A] time = 1.54976, size = 39, normalized size = 0.91

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) -
 1)
```

Fricas [A] time = 1.48442, size = 105, normalized size = 2.44

$$-\frac{2}{5}\left(x^2+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) - 1
)
```

Sympy [A] time = 2.91003, size = 36, normalized size = 0.84

$$-\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}-\log\left(\sqrt{x}-1\right)+\frac{\log\left(x+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(7/2))/(-x**2+1),x)
```

```
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + atan(sqrt(x))
```

Giac [A] time = 1.17964, size = 41, normalized size = 0.95

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(abs(sqrt(
x) - 1))
```

3.712 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=116

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18$$

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 +
 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 +
 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

Rubi [A] time = 0.138134, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1620}

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 +
 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 +
 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^3\left(5+x^6\right)}{1+x}\,dx, x, \sqrt[6]{-1+2x}\right)\\ &= 3\,\text{Subst}\left(\int \left(6-6x+6x^2-x^3+x^4-x^5+x^6-x^7+x^8-\frac{6}{1+x}\right)dx, x, \sqrt[6]{-1+2x}\right)\\ &= -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x}-\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{5/6}+\frac{3}{7}(-1\end{aligned}$$

Mathematica [A] time = 0.0803036, size = 127, normalized size = 1.09

$$2\left(x\left(\frac{1}{3}\sqrt{2x-1}-\frac{3}{8}\sqrt[3]{2x-1}+\frac{3}{7}\sqrt[6]{2x-1}-\frac{1}{2}\right)+\frac{3}{10}(2x-1)^{5/6}-\frac{3}{8}(2x-1)^{2/3}+\frac{17}{6}\sqrt{2x-1}-\frac{69}{16}\sqrt[3]{2x-1}+\frac{123}{14}\sqrt[6]{2x-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] 2*((123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1 + 2*x
])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 + (3*(-1
```

```
+ 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*Log[1 + (-
1 + 2*x)^(1/6)])
```

Maple [A] time = 0.004, size = 90, normalized size = 0.8

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{1}{2}+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x)
```

```
[Out] 1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(2*x-1)^(5/
6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18*(2*x-1)^(1/6)-18*ln
(1+(2*x-1)^(1/6))
```

Maxima [A] time = 1.02375, size = 120, normalized size = 1.03

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}+18(2x-1)^{\frac{1}{6}}-18$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

Fricas [A] time = 1.46231, size = 235, normalized size = 2.03

$$\frac{1}{3}(2x+17)\sqrt{2x-1}-\frac{3}{8}(2x+23)(2x-1)^{\frac{1}{3}}+\frac{3}{7}(2x+41)(2x-1)^{\frac{1}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}-18\log\left((2x-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3/7*(2*x +
41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) - 18*lo
g((2*x - 1)^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\left(\int\frac{x}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx+\int\frac{2}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] 2*(Integral(x/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x) + Integral(2/((2*x - 1
)**(1/3) + sqrt(2*x - 1)), x))
```

Giac [A] time = 1.22167, size = 120, normalized size = 1.03

$$\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{3}{8}(2x-1)^{\frac{4}{3}} + \frac{3}{7}(2x-1)^{\frac{7}{6}} - x + \frac{3}{5}(2x-1)^{\frac{5}{6}} - \frac{3}{4}(2x-1)^{\frac{2}{3}} + 6\sqrt{2x-1} - 9(2x-1)^{\frac{1}{3}} + 18(2x-1)^{\frac{1}{6}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

3.713 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} \, dx$

Optimal. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rubi [A] time = 0.0588152, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\,\mathrm{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\,\mathrm{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}} + 11\sqrt{2+x} - 6(2+x)^{3/2} + (2+x)^{5/2}\right) dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}} + \frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} + \frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

Mathematica [A] time = 0.0598087, size = 58, normalized size = 0.7

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A] time = 0.01, size = 54, normalized size = 0.7

$$\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}} + \frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}} - 48\sqrt{2+\sqrt{1+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Maxima [A] time = 1.0017, size = 72, normalized size = 0.87

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Fricas [A] time = 1.50503, size = 124, normalized size = 1.49

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.714 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

Optimal. Leaf size=64

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7
 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rubi [A] time = 0.0491043, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7
 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{2+\sqrt{4+x}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \sqrt{2+\sqrt{x}}(-4+x)\,dx, x, 4+\sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int x\sqrt{2+x}\left(-4+x^2\right)\,dx, x, \sqrt{4+\sqrt{x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(8(2+x)^{3/2}-6(2+x)^{5/2}+(2+x)^{7/2}\right)\,dx, x, \sqrt{4+\sqrt{x}}\right)$$
$$= \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

Mathematica [A] time = 0.0291675, size = 43, normalized size = 0.67

$$-\frac{8}{315}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}\left(130\sqrt{\sqrt{x}+4}-35\sqrt{x}-244\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (-8*(2 + Sqrt[4 + Sqrt[x]])^(5/2)*(-244 + 130*Sqrt[4 + Sqrt[x]] - 35*Sqrt[x
]))/315
```

Maple [A] time = 0.01, size = 41, normalized size = 0.6

$$\frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(4+x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(4
+x^(1/2))^(1/2))^(9/2)
```

Maxima [A] time = 1.0407, size = 54, normalized size = 0.84

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 64
/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

Fricas [A] time = 1.46157, size = 134, normalized size = 2.09

$$\frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt(
sqrt(sqrt(x) + 4) + 2)
```

Sympy [B] time = 2.26442, size = 216, normalized size = 3.38

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi}-\frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}-\frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4
)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4)
+ 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) + 4) +
 2)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.715 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

Optimal. Leaf size=82

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 +
5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45
```

Rubi [A] time = 0.0801328, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {371, 1398, 772}

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 +
5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx &= \frac{2}{5}\operatorname{Subst}\left(\int x\sqrt{2-\sqrt{4+x}}\,dx,x,\sqrt{-9+5x}\right)\\ &= \frac{2}{5}\operatorname{Subst}\left(\int \sqrt{2-\sqrt{x}}(-4+x)\,dx,x,4+\sqrt{-9+5x}\right)\\ &= \frac{4}{5}\operatorname{Subst}\left(\int \sqrt{2-x}x\left(-4+x^2\right)\,dx,x,\sqrt{4+\sqrt{-9+5x}}\right)\\ &= \frac{4}{5}\operatorname{Subst}\left(\int \left(-8(2-x)^{3/2}+6(2-x)^{5/2}-(2-x)^{7/2}\right)dx,x,\sqrt{4+\sqrt{-9+5x}}\right)\\ &= \frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right.\end{aligned}$$

Mathematica [A] time = 0.0475984, size = 57, normalized size = 0.7

$$\frac{8\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}\left(35\sqrt{5x-9}+130\sqrt{\sqrt{5x-9}+4}+244\right)}{1575}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2)*(244 + 35*Sqrt[-9 + 5*x] + 130*Sqrt
[4 + Sqrt[-9 + 5*x]]))/1575
```

Maple [A] time = 0.012, size = 59, normalized size = 0.7

$$\frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Maxima [A] time = 0.996798, size = 78, normalized size = 0.95

$$\frac{8}{45}\left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{9}{2}}-\frac{48}{35}\left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{7}{2}}+\frac{64}{25}\left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4)
 + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

Fricas [A] time = 1.43526, size = 171, normalized size = 2.09

$$-\frac{8}{1575}\left(2\left(5\sqrt{5x-9}-32\right)\sqrt{\sqrt{5x-9}+4}-175x-4\sqrt{5x-9}+443\right)\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt(
5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{2-\sqrt{\sqrt{5x-9}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2 - sqrt(sqrt(5*x - 9) + 4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.716 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rubi [A] time = 0.0465592, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\,\mathrm{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\,\mathrm{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}} + 11\sqrt{2+x} - 6(2+x)^{3/2} + (2+x)^{5/2}\right) dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}} + \frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} + \frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

Mathematica [A] time = 0.0204362, size = 58, normalized size = 0.7

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A] time = 0., size = 54, normalized size = 0.7

$$\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}} + \frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}} - 48\sqrt{2+\sqrt{1+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Maxima [A] time = 1.01124, size = 72, normalized size = 0.87

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Fricas [A] time = 1.52566, size = 124, normalized size = 1.49

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.717 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=190

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right.$$

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1
+ Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (
320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/
15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17
```

Rubi [A] time = 0.366168, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1618, 1620}

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1
+ Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (
320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/
15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17
```

Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int x\left(-1+x^2\right)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 8\,\text{Subst}\left(\int x^3\sqrt{1+x}\left(-2+x^2\right)\left(-1+x^2\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)$$
$$= 8\,\text{Subst}\left(\int x^3(1+x)^{3/2}\left(2-2x-x^2+x^3\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)$$
$$= 8\,\text{Subst}\left(\int \left(-2(1+x)^{3/2}+3(1+x)^{5/2}+7(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{11/2}-7(1$$
$$= -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2}+\frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right.$$

Mathematica [A] time = 0.103578, size = 135, normalized size = 0.71

$$\frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}\left(231\sqrt{x}\left(-377\sqrt{\sqrt{\sqrt{x}+1}+1}+195\sqrt{\sqrt{x}+1}+365\right)+8\left(252\sqrt{\sqrt{x}+1}\sqrt{\sqrt{\sqrt{x}+1}+1}+8\right.\right.}{765765}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2)*(8*(-8221 + 8642*Sqrt[1 + Sqrt[
1 + Sqrt[x]]] - 4865*Sqrt[1 + Sqrt[x]] + 252*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sq
rt[1 + Sqrt[x]]) + 231*(365 - 377*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 195*Sqrt[1
+ Sqrt[x]])*Sqrt[x]))/765765
```

Maple [A] time = 0.013, size = 121, normalized size = 0.6

$$-\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}+\frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}-\frac{320}{11}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x^(
1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(17/2)
```

Maxima [A] time = 1.00248, size = 162, normalized size = 0.85

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
+ 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) - 3
20/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32/5*(
sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

Fricas [A] time = 1.54633, size = 293, normalized size = 1.54

$$\frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+1}+45045\,x+4613\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)*
sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 4504
5*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.718 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

Optimal. Leaf size=233

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}\cdot\right.$$

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7
/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqr
t[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 +
 (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rubi [A] time = 0.38133, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1620}

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}\cdot\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7
/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqr
t[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 +
 (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{2+\sqrt{3+\sqrt{-1+2x}}}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int x\left(1+x^2\right)\sqrt{2+\sqrt{3+x}}\,dx, x, \sqrt{-1+2\sqrt{x}}\right)$$
$$= 2\,\text{Subst}\left(\int x\sqrt{2+x}\left(-3+x^2\right)\left(1+\left(-3+x^2\right)^2\right)dx, x, \sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-4\sqrt{2+x}+34(2+x)^{3/2}-120(2+x)^{5/2}+228(2+x)^{7/2}-190(2+x)^{9/2}+\right.\right.$$
$$= -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2}-\frac{480}{7}\left(2+\sqrt{3+\sqrt{\right.\right.}}$$

Mathematica [A] time = 0.130451, size = 183, normalized size = 0.79

$$\frac{8\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{3/2}\left(7\sqrt{x}\left(2145\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3}+1452\sqrt{\sqrt{2\sqrt{x}-1}+3}-4004\sqrt{2\sqrt{x}-1}-3576\right)+4\right.}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (8*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2)*(4*(-9786 - 2535*Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]] - 4286*Sqrt[-1 + 2*Sqrt[x]] + 3843*Sqrt[3 + Sqrt[-1 + 2*
Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-3576 + 1452*Sqrt[3 + Sqrt[-1 + 2*Sqrt
[x]]] - 4004*Sqrt[-1 + 2*Sqrt[x]] + 2145*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqr
t[-1 + 2*Sqrt[x]])*Sqrt[x]))/255255
```

Maple [A] time = 0.017, size = 154, normalized size = 0.7

$$-\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}-\frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}+\frac{304}{3}\left(2+\sqrt{3+\sqrt{-\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

Maxima [A] time = 1.01166, size = 207, normalized size = 0.89

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(x
) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/2)
 - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(2*s
qrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(7/
2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt(2*sq
rt(x) - 1) + 3) + 2)^(3/2)
```

Fricas [A] time = 1.49924, size = 309, normalized size = 1.33

$$-\frac{8}{255255}\left((847\sqrt{x}-1688)\sqrt{2\sqrt{x}-1}-2\left((1001\sqrt{x}+6800)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2\sqrt{x}-1}+3}-3003\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 68
00)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1) +
3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.719 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$

Optimal. Leaf size=160

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+8\Big(\sqrt{}$$

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt
[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*
(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(
17/2))/17
```

Rubi [A] time = 0.275713, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1618, 1620}

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+8\Big(\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```


```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt
[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*
(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(
17/2))/17
```


Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = 2\,\mathrm{Subst}\left(\int x\left(1+x^2\right)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{-1+x}\right)$$
$$= 4\,\mathrm{Subst}\left(\int x\sqrt{1+x}\left(-1+x^2\right)\left(1+\left(-1+x^2\right)^2\right)dx, x, \sqrt{1+\sqrt{-1+x}}\right)$$
$$= 4\,\mathrm{Subst}\left(\int x(1+x)^{3/2}\left(-2+2x+2x^2-2x^3-x^4+x^5\right)dx, x, \sqrt{1+\sqrt{-1+x}}\right)$$
$$= 4\,\mathrm{Subst}\left(\int \left(2(1+x)^{3/2}-3(1+x)^{5/2}+9(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{11/2}-7(1+\right.\right.$$
$$= \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2}-\frac{1}{}$$

Mathematica [A] time = 0.0896872, size = 103, normalized size = 0.64

$$\frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}\left(8\left(84\sqrt{x-1}\sqrt{\sqrt{x-1}+1}-3030\sqrt{\sqrt{x-1}+1}+1715\sqrt{x-1}+2591\right)+77\left(-377\sqrt{\sqrt{x-1}+\right.\right.}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2)*(8*(2591 - 3030*Sqrt[1 + Sqrt[-1 + x]
] + 1715*Sqrt[-1 + x] + 84*Sqrt[1 + Sqrt[-1 + x]]*Sqrt[-1 + x]) + 77*(365 -
 377*Sqrt[1 + Sqrt[-1 + x]] + 195*Sqrt[-1 + x])*x))/255255
```

Maple [A] time = 0.006, size = 107, normalized size = 0.7

$$\frac{16}{5}\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{9/2}-\frac{160}{11}\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}+\frac{144}{13}\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(1+(x-1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 16/5*(1+(1+(x-1)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(x-1)^(1/2))^(1/2))^(7/2)+8
*(1+(1+(x-1)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(x-1)^(1/2))^(1/2))^(11/2)+14
4/13*(1+(1+(x-1)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(x-1)^(1/2))^(1/2))^(15/2
)+8/17*(1+(1+(x-1)^(1/2))^(1/2))^(17/2)
```

Maxima [A] time = 1.00873, size = 143, normalized size = 0.89

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}+8\left(\sqrt{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1)
^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x -
 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(sqr
t(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

Fricas [A] time = 1.51213, size = 228, normalized size = 1.42

$$\frac{8}{255255}\left(15015\,x^2 + (77\,x + 1032)\sqrt{x-1} + \left((1001\,x + 4544)\sqrt{x-1} - 1176\,x - 7696\right)\sqrt{\sqrt{x-1}+1} - 1799\,x - 22088\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x -
 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sqrt
(x - 1) + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.720 $\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}} dx$

Optimal. Leaf size=20

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rubi [A] time = 0.0778544, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {619, 215}

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}} dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}} dx, x, \sqrt{-1+x}\right) \\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}} \\
&= -2\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0206398, size = 20, normalized size = 1.

$$2\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] 2*ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

Maple [A] time = 0.001, size = 16, normalized size = 0.8

$$2 \operatorname{Arcsinh}\left(2/3 \sqrt{3} \left(\sqrt{x-1} - 1/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)

[Out] 2*arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x - \sqrt{x-1}}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Fricas [B] time = 3.70194, size = 103, normalized size = 5.15

$$\log\left(4\sqrt{x - \sqrt{x-1}}\left(2\sqrt{x-1} - 1\right) + 8x - 8\sqrt{x-1} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")

[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x - \sqrt{x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)

[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)
```

Giac [A] time = 1.22316, size = 34, normalized size = 1.7

$$-2\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

3.721 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=44

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2*x])/Sqrt[
2]]
```

Rubi [A] time = 0.0351152, antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 215}

$$\sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}}\,dx, x, 1+\sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \sqrt{2}\sinh^{-1}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.0175189, size = 44, normalized size = 1.

$$2\sqrt{x+\sqrt{2x-1}+1} - \sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]

[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2*x])/Sqrt[
2]]
```

Maple [A] time = 0.009, size = 38, normalized size = 0.9

$$\sqrt{4x+4+4\sqrt{2x-1}} - \text{Arcsinh}\left(\frac{\sqrt{2}}{2}\left(1+\sqrt{2x-1}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+(2*x-1)^(1/2))^(1/2),x)

[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

Fricas [B] time = 5.8144, size = 246, normalized size = 5.59

$$\frac{1}{4}\sqrt{2}\log\left(-8x^2 - 8(2x+1)\sqrt{2x-1} + 2\left(\sqrt{2}(2x+3)\sqrt{2x-1} + \sqrt{2}(6x-1)\right)\sqrt{x+\sqrt{2x-1}+1} - 24x + 7\right) + 2\sqrt{x+\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*s
qrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7) +
 2*sqrt(x + sqrt(2*x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

Giac [A] time = 1.22226, size = 92, normalized size = 2.09

$$-\sqrt{2}\left(\sqrt{3}+\log\left(\sqrt{3}-1\right)\right) + \sqrt{2}\log\left(\sqrt{2x+2\sqrt{2x-1}+2} - \sqrt{2x-1} - 1\right) + \sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(3) + log(sqrt(3) - 1)) + sqrt(2)*log(sqrt(2*x + 2*sqrt(2*x -
 1) + 2) - sqrt(2*x - 1) - 1) + sqrt(2)*sqrt(2*x + 2*sqrt(2*x - 1) + 2)
```

3.722 $\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)} dx$

Optimal. Leaf size=54

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b
 + a*x]])/a^2
```

Rubi [A] time = 0.380631, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {697}

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b
 + a*x]])/a^2
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}dx &= \frac{2\,\text{Subst}\left(\int \frac{-bp+aq+px^2}{f+x}dx,x,\sqrt{b+ax}\right)}{a^2}\\ &= \frac{2\,\text{Subst}\left(\int \left(-fp+px+\frac{-bp+f^2p+aq}{f+x}\right)dx,x,\sqrt{b+ax}\right)}{a^2}\\ &= \frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2\left(bp-f^2p-aq\right)\log\left(f+\sqrt{b+ax}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0828806, size = 50, normalized size = 0.93

$$\frac{2\left(aq-bp+f^2p\right)\log\left(\sqrt{ax+b}+f\right)+p\left(ax-2f\sqrt{ax+b}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*(a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b + a*
x]])/a^2
```

Maple [A] time = 0.005, size = 80, normalized size = 1.5

$$\frac{px}{a}+\frac{bp}{a^2}-2\,\frac{fp\sqrt{ax+b}}{a^2}+2\,\frac{\ln\left(f+\sqrt{ax+b}\right)f^2p}{a^2}+2\,\frac{\ln\left(f+\sqrt{ax+b}\right)q}{a}-2\,\frac{\ln\left(f+\sqrt{ax+b}\right)bp}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x)
```

```
[Out] p*x/a+1/a^2*b*p-2*f*p*(a*x+b)^(1/2)/a^2+2/a^2*ln(f+(a*x+b)^(1/2))*f^2*p+2/a
*ln(f+(a*x+b)^(1/2))*q-2/a^2*ln(f+(a*x+b)^(1/2))*b*p
```

Maxima [A] time = 0.973133, size = 78, normalized size = 1.44

$$\frac{\frac{2\left(\left(f^2-b\right)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a}-\frac{2\sqrt{ax+b}fp-(ax+b)p}{a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")
```

```
[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b)*f*p - (a
*x + b)*p)/a)/a
```

Fricas [A] time = 1.48291, size = 111, normalized size = 2.06

$$\frac{apx-2\sqrt{ax+b}fp+2\left(\left(f^2-b\right)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))
)/a^2
```

Sympy [A] time = 21.2289, size = 99, normalized size = 1.83

$$-\frac{2fp\sqrt{ax+b}}{a^2}-\frac{2f\left(-aq+bp-f^2p\right)\left(\begin{cases}\frac{1}{\sqrt{ax+b}} & \text{for}\ f=0\\ \frac{\log\left(\frac{f}{\sqrt{ax+b}}+1\right)}{f} & \text{otherwise}\end{cases}\right)}{a^2}+\frac{p\left(ax+b\right)}{a^2}+\frac{2\left(-aq+bp-f^2p\right)\log\left(\frac{1}{\sqrt{ax+b}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

```
[Out] -2*f*p*sqrt(a*x + b)/a**2 - 2*f*(-a*q + b*p - f**2*p)*Piecewise((1/sqrt(a*x
 + b), Eq(f, 0)), (log(f/sqrt(a*x + b) + 1)/f, True))/a**2 + p*(a*x + b)/a*
```

```
*2 + 2*(-a*q + b*p - f**2*p)*log(1/sqrt(a*x + b))/a**2
```

Giac [A] time = 1.21065, size = 119, normalized size = 2.2

$$\frac{2\left(f^2p-bp+aq\right)\log\left(\left|f+\sqrt{ax+b}\right|\right)}{a^2}-\frac{2\left(f^2p\log\left(\left|f\right|\right)-bp\log\left(\left|f\right|\right)+aq\log\left(\left|f\right|\right)\right)}{a^2}-\frac{2\sqrt{ax+b}a^2fp-(ax+b)a^2p}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - 2*(f^2*p*log(abs(f)
) - b*p*log(abs(f)) + a*q*log(abs(f)))/a^2 - (2*sqrt(a*x + b)*a^2*f*p - (a*
x + b)*a^2*p)/a^4
```

3.723 $\int \sqrt{1-\sqrt{x}-x}\,dx$

Optimal. Leaf size=70

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)^(3/2))/3
- (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rubi [A] time = 0.0347816, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1341, 640, 612, 619, 216}

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)^(3/2))/3
- (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} + \frac{1}{8}\sqrt{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx, x, -1-2\sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0281065, size = 53, normalized size = 0.76

$$\frac{1}{12}\sqrt{-x-\sqrt{x}+1}\left(8x+2\sqrt{x}-11\right)+\frac{5}{8}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 + (5*ArcSin[(-1 - 2*Sqrt
[x])/Sqrt[5]])/8
```

Maple [A] time = 0.006, size = 50, normalized size = 0.7

$$-\frac{2}{3}\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}+\frac{1}{4}\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}-\frac{5}{8}\arcsin\left(\frac{2\sqrt{5}}{5}\left(\sqrt{x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-x^(1/2))^(1/2),x)
```

```
[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin(
2/5*5^(1/2)*(x^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x - sqrt(x) + 1), x)
```

Fricas [A] time = 7.01037, size = 231, normalized size = 3.3

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1}+\frac{5}{16}\,\arctan\left(-\frac{\left(8\,x^2-\left(16\,x^2-38\,x+11\right)\sqrt{x}-9\,x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4\,x^3-13\,x^2+7\,x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x^
2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 -
 13*x^2 + 7*x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) - x + 1), x)
```

Giac [A] time = 1.23041, size = 59, normalized size = 0.84

$$\frac{1}{12}\left(2\,\sqrt{x}\left(4\,\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\,\arcsin\left(\frac{1}{5}\,\sqrt{5}\left(2\,\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(1
/5*sqrt(5)*(2*sqrt(x) + 1))
```

3.724 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx$

Optimal. Leaf size=19

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Rubi [A] time = 0.0207082, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 1397, 771}

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 1397

```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g
 - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x
]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}
\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx &= \int \frac{\left(3+\sqrt{x}\right)^2}{4\sqrt{x}+x}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{x(3+x)^2}{4x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(2+x+\frac{1}{4+x}\right)dx, x, \sqrt{x}\right)\\
&= 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0113331, size = 19, normalized size = 1.

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x)
```

```
[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)
```

Maxima [A] time = 1.03659, size = 20, normalized size = 1.05

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Fricas [A] time = 1.43486, size = 49, normalized size = 2.58

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Sympy [A] time = 0.14789, size = 17, normalized size = 0.89

$$4\sqrt{x}+x+2\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

Giac [A] time = 1.20554, size = 20, normalized size = 1.05

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

3.725 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

Optimal. Leaf size=77

$$\frac{2x^4}{9}+\frac{80x^{7/2}}{567}+\frac{200x^3}{2187}+\frac{400x^{5/2}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{3/2}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323
 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/56
7 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

Rubi [A] time = 0.0656674, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1893, 190, 43, 266}

$$\frac{2x^4}{9}+\frac{80x^{7/2}}{567}+\frac{200x^3}{2187}+\frac{400x^{5/2}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{3/2}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In] Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323
 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/56
7 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \int \left(-\frac{6}{-5+9\sqrt{x}}+\frac{8x^{7/2}}{-5+9\sqrt{x}}\right)dx$$
$$= -\left(6\int \frac{1}{-5+9\sqrt{x}}\,dx\right)+8\int \frac{x^{7/2}}{-5+9\sqrt{x}}\,dx$$
$$= -\left(12\,\text{Subst}\left(\int \frac{x}{-5+9x}\,dx,x,\sqrt{x}\right)\right)+16\,\text{Subst}\left(\int \frac{x^8}{-5+9x}\,dx,x,\sqrt{x}\right)$$
$$= -\left(12\,\text{Subst}\left(\int \left(\frac{1}{9}+\frac{5}{9(-5+9x)}\right)dx,x,\sqrt{x}\right)\right)+16\,\text{Subst}\left(\int \left(\frac{78125}{43046721}+\frac{15625x}{4782969}+\frac{3125x^2}{531441}+\right.\right.$$
$$= -\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049}+\frac{400x^{5/2}}{6561}+\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}-\frac{28072814}{38}$$

Mathematica [A] time = 0.0556381, size = 66, normalized size = 0.86

$$\frac{2\left(9\left(33480783x^4+21257640x^{7/2}+13778100x^3+9185400x^{5/2}+6378750x^2+4725000x^{3/2}+3937500x-19650969\right.\right.}{2711943423}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*(9*(-196509698*Sqrt[x] + 3937500*x + 4725000*x^(3/2) + 6378750*x^2 + 918
5400*x^(5/2) + 13778100*x^3 + 21257640*x^(7/2) + 33480783*x^4) - 982548490*
Log[5 - 9*Sqrt[x]]))/2711943423
```

Maple [A] time = 0.005, size = 50, normalized size = 0.7

$$\frac{2\,x^4}{9}+\frac{80}{567}x^{\frac{7}{2}}+\frac{200\,x^3}{2187}+\frac{400}{6561}x^{\frac{5}{2}}+\frac{2500\,x^2}{59049}+\frac{50000}{1594323}x^{\frac{3}{2}}+\frac{125000\,x}{4782969}-\frac{56145628}{43046721}\sqrt{x}-\frac{280728140}{387420489}\ln\left(-5+9\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6-8*x^(7/2))/(5-9*x^(1/2)),x)
```

```
[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/1
594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/3874204
89*ln(-5+9*x^(1/2))
```

Maxima [A] time = 1.0145, size = 66, normalized size = 0.86

$$\frac{2}{9}x^4+\frac{80}{567}x^{\frac{7}{2}}+\frac{200}{2187}x^3+\frac{400}{6561}x^{\frac{5}{2}}+\frac{2500}{59049}x^2+\frac{50000}{1594323}x^{\frac{3}{2}}+\frac{125000}{4782969}x-\frac{56145628}{43046721}\sqrt{x}-\frac{280728140}{387420489}\log\left(9\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(9*sqrt(x) - 5)
```

Fricas [A] time = 1.4554, size = 236, normalized size = 3.06

$$\frac{2}{9}x^4 + \frac{200}{2187}x^3 + \frac{2500}{59049}x^2 + \frac{4}{301327047}\left(10628820\,x^3 + 4592700\,x^2 + 2362500\,x - 98254849\right)\sqrt{x} + \frac{125000}{4782969}x - \frac{2807}{3874}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 45927
00*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/38742
0489*log(9*sqrt(x) - 5)
```

Sympy [A] time = 27.6859, size = 71, normalized size = 0.92

$$\frac{80x^{\frac{7}{2}}}{567} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{50000x^{\frac{3}{2}}}{1594323} - \frac{56145628\sqrt{x}}{43046721} + \frac{2x^4}{9} + \frac{200x^3}{2187} + \frac{2500x^2}{59049} + \frac{125000x}{4782969} - \frac{280728140\log\left(\sqrt{x} - \frac{5}{9}\right)}{387420489}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sqr
t(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/47829
69 - 280728140*log(sqrt(x) - 5/9)/387420489
```

Giac [A] time = 1.15878, size = 68, normalized size = 0.88

$$\frac{2}{9}x^4 + \frac{80}{567}x^{\frac{7}{2}} + \frac{200}{2187}x^3 + \frac{400}{6561}x^{\frac{5}{2}} + \frac{2500}{59049}x^2 + \frac{50000}{1594323}x^{\frac{3}{2}} + \frac{125000}{4782969}x - \frac{56145628}{43046721}\sqrt{x} - \frac{280728140}{387420489}\log\left(\left|9\sqrt{x} -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(abs(9*sqrt(x) - 5))
```

3.726 $\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx$

Optimal. Leaf size=80

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 + (1 - I)^(3/2)*
ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sqrt[1
 + I]]
```

Rubi [B] time = 0.285622, antiderivative size = 224, normalized size of antiderivative = 2.8, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1625, 1629, 825, 12, 708, 1094, 634, 618, 204, 628}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+\right.}{2\sqrt{1+\sqrt{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt[
2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]]
+ Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/
(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1
+ x]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 1625

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x}\left(1+x^3\right)}{1+x^2}\,dx &= \int \frac{(1+x)^{3/2}\left(1-x+x^2\right)}{1+x^2}\,dx\\
&= \int \left((1+x)^{3/2}-\frac{x(1+x)^{3/2}}{1+x^2}\right)dx\\
&= \frac{2}{5}(1+x)^{5/2}-\int \frac{x(1+x)^{3/2}}{1+x^2}\,dx\\
&= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}-\int \frac{(-1+x)\sqrt{1+x}}{1+x^2}\,dx\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}-\int -\frac{2}{\sqrt{1+x}\left(1+x^2\right)}\,dx\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}+2\int \frac{1}{\sqrt{1+x}\left(1+x^2\right)}\,dx\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}+4\,\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx,x,\sqrt{1+x}\right)\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}+\frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\text{Subst}\left(\int\right.}{}\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+x}\right)}{\sqrt{2}}+\frac{\text{Subst}\left(\int\right.}{}\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}-\frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(1+\right.}{}\\
&= -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}-\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}}-
\end{aligned}
$$

Mathematica [A] time = 0.103947, size = 68, normalized size = 0.85

$$
\frac{2}{15}\sqrt{x+1}\left(3x^2+x-17\right)+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 + (1 - I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sq
rt[1 - I]] + (1 + I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]
```

Maple [B] time = 0.033, size = 443, normalized size = 5.5

$$
\frac{2}{5}(1+x)^{\frac{5}{2}}-\frac{2}{3}(1+x)^{\frac{3}{2}}-2\sqrt{1+x}-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}}{4}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)+\frac{\sqrt{2+2\sqrt{2}}}{2}\ln\left(1+x+\sqrt{2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(1+x)^(1/2)/(x^2+1),x)
```

```
[Out] 2/5*(1+x)^(5/2)-2/3*(1+x)^(3/2)-2*(1+x)^(1/2)-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2
)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)+1/2*ln(1+x+2^(1/2)+(1+x)
^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2)
)*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)
^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2
)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)-1/2*ln(1+x+2
^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/
2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*
(2+2*2^(1/2))*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arc
tan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3+1)\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

Fricas [B] time = 1.60087, size = 986, normalized size = 12.32

$$-\frac{1}{8} \cdot 8^{\frac{1}{4}} \sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2 \cdot 8^{\frac{1}{4}} \sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)+\frac{1}{8} \cdot 8^{\frac{1}{4}} \sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2 \cdot 8^{\frac{1}{4}} \sqrt{x\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*8^(1/4)*sqrt(x + 1)*sq
rt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*
(sqrt(2) - 2)*log(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt
(2) + 4) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*sqrt
(2)*sqrt(2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*s
qrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*
sqrt(2)*sqrt(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) +
 4)*sqrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) +
4) + sqrt(2) + 1) + 2/15*(3*x^2 + x - 17)*sqrt(x + 1)
```

Sympy [A] time = 7.36695, size = 56, normalized size = 0.7

$$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}+4\operatorname{RootSum}\left(512t^4+32t^2+1,\left(t \mapsto t\log\left(-128t^3+\sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) + 4*RootSum(512*_t*
*4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*_t**3 + sqrt(x + 1))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3+1)\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

3.727 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

Optimal. Leaf size=89

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

Rubi [A] time = 0.263023, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {990, 621, 206, 1033, 724, 204}

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```


```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```


Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{-1-x+x^2}}{-1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) - 2\,\text{Subst}\left(\int \frac{x}{\left(-1+x^2\right)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+2\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) - \text{Sul}$$
$$= -2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3+\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{4-}\right.$$
$$= \tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - \tanh^{-1}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)$$

Mathematica [A] time = 0.0621889, size = 89, normalized size = 1.

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right) - \tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

Maple [A] time = 0.013, size = 130, normalized size = 1.5

$$\sqrt{\left(-1+\sqrt{x}\right)^2+\sqrt{x}-2} + \frac{1}{2}\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{\left(-1+\sqrt{x}\right)^2+\sqrt{x}-2}\right) - \arctan\left(\frac{1}{2}\left(\sqrt{x}-3\right)\frac{1}{\sqrt{\left(-1+\sqrt{x}\right)^2+\sqrt{x}-2}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x-x^(1/2))^(1/2)/(x-1)/x^(1/2),x)
```

```
[Out] ((-1+x^(1/2))^2+x^(1/2)-2)^(1/2)+1/2*ln(x^(1/2)-1/2+((-1+x^(1/2))^2+x^(1/2)
-2)^(1/2))-arctan(1/2*(x^(1/2)-3)/((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))-((1+x^(
1/2))^2-3*x^(1/2)-2)^(1/2)+3/2*ln(x^(1/2)-1/2+((1+x^(1/2))^2-3*x^(1/2)-2)^(
1/2))+arctanh(1/2*(-1-3*x^(1/2))/((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)
```

Fricas [A] time = 23.5296, size = 248, normalized size = 2.79

$$-\arctan\left(\frac{\left((x-4)\sqrt{x}-2x+3\right)\sqrt{x-\sqrt{x}-1}}{2\left(x^2-3x+1\right)}\right)+\log\left(-\frac{8x^2+2\left((4x-5)\sqrt{x}+2x-1\right)\sqrt{x-\sqrt{x}-1}-17x-2\sqrt{x}+11}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x +
1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1) -
 17*x - 2*sqrt(x) + 11)/(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\sqrt{x}+x-1}}{\sqrt{x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.728 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=61

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rubi [A] time = 0.512715, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1033, 724, 206, 204}

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{1+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$= 3\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx,$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\right) - 6\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}$$

$$= -\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + 3\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A] time = 0.051333, size = 61, normalized size = 1.

$$\tan^{-1}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right) - 3\tanh^{-1}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[(-3 - Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] - 3*ArcTanh[(-1 + 3*Sq
rt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Maple [A] time = 0.015, size = 68, normalized size = 1.1

$$-3\,\text{Artanh}\left(1/2\,\frac{3\,\sqrt{1+x}-1}{\sqrt{\left(\sqrt{1+x}-1\right)^2+3\,\sqrt{1+x}-2}}\right) + \arctan\left(\frac{1}{2}\left(-3-\sqrt{1+x}\right)\frac{1}{\sqrt{\left(1+\sqrt{1+x}\right)^2-\sqrt{1+x}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] -3*arctanh(1/2*(3*(1+x)^(1/2)-1)/(((1+x)^(1/2)-1)^2+3*(1+x)^(1/2)-2)^(1/2))
+arctan(1/2*(-3-(1+x)^(1/2))/((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\sqrt{x+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)
```

Fricas [A] time = 15.3547, size = 189, normalized size = 3.1

$$\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right)+3\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3x-2\sqrt{x+1}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="fricas")
```

```
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(x
 + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.729 $\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0018681, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {54, 215}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + x]),x]

[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0023085, size = 8, normalized size = 1.

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]

[Out] 2*ArcSinh[Sqrt[x]]
```

Maple [B] time = 0.003, size = 28, normalized size = 3.5

$$\sqrt{x(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (x*(1+x))^(1/2)/x^(1/2)/(1+x)^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [B] time = 0.980169, size = 36, normalized size = 4.5

$$\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)-\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [B] time = 1.58046, size = 53, normalized size = 6.62

$$-\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [A] time = 0.98345, size = 26, normalized size = 3.25

$$\begin{cases} 2\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1 \\ -2i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
True))
```

Giac [B] time = 1.21662, size = 20, normalized size = 2.5

$$-2\log\left(\left|-\sqrt{x+1}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(x + 1) + sqrt(x)))
```

3.730 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} dx$

Optimal. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0113483, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1958, 54, 215}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)]/x,x]

[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{x}{1+x}}}{x} dx &= \int \frac{1}{\sqrt{x}\sqrt{1+x}} dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x}\right) \\
&= 2\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0057167, size = 8, normalized size = 1.

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Maple [B] time = 0.01, size = 32, normalized size = 4.

$$(1+x)\sqrt{\frac{x}{1+x}}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(1+x))^(1/2)/x,x)
```

```
[Out] (x/(1+x))^(1/2)/(x*(1+x))^(1/2)*(1+x)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [B] time = 1.01792, size = 36, normalized size = 4.5

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Fricas [B] time = 1.5946, size = 72, normalized size = 9.

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

<u>Giac</u> [B] time = 1.13737, size = 30, normalized size = 3.75

$$-\log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right)\operatorname{sgn}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

3.731 $\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0033112, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 + x],x]

[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx &= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}} \, dx \\ &= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sqrt{x}\right) \\ &= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right) \end{aligned}$$

Mathematica [A] time = 0.0161803, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

Maple [B] time = 0.003, size = 39, normalized size = 1.8

$$\sqrt{x}\sqrt{1+x}-\frac{1}{2}\sqrt{x(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x)^(1/2),x)
```

```
[Out] x^(1/2)*(1+x)^(1/2)-1/2*(x*(1+x))^(1/2)/x^(1/2)/(1+x)^(1/2)*ln(1/2+x+(x^2+x
)^(1/2))
```

Maxima [B] time = 0.983564, size = 66, normalized size = 3.

$$\frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)}-\frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 1.78441, size = 86, normalized size = 3.91

$$\sqrt{x+1}\sqrt{x}+\frac{1}{2}\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [A] time = 1.54248, size = 60, normalized size = 2.73

$$\begin{cases} -\operatorname{acosh}\left(\sqrt{x+1}\right)+\frac{(x+1)^{\frac{3}{2}}}{\sqrt{x}}-\frac{\sqrt{x+1}}{\sqrt{x}} & \text{for } |x+1|>1 \\ i\sqrt{-x}\sqrt{x+1}+i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1)) + (x + 1)**(3/2)/sqrt(x) - sqrt(x + 1)/sqrt(
x), Abs(x + 1) > 1), (I*sqrt(-x)*sqrt(x + 1) + I*asin(sqrt(x + 1)), True))
```

Giac [A] time = 1.2386, size = 31, normalized size = 1.41

$$\sqrt{x+1}\sqrt{x} + \log\left(\left|-\sqrt{x+1} + \sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x) + log(abs(-sqrt(x + 1) + sqrt(x)))
```

3.732 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0047217, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0020801, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

Maple [B] time = 0.003, size = 45, normalized size = 2.1

$$\frac{1+x}{2}\sqrt{\frac{x}{1+x}}\left(2\sqrt{x^2+x} - \ln\left(\frac{1}{2} + x + \sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(1+x))^(1/2),x)
```

```
[Out] 1/2*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)))/(x*(1+x
))^(1/2)
```

Maxima [B] time = 0.975809, size = 69, normalized size = 3.14

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1} - 1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

Fricas [B] time = 1.70566, size = 117, normalized size = 5.32

$$(x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

Giac [B] time = 1.21096, size = 47, normalized size = 2.14

$$\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+\sqrt{x^2+x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

3.733 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rubi [A] time = 0.0038911, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\sqrt{1+x}\right)\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0179896, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(-x^2+\sqrt{x^2-1}x\tan^{-1}\left(\sqrt{x^2-1}\right)+1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])
)/((-1 + x)*x)
```

Maple [A] time = 0.014, size = 43, normalized size = 1.2

$$\frac{1}{x}\left(-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x-\sqrt{x^2-1}\right)\sqrt{x-1}\sqrt{1+x}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/x^2/(1+x)^(1/2),x)
```

```
[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(1+x)^(1/2)/x/(x^2-1
)^(1/2)
```

Maxima [A] time = 1.49192, size = 27, normalized size = 0.75

$$-\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

Fricas [A] time = 1.69141, size = 101, normalized size = 2.81

$$\frac{2x\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

Giac [A] time = 1.146, size = 57, normalized size = 1.58

$$-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}-2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(x
 - 1))^2)
```

3.734 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rubi [A] time = 0.0144775, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1958, 94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$$
$$= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx$$
$$= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\sqrt{1+x}\right)$$
$$= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

Mathematica [A] time = 0.0044747, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(-x^2+\sqrt{x^2-1}x\tan^{-1}\left(\sqrt{x^2-1}\right)+1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])
)/((-1 + x)*x)
```

Maple [B] time = 0.013, size = 60, normalized size = 1.7

$$-\frac{1+x}{x}\sqrt{\frac{x-1}{1+x}}\left(-\left(x^2-1\right)^{\frac{3}{2}}+x^2\sqrt{x^2-1}+\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x\right)\frac{1}{\sqrt{(x-1)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/(1+x))^(1/2)/x^2,x)
```

```
[Out] -((x-1)/(1+x))^(1/2)*(1+x)*(-(x^2-1)^(3/2)+x^2*(x^2-1)^(1/2)+arctan(1/(x^2-
1)^(1/2))*x)/((x-1)*(1+x))^(1/2)/x
```

Maxima [A] time = 1.4739, size = 55, normalized size = 1.53

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}+2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x +
 1)))
```

Fricas [A] time = 1.78008, size = 96, normalized size = 2.67

$$\frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-(x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

Giac [A] time = 1.15743, size = 69, normalized size = 1.92

$$-\frac{1}{2}\,(\pi-2)\mathrm{sgn}\,(x+1)+2\,\arctan\left(-x+\sqrt{x^2-1}\right)\mathrm{sgn}\,(x+1)-\frac{2\,\mathrm{sgn}\,(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn(
x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

3.735 $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rubi [A] time = 0.012763, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}x}{\sqrt{1+x}}\,dx\\ &= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx\\ &= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\\ &= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0741051, size = 76, normalized size = 1.1

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(6x^5-8x^4+3x^3-8x^2-18\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-9x+16\right)}{24(x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))
```

Maple [A] time = 0.01, size = 76, normalized size = 1.1

$$\frac{1}{24}\sqrt{x-1}\sqrt{1+x}\left(6\,x^3\sqrt{x^2-1}-8\,x^2\sqrt{x^2-1}+9\,x\sqrt{x^2-1}+9\,\ln\left(x+\sqrt{x^2-1}\right)-16\,\sqrt{x^2-1}\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x-1)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 1/24*(x-1)^(1/2)*(1+x)^(1/2)*(6*x^3*(x^2-1)^(1/2)-8*x^2*(x^2-1)^(1/2)+9*x*(
x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))-16*(x^2-1)^(1/2))/(x^2-1)^(1/2)
```

Maxima [A] time = 1.00543, size = 74, normalized size = 1.07

$$\frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x-\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{x^2-1}x-\sqrt{x^2-1}+\frac{3}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x^
2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 1.71221, size = 130, normalized size = 1.88

$$\frac{1}{24}\left(6x^{3}-8x^{2}+9x-16\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{8}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
1)*sqrt(x - 1) - x)
```

Sympy [A] time = 14.2676, size = 83, normalized size = 1.2

$$\frac{(x-1)^{\frac{7}{2}}\sqrt{x+1}}{4}+\frac{5(x-1)^{\frac{5}{2}}\sqrt{x+1}}{12}+\frac{11(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\sqrt{x+1}}{8}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] (x - 1)**(7/2)*sqrt(x + 1)/4 + 5*(x - 1)**(5/2)*sqrt(x + 1)/12 + 11*(x - 1)
**(3/2)*sqrt(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*asinh(sqrt(2)*sqrt
(x - 1)/2)/4
```

Giac [A] time = 1.18659, size = 65, normalized size = 0.94

$$\frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{4}\log\left(\left|-\sqrt{x+1}+\sqrt{x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) - 3
/4*log(abs(-sqrt(x + 1) + sqrt(x - 1)))
```

3.736 $\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rubi [A] time = 0.0248089, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1958, 100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx &= \int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx\\
&= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}x}{\sqrt{1+x}}\,dx\\
&= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx\\
&= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\\
&= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0376549, size = 76, normalized size = 1.1

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(6x^5-8x^4+3x^3-8x^2-18\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-9x+16\right)}{24(x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))
```

Maple [A] time = 0.009, size = 79, normalized size = 1.1

$$\frac{1+x}{24}\sqrt{\frac{x-1}{1+x}}\left(6x\left(x^2-1\right)^{3/2}-8\left((x-1)(1+x)\right)^{3/2}+15x\sqrt{x^2-1}-24\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{(x-1)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x-1)/(1+x))^(1/2),x)
```

```
[Out] 1/24*((x-1)/(1+x))^(1/2)*(1+x)*(6*x*(x^2-1)^(3/2)-8*((x-1)*(1+x))^(3/2)+15*
x*(x^2-1)^(1/2)-24*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2)))/((x-1)*(1+x))^(1/2)
```

Maxima [B] time = 1.02151, size = 186, normalized size = 2.7

$$-\frac{39\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-31\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+49\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x - 1
)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^
2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 1.67558, size = 182, normalized size = 2.64

$$\frac{1}{24}\left(6x^4-2x^3+x^2-7x-16\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt(
(x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{\frac{x-1}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((-1+x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

Giac [A] time = 1.12025, size = 84, normalized size = 1.22

$$-\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\mathrm{sgn}\left(x+1\right)+\frac{1}{24}\left(\left(2\left(3x\mathrm{sgn}\left(x+1\right)-4\,\mathrm{sgn}\left(x+1\right)\right)x+9\,\mathrm{sgn}\left(x+1\right)\right)x-16\,\mathrm{sgn}\left(x+1\right)\right)\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] -3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) - 4
*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)
```

3.737 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx$

Optimal. Leaf size=15

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rubi [A] time = 0.012346, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 204}

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))]/x,x]

[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)\\ &= 2\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)\end{aligned}$$

Mathematica [B] time = 0.013448, size = 32, normalized size = 2.13

$$\frac{2\sqrt{-\frac{x}{x+1}}\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))]/x,x]
```

```
[Out] (2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x]
```

Maple [B] time = 0.004, size = 33, normalized size = 2.2

$$(1+x)\sqrt{-\frac{x}{1+x}}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(1+x))^(1/2)/x,x)
```

```
[Out] (-x/(1+x))^(1/2)*(1+x)/(x*(1+x))^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 1.47703, size = 18, normalized size = 1.2

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Fricas [A] time = 1.68155, size = 38, normalized size = 2.53

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-x/(x + 1))/x, x)
```

Giac [A] time = 1.138, size = 27, normalized size = 1.8

$$-\frac{1}{2}\,\pi\mathrm{sgn}\,(x+1)-\arcsin\,(2\,x+1)\,\mathrm{sgn}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

3.738 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx$

Optimal. Leaf size=18

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi [A] time = 0.0210876, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1961, 204}

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-2x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0139678, size = 34, normalized size = 1.89

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{1-x^2}\sin^{-1}(x)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcSin[x])/(-1 + x)
```

Maple [A] time = 0.01, size = 30, normalized size = 1.7

$$-(1+x)\arcsin(x)\sqrt{-\frac{x-1}{1+x}}\frac{1}{\sqrt{-(x-1)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(1+x))^(1/2)/(x-1),x)
```

```
[Out] -(-(x-1)/(1+x))^(1/2)*(1+x)/(-(x-1)*(1+x))^(1/2)*arcsin(x)
```

Maxima [A] time = 1.44567, size = 20, normalized size = 1.11

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Fricas [A] time = 1.6908, size = 46, normalized size = 2.56

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
```

```
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

Giac [A] time = 1.18225, size = 22, normalized size = 1.22

$$-\frac{1}{2}\pi \operatorname{sgn}(x+1) - \arcsin(x)\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

3.739 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} \, dx$

Optimal. Leaf size=24

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

```
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rubi [A] time = 0.0600706, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1961, 12, 203}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]

[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} \, dx &= (2b(a+c))\,\text{Subst}\left(\int \frac{1}{b^2(a+c)\left(1+x^2\right)} \, dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)}{b}\\
&= \frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}
\end{aligned}$$

Mathematica [B] time = 0.200211, size = 93, normalized size = 3.88

$$\frac{2b\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\sin^{-1}\left(\frac{b\sqrt{c-bx}}{\sqrt{-b}\sqrt{-b(a+c)}}\right)}{(-b)^{3/2}\sqrt{-b(a+c)}\sqrt{\frac{a+bx}{a+c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*b*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcSin[(b*Sqrt[c - b*x])/(Sqrt
[-b]*Sqrt[-(b*(a + c))]])])/((-b)^(3/2)*Sqrt[-(b*(a + c))]*Sqrt[(a + b*x)/(a
 + c)])
```

Maple [B] time = 0.029, size = 85, normalized size = 3.5

$$-(bx-c)\arctan\left(\frac{2\,bx+a-c}{2\,b}\sqrt{b^2}\frac{1}{\sqrt{-(bx+a)(bx-c)}}\right)\sqrt{-\frac{bx+a}{bx-c}}\frac{1}{\sqrt{b^2}}\frac{1}{\sqrt{-(bx+a)(bx-c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-(
b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

Maxima [A] time = 1.45854, size = 32, normalized size = 1.33

$$\frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Fricas [A] time = 1.71513, size = 54, normalized size = 2.25

$$\frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20831, size = 55, normalized size = 2.29

$$-\frac{\arcsin\left(-\frac{2\,bx+a-c}{a+c}\right)\operatorname{sgn}\left(-ab-bc\right)\operatorname{sgn}\left(bx-c\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

3.740 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.066614, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1961, 12, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]

[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = (2(bc-ad))\,\text{Subst}\left(\int \frac{1}{(bc-ad)\left(b-dx^2\right)}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [B] time = 0.0733484, size = 97, normalized size = 2.37

$$\frac{2\sqrt{bc-ad}\sqrt{\frac{a+bx}{c+dx}}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*Sqrt[b*c - a*d]*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[(b*(c + d*x))/(b*c - a*d)
]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[a + b*x
])
```

Maple [B] time = 0.02, size = 80, normalized size = 2.

$$(dx+c)\ln\left(\frac{1}{2}\left(2\,bdx+2\,\sqrt{(bx+a)\,(dx+c)}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{\frac{bx+a}{dx+c}}\frac{1}{\sqrt{(bx+a)\,(dx+c)}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72651, size = 251, normalized size = 6.12

$$\left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(
d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c))/(b*d*x + a*d))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25758, size = 100, normalized size = 2.44

$$-\frac{\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}bc-\sqrt{bd}ad\right|\right)\operatorname{sgn}\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))*sgn(d*x + c)/(b*d)
```

3.741 $\int \sqrt{-\frac{x}{1+x}} \, dx$

Optimal. Leaf size=32

$$\sqrt{-\frac{x}{x+1}}(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rubi [A] time = 0.0116393, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1959, 288, 204}

$$\sqrt{-\frac{x}{x+1}}(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))],x]

[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-\frac{x}{1+x}} \, dx &= -\left(2 \operatorname{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2} \, dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)\\
&= \sqrt{-\frac{x}{1+x}}(1+x)+\operatorname{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, \sqrt{-\frac{x}{1+x}}\right)\\
&= \sqrt{-\frac{x}{1+x}}(1+x)-\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0137088, size = 43, normalized size = 1.34

$$\frac{\sqrt{-\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))],x]
```

```
[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[
x]
```

Maple [A] time = 0.003, size = 46, normalized size = 1.4

$$\frac{1+x}{2}\sqrt{-\frac{x}{1+x}}\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(1+x))^(1/2),x)
```

```
[Out] 1/2*(-x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)))/(x*(1+
x))^(1/2)
```

Maxima [A] time = 1.50875, size = 50, normalized size = 1.56

$$-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

Fricas [A] time = 1.7115, size = 72, normalized size = 2.25

$$(x+1)\sqrt{-\frac{x}{x+1}}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(-x/(x + 1)), x)
```

Giac [A] time = 1.1992, size = 49, normalized size = 1.53

$$\frac{1}{4}\,\pi \mathrm{sgn}\left(x+1\right)+\frac{1}{2}\,\arcsin\left(2\,x+1\right)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^{2}-x}\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x +
 1)
```

3.742 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi [A] time = 0.0136912, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1959, 288, 204}

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1-x}{1+x}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x)+2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x)-2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0216509, size = 67, normalized size = 1.76

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{x+1}\left(\sqrt{x+1}(x-1)+2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*Ar
cSin[Sqrt[1 - x]/Sqrt[2]]))/(-1 + x)
```

Maple [A] time = 0.004, size = 39, normalized size = 1.

$$(1+x)\sqrt{-\frac{x-1}{1+x}}\left(\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-(x-1)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(1+x))^(1/2),x)
```

```
[Out] (-(x-1)/(1+x))^(1/2)*(1+x)/(-(x-1)*(1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

Maxima [A] time = 1.49234, size = 58, normalized size = 1.53

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

Fricas [A] time = 1.77437, size = 90, normalized size = 2.37

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1-x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

Giac [A] time = 1.12657, size = 39, normalized size = 1.03

$$\frac{1}{2}\,\pi \mathrm{sgn}\left(x+1\right)+\arcsin\left(x\right)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^2+1}\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

3.743 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rubi [A] time = 0.0161238, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1959, 288, 203}

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+x}{a-x}}\,dx &= (4a)\operatorname{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}}+(2a)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}}+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0423336, size = 78, normalized size = 1.86

$$\frac{\sqrt{\frac{a+x}{a-x}}\left(-2a^{3/2}\sqrt{a-x}\sqrt{\frac{a+x}{a}}\sin^{-1}\left(\frac{\sqrt{a-x}}{\sqrt{2}\sqrt{a}}\right)-a^2+x^2\right)}{a+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (Sqrt[(a + x)/(a - x)]*(-a^2 + x^2 - 2*a^(3/2)*Sqrt[a - x]*Sqrt[(a + x)/a]*
ArcSin[Sqrt[a - x]/(Sqrt[2]*Sqrt[a])]))/(a + x)
```

Maple [A] time = 0.014, size = 64, normalized size = 1.5

$$-(-a+x)\sqrt{-\frac{a+x}{-a+x}}\left(a\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)\frac{1}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x)
```

```
[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))
/(-(a+x)*(-a+x))^(1/2)
```

Maxima [A] time = 1.46206, size = 66, normalized size = 1.57

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

Fricas [A] time = 1.65947, size = 90, normalized size = 2.14

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

Giac [A] time = 1.14373, size = 49, normalized size = 1.17

$$a \arcsin\left(\frac{x}{a}\right) \operatorname{sgn}(a-x) \operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

3.744 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rubi [A] time = 0.0192864, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1959, 288, 206}

$$\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+x}{a+x}}\,dx &= (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)\\ &= \sqrt{-\frac{a-x}{a+x}}(a+x)-(2a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)\\ &= \sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0749607, size = 78, normalized size = 1.9

$$\frac{\sqrt{\frac{x-a}{a+x}}\left(\sqrt{x-a}(a+x)-2a^{3/2}\sqrt{\frac{a+x}{a}}\sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right)\right)}{\sqrt{x-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a^(3/2)*Sqrt[(a + x)/a]*A
rcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/Sqrt[-a + x]
```

Maple [A] time = 0.01, size = 60, normalized size = 1.5

$$-(a+x)\sqrt{\frac{-a+x}{a+x}}\left(a\ln\left(x+\sqrt{-a^2+x^2}\right)-\sqrt{-a^2+x^2}\right)\frac{1}{\sqrt{(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a+x)/(a+x))^(1/2),x)
```

```
[Out] -((-a+x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/((a
+x)*(-a+x))^(1/2)
```

Maxima [A] time = 0.988935, size = 95, normalized size = 2.32

$$a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

Fricas [A] time = 1.80178, size = 142, normalized size = 3.46

$$-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (a
 + x)*sqrt(-(a - x)/(a + x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)/(a + x)), x)
```

Giac [A] time = 1.15938, size = 54, normalized size = 1.32

$$a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x)+\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

3.745 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rubi [A] time = 0.039146, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1959, 288, 208}

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = (2(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(b-dx^2\right)^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= \frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)}{d}$$
$$= \frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

Mathematica [A] time = 0.273589, size = 123, normalized size = 1.62

$$\frac{\sqrt{\frac{a+bx}{c+dx}}\left(b\sqrt{d}(a+bx)(c+dx)-\sqrt{a+bx}(bc-ad)^{3/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{bd^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(b*Sqrt[d]*(a + b*x)*(c + d*x) - (b*c - a*d)^(3/
2)*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[b*c - a*d]]))/(b*d^(3/2)*(a + b*x))
```

Maple [B] time = 0.004, size = 152, normalized size = 2.

$$\frac{dx+c}{2\,d}\sqrt{\frac{bx+a}{dx+c}}\left(\ln\left(\frac{1}{2}\left(2\,bdx+2\,\sqrt{(bx+a)\,(dx+c)}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)ad-\ln\left(\frac{1}{2}\left(2\,bdx+2\,\sqrt{(bx+a)\,(dx+c)}\sqrt{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2),x)
```

```
[Out] 1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)*(b
*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78148, size = 424, normalized size = 5.58

$$\left[-\frac{(bc-ad)\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\left(bd^2x+bcd\right)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2},\ \frac{(bc-ad)\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{}}{bdx+ad}\right.}{bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c
)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)
))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x +
 a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c))
)/(b*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21165, size = 161, normalized size = 2.12

$$\frac{\sqrt{bdx^2+bcx+adx+ac}\,\mathrm{sgn}\left(dx+c\right)}{d}+\frac{\left(bc\,\mathrm{sgn}\left(dx+c\right)-ad\,\mathrm{sgn}\left(dx+c\right)\right)\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right.\right.}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d + 1/2*(b*c*sgn(d*x + c)
- a*d*sgn(d*x + c))*sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*
x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^2)
```

3.746 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3
*Sqrt[3])
```

Rubi [A] time = 0.0123619, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1958, 50, 54, 215}

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3
*Sqrt[3])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-1+x}{5+3x}}\,dx &= \int \frac{\sqrt{-1+x}}{\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x}-\frac{4}{3}\int \frac{1}{\sqrt{-1+x}\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x}-\frac{8}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{8+3x^2}}\,dx,x,\sqrt{-1+x}\right)\\ &= \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{-1+x}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0455404, size = 76, normalized size = 1.55

$$\frac{3(x-1)\sqrt{3x+5}-8\sqrt{3}\sqrt{x-1}\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{9\sqrt{\frac{x-1}{3x+5}}\sqrt{3x+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (3*(-1 + x)*Sqrt[5 + 3*x] - 8*Sqrt[3]*Sqrt[-1 + x]*ArcSinh[(Sqrt[3/2]*Sqrt[
-1 + x])/2])/(9*Sqrt[(-1 + x)/(5 + 3*x)]*Sqrt[5 + 3*x])
```

Maple [B] time = 0.01, size = 76, normalized size = 1.6

$$-\frac{5+3x}{9}\sqrt{\frac{x-1}{5+3x}}\left(4\ln\left(x\sqrt{3}+1/3\sqrt{3}+\sqrt{3x^2+2x-5}\right)\sqrt{3}-3\sqrt{3x^2+2x-5}\right)\frac{1}{\sqrt{(5+3x)(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/(5+3*x))^(1/2),x)
```

```
[Out] -1/9*((x-1)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(x*3^(1/2)+1/3*3^(1/2)+(3*x^2+2*x-5
)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(x-1))^(1/2)
```

Maxima [B] time = 1.47067, size = 108, normalized size = 2.2

$$\frac{4}{9}\sqrt{3}\log\left(-\frac{\sqrt{3}-3\sqrt{\frac{x-1}{3x+5}}}{\sqrt{3}+3\sqrt{\frac{x-1}{3x+5}}}\right)-\frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3(x-1)}{3x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((x
 - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1)
```

Fricas [A] time = 1.85112, size = 149, normalized size = 3.04

$$\frac{1}{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}+\frac{4}{9}\,\sqrt{3}\log\left(\sqrt{3}(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}-3\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*s
qrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x-1}{3x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/(3*x + 5)), x)
```

Giac [B] time = 1.16954, size = 100, normalized size = 2.04

$$-\frac{8}{9}\,\sqrt{3}\log\left(2\right)\mathrm{sgn}\left(3\,x+5\right)+\frac{4}{9}\,\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^{2}+2\,x-5}\right)-1\right|\right)\mathrm{sgn}\left(3\,x+5\right)+\frac{1}{3}\,\sqrt{3\,x^{2}+2\,x-5}\mathrm{sgn}\left(3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] -8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*sgn
(3*x + 5)
```

3.747 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}-12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x
]]
```

Rubi [A] time = 0.0226765, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1958, 94, 93, 204}

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}-12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x
]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \int \frac{\sqrt{-1+5x}}{x^2\sqrt{1+7x}}\,dx$$
$$= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} + 6\int \frac{1}{x\sqrt{-1+5x}\sqrt{1+7x}}\,dx$$
$$= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} + 12\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$
$$= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} - 12\tan^{-1}\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$

Mathematica [A] time = 0.0362176, size = 79, normalized size = 1.72

$$\frac{\sqrt{\frac{5x-1}{7x+1}}\left(12x\sqrt{7x+1}\tan^{-1}\left(\frac{\sqrt{5x-1}}{\sqrt{7x+1}}\right) - \sqrt{5x-1}(7x+1)\right)}{x\sqrt{5x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] (Sqrt[(-1 + 5*x)/(1 + 7*x)]*(-(Sqrt[-1 + 5*x]*(1 + 7*x)) + 12*x*Sqrt[1 + 7*
x]*ArcTan[Sqrt[-1 + 5*x]/Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x])
```

Maple [B] time = 0.02, size = 106, normalized size = 2.3

$$-\frac{1+7x}{x}\sqrt{\frac{-1+5x}{1+7x}}\left(-\left(35x^2-2x-1\right)^{\frac{3}{2}} + 35\sqrt{35x^2-2x-1}x^2 + 6\arctan\left(\frac{1+x}{\sqrt{35x^2-2x-1}}\right)x - 2\sqrt{35x^2-2x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x)
```

```
[Out] -((-1+5*x)/(1+7*x))^(1/2)*(1+7*x)*(-(35*x^2-2*x-1)^(3/2)+35*(35*x^2-2*x-1)^
(1/2)*x^2+6*arctan((1+x)/(35*x^2-2*x-1)^(1/2))*x-2*(35*x^2-2*x-1)^(1/2)*x)/
((-1+5*x)*(1+7*x))^(1/2)/x
```

Maxima [A] time = 1.51165, size = 72, normalized size = 1.57

$$-\frac{12\sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1}+1} + 12\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((5
*x - 1)/(7*x + 1)))
```

Fricas [A] time = 1.87202, size = 111, normalized size = 2.41

$$\frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right)-(7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x + 1
)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.748 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx$

Optimal. Leaf size=20

$$-\sqrt{\frac{1-x}{x+1}}(x+1)$$

```
[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rubi [A] time = 0.055983, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1962, 12, 383}

$$-\sqrt{\frac{1-x}{x+1}}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

```
[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rule 1962

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c
- a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^
((m + 1)/n - 1)*(u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(
b*e - d*x^q)^((m + 1)/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1-x^2}{2\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= -\sqrt{\frac{1-x}{1+x}}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0089171, size = 19, normalized size = 0.95

$$\frac{x-1}{\sqrt{\frac{1-x}{x+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

```
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$(x-1)\frac{1}{\sqrt{-\frac{x-1}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/((1-x)/(1+x))^(1/2),x)
```

```
[Out] (x-1)/(-(x-1)/(1+x))^(1/2)
```

Maxima [A] time = 0.971723, size = 36, normalized size = 1.8

$$\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Fricas [A] time = 1.68215, size = 45, normalized size = 2.25

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Giac [A] time = 1.15837, size = 39, normalized size = 1.95

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

3.749 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

Optimal. Leaf size=18

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

```
[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rubi [A] time = 0.0306565, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {512, 514, 375, 74}

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]

[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rule 512

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIn
tegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x], x
, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m]
 && NeQ[v, x]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \text{Subst}\left(\int \frac{-1+x}{\sqrt{-1+\frac{2}{x}}x}\,dx, x, 1+x\right)$$
$$= \text{Subst}\left(\int \frac{1-\frac{1}{x}}{\sqrt{-1+\frac{2}{x}}}\,dx, x, 1+x\right)$$
$$= -\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{-1+2x}}\,dx, x, \frac{1}{1+x}\right)$$
$$= -(1+x)\sqrt{-1+\frac{2}{1+x}}$$

Mathematica [A] time = 0.0067792, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{\frac{2}{x+1}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

```
[Out] (-1 + x)/Sqrt[-1 + 2/(1 + x)]
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$(x-1)\frac{1}{\sqrt{-\frac{x-1}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(-1+2/(1+x))^(1/2),x)
```

```
[Out] (x-1)/(-(x-1)/(1+x))^(1/2)
```

Maxima [A] time = 1.02281, size = 22, normalized size = 1.22

$$\frac{\sqrt{x+1}(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

Fricas [A] time = 1.74462, size = 45, normalized size = 2.5

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Giac [A] time = 1.23699, size = 39, normalized size = 2.17

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}} + \frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

3.750 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx$

Optimal. Leaf size=54

$$\sqrt{x+2}\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rubi [A] time = 0.0567202, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1958, 154, 157, 54, 215, 93, 207}

$$\sqrt{x+2}\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx &= \int \frac{x\sqrt{3+x}}{(1+x)\sqrt{2+x}}\,dx\\
&= \sqrt{2+x}\sqrt{3+x}+\int \frac{-\frac{5}{2}-\frac{x}{2}}{(1+x)\sqrt{2+x}\sqrt{3+x}}\,dx\\
&= \sqrt{2+x}\sqrt{3+x}-\frac{1}{2}\int \frac{1}{\sqrt{2+x}\sqrt{3+x}}\,dx-2\int \frac{1}{(1+x)\sqrt{2+x}\sqrt{3+x}}\,dx\\
&= \sqrt{2+x}\sqrt{3+x}-4\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\frac{\sqrt{2+x}}{\sqrt{3+x}}\right)-\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+x}\right)\\
&= \sqrt{2+x}\sqrt{3+x}-\sinh^{-1}\left(\sqrt{2+x}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2+x}}{\sqrt{3+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0753734, size = 106, normalized size = 1.96

$$\frac{\sqrt{x+3}\left(x^2+5x+6\right)+2\sqrt{2}\sqrt{x+2}\sqrt{-(x+3)^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{-x-3}}\right)-\sqrt{x+2}(x+3)\sinh^{-1}\left(\sqrt{x+2}\right)}{\sqrt{\frac{x+2}{x+3}}(x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] (Sqrt[3 + x]*(6 + 5*x + x^2) - Sqrt[2 + x]*(3 + x)*ArcSinh[Sqrt[2 + x]] + 2
*Sqrt[2]*Sqrt[2 + x]*Sqrt[-(3 + x)^2]*ArcTan[(Sqrt[2]*Sqrt[2 + x])/Sqrt[-3
- x]])/(Sqrt[(2 + x)/(3 + x)]*(3 + x)^(3/2))
```

Maple [A] time = 0.015, size = 79, normalized size = 1.5

$$-\frac{2+x}{2}\left(-2\sqrt{2}\text{Artanh}\left(1/4\,\frac{(7+3x)\sqrt{2}}{\sqrt{x^2+5x+6}}\right)+\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)-2\sqrt{x^2+5x+6}\right)\frac{1}{\sqrt{\frac{2+x}{3+x}}}\frac{1}{\sqrt{(3+x)(2+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/((2+x)/(3+x))^(1/2),x)
```

```
[Out] -1/2*(2+x)*(-2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+ln(5/
2+x+(x^2+5*x+6)^(1/2))-2*(x^2+5*x+6)^(1/2))/((2+x)/(3+x))^(1/2)/((3+x)*(2+x
))^(1/2)
```

Maxima [B] time = 1.46524, size = 139, normalized size = 2.57

$$-\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)
/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x
 + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

Fricas [B] time = 1.75325, size = 243, normalized size = 4.5

$$(x+3)\sqrt{\frac{x+2}{x+3}}+\sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}}+3x+7}{x+1}\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2)
/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*lo
g(sqrt((x + 2)/(x + 3)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

Giac [B] time = 1.21892, size = 144, normalized size = 2.67

$$-\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sqrt{\frac{x+2}{x+3}}\right|}{2\left(\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}\right)}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\left|\sqrt{\frac{x+2}{x+3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 4*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqr
t((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*lo
g(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(abs(sqrt((x + 2)/(x + 3)) - 1))
```

3.751 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx$

Optimal. Leaf size=11

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

```
[Out] 2/Sqrt[1 + x^(-1)]
```

Rubi [A] time = 0.0034446, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {25, 261}

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

```
[Out] 2/Sqrt[1 + x^(-1)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx &= \int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2} x^2} \, dx \\
&= \frac{2}{\sqrt{1+\frac{1}{x}}}
\end{aligned}$$

Mathematica [A] time = 0.0052393, size = 11, normalized size = 1.

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

```
[Out] 2/Sqrt[1 + x^(-1)]
```

Maple [A] time = 0.003, size = 18, normalized size = 1.6

$$2\frac{x}{1+x}\sqrt{\frac{1+x}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(1+x)^2,x)
```

```
[Out] 2*x/(1+x)*((1+x)/x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

Fricas [A] time = 1.62278, size = 39, normalized size = 3.55

$$\frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 2*x*sqrt((x + 1)/x)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(1+x)**2,x)
```

```
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

Giac [B] time = 1.10869, size = 31, normalized size = 2.82

$$\frac{2\,\mathrm{sgn}\,(x)}{x-\sqrt{x^2+x}+1}-2\,\mathrm{sgn}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

3.752 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

```
[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rubi [A] time = 0.0122943, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1448, 26, 53, 619, 216}

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rule 1448

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1
 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p
, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&  !Integer
Q[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{\sqrt{1+x}}{\sqrt{x}\sqrt{1-x^2}}\,dx}{\sqrt{1+x}}$$
$$= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx}{\sqrt{1+x}}$$
$$= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x-x^2}}\,dx}{\sqrt{1+x}}$$
$$= -\frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)}{\sqrt{1+x}}$$
$$= -\frac{\sqrt{1+\frac{1}{x}}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{1+x}}$$

Mathematica [A] time = 0.228729, size = 41, normalized size = 1.41

$$-\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{x}}(2x-1)\sqrt{1-x^2}}{2\left(x^2-1\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

Maple [A] time = 0.018, size = 40, normalized size = 1.4

$$\frac{x\arcsin(2x-1)}{1+x}\sqrt{\frac{1+x}{x}}\sqrt{-x^2+1}\frac{1}{\sqrt{-x(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] ((1+x)/x)^(1/2)*x*(-x^2+1)^(1/2)/(1+x)/(-x*(x-1))^(1/2)*arcsin(2*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Fricas [A] time = 1.84845, size = 82, normalized size = 2.83

$$-\arctan\left(\frac{2\sqrt{-x^2+1}x\sqrt{\frac{x+1}{x}}}{2x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

3.753 $\int \frac{1}{x+\sqrt{3-2x-x^2}} dx$

Optimal. Leaf size=180

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}\left(7-\sqrt{7}\right.$$

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 -
 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*
(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] +
Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14
```

Rubi [A] time = 0.195066, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1074, 632, 31, 635, 203, 260}

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}\left(7-\sqrt{7}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```


```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 -
 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*
(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] +
Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14
```


Rule 1074

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{3}-2x-\sqrt{3}x^2}{\left(1+x^2\right)\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{1}{16}\,\text{Subst}\left(\int \frac{-6+2\sqrt{3}\left(2-\sqrt{3}\right)-4\left(1+\sqrt{3}\right)-\left(-2\sqrt{3}+2\left(2-\sqrt{3}\right)+4\sqrt{3}\left(1+\sqrt{3}\right)\right)x}{1+x^2}\,dx, x$$

$$= -\left(\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(1-\sqrt{7}\right)\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{3}+\sqrt{7}+\sqrt{3}x}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)\right)+\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(\right.\right.$$

$$= -\tan^{-1}\left(\frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)-\frac{1}{2}\log\left(\frac{-3+x+\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(1+\right.$$

Mathematica [A] time = 0.388879, size = 197, normalized size = 1.09

$$\frac{1}{28}\left(-\sqrt{14\left(4+\sqrt{7}\right)}\tanh^{-1}\left(\frac{\left(\sqrt{7}-1\right)x+\sqrt{7}+7}{\sqrt{2\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}}\right)-\sqrt{56-14\sqrt{7}}\tanh^{-1}\left(\frac{\sqrt{7}x+x+\sqrt{7}-7}{\sqrt{2}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-3\right)}}\right)-\sqrt{7}\,\text{l}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] (14*ArcSin[(1 + x)/2] - Sqrt[14*(4 + Sqrt[7])]*ArcTanh[(7 + Sqrt[7] + (-1 +
 Sqrt[7])*x)/(Sqrt[2*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2])] - Sqrt[56 - 14*Sq
rt[7]]*ArcTanh[(-7 + Sqrt[7] + x + Sqrt[7]*x)/(Sqrt[2]*Sqrt[(-4 + Sqrt[7])*
(-3 + 2*x + x^2)])] + 7*Log[1 - Sqrt[7] + 2*x] - Sqrt[7]*Log[1 - Sqrt[7] +
2*x] + 7*Log[1 + Sqrt[7] + 2*x] + Sqrt[7]*Log[1 + Sqrt[7] + 2*x])/28
```

Maple [B] time = 0.059, size = 551, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2)),x)
```

```
[Out] 1/28*7^(1/2)*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8
+2*7^(1/2))^(1/2)-1/28*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1
```

```
+x))*7^(1/2)+1/4*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))-1
/7/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(
1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2
))+8+2*7^(1/2))^(1/2))*7^(1/2)-1/4/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1
+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+
4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2))-1/28*7^(1/2)*(-4*(x+
1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/
28*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(1+x))*7^(1/2)+1/4*arc
sin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(1+x))+1/7/(-1/2+1/2*7^(1/2)
)*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(
-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1
/2))*7^(1/2)-1/4/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-
1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(
x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))+1/4*ln(2*x^2+2*x-3)+1/14*7^(1/2)*arc
tanh(1/14*(4*x+2)*7^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

Fricas [B] time = 1.84741, size = 972, normalized size = 5.4

$$\frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2+2\sqrt{7}\left(3x^4+x^3-45x^2+45x\right)-\left(14x^3-84x^2+\sqrt{7}\left(8x^3-30x^2+27x-27\right)+\right.}{4x^4+8x^3-8x^2-12x+9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*x
^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*
x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9))
+ 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3 - 45
*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 12
6*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
) + 1/28*sqrt(7)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3
)) - 1/2*arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 1/4*log(2*x
^2 + 2*x - 3) - 1/8*log((2*sqrt(-x^2 - 2*x + 3)*x + 2*x - 3)/x^2) + 1/8*log
(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*x + 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

Giac [B] time = 1.29647, size = 387, normalized size = 2.15

$$-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 1/28
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 1/28*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) + 1/4*
log(abs(2*x^2 + 2*x - 3)) + 1/4*log(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1
) + 3*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*log(abs(-4*(sqrt(-
x^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 3))
```

3.754 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqr
t[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*Sqrt[7])
```

Rubi [A] time = 0.143687, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1660, 12, 618, 206}

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```


```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqr
t[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*Sqrt[7])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{-\sqrt{3}+2x+\sqrt{3}x^2}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{1}{14}\,\text{Subst}\left(\int -\frac{16}{2-\sqrt{3}+2\left(1+\sqrt{3}\right.}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8}{7}\,\text{Subst}\left(\int \frac{1}{2-\sqrt{3}+2\left(1+\sqrt{3}\right)x}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{16}{7}\,\text{Subst}\left(\int \frac{1}{28-x^2}\,dx, x, \frac{2\left(-3\right.}{}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8\tanh^{-1}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

Mathematica [A] time = 0.465425, size = 306, normalized size = 1.78

$$\frac{1}{98}\left(\frac{7(3-8x)}{2x^2+2x-3}-\frac{14(x-3)\sqrt{-x^2-2x+3}}{2x^2+2x-3}-2\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x+7\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] ((7*(3 - 8*x))/(-3 + 2*x + 2*x^2) - (14*(-3 + x)*Sqrt[3 - 2*x - x^2])/(-3 +
 2*x + 2*x^2) - 4*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + (2*(-1 + Sqrt[7])*Sqrt[
14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x])/3 + 4*Sqrt[7]*Log[1 + Sqrt[7] + 2
*x] + 2*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 2*(1
+ Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqr
t[14*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - (2*(-1 + Sqrt[7])*Sqrt[14*(4 + S
qrt[7])]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])
*(-3 + 2*x + x^2)]])/3)/98
```

Maple [B] time = 0.027, size = 1066, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^2,x)
```

```
[Out] -3/28*(4*x+2)/(2*x^2+2*x-3)+4/49*7^(1/2)*arctanh(1/14*(4*x+2)*7^(1/2))+1/14
*(-2*x+6)/(2*x^2+2*x-3)-2*(-1/14+1/14*7^(1/2))*(-1/4/(2-1/2*7^(1/2))/(x+1/2
-1/2*7^(1/2))*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/
2*7^(1/2))^(3/2)+1/8*(-1-7^(1/2))/(2-1/2*7^(1/2))*(1/2*(-4*(x+1/2-1/2*7^(1/
2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/2*(-1-7^(1/2)
)*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(1+x))-(2-1/2*7^(1/2))/
(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1
/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)
)+8-2*7^(1/2))^(1/2)))-1/2/(2-1/2*7^(1/2))*(-1/4*(-2*x-2)*(-(x+1/2-1/2*7^(1
/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(1/2)-1/8*(-8+2*7^(1
/2)-(-1-7^(1/2))^2)*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(1+x)
)))+1/49*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7
^(1/2))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1
+7^(1/2))^2)^(1/2)*(1+x))-1/2*(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*arctanh((4+
7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*
7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-1/49*7^(
1/2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*
7^(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)
^(1/2)*(1+x))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1
-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2
+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-2*(-1/14-1/14*7^(1
/2))*(-1/4/(2+1/2*7^(1/2))/(x+1/2+1/2*7^(1/2))*(-(x+1/2+1/2*7^(1/2))^2+(-1+
7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(3/2)+1/8*(-1+7^(1/2))/(2+1/2*7
^(1/2))*(1/2*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8
+2*7^(1/2))^(1/2)+1/2*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))
^2)^(1/2)*(1+x))-(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7
^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*
(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-1/2/(2+1/2*7^(1/2))*(
-1/4*(-2*x-2)*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/
2*7^(1/2))^(1/2)-1/8*(-8-2*7^(1/2)-(-1+7^(1/2))^2)*arcsin(1/(2+1/2*7^(1/2)+
1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

Fricas [A] time = 1.78358, size = 440, normalized size = 2.56

$$\frac{2\sqrt{7}\left(2x^2+2x-3\right)\log\left(\frac{x^4+44x^3-\sqrt{7}\left(3x^3+x^2-45x+45\right)\sqrt{-x^2-2x+3}+26x^2-276x+207}{4x^4+8x^3-8x^2-12x+9}\right)+4\sqrt{7}\left(2x^2+2x-3\right)\log\left(\frac{2x^2+\sqrt{7}(2x+1)+2x+}{2x^2+2x-3}\right.}{98\left(2x^2+2x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/98*(2*sqrt(7)*(2*x^2 + 2*x - 3)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2
- 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 -
8*x^2 - 12*x + 9)) + 4*sqrt(7)*(2*x^2 + 2*x - 3)*log((2*x^2 + sqrt(7)*(2*x
+ 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*sqrt(-x^2 - 2*x + 3)*(x - 3) - 56*x
 + 21)/(2*x^2 + 2*x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

Giac [B] time = 1.23232, size = 473, normalized size = 2.75

$$-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2/49*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 2/49
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 2/49*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x -
3) - 8/21*(5*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3)
 - 2)^2/(x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8*(sqrt(
-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 +
8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(
x + 1)^4 - 3)
```

3.755 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

Optimal. Leaf size=307

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+\right.}$$

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))
/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (
Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] -
((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sq
rt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rubi [A] time = 0.250719, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1660, 12, 618, 206}

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))
/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (
Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] -
((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sq
rt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{3}-2x-2x^3-\sqrt{3}x^4}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)^3}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} - \frac{1}{28}\,\text{Subst}\left(\int \frac{-\frac{8}{3}\left(21+16\sqrt{3}\right) \cdots}{\left(2-\sqrt{3}+2\left(1+\cdots\right.\right.}\right.$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\cdots\right.}{\cdots}\right.}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x\cdots}\right)}{x}\cdots\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\cdots\right.}{\cdots}\right.}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x\cdots}\right)}{x}\cdots\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\cdots\right.}{\cdots}\right.}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x\cdots}\right)}{x}\cdots\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\cdots\right.}{\cdots}\right.}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x\cdots}\right)}{x}\cdots\right.}$$

Mathematica [A] time = 1.15821, size = 333, normalized size = 1.08

$$\frac{7(37-24x)}{2x^2+2x-3}-\frac{14\sqrt{-x^2-2x+3}\left(34x^3+58x^2-83x-15\right)}{\left(2x^2+2x-3\right)^2}+\frac{98(11x-12)}{\left(2x^2+2x-3\right)^2}-6\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] ((98*(-12 + 11*x))/(-3 + 2*x + 2*x^2)^2 + (7*(37 - 24*x))/(-3 + 2*x + 2*x^2
) - (14*Sqrt[3 - 2*x - x^2]*(-15 - 83*x + 58*x^2 + 34*x^3))/(-3 + 2*x + 2*x
^2)^2 - 12*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + 2*(-1 + Sqrt[7])*Sqrt[14*(4 +
Sqrt[7])]*Log[1 - Sqrt[7] + 2*x] + 12*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 6*(1
 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 6*(1 + Sqrt[7])
*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 +
Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - 2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Lo
g[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x
+ x^2)]])/1372
```

Maple [B] time = 0.038, size = 5984, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

Fricas [A] time = 1.78163, size = 576, normalized size = 1.88

$$-\frac{336\,x^3-6\,\sqrt{7}\left(4\,x^4+8\,x^3-8\,x^2-12\,x+9\right)\log\left(\frac{x^4+44\,x^3-\sqrt{7}\left(3\,x^3+x^2-45\,x+45\right)\sqrt{-x^2-2\,x+3}+26\,x^2-276\,x+207}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right)-12\,\sqrt{7}\left(4\,x^4+8\right.}{1372\,\left(4\,x^4+8\,x^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 +
44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 -
276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8*x^
3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*
x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x + 3) -
```

```
2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23195, size = 610, normalized size = 1.99

$$-\frac{3}{343}\sqrt{7}\log\left(\frac{|4x-2\sqrt{7}+2|}{|4x+2\sqrt{7}+2|}\right)+\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-\right.}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/3
43*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7)
 + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 30
0*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x +
 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 12487
*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3) - 2)^6/
(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8*(sqrt(-x^
2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(
sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x +
 1)^4 - 3)^2
```

3.756 $\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (
3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rubi [A] time = 0.0300388, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (
3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x(-2+2x)^2}\,dx,x,x+\sqrt{-3-2x+x^2}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}-\frac{3}{4x}\right)dx,x,x+\sqrt{-3-2x+x^2}\right)\\ &= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+2\log\left(1-x-\sqrt{-3-2x+x^2}\right)-\frac{3}{2}\log\left(x+\sqrt{-3-2x+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0266081, size = 59, normalized size = 0.91

$$2\left(\frac{1}{\sqrt{x^2-2x-3}+x-1}+\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{4}\log\left(\sqrt{x^2-2x-3}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] 2*((-1 + x + Sqrt[-3 - 2*x + x^2])^(-1) + Log[1 - x - Sqrt[-3 - 2*x + x^2]]
 - (3*Log[x + Sqrt[-3 - 2*x + x^2]])/4)
```

Maple [A] time = 0.007, size = 71, normalized size = 1.1

$$-\frac{1}{4}\sqrt{4\,(x+3/2)^2-20\,x-21}+\frac{5}{4}\ln\left(-1+x+\sqrt{\left(x+\frac{3}{2}\right)^2-5\,x-\frac{21}{4}}\right)+\frac{3}{4}\mathrm{Artanh}\left(\frac{-6-10\,x}{3}\frac{1}{\sqrt{4\,(x+3/2)^2-20\,x-21}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2)),x)
```

```
[Out] -1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+3/
4*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2*x-3/4*ln(3+2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{x^2-2\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

Fricas [A] time = 1.77213, size = 227, normalized size = 3.49

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{3}{4}\,\log\left(2\,x+3\right)-\frac{5}{4}\,\log\left(-x+\sqrt{x^2-2\,x-3}+1\right)+\frac{3}{4}\,\log\left(-x+\sqrt{x^2-2\,x-3}\right)-\frac{3}{4}\,\log\left(-x+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(2*x + 3) - 5/4*log(-x + sqrt(x^2
- 2*x - 3) + 1) + 3/4*log(-x + sqrt(x^2 - 2*x - 3)) - 3/4*log(-x + sqrt(x^2
 - 2*x - 3) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{x^2-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

Giac [A] time = 1.17132, size = 109, normalized size = 1.68

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2 - 2x - 3} - \frac{3}{4}\log(|2x + 3|) - \frac{5}{4}\log\left(\left|-x + \sqrt{x^2 - 2x - 3} + 1\right|\right) + \frac{3}{4}\log\left(\left|-x + \sqrt{x^2 - 2x - 3}\right|\right) - \frac{3}{4}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(abs(2*x + 3)) - 5/4*log(abs(-x +
sqrt(x^2 - 2*x - 3) + 1)) + 3/4*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3/4*lo
g(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.757 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rubi [A] time = 0.0330327, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x^2(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}-\frac{3}{4x^2}-\frac{2}{x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)}+4\log\left(1-x-\sqrt{-3-2x+x^2}\right)-4\log\end{aligned}$$

Mathematica [A] time = 0.0270093, size = 79, normalized size = 0.95

$$\frac{2}{\sqrt{x^2-2x-3}+x-1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Maple [A] time = 0.015, size = 118, normalized size = 1.4

$$-2\ln(3+2x)+\frac{x}{2}-\frac{9}{12+8x}-\frac{2}{3}\sqrt{4(x+3/2)^2-20x-21}+2\ln\left(-1+x+\sqrt{(x+3/2)^2-5x-\frac{21}{4}}\right)+2\,\mathrm{Artanh}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^2,x)
```

```
[Out] -2*ln(3+2*x)+1/2*x-9/4/(3+2*x)-2/3*(4*(x+3/2)^2-20*x-21)^(1/2)+2*ln(-1+x+((
x+3/2)^2-5*x-21/4)^(1/2))+2*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2
))-1/3/(x+3/2)*((x+3/2)^2-5*x-21/4)^(3/2)+1/6*(2*x-2)*((x+3/2)^2-5*x-21/4)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)
```

Fricas [A] time = 1.76576, size = 262, normalized size = 3.16

$$\frac{4x^2-8(2x+3)\log\left(x^2-\sqrt{x^2-2x-3}(x+1)-3\right)-8(2x+3)\log(2x+3)+8(2x+3)\log\left(-x+\sqrt{x^2-2x-3}\right)-}{4(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2*
x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(x^
2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

Giac [B] time = 1.21153, size = 193, normalized size = 2.33

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{3\left(5\,x-5\,\sqrt{x^2-2\,x-3}+3\right)}{4\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)}-\frac{9}{4\,(2\,x+3)}-2\,\log\left(|2\,x+3|\right)-2\,\log\left(\left|-x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((x
 - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3) -
2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(abs(
-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.758 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-\right.$$

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Rubi [A] time = 0.037457, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x^3(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{3}{-1+x}-\frac{3}{4x^3}-\frac{2}{x^2}-\frac{3}{x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)\\
&= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-2x+x^2}}+6\log\left(1-\right.
\end{aligned}$$

Mathematica [A] time = 0.0381865, size = 97, normalized size = 0.96

$$\frac{2}{\sqrt{x^2-2x-3}+x-1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Maple [A] time = 0.022, size = 146, normalized size = 1.5

$$-9\,(3+2\,x)^{-1}-3\,\ln(3+2\,x)+\frac{x}{2}+\frac{27}{8\,(3+2\,x)^2}-\frac{1}{2}\left(\left(x+\frac{3}{2}\right)^2-5\,x-\frac{21}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{2}\right)^{-1}-\sqrt{4\,(x+3/2)^2-20\,x-21}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^3,x)
```

```
[Out] -9/(3+2*x)-3*ln(3+2*x)+1/2*x+27/8/(3+2*x)^2-1/2/(x+3/2)*((x+3/2)^2-5*x-21/4
)^(3/2)-(4*(x+3/2)^2-20*x-21)^(1/2)+3*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*
x-21)^(1/2))+1/4*(2*x-2)*((x+3/2)^2-5*x-21/4)^(1/2)+3*ln(-1+x+((x+3/2)^2-5*
x-21/4)^(1/2))+1/4/(x+3/2)^2*((x+3/2)^2-5*x-21/4)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

Fricas [A] time = 1.71949, size = 347, normalized size = 3.44

$$\frac{8\,x^3-10\,x^2-12\left(4\,x^2+12\,x+9\right)\log\left(x^2-\sqrt{x^2-2\,x-3}(x+1)-3\right)-12\left(4\,x^2+12\,x+9\right)\log(2\,x+3)+12\left(4\,x^2+12\,x\right.}{4\left(4\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3)*(
x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*lo
```

```
g(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3) - 1
56*x - 171)/(4*x^2 + 12*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

Giac [B] time = 1.14482, size = 248, normalized size = 2.46

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{104\left(x-\sqrt{x^2-2\,x-3}\right)^3+315\left(x-\sqrt{x^2-2\,x-3}\right)^2+162\,x-162\,\sqrt{x^2-2\,x-3}+27}{8\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)^2}-\frac{9\,(16}{8\,(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 31
5*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x -
 sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 21)/
(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)
) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2*x -
3) - 3))
```

3.759 $\int \frac{1}{x+\sqrt{-3-4x-x^2}} \, dx$

Optimal. Leaf size=108

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}x+\sqrt{x+3}x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sq
rt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x +
 x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rubi [A] time = 0.101555, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {12, 1023, 634, 618, 204, 628, 635, 203, 260}

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}x+\sqrt{x+3}x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sq
rt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x +
 x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{2x}{\left(1+x^2\right)\left(1-2x+3x^2\right)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= 4\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(1-2x+3x^2\right)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{-2-2x}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) -\\
&= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}x+x\sqrt{3+x}}{(3+x)^{3/2}}\right) - 4\,\text{Subst}\\
&= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) - \sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-}}{(3+x)^{3}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.414437, size = 187, normalized size = 1.73

$$
\frac{1}{4}\left(\log\left(2x^2+4x+3\right) + i\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - i\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4}}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] (2*ArcSin[2 + x] - 2*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + I*Sqrt[1 - (2*I)*Sqr
t[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[
2]]*Sqrt[-3 - 4*x - x^2])] - I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*S
qrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])]
 + Log[3 + 4*x + 2*x^2])/4
```

Maple [B] time = 0.026, size = 370, normalized size = 3.4

$$\frac{\arcsin(2+x)}{2}-\frac{\sqrt{3}\sqrt{4}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-\mathrm{Artanh}\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2)),x)
```

```
[Out] 1/2*arcsin(2+x)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2
-x)+1)+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(
-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2
-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2
)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-
3/2-x)+1)+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4*x+4)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [B] time = 1.82082, size = 529, normalized size = 4.9

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x +
3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(
```

```
2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 -
4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(-(2*
sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x
- 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

Giac [B] time = 1.14987, size = 266, normalized size = 2.46

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-}}{x+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x^2
+ 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.760 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/
Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0651296, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 638, 618, 204}

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/
Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = 2\,\text{Subst}\left(\int -\frac{2x}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{x}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\right)$$
$$= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} - \text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$
$$= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + 2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right)$$
$$= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 1.62129, size = 881, normalized size = 10.13

$$\frac{1}{16}\left(\frac{8(x+3)}{2x^2+4x+3} + 4\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right) - \frac{2i\left(-2i+\sqrt{2}\right)\tan^{-1}\left(\frac{(x+2)\left(2\left(9+2i\sqrt{2}\right)x^2+16\right.\cdots}{\left(8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}+36i\right)x^2+\left(-12\sqrt{1+2\cdots\right.}\right.}{\sqrt{1+2i\sqrt{2}}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] ((8*(3 + x))/(3 + 4*x + 2*x^2) + (8*(3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(3 + 4*
x + 2*x^2) + 4*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] - ((2*I)*(-2*I + Sqrt[2])*Ar
cTan[((2 + x)*(3*(5 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*
Sqrt[2])*x^2))/(12*I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)
*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sq
rt[2]]*Sqrt[-3 - 4*x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqr
t[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] + (2*(2*I + Sqrt[2])
*ArcTanh[((2 + x)*(3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*
Sqrt[2])*x^2))/(-5*(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 -
 (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1
- (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2
*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] - ((-2*I + Sq
rt[2])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] - ((2*I + Sqrt[2
])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + Sqrt[2])*L
og[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[
2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 -
 (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + ((-2*I +
 Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^
2 - 2*Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2
] + Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]
])/16
```

Maple [B] time = 0.079, size = 2407, normalized size = 27.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^2,x)
```

```
[Out] -3/8*(4*x+4)/(2*x^2+4*x+3)+1/4*2^(1/2)*arctan(1/4*(4*x+4)*2^(1/2))-1/2*(-4*
x-6)/(2*x^2+4*x+3)+1/36*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(7*2^(1
/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+4*arctanh(3*x/(-3/2-x)/
(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(
x/(-3/2-x)+1)+1/72*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(arctan(1/6*
(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^2/(-3/2-x)^2-8*arctanh(3*x/(
-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2+2*2^(1/2)*arctan(1/6*(3
*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-16*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^
2-12)^(1/2))-6*(3*x^2/(-3/2-x)^2-12)^(1/2))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)
+1)^2)^(1/2)/(x/(-3/2-x)+1)/(x^2/(-3/2-x)^2+2)-2/9*3^(1/2)*4^(1/2)*(3*x^2/(
-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)
)+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x
/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)-2/9*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-
12)^(1/2)*(3*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^6/(-
3/2-x)^6+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^6/(-3/2-x)^6
-2*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-
x)^2-4))*x^6/(-3/2-x)^6+2*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-
3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^6/(-3/2-x)^6+(3*x^2/(-3/2-x)^2-12)^(1/2)*
x^5/(-3/2-x)^5-(3*x^2/(-3/2-x)^2-12)^(3/2)*x^2/(-3/2-x)^2+(3*x^2/(-3/2-x)^2
-12)^(1/2)*x^4/(-3/2-x)^4-36*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)
)*2^(1/2)*x^2/(-3/2-x)^2-2*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^3/(-3/2-x)^3-48*ar
ctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2-8*(3*x^2/(-3
/2-x)^2-12)^(1/2)*x^2/(-3/2-x)^2+24*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2
-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2-x)^2-24*ln(((3*x^2/(-3/
2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2
-x)^2-48*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-8*(3*x^2/(
-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-64*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12
)^(1/2))+16*(3*x^2/(-3/2-x)^2-12)^(1/2)+32*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*
x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))-32*ln(((3*x^2/(-3/2-x)^2-1
2)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4)))/((x^2/(-3/2-x)^2
-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)/(x^2/(-3/2-x)^2+2)/((3*x^2/(-3/2
-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/((3*x^2/(-3/2-x)^2-12)^(1/2)*x
/(-3/2-x)-x^2/(-3/2-x)^2+4)+1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2
)*(11*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^6/(-3/2-x)^
6+24*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^6/(-3/2-x)^6-8*ln(
((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4
))*x^6/(-3/2-x)^6+8*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)
^2+4)/(x^2/(-3/2-x)^2-4))*x^6/(-3/2-x)^6+4*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^5/
(-3/2-x)^5-(3*x^2/(-3/2-x)^2-12)^(3/2)*x^2/(-3/2-x)^2+(3*x^2/(-3/2-x)^2-12)
^(1/2)*x^4/(-3/2-x)^4-132*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2
^(1/2)*x^2/(-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^3/(-3/2-x)^3-288*arct
anh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2-8*(3*x^2/(-3/2
-x)^2-12)^(1/2)*x^2/(-3/2-x)^2+96*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x
)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2-x)^2-96*ln(((3*x^2/(-3/2-
x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2-x
)^2-176*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-32*(3*x^2/(
-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-384*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-1
2)^(1/2))+16*(3*x^2/(-3/2-x)^2-12)^(1/2)+128*ln(((3*x^2/(-3/2-x)^2-12)^(1/2
)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))-128*ln(((3*x^2/(-3/2-x)^
2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4)))/((x^2/(-3/2-x
)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)/(x^2/(-3/2-x)^2+2)/((3*x^2/(-
```

```
3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/((3*x^2/(-3/2-x)^2-12)^(1/2
)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

Fricas [A] time = 1.71634, size = 324, normalized size = 3.72

$$\frac{2\sqrt{2}\left(2x^2+4x+3\right)\arctan\left(\sqrt{2}(x+1)\right)-\sqrt{2}\left(2x^2+4x+3\right)\arctan\left(\frac{\sqrt{2}\left(6x^2+20x+15\right)\sqrt{-x^2-4x-3}}{4\left(2x^3+11x^2+18x+9\right)}\right)+4\sqrt{-x^2-4x-3}(2}{8\left(2x^2+4x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2 +
 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^
3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(2*x
^2 + 4*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

Giac [B] time = 1.13715, size = 355, normalized size = 4.08

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}}{x+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) - 1/3
*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
 + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x
 - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x
^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 +
 3)
```

3.761 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x) - (2*Sqr
t[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x]/Sqrt[3 + x]))/(9*(1 - (3*(1
 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[
-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])
```

Rubi [A] time = 0.09538, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 1660, 638, 618, 204}

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x) - (2*Sqr
t[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x]/Sqrt[3 + x]))/(9*(1 - (3*(1
 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[
-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
```

```
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{2x\left(1+x^2\right)}{\left(1-2x+3x^2\right)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\left(1-2x+3x^2\right)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{\frac{56}{9}+\frac{16x}{3}}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{}}{}\right.$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}-3\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6}{}\right.$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 2.40863, size = 914, normalized size = 6.13

$$\frac{1}{32}\left(\frac{8(2x-3)}{\left(2x^2+4x+3\right)^2}-\frac{8\sqrt{-x^2-4x-3}\left(8x^3+22x^2+26x+15\right)}{\left(2x^2+4x+3\right)^2}-12\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)+\frac{6\left(2+i\sqrt{2}\right)\tan^{-1}\left(\frac{}{\left(8i+6\sqrt{2}\right.}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] ((8*(-3 + 2*x))/(3 + 4*x + 2*x^2)^2 - (8*(2 + 3*x))/(3 + 4*x + 2*x^2) - (8*
Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2 + 8*x^3))/(3 + 4*x + 2*x^2)^2 - 12
```

```
*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + (6*(2 + I*Sqrt[2])*ArcTan[((2 + x)*(3*(5
 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2))/(12*
I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 -
 4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*
x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x
 - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] - (6*(2*I + Sqrt[2])*ArcTanh[((2 + x)*(
3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*Sqrt[2])*x^2))/(-5*
(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*S
qrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sq
rt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-
3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + (3*(-2*I + Sqrt[2])*Log[4*(3 +
 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] + (3*(2*I + Sqrt[2])*Log[4*(3 + 4
*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(2*I + Sqrt[2])*Log[(3 + 4*x +
 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 - (4*I)*Sqr
t[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2
]]*Sqrt[-3 - 4*x - x^2])))])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(-2*I + Sqrt[2])*L
og[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[
2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sqrt[2
+ (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]])/32
```

Maple [B] time = 0.233, size = 14529, normalized size = 97.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 4x - 3}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

Fricas [A] time = 1.80806, size = 458, normalized size = 3.07

$$-\frac{24\,x^3 + 6\,\sqrt{2}\left(4\,x^4 + 16\,x^3 + 28\,x^2 + 24\,x + 9\right)\arctan\left(\sqrt{2}(x+1)\right) - 3\,\sqrt{2}\left(4\,x^4 + 16\,x^3 + 28\,x^2 + 24\,x + 9\right)\arctan\left(\cdots\right.}{16\left(4\,x^4 + 16\,x^3 + 28\,x^2 + 24\,x \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(
2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqr
t(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))
+ 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x + 36)
/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29067, size = 495, normalized size = 3.32

$$-\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-}{x+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)/(2
*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x
+ 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*
x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 + 6*(s
qrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(
x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3
) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)^2
```

3.762 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

```
[Out] -((1 - x^2 - 2*x^3 - x^4)^(3/2)*(2 + 3*x^2 + 6*x^3 + 3*x^4))/15
```

Rubi [A] time = 0.215117, antiderivative size = 59, normalized size of antiderivative = 1.4, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
```

```
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{256}\text{Subst}\left(\int(-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\ &= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}+\frac{1}{40}\text{Subst}\left(\int(-1+4x)\sqrt{15+8x-16\right.\\ &= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.34547, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 + 18*x^6 +
12*x^7 + 3*x^8))/15
```

Maple [A] time = 0.007, size = 51, normalized size = 1.2

$$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)}{15}\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)
```

Maxima [A] time = 1.15548, size = 80, normalized size = 1.9

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

Fricas [A] time = 1.64164, size = 130, normalized size = 3.1

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

Sympy [B] time = 0.774672, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Giac [A] time = 1.17079, size = 69, normalized size = 1.64

$$\frac{1}{15}\,\sqrt{-x^4-2\,x^3-x^2+1}\left((((3\,(((x+4)x+6)x+4)x+2)x-2)x-1)x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(-x^4 - 2*x^3 - x^2 + 1)*((((3*(((x + 4)*x + 6)*x + 4)*x + 2)*x -
2)*x - 1)*x^2 - 2)
```

3.763 $\int (1+2x)\left(x+x^2\right)^3 \sqrt{1-\left(x+x^2\right)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

```
[Out] -((1 - x^2 - 2*x^3 - x^4)^(3/2)*(2 + 3*x^2 + 6*x^3 + 3*x^4))/15
```

Rubi [A] time = 0.238925, antiderivative size = 59, normalized size of antiderivative = 1.4, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1593, 1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
```

```
Q[p]) || OddQ[m])
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx &= \int x^3(1+x)^3(1+2x)\sqrt{1-\left(x+x^2\right)^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\ &= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}+\frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+8x-16\right.\\ &= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.302387, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 + 18*x^6 +
12*x^7 + 3*x^8))/15
```

Maple [A] time = 0.006, size = 51, normalized size = 1.2

$$\frac{\left(3\,x^4+6\,x^3+3\,x^2+2\right)\left(x^2+x+1\right)\left(x^2+x-1\right)}{15}\sqrt{-x^4-2\,x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)
```

Maxima [A] time = 1.1783, size = 80, normalized size = 1.9

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

Fricas [A] time = 1.7886, size = 130, normalized size = 3.1

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

Sympy [B] time = 12.3649, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Giac [A] time = 1.17992, size = 69, normalized size = 1.64

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left((((3(((x+4)x+6)x+4)x+2)x-2)x-1)x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(-x^4 - 2*x^3 - x^2 + 1)*((((3*(((x + 4)*x + 6)*x + 4)*x + 2)*x -
2)*x - 1)*x^2 - 2)
```

3.764 $\int \left(8x - 8x^2 + 4x^3 - x^4\right)^{3/2} dx$

Optimal. Leaf size=102

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x\right.$$

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rubi [A] time = 0.0712901, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1106, 1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x\right.$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```


```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$
$$= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\,\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3-2x^2-x^4}\,dx, x,\right.$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}$$

Mathematica [C] time = 0.679792, size = 278, normalized size = 2.73

$$\frac{5x^9-45x^8+206x^7-602x^6+1152x^5-1420x^4+848x^3+352x^2-304i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2}{-i}\right.}{35\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6 + 206*x^7
 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]*E
llipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
```

```
/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] - (304*I)*Sqrt[2
]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*El
lipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-I + Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] time = 0.109, size = 1050, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-
66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+8*x)^(1/2
)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+32/
7*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2
+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3
^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+64/5*(-I*3^(1/2)-1)*((I*3^(1/
2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))
/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(
-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*3^(1/2)-1
)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)
/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^
(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-
1)/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-I*3^(
1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/
2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2
)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2
))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))
^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^
(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(
I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(-I*3^
(1/2)-1)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(
1/2)))^(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

3.765 $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rubi [A] time = 0.0490289, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1106, 1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx,x,-1+x\right)$$
$$= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx,x,-1+x\right)$$
$$= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx,x,-1+x\right)$$
$$= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)-\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx,x,-1+x\right)+8\,\text{Subst}\Bigg($$
$$= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.597811, size = 256, normalized size = 4.13

$$-\frac{x^5-5x^4+14x^3-24x^2+8i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)-\frac{2i\sqrt{2}(x-2)\sqrt{\frac{x^2-2x+4}{x^2}}xE\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{}}{-i+}\right.}{\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}}}{3\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2 + x)*x*Sq
rt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[
(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(3*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3
))])
```

Maple [B] time = 0.024, size = 946, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+8/3*(-I
*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3
^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^
(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ellip
ticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2
)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+8/3*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*
x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x
))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2
+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*3^(1/2)-1)*x/(1
+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*
3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),
(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-
I*3^(1/2)))^(1/2)))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-I*3^(1/2)-1)
*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-
I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)*(1/2*
(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+
x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(
(1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/
2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(-I*3^(1/2)-1
)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^
(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.766 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}} \, dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rubi [A] time = 0.0135043, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1106, 1095, 419}

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}} \, dx, x, -1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}} \, dx, x, -1+x\right)\\
&= -\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.149922, size = 156, normalized size = 9.18

$$\frac{\sqrt{\frac{4i}{x}+\sqrt{3}-i}\sqrt{-\frac{i(x-2)}{(\sqrt{3}-i)x}}x\left(-i\sqrt{3}x+x-4\right)F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{-\frac{4i}{x}+\sqrt{3}+i}\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x*(-
4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt[2]*Sqrt[I + Sqrt[3] - (4*I)/x
]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] time = 0.026, size = 200, normalized size = 11.8

$$2\frac{\left(-i\sqrt{3}-1\right)(-2+x)^2}{\left(i\sqrt{3}-1\right)\sqrt{-x(-2+x)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}\sqrt{\frac{\left(i\sqrt{3}-1\right)x}{\left(1+i\sqrt{3}\right)(-2+x)}}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(-2+x)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(-2+x)}}\mathrm{El}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

```
[Out] 2*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2
+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3
^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^4-4x^3+8x^2-8x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^4 - 4*x^3 + 8*x^2 - 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.767 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rubi [A] time = 0.0514859, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1106, 1092, 1180, 524, 424, 419}

$$\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\,\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\,\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\,\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right) + \frac{1}{2}\,\text{Subst}\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}
\end{aligned}
$$

Mathematica [C] time = 0.869394, size = 261, normalized size = 3.58

$$
\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\frac{\sqrt{2}\left(\sqrt{3}-i\right)\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{\frac{x^2-2x+4}{x^2}}} - \frac{x^2-4i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}\,x^2 F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+2}{x^2-2x+4}\right)}{24(x-2)x}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-
2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[(4 - 2*x + x^2)/x^2] -
(2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[
(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x)*x)
```

Maple [B] time = 0.032, size = 963, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

```
[Out] -1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1/192*x^2)/
(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/
2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x
-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3
^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2
+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)
)+1/6*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*
((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))
/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1
/2)*(2*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2)
)*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/
2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/
2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)))-1/24*(x*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2))+2*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x)
)^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/
2))/(1+I*3^(1/2))/(-2+x))^(1/2)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*Elliptic
F(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1
)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/
2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2
)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I
*3^(1/2))/(-2+x))^(1/2),(-I*3^(1/2)-1)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(
1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x
-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}}{x^8 - 8\,x^7 + 32\,x^6 - 80\,x^5 + 128\,x^4 - 128\,x^3 + 64\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 +
128*x^4 - 128*x^3 + 64*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

3.768 $\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rubi [A] time = 0.0732775, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1106, 1092, 1178, 1180, 524, 424, 419}

$$\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```



```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{192}{\sqrt{3-}}\right.}{}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-}}\right.}{}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{7}{432}\text{Subst}\left(\int\right.$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{7E\left(\sin^{-1}(1-\right.}{144\sqrt{}}$$

Mathematica [C] time = 1.0467, size = 298, normalized size = 2.73

$$\frac{\frac{7x^6-37x^5+115x^4-226x^3+274x^2-19i\sqrt{2}\sqrt{-\frac{i(x-2)}{(\sqrt{3}-i)x}}\sqrt{\frac{x^2-2x+4}{x^2}}(x^3-4x^2+8x-8)x^3F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)-232x+36}{x^3-4x^2+8x-8}+\frac{7i\sqrt{2}(x-2)\sqrt{\frac{x^2-2x+4}{x^2}}x^2E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}}}{\sqrt{}}\right.\right.}{\sqrt{-\frac{i(x-2)}{(\sqrt{3}-i)x}}}}{432x\sqrt{-x(x^3-4x^2+8x-8)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] (((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])]])/S
qrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36 - 232*x + 274*x^2 - 226*x^3 +
 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)]*x^3*Sqrt[(4 - 2*x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*EllipticF[A
rcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqr
t[3])]])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))
])
```

Maple [B] time = 0.033, size = 1039, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x)
```

```
[Out] (1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*
x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/768*(-x^4+4
*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1
/2)+5/216*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x
)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1
/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
)^(1/2)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2
))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+7/108*(-I*3^(1/2)-1)*
((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I
*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(
1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*
3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3
^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))
/(-2+x))^(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I
*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
+2*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x
-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-
2+x))^(1/2)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(
1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I
*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2)
)/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^
(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1
/2),(-I*3^(1/2)-1)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1
)/(1-I*3^(1/2)))^(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}{x^{12}-12\,x^{11}+72\,x^{10}-280\,x^9+768\,x^8-1536\,x^7+2240\,x^6-2304\,x^5+1536\,x^4-512\,x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^12 - 12*x^11 + 72*x^10 - 280*
x^9 + 768*x^8 - 1536*x^7 + 2240*x^6 - 2304*x^5 + 1536*x^4 - 512*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

3.769 $\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)|-\right.$$

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rubi [A] time = 0.069017, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1106, 1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)|-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$
$$= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3-2x^2-x^4}\right.$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+\right.$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+\right.$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+\right.$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+\right.$$

Mathematica [C] time = 0.988792, size = 278, normalized size = 2.73

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\sqrt{\frac{x^2-2x+4}{x^2}}\left(-5x^7+35x^6-116x^5+230x^4-228x^3+44x^2+152x-224\right)+304i\sqrt{2}\sqrt{-\frac{i(x-2}{(\sqrt{3}-i}}\right.}{35(x-2)x\sqrt{\frac{x^2-2x+4}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-224 + 152
*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*x^7) + 112*Sqrt[2]*(
-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])] +
(304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[ArcSin[S
```

```
qrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])
)/(35*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] time = 0.032, size = 1050, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-
66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+8*x)^(1/2
)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+32/
7*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2
+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3
^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+64/5*(-I*3^(1/2)-1)*((I*3^(1/
2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))
/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(
-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*3^(1/2)-1
)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)
/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^
(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-
1)/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-I*3^(
1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/
2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2
)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2
))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))
^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^
(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(
I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(-I*3^
(1/2)-1)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(
1/2)))^(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2 - 2\,x + 4\right)(x - 2)x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

3.770 $\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rubi [A] time = 0.0486551, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1106, 1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx &= \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)-\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)+8\,\text{Su}\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x)\middle|-\right.}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.746342, size = 256, normalized size = 4.13

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\sqrt{\frac{x^2-2x+4}{x^2}}\left(x^3-3x^2+4x-4\right)+8i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+2\sqrt{2}\left(\sqrt{3}-i\right)\right.}{3(x-2)x\sqrt{\frac{x^2-2x+4}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-4 + 4*x -
 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] time = 0.026, size = 946, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+8/3*(-I
*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3
^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^
(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ellip
ticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2
)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+8/3*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*
x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x
))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2
+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*3^(1/2)-1)*x/(1
+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*
3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),
(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-
I*3^(1/2)))^(1/2)))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-I*3^(1/2)-1)
*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-
I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)*(1/2*
(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+
x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(
(1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/
2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(-I*3^(1/2)-1
)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^
(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\,x+4\right)(x-2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(2-x)\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2x+4\right)(x-2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.771 $\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}} dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rubi [A] time = 0.0118359, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1106, 1095, 419}

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)$$
$$= -\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.262311, size = 100, normalized size = 5.88

$$-\frac{\sqrt[3]{-1}(x-2)^2\sqrt{\frac{x(x+i\sqrt{3}-1)}{(x-2)^2}}\sqrt{\frac{-\sqrt[3]{-1}x+x-2}{x-2}}F\left(\sin^{-1}\left(\sqrt{-\frac{(-1)^{2/3}x}{x-2}}\right)\middle|(-1)^{2/3}\right)}{\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2]*Sqrt[(-2
 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*x)/(-2 +
 x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] time = 0.025, size = 200, normalized size = 11.8

$$2\,\frac{\left(-i\sqrt{3}-1\right)(-2+x)^2}{\left(i\sqrt{3}-1\right)\sqrt{-x(-2+x)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}\sqrt{\frac{\left(i\sqrt{3}-1\right)x}{\left(1+i\sqrt{3}\right)(-2+x)}}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(-2+x)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(-2+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

```
[Out] 2*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2
+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3
^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^2-2\,x+4\right)(x-2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^4-4x^3+8x^2-8x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^4 - 4*x^3 + 8*x^2 - 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(2-x)\left(x^2-2x+4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^2-2x+4\right)(x-2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.772 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rubi [A] time = 0.0505107, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1106, 1092, 1180, 524, 424, 419}

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right) + \frac{1}{2}\text{Subst}\left(\right.\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.97131, size = 298, normalized size = 4.08

$$\frac{(x-2)^2x\left(x^2-2x+4\right)\left(-\frac{3\left(x^2-2x+4\right)x}{x-2}-3\left(x^2-2x+4\right)-4(2-x)\sqrt{\frac{x^2-2x+4}{(x-2)^2}}\left(\sqrt{\frac{x^2-2x+4}{(x-2)^2}}x+4i\sqrt{2}\sqrt{\frac{ix}{\left(\sqrt{3}+i\right)(x-2)}}F\left(\sin^{-1}\left(\right.\right.\right.\right.}{96\left(-x\left(x^3-4x^2+8x-8\right)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) - (3*x*(4 -
 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*(x*Sqrt[
(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sqrt[3])*Sqrt[(I*x)/((I + Sqrt[3
])*(-2 + x))]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I + Sqr
t[3])*(-2 + x))]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*x^2 + x
^3)))^(3/2))
```

Maple [B] time = 0.03, size = 963, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

```
[Out] -1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1/192*x^2)/
(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/
2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x
-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3
^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2
+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)
)+1/6*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*
((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))
/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1
/2)*(2*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2)
)*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/
2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/
2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)))-1/24*(x*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2))+2*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x)
)^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/
2))/(1+I*3^(1/2))/(-2+x))^(1/2)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*Elliptic
F(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1
)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/
2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2
)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I
*3^(1/2))/(-2+x))^(1/2),(-I*3^(1/2)-1)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(
1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x
-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}{x^8-8\,x^7+32\,x^6-80\,x^5+128\,x^4-128\,x^3+64\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 +
128*x^4 - 128*x^3 + 64*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x\left(2-x\right)\left(x^{2}-2x+4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^{2}-2\,x+4\right)\left(x-2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

3.773 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rubi [A] time = 0.0704578, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1106, 1092, 1178, 1180, 524, 424, 419}

$$\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{192-\cdots}{\sqrt{3-2\cdots}}\right.}{\cdots}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{192\cdots}{\sqrt{2-2\cdots}}\right.}{\cdots}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{7}{432}\text{Subst}\left(\int\cdots\right.$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{7E\left(\sin^{-1}(1-\cdots\right.}{144\sqrt{3}\cdots}$$

Mathematica [C] time = 1.0615, size = 327, normalized size = 3.

$$\frac{(x-2)^3x^2\left(x^2-2x+4\right)^2\left(-\frac{7x\left(x^2-2x+4\right)}{x-2}+\frac{7x^7-49x^6+187x^5-445x^4+670x^3-622x^2+216x+36}{(x-2)^2x\left(x^2-2x+4\right)}-19i\sqrt{2}(x-2)\sqrt{\frac{ix}{\left(\sqrt{3}+i\right)(x-2)}}\sqrt{\frac{x^2-2x+4}{(x-2)^2}}\right.}{432\left(-x\left(x^3-4x^2+8x-8\right)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x) + (36 +
216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6 + 7*x^7)/((-2 + x)^2
*x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*Ell
ipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(I + Sqrt[3])])/Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*Sqrt[2
]*(-2 + x)*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/(-2 +
x)^2]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2 + x^3)))^(5/2)
)
```

Maple [B] time = 0.032, size = 1039, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x)
```

```
[Out] (1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*
x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/768*(-x^4+4
*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1
/2)+5/216*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x
)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1
/2))/(-2+x))^(1/2)/(I*3^(1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
)^(1/2)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2
))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+7/108*(-I*3^(1/2)-1)*
((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x-1+I*3^(1/2))/(1-I
*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-2+x))^(1/2)/(I*3^(
1/2)-1)/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*
3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3
^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))
/(-2+x))^(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I
*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
+2*(-I*3^(1/2)-1)*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1/2)*(-2+x)^2*((x
-1+I*3^(1/2))/(1-I*3^(1/2))/(-2+x))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(-
2+x))^(1/2)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(
1+I*3^(1/2))/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I
*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2)
)/(-2+x))^(1/2),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1)/(1-I*3^(1/2)))^
(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(-2+x))^(1
/2),(-I*3^(1/2)-1)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-I*3^(1/2)-1)/(I*3^(1/2)-1
)/(1-I*3^(1/2)))^(1/2))))/(-x*(-2+x)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^{12}-12x^{11}+72x^{10}-280x^9+768x^8-1536x^7+2240x^6-2304x^5+1536x^4-512x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^12 - 12*x^11 + 72*x^10 - 280*
x^9 + 768*x^8 - 1536*x^7 + 2240*x^6 - 2304*x^5 + 1536*x^4 - 512*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

3.774 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=730

$$-\frac{16c^3\left(8ad^2+c^3\right)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(20ad^2+7c^3-3c\right.}{35d^2}$$

```
[Out] ((c/d + x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/7 + (2*c*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(7*c^3 + 20*a*d^2 - 3*c*d
^2*(c/d + x)^2))/(35*d^2) - (16*c^3*(c^3 + 8*a*d^2)*(c/d + x)*Sqrt[4*a*c +
4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(35*d^2*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*
(c^3 + 8*a*d^2)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/
4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*S
qrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (8*c^(7/4)*(c^3 + 4*a*d^2)^
(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2) - c^(3/2)*(c^3 + 8*a*d^2))*Sqrt[
(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1
/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 0.903018, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1106, 1091, 1176, 1197, 1103, 1195}

$$-\frac{16c^3\left(8ad^2+c^3\right)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(20ad^2+7c^3-3c\right.}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] ((c/d + x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/7 + (2*c*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(7*c^3 + 20*a*d^2 - 3*c*d
^2*(c/d + x)^2))/(35*d^2) - (16*c^3*(c^3 + 8*a*d^2)*(c/d + x)*Sqrt[4*a*c +
4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(35*d^2*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*
(c^3 + 8*a*d^2)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/
4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*S
qrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (8*c^(7/4)*(c^3 + 4*a*d^2)^
(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2) - c^(3/2)*(c^3 + 8*a*d^2))*Sqrt[
(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] +
```

```
(d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1
/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}dx = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{3}{7}\text{Subst}\left(\int \left(2c\left(4a+\frac{c^3}{d^2}\right)-2\right.\right.$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx}}{}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx}}{}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx}}{}$$

Mathematica [C] time = 6.19849, size = 10468, normalized size = 14.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.181, size = 5229, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

3.775 $\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx$

Optimal. Leaf size=622

$$-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}+\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2\cdots\right.}{\cdots}$$

```
[Out] ((c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/3 - (2*c^2*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*Sqrt[c^3 + 4*a*d^2]*(
Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (2*c^(9/4)*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*
d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^
3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a
*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a*d^2)^(1/4)*(c^
3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4
*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elliptic
F[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^
3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 0.660194, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1106, 1091, 1197, 1103, 1195}

$$-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}+\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/3 - (2*c^2*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*Sqrt[c^3 + 4*a*d^2]*(
Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (2*c^(9/4)*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*
d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^
3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a
*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a*d^2)^(1/4)*(c^
3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4
*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elliptic
F[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^
3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}+\frac{1}{3}\text{Subst}\left(\int \frac{2c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2}}\right.$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}+\frac{\left(2c^{5/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1-\frac{d^2}{\sqrt{c}\sqrt{c^3}}}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-}}\right.}{3d^2}$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x}}{3d\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)}$$

Mathematica [C] time = 6.0953, size = 5218, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.03, size = 4890, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

```
[Out] 1/3*x*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/3*c/d*(d^2*x^4+4*c*d*x^3+
4*c^2*x^2+4*a*c)^(1/2)+16/3*a*c*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(
x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/
2))-8/3*c^3/d*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2
)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(
```

```
1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d*Ellip
ticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(
c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)))-2/3*c^2*((x-(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))^2/d^2)/(-(c+(-2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticF(((-(c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(
1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)*EllipticE(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)+4*c/d/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-c+(2*d
```

```
*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))))/(d
^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.776 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

Optimal. Leaf size=227

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))
/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sq
rt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2
*c^(1/4)*d*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 0.173098, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1106, 1103}

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))
/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sq
rt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2
*c^(1/4)*d*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\right.}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Mathematica [C] time = 2.04817, size = 822, normalized size = 3.62

$$\frac{2\left(-c-dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)\left(c+dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c+dx-\sqrt{c^2+2i\sqrt{a}d\sqrt{c}}\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}d\sqrt{c}}\right)\left(-c-dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)}}\sqrt{-\frac{}{\left(\sqrt{c^2-2i\cdots\right.}}}{d\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\sqrt{\frac{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-\sqrt{c^2+2i\cdots\right.}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] (2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c - S
qrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[
c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - d*x)))]*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c + Sqrt
[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*
d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]
*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*S
qrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x))]], (Sq
rt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])^2/
(Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])
^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sq
rt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 +
 (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)
)]*Sqrt[4*a*c + x^2*(2*c + d*x)^2])
```

Maple [B] time = 0.026, size = 1056, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

```
[Out] 2*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
```

```
*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)
/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/
(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(
d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.777 $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=674

$$-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+\right.}{}$$

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(8*a*c*(c^3 + 4*a*d^2)*Sqr
t[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d^2*(c/d + x)*Sqrt[4*a*c + 4
*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 + 4*a*d^2)^(3/2)*(Sqrt[c] + (d^2
*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (c^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt
[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elli
pticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqr
t[c^3 + 4*a*d^2])/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*c + 4*c^2*x^2 +
 4*c*d*x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqr
t[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c]
 + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/
Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^
(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(16*a*c^(5/4)*d*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 0.683077, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1106, 1092, 1197, 1103, 1195}

$$-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```


```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(8*a*c*(c^3 + 4*a*d^2)*Sqr
t[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d^2*(c/d + x)*Sqrt[4*a*c + 4
*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 + 4*a*d^2)^(3/2)*(Sqrt[c] + (d^2
*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (c^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt
[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elli
pticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqr
t[c^3 + 4*a*d^2])/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*c + 4*c^2*x^2 +
 4*c*d*x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqr
t[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c]
 + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/
Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^
(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(16*a*c^(5/4)*d*(c^3 + 4*a*d
```

```
^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\text{Subst}\left(\int \frac{2c\left(4a+\frac{c^3}{d^2}\right)d^2-2c^2d^2x^2}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}} \cdots\right.}{16ac^2\left(c^3+4ad^2 \cdots\right.}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1-\frac{d^2x^2}{\sqrt{c}\sqrt{c^3+4ad^2}}}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+} \cdots}\right.}{8a\sqrt{c^3+4ad \cdots}}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} - \frac{d(c+dx)\sqrt{4ac+4c^2x^2+4cd \cdots}}{8a\left(c^3+4ad^2\right)^{3/2}\left(\sqrt{c}+\frac{(c \cdots}{\sqrt{c \cdots}}\right.}$$

Mathematica [C] time = 6.13315, size = 5276, normalized size = 7.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.04, size = 5024, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}{d^4x^8 + 8\,cd^3x^7 + 24\,c^2d^2x^6 + 32\,c^3dx^5 + 32\,ac^2dx^3 + 32\,ac^3x^2 + 8\left(2\,c^4 + acd^2\right)x^4 + 16\,a^2c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)/(d^4*x^8 + 8*c*d^3*x
^7 + 24*c^2*d^2*x^6 + 32*c^3*d*x^5 + 32*a*c^2*d*x^3 + 32*a*c^3*x^2 + 8*(2*c
^4 + a*c*d^2)*x^4 + 16*a^2*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

3.778 $\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx$

Optimal. Leaf size=663

$$-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}+\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256a\right.}{}$$

```
[Out] ((d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/3 - (2*d^2*
(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/(Sqrt[5*d^4
+ 256*a*e^3]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) + (d^2
*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(8*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4]) + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 2
56*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 2
56*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (
16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*
e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/
(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rubi [A] time = 0.810476, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1106, 1091, 1197, 1103, 1195}

$$-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}+\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```


```
[Out] ((d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/3 - (2*d^2*
(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/(Sqrt[5*d^4
+ 256*a*e^3]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) + (d^2
*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(8*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4]) + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 2
56*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 2
56*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (
16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*
e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/
```

```
(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\text{Subst}\left(\int \sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx,x,\frac{d}{4e}+x\right)$$

$$=\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{1}{3}\text{Subst}\left(\int \frac{\frac{1}{16}\left(\frac{5d^4}{e}+256ae^2\right)}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3}}\right.$$

$$=\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{\left(d^2\sqrt{5d^4+256ae^3}\right)\text{Subst}\left(\int \frac{}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}\right.}}\right.}{16e}$$

$$=\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}-\frac{d^2(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8}}{2e\sqrt{5d^4+256ae^3}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right.}$$

Mathematica [B] time = 6.12362, size = 7543, normalized size = 11.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.24, size = 7887, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.779 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rubi [A] time = 0.177037, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1106, 1103}

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{\sqrt[4]{5d^4+256ae^3}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(5d^4+256ae^3\right)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)F\left(2\tan^{-1}\left(\frac{d+\cdots}{\sqrt[4]{5d^4+\cdots}}\right.\right.}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Mathematica [B] time = 2.2346, size = 1065, normalized size = 4.53

$$-\frac{\left(-d-4ex+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\right)\left(d+4ex-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\left(d+4ex+\sqrt{3d^2+2\sqrt{d^4-\cdots}}\right.}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d-4ex+\cdots\right.}}}{2e\left(\sqrt{3d^2\cdots}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] -((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3*d^2 + 2*S
qrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*
(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d
^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 -
2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*a*e^3] -
Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] +
 d*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e
^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4
 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*S
qrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]
*EllipticF[ArcSin[Sqrt[((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2
+ 2*Sqrt[d^4 - 64*a*e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*
x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a
*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]], (Sqrt[3*d^2
 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])^2/(Sqrt[
3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])^2])
/(2*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*
a*e^3]])*Sqrt[(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d + Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt
[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]
] - 4*e*x))]*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Maple [B] time = 0.024, size = 1704, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

```
[Out] 1/2*(1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+
(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-1/4*(d*e+(3*e^2*d^2-
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e
^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*
e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*(d*e+(3
*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(d*e+(3*e^2*d^2+
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d
^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*
e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/
e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d
*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*((-1/4*(d*e+(
3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-
64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2
)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/
(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)/(-1/
4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d
^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2+2*(-64*a*
e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)
*e^2)^(1/2))/e^2)*2^(1/2)/(e^3*(x-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1
/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1
/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*
(x+1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*Elli
pticF(((-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d
*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*
d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a
*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2
)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2
))/e^2))^(1/2),((-1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(1/4*(-d*e
+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d^2-2*(
-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^
(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1
/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.780 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=748

$$2\sqrt{2}$$

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}-$$

```
[Out] (4*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e
^3*x^4]) + (384*d^2*e^2*(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 +
8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(3/2)*(1 + (16*e^2*(d/(4*
e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) - (12*Sqrt[2]*d^2*Sqrt[(e*(8*a*e^2 - d
^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
 + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*
d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)]
, (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*
a*e^3)^(1/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*
(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*
x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) +
x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4
 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e
^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rubi [A] time = 0.786038, antiderivative size = 748, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1106, 1092, 1197, 1103, 1195}

$$2\sqrt{2}$$

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] (4*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e
^3*x^4]) + (384*d^2*e^2*(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 +
8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(3/2)*(1 + (16*e^2*(d/(4*
e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) - (12*Sqrt[2]*d^2*Sqrt[(e*(8*a*e^2 - d
^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
 + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*
d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)]
, (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*
a*e^3)^(1/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*
```

```
(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*
x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) +
x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4
 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e
^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^{3/2}}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{8\,\text{Subst}\left(\int \frac{\frac{1}{2}\cdots}{\sqrt{\frac{1}{32}\cdots}}\cdots\right)}{e\left(5d\cdots\right.}$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{\left(12d^2e\right)\text{Subst}\left(\cdots\right.}{\left(\cdots\right.}$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} + \frac{96d^2e(d+4ex\cdots}{\left(d^4-64ae^3\right)\left(5\cdots\right.}$$

Mathematica [B] time = 6.15134, size = 7629, normalized size = 10.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.04, size = 8103, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}}{64\,e^6x^8+128\,de^5x^7+64\,d^2e^4x^6-16\,d^3e^3x^5+128\,ade^4x^3+d^6x^2-16\,ad^3e^2x+64\,a^2e^4-16\left(d^4e^2-8\,ae^5\right)x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)/(64*e^6*x^8 + 128*
d*e^5*x^7 + 64*d^2*e^4*x^6 - 16*d^3*e^3*x^5 + 128*a*d*e^4*x^3 + d^6*x^2 - 1
6*a*d^3*e^2*x + 64*a^2*e^4 - 16*(d^4*e^2 - 8*a*e^5)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

```
[Out] Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError
```

3.781 $\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=452

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)\left(1-\right.}{35\sqrt{a-}}$$

```
[Out] (-16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x
))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)
^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt
[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[
1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.57864, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1106, 1091, 1176, 1202, 531, 418, 492, 411}

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)\left(1-\right.}{35\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (-16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x
))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)
^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt
[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[
1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
```

```
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2} dx, x, -1+x\right)$$

$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(2(3+a)-2x^2\right)\sqrt{3+\right.$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3+a-2(-1+\right.$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3+a-2(-1+\right.$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3+a-2(-1+\right.$$

$$= \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2\right.$$

$$= \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2\right.$$

Mathematica [B] time = 6.12211, size = 6287, normalized size = 13.91

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.063, size = 2655, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+a+8
*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-3/7*a-4/7)*(-x^4
+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7*a-32/35)*a+12/7*a+16/7)*((-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))
```

```
^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1
+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1
/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+
(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(
4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2)
)^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*
(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1
/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*E
llipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-
2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a
)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(
1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(
4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1
-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+
```

```
(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+
a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.782 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=397

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\Big(\text{ta}}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4}}$$

```
[Out] (-2*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]*(-1 + x))/3 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^
2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*
Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))
/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) + (2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*El
lipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + S
qrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.400912, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1106, 1091, 1202, 531, 418, 492, 411}

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\Big(\text{ta}}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```


```
[Out] (-2*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]*(-1 + x))/3 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^
2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*
Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))
/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) + (2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*El
lipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + S
qrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx,x,-1+x\right)$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{2(3+a)-2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx,x,\right.$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subs}}{3\sqrt{3+a-2(-}}$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)-\frac{\left(2\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Sub}}{3\sqrt{3+a-2(-}}$$

$$= \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)-$$

$$= \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)-$$

Mathematica [B] time = 6.05808, size = 3470, normalized size = 8.74

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], (
(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4
+ a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sq
rt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
```

```
qrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellipt
icF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
 Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 +
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
- Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1
+ Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[A
rcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1
 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-
1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x
))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4
 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3
 - x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]]
+ x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 -
Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^
2])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]))/3
```

Maple [B] time = 0.016, size = 2519, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/
3*a+4/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1
+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(
1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(
4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-4/3*((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2)
)^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(
1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1
+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(
1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-2/3*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(
-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+
a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^
(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+
(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(
1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
```

```
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))
^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1
+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4
+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.783 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.103659, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1106, 1104, 418}

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```


```
[Out] (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}
\end{aligned}$$

Mathematica [B] time = 1.4009, size = 540, normalized size = 3.75

$$\frac{2\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(\sqrt{\sqrt{a+4}-1}-x+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)}}\left(\sqrt{-\sqrt{a+4}-1}+x-1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(\sqrt{\sqrt{a+4}-1}+\cdots\right.}{\left(\sqrt{\sqrt{a+4}-1}-\sqrt{-\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{\cdots}}\right.}}}{\sqrt{-\sqrt{a+4}-1}\sqrt{\frac{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}+x-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)}}\sqrt{a-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Ellip
ticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(Sq
rt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8 + 8*x - 4*x^
2 + x^3)])
```

Maple [B] time = 0.015, size = 530, normalized size = 3.7

$$-\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(x-1-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] -((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+
a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(
x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}{x^4-4\,x^3+8\,x^2-a-8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^4 - 4*x^3 + 8*x^2 - a - 8
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.784 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=437

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\cdots\right.}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\cdots}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)
^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(
-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 +
a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]
))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.415938, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1106, 1092, 1202, 531, 418, 492, 411}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\cdots\right.}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)
^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(
-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 +
a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]
))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
```

```
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{-2(3+a)+2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1\right.}{4\left(12+7a+a^2\right)}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right.}{4\left(12+7a+a^2\right.}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right.}{2\left(12+7a+a^2\right.}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+}}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+}}$$

Mathematica [B] time = 6.08066, size = 3526, normalized size = 8.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])/(2*
(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 +
a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
```

]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt
[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqr
t[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Elliptic
F[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^
2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1
- Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))
/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 +
a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 -
x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] +
x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sq
rt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]
)/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2

```
+ 4*x^3 - x^4])/(2*(3 + a)*(4 + a))
```

Maple [B] time = 0.018, size = 2601, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(8+a)/(a^2+7*a+12)*x-1/4*(
6+a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5)/(a^2+7*a+12)-1/2*(
8+a)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a
)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*
(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(
((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/(a^2+7
*a+12)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/
2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+
a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(
1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))
^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-1/2/(a^2+7*a+12
)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)
^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^
(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
```

```
(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-
(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(
1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(
-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(
1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}{x^8-8\,x^7+32\,x^6-2\,(a-64)x^4-80\,x^5+8\,(a-16)x^3-16\,(a-4)x^2+a^2+16\,ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 2*(a
- 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.785 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=517

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)\left(1-\cdots\right.}{3(a+3)^2(a+4)\cdots}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 +
x))/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7
+ 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3
 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
+ (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
 ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
 a])])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (
-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.559856, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1106, 1092, 1178, 1202, 531, 418, 492, 411}

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)\left(1-\cdots\right.}{3(a+3)^2(a+4)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 +
x))/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7
+ 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3
 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
+ (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
 ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
 a])])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (
-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-3(4+4(3-}{\left(3+a-2x^2-x^4\right)}\right.}{12\left(12+7a\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+\right.}{6\left(12+7a+a^2\right)\left(3+a-2(\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+\right.}{6\left(12+7a+a^2\right)\left(3+a-2(\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+\right.}{6\left(12+7a+a^2\right)\left(3+a-2(\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1-\right.}{3\left(12+7a+a^2\right)^2\sqrt{3+a-}}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1-\right.}{3\left(12+7a+a^2\right)^2\sqrt{3+a-}}$$

Mathematica [B] time = 6.21135, size = 6386, normalized size = 12.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.025, size = 2757, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)

[Out] (1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(8+a)/(a^2+7*a+12)*x-1/6*(6+
a)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2
*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3-1/2*(7+2*a)/(a^2+7*a+12)^2*x^2+1/24*(5*a^2
+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4+4*x
^3-8*x^2+a+8*x)^(1/2)-(1/6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2+71*a
+188)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-2/3*(
7+2*a)/(a^2+7*a+12)^2*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1
+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-
1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/
2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*
(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1
+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/
2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2
))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/
2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-
(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x
-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
```

```
)^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(
((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a
)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticP
i(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2)
)*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}{x^{12}-12\,x^{11}+72\,x^{10}-3\,(a-256)x^8-280\,x^9+24\,(a-64)x^7-32\,(3\,a-70)x^6+48\,(5\,a-48)x^5+3\left(a^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^12 - 12*x^11 + 72*x^10 -
3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a -
 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 - 2
4*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.786 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=558

$$\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{7}(x-1)\left(a\right.$$

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/16 + (
(1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/8 - (16*(7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(35*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 +
 a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)*(-1 + x))/7 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4]])/16 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4
+ a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
 Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.565067, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1680, 1673, 1091, 1176, 1202, 531, 418, 492, 411, 1107, 612, 621, 204}

$$\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{7}(x-1)\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```


```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/16 + (
(1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/8 - (16*(7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(35*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 +
 a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)*(-1 + x))/7 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4]])/16 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4
+ a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
 Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
```

```
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}dx = \text{Subst}\left(\int (1+x)\left(3+a-2x^2-x^4\right)^{3/2}dx,x,-1+x\right)$$
$$= \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2}dx,x,-1+x\right)+\text{Subst}\left(\int x\left(3+a-2x^2-x^4\right)^{3/2}$$
$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(2(3+a)-2x^2\right)\sqrt{3}$$
$$= \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right)-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+}$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{8}\left(3+a-2(1-x)^2-(1\right.$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{8}\left(3+a-2(1-x)^2-(1\right.$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{8}\left(3+a-2(1-x)^2-(1\right.$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{8}\left(3+a-2(1-x)^2-(1\right.$$

Mathematica [B] time = 6.12917, size = 7235, normalized size = 12.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.018, size = 2694, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^
(1/2)-43/28*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+74/35*x^3*(-x^4+4*x^3-8*x^2+
a+8*x)^(1/2)+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-11/56*a-29/
70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+a+8*
x)^(1/2)-(-(-11/56*a-29/70)*a-11/14*a-26/7)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+
```

```
a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+
(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)
^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1
/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))
^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*
(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5)*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2
*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)
^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(
1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4
+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(
1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2)))^(1/2))+(-32/35*a-16/5)*((x-1-(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2)
)^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(
4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1
+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/
2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
```

```
/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(
1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(x^5-4\,x^4+8\,x^3-ax-8\,x^2\right)\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 + a
 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

3.787 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=466

$$\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/4 - (2*(1 - Sqrt
[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))
/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]])/4 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 +
a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(
3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a
]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.454577, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1680, 1673, 1091, 1202, 531, 418, 492, 411, 1107, 612, 621, 204}

$$\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```


```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/4 - (2*(1 - Sqrt
[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))
/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]])/4 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 +
a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(
3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a
]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
```

```
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int (1+x)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int x\sqrt{3+a-2x^2-x^4}\,dx, x, -1\right.$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{3}\text{Subst}\left(\int \frac{2(3+a)-2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1\right.$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] time = 6.08799, size = 4389, normalized size = 9.42

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]

[Out] (1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
qrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellip
ticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 +
Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt
[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt
[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqr
t[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[
-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a
 + 8*x - 8*x^2 + 4*x^3 - x^4]) + (40*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]
*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (6*a*(-Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqr
```

```
t[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sq
rt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4
]) - (4*((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x
)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqr
t[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])
/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))))/Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]))/(6*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])
```

Maple [B] time = 0.016, size = 2551, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] 1/4*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/6*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)
+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(
4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2)
```

)^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*
(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1
/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1
/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(
-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-
1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(
1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
)))^(1/2)))-2/3*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*
(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(
x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^
(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-
(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2
)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*E
llipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(
1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))
-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1
/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))

```
*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/
2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-
(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

3.788 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=179

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.152437, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1680, 1673, 1104, 418, 1107, 621, 204}

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```


```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4])
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{1+x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1\right.$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Su}}{\sqrt{3+a-2(}}$$

$$= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,\right.$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{}{\sqrt{1}}\right.\right.}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2}}$$

Mathematica [B] time = 2.73358, size = 813, normalized size = 4.54

$$\frac{2\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(-x+\sqrt{\sqrt{a+4}-1}+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\left(x+\sqrt{-\sqrt{a+4}-1}-1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(x+\sqrt{\sqrt{a}\right.}}{\left(\sqrt{\sqrt{a+4}-1}-\sqrt{-\sqrt{a+4}-1}\right)\left(-x\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*((1 +
 Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)])
```

Maple [B] time = 0.016, size = 788, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] -((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+
a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(
x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*
EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
```

```
)+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),
((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x}{x^4-4\,x^3+8\,x^2-a-8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^4 - 4*x^3 + 8*x^2 - a -
 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.789 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}} dx$

Optimal. Leaf size=474

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(}{1-}\right.}{2(a+3)(a+4)\sqrt{a-(x-1)^4}}$$

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 +
 x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 + a)*S
qrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.43514, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1680, 1673, 1092, 1202, 531, 418, 492, 411, 1107, 613}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(}{1-}\right.}{2(a+3)(a+4)\sqrt{a-(x-1)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 +
 x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 + a)*S
qrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
```

```
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
```

```
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x^4\right)^{3/2}}\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(3+a-2x-x\right.}\right.$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1}}$$

Mathematica [B] time = 6.07987, size = 3593, normalized size = 7.58

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2)/(2*(3 +
a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^3)
)^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((4*(-Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
```

] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a
]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
 Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]
]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 +
 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-
1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[
-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[
ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sq
rt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 -
Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a
]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x
^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)
(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]

```
)^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt
[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/
(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 +
 4*x^3 - x^4]))/(2*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

Maple [B] time = 0.016, size = 2616, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3+1/4*a/(a^2+7*a+12)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4
*a/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)+1/2*(a-2)/(
a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2)
)/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2
))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(
-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-((1+a)/(a^2+7*
a+12)-a/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1
+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
```

```
^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-
1/2/(a^2+7*a+12)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))
*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*
(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)
)/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)
^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1
-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^
2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*
EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(
1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)
)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1
/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-x^4+4x^3-8x^2+a+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}x}{x^8-8x^7+32x^6-2(a-64)x^4-80x^5+8(a-16)x^3-16(a-4)x^2+a^2+16ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^8 - 8*x^7 + 32*x^6 - 2*(
a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.790 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}} \, dx$

Optimal. Leaf size=591

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(x-1}{3(a+4)^2\sqrt{a-(x-}}$$

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (1
 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a
)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
6 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])
])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.584279, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1680, 1673, 1092, 1178, 1202, 531, 418, 492, 411, 1107, 614, 613}

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(x-1}{3(a+4)^2\sqrt{a-(x-}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (1
 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a
)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
6 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])
])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
```

```
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x^4\right)^{5/2}} \cdots\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(3+a-2x \cdots\right.} \cdots\right.$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} - \frac{\left(104+47a+5a^2+4(7+2a)(1-x \cdots\right.}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2 \cdots}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x \cdots}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x \cdots}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x \cdots}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x \cdots}}$$

Mathematica [B] time = 6.11995, size = 6452, normalized size = 10.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.023, size = 2777, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3+1/6*a/(a^2+7*a+12)*x^2-1/6*(a-2)/(a^2+7*a+12)*x+1/6*a
/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(
1/6*(7+2*a)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(a^2+7*a+12)^2*x^2-1/24*(3*a^2-
23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(1/6*(a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-116)/
(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(2/3*(a^2+2
*a-5)/(a^2+7*a+12)^2-2/3*(a^2-12)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1
+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/
2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2)
)*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^
(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)
*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(
4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2)))^(1/2)))-1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2)
)^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)
*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))
^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((
1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1
/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/
2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^
```

```
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))
*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(
1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x}{x^{12}-12\,x^{11}+72\,x^{10}-3\,(a-256)x^8-280\,x^9+24\,(a-64)x^7-32\,(3\,a-70)x^6+48\,(5\,a-48)x^5+3\left(a^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^12 - 12*x^11 + 72*x^10
- 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a
 - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 -
 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.791 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx$

Optimal. Leaf size=585

$$\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\right.}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x}}$$

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/8 + ((
1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/4 + (4*(140 + 11
1*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x)
)/(315*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(2*(80 + 27*a) + 3*(20
 + 7*a)*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/315 +
 ((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
3 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4]])/8 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sq
rt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2
/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]) + (4*(3 + a)*(100 + 33*a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a
]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rubi [A] time = 0.678902, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1680, 1673, 1176, 1202, 531, 418, 492, 411, 12, 1107, 612, 621, 204}

$$\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\right.}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/8 + ((
1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/4 + (4*(140 + 11
1*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x)
)/(315*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(2*(80 + 27*a) + 3*(20
 + 7*a)*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/315 +
 ((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
3 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4]])/8 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sq
rt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2
/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]) + (4*(3 + a)*(100 + 33*a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a
]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2/(1 - Sq
```

```
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}dx = \text{Subst}\left(\int (1+x)^2\left(3+a-2x^2-x^4\right)^{3/2}dx,x,-1+x\right)$$
$$= \text{Subst}\left(\int 2x\left(3+a-2x^2-x^4\right)^{3/2}dx,x,-1+x\right)+\text{Subst}\left(\int \left(1+x^2\right)\left(3+a-2\right.\right.$$
$$= \frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)-\frac{1}{21}\text{Subst}\left(\int\right.$$
$$= -\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)+\frac{}{6}$$
$$= \frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}-\frac{2}{315}\left(2(80+27a)+3(20\right.$$
$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a\right.$$
$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a\right.$$
$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a\right.$$

Mathematica [B] time = 6.16396, size = 8500, normalized size = 14.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.019, size = 2733, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^
(1/2)-163/126*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+71/42*x^4*(-x^4+4*x^3-8*x^
2+a+8*x)^(1/2)+(11/45*a-16/63)*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-13/120*
a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^3-8*
```

```
x^2+a+8*x)^(1/2)+(107/252*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(9/140
*a+23/63)*a-107/63*a-404/63)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(
x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*
EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)-(-2*(-13/120*a-5/18)*a+827/315*a+76/9)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1
/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)
*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-
1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1
/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*Elliptic
Pi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)))^(1/2))+(a^2-3*(11/45*a-16/63)*a+68/105*a+16/9)*((x-1-(-1+(4+a)^
(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(
1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/
2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2)
)^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)
^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-
1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1
```

```
+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))/(-(x-1-(-1+(4+a)^(1/2))^(1
/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+
a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^6-4\,x^5+8\,x^4-ax^2-8\,x^3\right)\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3 - 8*x^2 +
 a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

3.792 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=485

$$\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{2(3a+8)\left(1-\sqrt{\cdots}\right.}{15\sqrt{a-(x\cdots}}$$

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/2 + (2*(8 + 3*a)
*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(15*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]*(-1 + x))/15 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 - (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
 x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(
1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (8*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rubi [A] time = 0.529802, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1680, 1673, 1176, 1202, 531, 418, 492, 411, 12, 1107, 612, 621, 204}

$$\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{2(3a+8)\left(1-\sqrt{\cdots}\right.}{15\sqrt{a-(x\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```


```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/2 + (2*(8 + 3*a)
*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(15*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]*(-1 + x))/15 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 - (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
 x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(
1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (8*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int (1+x)^2\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int 2x\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \left(1+x^2\right)\sqrt{3+a-2x^2$$
$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) - \frac{1}{15}\,\text{Subst}\left(\int \frac{-8$$
$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\right.}{}$$
$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a$$
$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\right.}{15\sqrt{3+a-2(1-x)^2}}$$
$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\right.}{15\sqrt{3+a-2(1-x)^2}}$$

Mathematica [B] time = 6.11967, size = 5647, normalized size = 11.64

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.019, size = 2582, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] 1/5*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/10*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)+1/15*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)-(-1/15*a-4/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
```

```
a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(1/5*a+28/1
5)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(
4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2)
)/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(
-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(
1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2)
)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(
4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/
2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))+(2/5*a+16/15)*((x-1
-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))
^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+
(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^
(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-
(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(
1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
```

```
/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a
)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(
x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

3.793 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=388

$$\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)}}$$

```
[Out] ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/Sqrt[3 + a
- 2*(-1 + x)^2 - (-1 + x)^4] + ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4]] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 +
x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (
-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a])
)/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.387102, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1680, 1673, 1202, 531, 418, 492, 411, 12, 1107, 621, 204}

$$\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/Sqrt[3 + a
- 2*(-1 + x)^2 - (-1 + x)^4] + ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4]] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 +
x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (
-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a])
)/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1 \ldots\right.$$

$$= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subs}\ldots}{\sqrt{3+a-2(-\ldots}}$$

$$= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \ldots$$

$$= -\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\right|\ldots}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x\ldots}}$$

$$= -\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) + \frac{\left(1-\sqrt{\ldots}\right.}{\ldots}$$

Mathematica [B] time = 5.8173, size = 1145, normalized size = 2.95

$$\frac{2\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(-x+\sqrt{\sqrt{a+4}-1}+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\sqrt{\frac{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(x+\sqrt{-\sqrt{a+4}-1}-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\sqrt{-\frac{\sqrt{-\sqrt{a+4}-1}\left(x+\sqrt{\sqrt{a+4}-1}-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\left(\left(\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 +
Sqrt[-1 + Sqrt[4 + a]] + x) + (2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)^2*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1
+ Sqrt[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[-
((Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
```

```
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)))]*((1
 + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - S
qrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - (1 + 2*Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticF[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 4*Sqrt[-1 - Sqr
t[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
- x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(1 + Sqrt[4 + a] + Sqrt[-1 - Sqrt[
4 + a]]*Sqrt[-1 + Sqrt[4 + a]]))/Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)]
```

Maple [B] time = 0.019, size = 1147, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] ((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(
1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2)
)/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1
-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/
2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1
```

```
+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/
2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2}{x^4-4\,x^3+8\,x^2-a-8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^4 - 4*x^3 + 8*x^2 - a
 - 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.794 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=311

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}\right.}{2(a+3)\sqrt{a-(x-1)^4-2(x-}}$$

```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((4 +
a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2
 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1
 + x))/(2*(3 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 +
 a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
2*(3 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt
[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.327172, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1680, 1673, 1178, 12, 1140, 492, 411, 1107, 613}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}\right.}{2(a+3)\sqrt{a-(x-1)^4-2(x-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```


```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((4 +
a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2
 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1
 + x))/(2*(3 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 +
 a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
2*(3 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt
[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1140

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/
(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[x^2/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqr
t[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x^4\right)^{3/2}}\right.$$
$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + 2\,\text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-\right.}\right.$$
$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x,\right.}{2(3+a)}$$
$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2}}$$
$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(3+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(3+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] time = 6.11133, size = 2941, normalized size = 9.46

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^2 + 4*x^3
- x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 +
8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((2*(-S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sq
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + S
qrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt
[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
```

```
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + ((-1 + Sqrt[-1 - S
qrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 +
 a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
 a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4
+ a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4
 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]]
, (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-((-1 - Sqr
t[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqr
t[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]))/(2*(3 + a
)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

Maple [B] time = 0.022, size = 2607, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(3+a)*x^3-1/4*(6+a)/(a^2+7*a+12)*x^2+1/4*(8+a)/(a^2+7*a+12)*x+1/4*a/
(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)-1/2*(8+a)/(a^2
```

```
+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-
1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2)
)^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^
(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-
(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(-2/(a^2+7*a+12)+
(6+a)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(
4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1
/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+
a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))
*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(
4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/
2/(3+a)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1
-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(
1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(
1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1
/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF
(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
```

```
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-
(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1
/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2}{x^8-8\,x^7+32\,x^6-2\,(a-64)x^4-80\,x^5+8\,(a-16)x^3-16\,(a-4)x^2+a^2+16\,ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^8 - 8*x^7 + 32*x^6 - 2
*(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.795 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=582

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{}{3(a+4)}$$

```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (2
*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)
^2 - (-1 + x)^4)^(3/2)) + ((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(12
*(3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((13 + 3*a)*(
1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(12*(3 + a)^2
*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((13 + 3*a)*(1 - Sqrt[4
 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(12*(3 + a)^2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
+ Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(12 + 7*a
 + a^2)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4
 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.674828, antiderivative size = 582, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1680, 1673, 1178, 1202, 531, 418, 492, 411, 12, 1107, 614, 613}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{}{3(a+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```


```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (2
*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)
^2 - (-1 + x)^4)^(3/2)) + ((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(12
*(3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((13 + 3*a)*(
1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(12*(3 + a)^2
*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((13 + 3*a)*(1 - Sqrt[4
 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(12*(3 + a)^2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
+ Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(12 + 7*a
 + a^2)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4
```

```
 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x^4\right)^{5/2}}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + 2\,\text{Subst}\left(\int \frac{x}{\left(3+a-2x^2\right.}\right.$$

$$= -\frac{\left(29+7a+(13+3a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-1+\right.}{6\left(12+7a+a^2\right)\left(3+a-2(-\right.}$$

$$= \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\left(29+7a+(13+3a)(1-x)^2\right.}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x}}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x\right.}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x\right.}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x\right.}$$

Mathematica [B] time = 6.1629, size = 5812, normalized size = 9.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.029, size = 2780, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(3+a)*x^3-1/6*(6+a)/(a^2+7*a+12)*x^2+1/6*(8+a)/(a^2+7*a+12)*x+1/6*a/(a
^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(1/2
4*(13+3*a)/(3+a)/(a^2+7*a+12)*x^3-1/24*(a^2+27*a+84)/(a^2+7*a+12)^2*x^2+1/6
*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(-1/6*(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2+7*a+
12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/
2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+
a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(1/3*(a^2-a-16)/(a^2
+7*a+12)^2+1/6*(a^2+27*a+84)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1
+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a
)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(
1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-
1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2)
)^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*Elli
pticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2)))^(1/2))-1/12*(13+3*a)/(3+a)/(a^2+7*a+12)*((x-1-(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2
))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*(
(1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(
1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1
/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2
```

```
))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2)
)*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^
(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2}{x^{12}-12\,x^{11}+72\,x^{10}-3\,(a-256)x^8-280\,x^9+24\,(a-64)x^7-32\,(3\,a-70)x^6+48\,(5\,a-48)x^5+3\,\left(a^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^12 - 12*x^11 + 72*x^1
0 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5
*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3
 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.796 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

```
[Out] -(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x
^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]*
29^(1/4)*x)], (29 + Sqrt[29])/58])/(8*Sqrt[3]*29^(1/4)*Sqrt[8 + 8*x - x^3 +
 8*x^4])
```

Rubi [A] time = 0.302434, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2069, 12, 6719, 1103}

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] -(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x
^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]*
29^(1/4)*x)], (29 + Sqrt[29])/58])/(8*Sqrt[3]*29^(1/4)*Sqrt[8 + 8*x - x^3 +
 8*x^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\left(1024\,\mathrm{Subst}\left(\int \frac{1}{2\sqrt{2}(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(256\sqrt{2}\right)\mathrm{Subst}\left(\int \frac{1}{(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1069056-393216x^2+1048576x}}\right.}{\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)F\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

Mathematica [C] time = 0.344436, size = 927, normalized size = 7.19

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0
])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 4, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^
2*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])^2)])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))
```

Maple [C] time = 1.415, size = 965, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^(1/2),x)
```

```
[Out] 1/2*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))
*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(
x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^
(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,i
ndex=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^
3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*El
lipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

3.797 $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

Optimal. Leaf size=431

$$-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}+\frac{(14-5\sqrt{}}{}$$

```
[Out] -((66 - (1 + 4/x)^2)*x^2)/(1008*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((216 - 7*(1
 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (7*(261 - 6
*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(432*Sqrt[29]*Sqrt[8 + 8*x - x^3
 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)
^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x
)^2)/x^2)*EllipticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])
/58])/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sqrt[29
])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/
x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]
*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^
3 + 8*x^4])
```

Rubi [A] time = 0.528148, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2069, 12, 6719, 1673, 1678, 1197, 1103, 1195, 1247, 636}

$$-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}+\frac{(14-5\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```


```
[Out] -((66 - (1 + 4/x)^2)*x^2)/(1008*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((216 - 7*(1
 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (7*(261 - 6
*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(432*Sqrt[29]*Sqrt[8 + 8*x - x^3
 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)
^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x
)^2)/x^2)*EllipticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])
/58])/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sqrt[29
])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/
x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]
*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^
3 + 8*x^4])
```


Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
```

```
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^{3/2}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{16\sqrt{2}(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{(8-32x)^4}{\left(1069056-393216x^2+10\right.}\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x\left(-65536-10485\right.}{\left(1069056-393216x^2+10\right.}\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right.}{37026344336426}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(7\sqrt{1069056-393216\left(\right.}\right.}{}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} + \frac{7\left(261-6\left(1+\frac{4}{x}\right)^2+\left(\right.\right.}{432\sqrt{29}\sqrt{8+8x-x^3+}}$$

Mathematica [C] time = 6.04711, size = 4865, normalized size = 11.29

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (
(28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(EllipticF[ArcSin[Sqr
t[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - R
```

oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]
)*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0]))))]*Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + EllipticPi[(-Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
])/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1
, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]))*Sqrt[((-Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8
#1 - #1^3 + 8#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*
Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0
] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4
]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 4, 0])) + (842*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2
, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))]], ((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8
#1^4 & , 2, 0] - Root[8 + 8#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0])*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8
#1 - #1^3 + 8#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]
))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((x - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0])) - (224*((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4
, 0]) + (x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*

```
#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x
- Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0]))]*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sq
rt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0]
+ Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*((EllipticE[ArcSin[S
qrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] -
 Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0]))/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + (EllipticF[ArcSin[Sqrt[((x - Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]
- Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]) - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 4, 0])))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])) - (EllipticPi[(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])/(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])
*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0]
 - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 3, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/(-Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))/Sqrt[8 +
 8*x - x^3 + 8*x^4])/6264
```

Maple [C] time = 0.019, size = 4426, normalized size = 10.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-x^3+8*x+8)^(3/2),x)

[Out] -16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2)
+421/12528*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-
_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3
+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(
1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3
+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,i
ndex=4)))^(1/2))+7/6264*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^
3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^
3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2
)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1
/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)))^(1/2)*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*EllipticF(((RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*EllipticPi((
(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-Roo
tOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1
/2),(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)),((
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(Roo
tOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)))-7/78
3*((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde

```
x=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))+(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=
1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)*RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)^2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*EllipticF(((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*EllipticE(((RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))/(RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))-1/8/(RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*EllipticPi(((R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2
),(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)),((Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))))*2^(1/
2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^4 - x^3 + 8\,x + 8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}{64\,x^8 - 16\,x^7 + x^6 + 128\,x^5 + 112\,x^4 - 16\,x^3 + 64\,x^2 + 128\,x + 64}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*x^4 - x^3 + 8*x + 8)/(64*x^8 - 16*x^7 + x^6 + 128*x^5 + 112
*x^4 - 16*x^3 + 64*x^2 + 128*x + 64), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8x^4 - x^3 + 8x + 8\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
```

```
[Out] Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^4 - x^3 + 8\,x + 8\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

3.798 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

```
[Out] -((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(
Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)],
(5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rubi [A] time = 0.216657, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2069, 6719, 1103}

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] -((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(
Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)],
(5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\left(16\,\text{Subst}\left(\int \frac{1}{(4-4x)^2\sqrt{\frac{1280-512x^2+256x^4}{(4-4x)^4}}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1280-512x^2+256x^4}}\,dx, x, 1+\frac{1}{x}\right)}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}\,x^2 F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

Mathematica [C] time = 0.52478, size = 249, normalized size = 2.31

$$\frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-2x+\sqrt{-1-2i}-i\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1-2i}+i\right)}}\left(2ix^2+2x+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\left(2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1+2i}-i\right)}{\sqrt{-1+2i}\left(2x+\sqrt{-1-2i}+i\right)}}}{\sqrt{2}}\right)|\frac{1}{2}\left(5-\sqrt{5}\right)\right)}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(2ix^2+2x+1\right)}{\left(2x+\sqrt{-1-2i}+i\right)^2}}\sqrt{4x^4+4x^2+4x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] ((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(-
I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I + S
qrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I +
Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*
I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2]])/(Sqrt[((1 + 2*I
)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] +
2*x)^2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] time = 0.654, size = 961, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x)
```

```
[Out] (-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)
)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,i
ndex=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z
^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1
/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1
```

```
,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))
/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)
)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.799 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}} dx$

Optimal. Leaf size=367

$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}+\frac{3\left(3-\sqrt{5}\right)\left(\left(\frac{1}{x}+\right.\right.}{}$$

```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((13 - 9*(1 +
 x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + (9*(5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1))*x^2)/(10*(Sqrt[5] + (1 + x^
(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqr
t[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2
*EllipticE[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sq
rt[1 + 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*
Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*
x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(4*5^(3/4)
*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rubi [A] time = 0.376027, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2069, 6719, 1673, 1678, 1197, 1103, 1195, 1247, 636}

$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}+\frac{3\left(3-\sqrt{5}\right)\left(\left(\frac{1}{x}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```


```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((13 - 9*(1 +
 x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + (9*(5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1))*x^2)/(10*(Sqrt[5] + (1 + x^
(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqr
t[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2
*EllipticE[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sq
rt[1 + 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*
Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*
x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(4*5^(3/4)
*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```


Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^{3/2}}\,dx = -\left(16\,\mathrm{Subst}\left(\int \frac{1}{(4-4x)^2\left(\frac{1280-512x^2+256x^4}{(4-4x)^4}\right)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\mathrm{Subst}\left(\int \frac{(4-4x)^4}{\left(1280-512x^2+256x^4\right)^{3/2}}\,dx, x, 1+\frac{1}{x}\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\mathrm{Subst}\left(\int \frac{x\left(-1024-1024x^2\right)}{\left(1280-512x^2+256x^4\right)^{3/2}}\,dx, x, 1+\frac{1}{x}\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\mathrm{Subst}\left(\int \ldots\right.}{1342177280\sqrt{1+4x+4x^2 \ldots}}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(9\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+\ldots}\right.}{\ldots}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} + \frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^{\ldots}\right.}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{1+4x\ldots}}$$

Mathematica [C] time = 4.31841, size = 602, normalized size = 1.64

$$\frac{36x^3-16x^2+\frac{(6-3i)\sqrt{-\frac{2}{5}+\frac{4i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-2x+\sqrt{-1-2i}-i\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1-2i}+i\right)}}\left(2ix^2+2x+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\left(2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1+2i}-i\right)}{\sqrt{-1+2i}\left(2x+\sqrt{-1-2i}+i\right)}}}{\sqrt{2}}\right)|\frac{1}{2}\left(5-\sqrt{5}\right)\right)}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(2ix^2+2x+1\right)}{\left(2x+\sqrt{-1-2i}+i\right)^2}}} - \frac{9i\sqrt{-\frac{2}{5}+\frac{4i}{5}}\ldots}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] (19 + 42*x - 16*x^2 + 36*x^3 + (9*(-I + Sqrt[-1 - 2*I] - 2*x)*(-I - Sqrt[-1
 + 2*I] + 2*x)*(-I + Sqrt[-1 + 2*I] + 2*x))/2 - ((9*I)*Sqrt[-2/5 + (4*I)/5]
*(-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(2*I + Sqrt[-1 - 2*I] + Sqrt[-1 +
 2*I])*((I + Sqrt[-1 - 2*I])/2 + x)^2*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1
 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2
*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I
])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2]*EllipticE[ArcSin[Sq
rt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(S
qrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/((-1
```

```
 + I) + Sqrt[-1 - 2*I]) + ((6 - 3*I)*Sqrt[-2/5 + (4*I)/5]*Sqrt[((2*I + Sqrt
[-1 - 2*I] - Sqrt[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1
- 2*I] + Sqrt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)
*EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[
-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5
 - Sqrt[5])/2])/Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I
)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2])/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] time = 0.016, size = 2564, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x)
```

```
[Out] -8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/5*(-RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*((Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4
)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4
*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2
))-9/5*((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))+(-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*((RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z
^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*
(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2
)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4
)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)^2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
```

```
=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*EllipticF(((RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))
)^(1/2))+(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=3))*EllipticE(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z
^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))/(RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))))/((x-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{4\,x^4+4\,x^2+4\,x+1}}{16\,x^8+32\,x^6+32\,x^5+24\,x^4+32\,x^3+24\,x^2+8\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^4 + 4*x^2 + 4*x + 1)/(16*x^8 + 32*x^6 + 32*x^5 + 24*x^4 +
 32*x^3 + 24*x^2 + 8*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4x^4+4x^2+4x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

```
[Out] Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4\,x^4 + 4\,x^2 + 4\,x + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

3.800 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[Out] -((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt
[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (5
17 + 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x
^4])
```

Rubi [A] time = 0.325988, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2069, 12, 6719, 1103}

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] -((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt
[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (5
17 + 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x
^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{2\sqrt{2}(24-32x)^2\sqrt{\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\sqrt{\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2117632-24903\cdots}}\right.}{\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

Mathematica [C] time = 0.355084, size = 1148, normalized size = 9.11

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
& , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 +
 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]], ((Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
, 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]*(x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]
))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24
*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1
^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2
- 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
 , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#
```

```
1^4 & , 2, 0])^2*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 24*x + 8
*x^2 - 15*x^3 + 8*x^4]*(-Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
0] + Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 4, 0]))
```

Maple [C] time = 0.76, size = 1180, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x)
```

```
[Out] 1/2*(-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)+RootOf(8*_Z^4-15*_Z^3+8
*_Z^2+24*_Z+8,index=1))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-Roo
tOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2
+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*
_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z
+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^2*((R
ootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))/(RootOf(
8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8
,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)*((RootO
f(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z
+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,ind
ex=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)/(RootOf(8*_
Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,in
dex=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^
3+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8
,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8
,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)
-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootO
f(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-Roo
tOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8,index=4)+RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(RootOf(8*_
Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,in
dex=3))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^
3+8*_Z^2+24*_Z+8,index=4)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.801 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+51\right.}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x}}$$

```
[Out] -((172 - 7*(3 + 4/x)^2)*x^2)/(208*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
+ ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x
^2)/(322608*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^
4]) - (2455*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/
4)*x)], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 -
15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(
517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Ellipt
icF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(2496*51
7^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rubi [A] time = 0.52758, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2069, 12, 6719, 1673, 1678, 1197, 1103, 1195, 1247, 636}

$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+51\right.}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -((172 - 7*(3 + 4/x)^2)*x^2)/(208*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
+ ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x
^2)/(322608*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^
4]) - (2455*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/
4)*x)], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 -
15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(
517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Ellipt
icF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(2496*51
7^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
```

```
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{3/2}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{16\sqrt{2}(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\,dx, x, \frac{3}{4} \cdots\right.\right.$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\,dx, x, \frac{3}{4}+\frac{1}{x} \cdots\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{\cdots}{(2117632\cdots}\right.}{8\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x(-\cdots}{(2117632\cdots}\right.}{8\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1\cdots}\right.}{454\cdots}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}} \cdots$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}} \cdots$$

Mathematica [C] time = 6.04897, size = 6019, normalized size = 13.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.017, size = 5421, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}}{64\,x^8 - 240\,x^7 + 353\,x^6 + 144\,x^5 - 528\,x^4 + 144\,x^3 + 704\,x^2 + 384\,x + 64}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(64*x^8 - 240*x^7 + 353*x^
6 + 144*x^5 - 528*x^4 + 144*x^3 + 704*x^2 + 384*x + 64), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8x^4 - 15x^3 + 8x^2 + 24x + 8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
```

```
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

3.802 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=577

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{543262997\left(\left(\frac{4}{x}+3\right)^4 \cdots\right.}{39028470624\left(\left(\frac{4}{x}+3\right)^2+ \cdots\right.}$$

```
[Out] -((124415 - 6308*(3 + 4/x)^2)*x^2)/(97344*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
8*x^4]) - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 - 38*(3 + 4/x)^2 + (3
+ 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((18932921731 - 108652
5994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(78056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4]) + ((11921698 - 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517
- 38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) +
(543262997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(39028470624
*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (5432
62997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 1
5*x^3 + 8*x^4]) + ((4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x
)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034
])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rubi [A] time = 0.688193, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2069, 12, 6719, 1673, 1678, 1197, 1103, 1195, 1663, 1660, 636}

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{543262997\left(\left(\frac{4}{x}+3\right)^4 \cdots\right.}{39028470624\left(\left(\frac{4}{x}+3\right)^2+ \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] -((124415 - 6308*(3 + 4/x)^2)*x^2)/(97344*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
8*x^4]) - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 - 38*(3 + 4/x)^2 + (3
+ 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((18932921731 - 108652
5994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(78056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4]) + ((11921698 - 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517
- 38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) +
(543262997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(39028470624
*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (5432
62997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 1
5*x^3 + 8*x^4]) + ((4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x
)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034
```

```
])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{128\sqrt{2}(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\,dx, x, \frac{3}{4}\right.\right.$$

$$= -\left(\left(4\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{(2117632\ldots}\right.}{64\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x(-11741\ldots}{}\right.}{64\sqrt{8+24x+8x^2-\ldots}}$$

$$= \frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{\ldots}\right.}{}$$

$$= -\frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{(1893\ldots}{78\ldots}$$

$$= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(64489-1399\left(3\ldots\right.\right.}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\ldots}$$

$$= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(64489-1399\left(3\ldots\right.\right.}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\ldots}$$

Mathematica [C] time = 6.05348, size = 6084, normalized size = 10.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.018, size = 5477, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{8x^4 - 15x^3 + 8x^2 + 24x + 8}}{512x^{12} - 2880x^{11} + 6936x^{10} - 4527x^9 - 8808x^8 + 16776x^7 + 5528x^6 - 17856x^5 - 384x^4 + 20160x^3 + 153}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(512*x^12 - 2880*x^11 + 69
36*x^10 - 4527*x^9 - 8808*x^8 + 16776*x^7 + 5528*x^6 - 17856*x^5 - 384*x^4
+ 20160*x^3 + 15360*x^2 + 4608*x + 512), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
```

```
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

3.803 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

```
[Out] -(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2
]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)]
, (613 + 91*Sqrt[613])/1226])/(12*613^(1/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3
+ 3*x^4])
```

Rubi [A] time = 0.260063, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2069, 12, 6719, 1096}

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] -(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2
]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)]
, (613 + 91*Sqrt[613])/1226])/(12*613^(1/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3
+ 3*x^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\left(1296\,\text{Subst}\left(\int \frac{1}{3(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\left(432\,\text{Subst}\left(\int \frac{1}{(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{\sqrt{794448-8491\cdots}}\right.}{\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

Mathematica [C] time = 0.122162, size = 826, normalized size = 6.35

$$-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}}\right)|\frac{(\text{Root}[3\#1^4\cdots}{(\text{Root}[3\#1^4\cdots}\right.}{\sqrt{\left(3x^4+15x^3-44x^2-6x+9\right)(\text{Ro}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 &
, 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x -
 Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 -
 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*
Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(x - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])])/Sqrt[(9 - 6*x - 44*x^2 +
15*x^3 + 3*x^4)*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2
```

```
 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 4, 0])]
```

Maple [C] time = 0.339, size = 1182, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x)
```

```
[Out] 2/3*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-4
4*_Z^2-6*_Z+9,index=1))*((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/
(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z
^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(-Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*
_Z+9,index=2)))^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^2*(
-(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(-RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(1/2)*(-(
x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(1/2)/(Root
Of(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_
Z+9,index=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+
15*_Z^3-44*_Z^2-6*_Z+9,index=1))*3^(1/2)/((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootO
f(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=4)))^(1/2)*EllipticF(((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,
index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*
_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index
=2))*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=2)))^(1/2),((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,inde
x=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(-RootOf(3*_Z^4+15*_Z^3
-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_
Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

3.804 $\int \frac{1}{\left(9-6x-44x^2+15x^3+3x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2\right.}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x^4+}}$$

```
[Out] -((176 - 23*(1 - 6/x)^2)*x^2)/(51759*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4
]) + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(31728267*Sqrt[9 - 6*x - 44
*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)*(1 -
 6/x)*x^2)/(31728267*(Sqrt[613] + (6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15
*x^3 + 3*x^4]) + (3722*Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[61
3] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(
6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sqrt[9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*Sqrt[(613 - 182
*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6
 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[
613])/1226])/(207036*613^(3/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rubi [A] time = 0.468024, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2069, 12, 6719, 1673, 1678, 1183, 1096, 1182, 1247, 636}

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2\right.}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] -((176 - 23*(1 - 6/x)^2)*x^2)/(51759*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4
]) + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(31728267*Sqrt[9 - 6*x - 44
*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)*(1 -
 6/x)*x^2)/(31728267*(Sqrt[613] + (6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15
*x^3 + 3*x^4]) + (3722*Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[61
3] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(
6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sqrt[9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*Sqrt[(613 - 182
*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6
 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[
613])/1226])/(207036*613^(3/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
```

```
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c
/a, 0] && LtQ[b/a, 0]
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 -
4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(9-6x-44x^2+15x^3+3x^4\right)^{3/2}}\,dx = -\left(1296\,\text{Subst}\left(\int \frac{1}{27(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right.\right.$$

$$= -\left(48\,\text{Subst}\left(\int \frac{1}{(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{(794448}\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{(794448}\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2}\right.}{}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x}{31728267\sqrt{9-6x-44x^2+15x^3+}}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x}{31728267\sqrt{9-6x-44x^2+15x^3+}}$$

Mathematica [C] time = 6.04799, size = 5428, normalized size = 12.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.016, size = 5427, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^4+15\,x^3-44\,x^2-6\,x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}{9\,x^8+90\,x^7-39\,x^6-1356\,x^5+1810\,x^4+798\,x^3-756\,x^2-108\,x+81},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)/(9*x^8 + 90*x^7 - 39*x^6 -
 1356*x^5 + 1810*x^4 + 798*x^3 - 756*x^2 - 108*x + 81), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^4+15x^3-44x^2-6x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
```

```
[Out] Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^4+15\,x^3-44\,x^2-6\,x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

3.805 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$

Optimal. Leaf size=56

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt
[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rubi [A] time = 0.210606, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {6742, 36, 29, 31, 105, 53, 619, 216, 93, 207}

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt
[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx &= \int \left(-4+\frac{12}{x}+\frac{9}{x(1+x)}+\frac{12\sqrt{3-x}}{x\sqrt{1+x}}\right)dx\\
&= -4x+12\log(x)+9\int \frac{1}{x(1+x)}\,dx+12\int \frac{\sqrt{3-x}}{x\sqrt{1+x}}\,dx\\
&= -4x+12\log(x)+9\int \frac{1}{x}\,dx-9\int \frac{1}{1+x}\,dx-12\int \frac{1}{\sqrt{3-x}\sqrt{1+x}}\,dx+36\int \frac{1}{\sqrt{3-x}x\sqrt{1}}\\
&= -4x+21\log(x)-9\log(1+x)-12\int \frac{1}{\sqrt{3+2x-x^2}}\,dx+72\,\mathrm{Subst}\left(\int \frac{1}{-1+3x^2}\,dx,x,\right.\\
&= -4x-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)+3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,\right.\\
&= -4x+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.102528, size = 57, normalized size = 1.02

$$-4x+21\log(x)-9\log(x+1)+24\sin^{-1}\left(\frac{\sqrt{3-x}}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{1-\frac{x}{3}}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 24*ArcSin[Sqrt[3 - x]/2] - 24*Sqrt[3]*ArcTanh[Sqrt[1 - x/3]/Sqrt[1 +
 x]] + 21*Log[x] - 9*Log[1 + x]
```

Maple [A] time = 0.019, size = 76, normalized size = 1.4

$$-4x+21\ln(x)+12\frac{\sqrt{3-x}\sqrt{1+x}}{\sqrt{-x^2+2x+3}}\left(-\arcsin(-1/2+x/2)-\sqrt{3}\mathrm{Artanh}\left(1/3\frac{(3+x)\sqrt{3}}{\sqrt{-x^2+2x+3}}\right)\right)-9\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x)
```

```
[Out] -4*x+21*ln(x)+12*(1+x)^(1/2)*(3-x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-arcsin(-1/2+1
/2*x)-3^(1/2)*arctanh(1/3*(3+x)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-9*ln(1+x)
```

Maxima [A] time = 1.60881, size = 77, normalized size = 1.38

$$-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right)-4x+12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*x
 + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

Fricas [A] time = 1.84008, size = 236, normalized size = 4.21

$$6\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right)-4x+12\arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2x-3}\right)-9\log(x+1)+21\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 - 6*x - 9)/x
^2) - 4*x + 12*arctan(sqrt(x + 1)*(x - 1)*sqrt(-x + 3)/(x^2 - 2*x - 3)) - 9
*log(x + 1) + 21*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2\sqrt{3-x}\sqrt{x+1}+3\right)^2}{x(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.806 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.158976, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6742, 277, 215, 1591, 190, 43, 195}

$$\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}+\frac{x^2}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)+\int \left(-1+\sqrt{1+x^2}\right)dx-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\
&= -\frac{1}{x}-x+\int \sqrt{1+x^2}\,dx-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}-x+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx-\int \frac{1}{\sqrt{1+x^2}}\,dx+\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\right.\\
&= -\frac{1}{x}-x+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0570924, size = 65, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-x-x^{-1}-\frac{x}{2}\sqrt{x^2+1}-\frac{\operatorname{Arcsinh}(x)}{2}+\sqrt{x^2+1}-\operatorname{Artanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{1}{x}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -x-1/x-1/2*x*(x^2+1)^(1/2)-1/2*arcsinh(x)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(
1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2x - 5\arctan\left(\frac{1}{2}x\right) + \int \frac{x^6 + x^5 - x^4}{3x^4 + 16x^2 + (x^4 + 8x^2 + 16)\sqrt{x^2 + 1} + 16}\,dx + \log\left(x^2 + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="maxima")
```

```
[Out] 2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
+ 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

Fricas [A] time = 1.73818, size = 225, normalized size = 3.46

$$-\frac{2x^2 + 2x\log(x) + 2x\log\left(-x + \sqrt{x^2 + 1} + 1\right) - x\log\left(-x + \sqrt{x^2 + 1}\right) - 2x\log\left(-x + \sqrt{x^2 + 1} - 1\right) - \left(x^2 + 2x + 2\right)\sqrt{x^2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sqr
t(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2 +
1) - 2*x + 2)/x
```

Sympy [A] time = 3.75104, size = 63, normalized size = 0.97

$$\frac{x\sqrt{x^2 + 1}}{2} - x + \frac{x}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1} - \log\left(\sqrt{x^2 + 1} + 1\right) - \frac{\operatorname{asinh}(x)}{2} - \frac{1}{x} + \frac{1}{x\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/((x**2+1)**(1/2)+1),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
+ 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

Giac [A] time = 1.15208, size = 120, normalized size = 1.85

$$\frac{1}{2}\sqrt{x^2 + 1}(x + 2) - x - \frac{2}{\left(x - \sqrt{x^2 + 1}\right)^2 - 1} - \frac{1}{x} + \frac{1}{2}\log\left(-x + \sqrt{x^2 + 1}\right) - \log\left(|x|\right) - \log\left(\left|-x + \sqrt{x^2 + 1} + 1\right|\right) + \log\left(\left|-x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*l
og(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + l
og(abs(-x + sqrt(x^2 + 1) - 1))
```

3.807 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{12}\left(2x^3+6x^2+\left(-2x^2-3x+4\right)\sqrt{x^2+1}-6\log\left(\sqrt{x^2+1}+1\right)-3\sinh^{-1}(x)\right)$$

```
[Out] (6*x^2 + 2*x^3 + (4 - 3*x - 2*x^2)*Sqrt[1 + x^2] - 3*ArcSinh[x] - 6*Log[1 +
 Sqrt[1 + x^2]])/12
```

Rubi [A] time = 0.214881, antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6742, 2117, 893, 195, 215, 261}

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}-\frac{1}{4}\sinh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x + S
qrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sqrt
[1 + x^2]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+x+\sqrt{1+x^2}}+\frac{x}{1+x+\sqrt{1+x^2}}+\frac{x^2}{1+x+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+x+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+x+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+x+\sqrt{1+x^2}}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{2-2x+x^2}{(1-x)^2x}\,dx,x,1+x+\sqrt{1+x^2}\right)\right)+\int \left(\frac{1}{2}+\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}\right)dx+\int \left(\frac{x}{2}+\right.\\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{2}\int \sqrt{1+x^2}\,dx-\frac{1}{2}\int x\sqrt{1+x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{(-1+x)^2}+\frac{2}{x}\right)\right.\\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)-\log\left(1\right.\\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)}-\frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}\log\left(x+\sqrt{1\right.
\end{aligned}$$

Mathematica [A] time = 0.143002, size = 88, normalized size = 1.66

$$\frac{1}{12}\left(2x^3+6x^2-2\left(x^2+1\right)^{3/2}+6\left(\frac{1}{\sqrt{x^2+1}+x}+\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{x^2+1}+x+1\right)\right)-3\left(\sqrt{x^2+1}x+\sinh^{\text{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] (6*x + 6*x^2 + 2*x^3 - 2*(1 + x^2)^(3/2) - 3*(x*Sqrt[1 + x^2] + ArcSinh[x])
 + 6*((x + Sqrt[1 + x^2])^(-1) + Log[x + Sqrt[1 + x^2]] - 2*Log[1 + x + Sqr
t[1 + x^2]]))/12
```

Maple [A] time = 0.005, size = 58, normalized size = 1.1

$$\frac{x^2}{2}-\frac{\ln(x)}{2}+\frac{x^3}{6}-\frac{x}{4}\sqrt{x^2+1}-\frac{\text{Arcsinh}(x)}{4}+\frac{1}{2}\sqrt{x^2+1}-\frac{1}{2}\text{Artanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\frac{1}{6}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*x*(x^2+1)^(1/2)-1/4*arcsinh(x)+1/2*(x^2+1)^(1
/2)-1/2*arctanh(1/(x^2+1)^(1/2))-1/6*(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^2 - \frac{3}{56}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+3)\right) + \frac{1}{4}x + \int \frac{x^4+x^3-x^2}{4x^5+12x^4+19x^3+19x^2+\left(4x^4+12x^3+17x^2+12x+4\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((x
^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*x^
2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)
```

Fricas [A] time = 1.71586, size = 228, normalized size = 4.3

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{12}\left(2x^2+3x-4\right)\sqrt{x^2+1} - \frac{1}{2}\log(x) - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}+1\right) + \frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right) + \frac{1}{2}\log\left(-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/2
*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x + s
qrt(x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

Giac [A] time = 1.12376, size = 108, normalized size = 2.04

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{12}((2x+3)x-4)\sqrt{x^2+1} + \frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right) - \frac{1}{2}\log(|x|) - \frac{1}{2}\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \frac{1}{2}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sqr
t(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*
log(abs(-x + sqrt(x^2 + 1) - 1))
```

3.808 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} dx$

Optimal. Leaf size=14

$$2\sqrt{x-1}+2\log(x)$$

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Rubi [A] time = 0.117406, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6688}

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} dx = \int \left(\frac{1}{\sqrt{-1+x}}+\frac{2}{x}\right) dx$$
$$= 2\sqrt{-1+x}+2\log(x)$$

Mathematica [A] time = 0.0057806, size = 14, normalized size = 1.

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$2\ln(x)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2*(x-1)^(1/2))/x/(x-1)^(1/2),x)
```

```
[Out] 2*ln(x)+2*(x-1)^(1/2)
```

Maxima [A] time = 1.71893, size = 16, normalized size = 1.14

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Fricas [A] time = 1.74099, size = 35, normalized size = 2.5

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Sympy [A] time = 0.138341, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Giac [A] time = 1.18945, size = 16, normalized size = 1.14

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

3.809 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx$

Optimal. Leaf size=61

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Rubi [A] time = 0.165853, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6741, 6742}

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx &= 6\,\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^2 dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^2 dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(a^2x^5+2acx^8+2abx^9+c^2x^{11}+2bcx^{12}+b^2x^{13}\right) dx, x, \sqrt[6]{x}\right)\\ &= a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}\end{aligned}$$

Mathematica [A] time = 0.0460942, size = 61, normalized size = 1.

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Maple [A] time = 0.003, size = 46, normalized size = 0.8

$$\frac{c^2x^2}{2} + 2\,c\left(\frac{6\,b}{13}x^{\frac{13}{6}} + 2/3\,ax^{3/2}\right) + xa^2 + \frac{3\,b^2}{7}x^{\frac{7}{3}} + \frac{6\,ab}{5}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^2,x)
```

```
[Out] 1/2*c^2*x^2+2*c*(6/13*b*x^(13/6)+2/3*a*x^(3/2))+x*a^2+3/7*b^2*x^(7/3)+6/5*a
*b*x^(5/3)
```

Maxima [A] time = 1.11709, size = 61, normalized size = 1.

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + a^2x + \frac{2}{15}\left(9\,bx^{\frac{5}{3}} + 10\,cx^{\frac{3}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(5
/3) + 10*c*x^(3/2))*a
```

Fricas [A] time = 1.70177, size = 130, normalized size = 2.13

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

Sympy [A] time = 1.90677, size = 60, normalized size = 0.98

$$a^2x + \frac{6abx^{\frac{5}{3}}}{5} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{3b^2x^{\frac{7}{3}}}{7} + \frac{12bcx^{\frac{13}{6}}}{13} + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

```
[Out] a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*x
**(13/6)/13 + c**2*x**2/2
```

Giac [A] time = 1.14186, size = 58, normalized size = 0.95

$$\frac{3}{7}b^2x^{\frac{7}{3}} + \frac{12}{13}bcx^{\frac{13}{6}} + \frac{1}{2}c^2x^2 + \frac{6}{5}abx^{\frac{5}{3}} + \frac{4}{3}acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

3.810 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx$

Optimal. Leaf size=114

$$\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+a^3x+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rubi [A] time = 0.192508, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6741, 6742}

$$\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+a^3x+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx &= 6\,\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^3\,dx,x,\sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^3\,dx,x,\sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(a^3x^5+3a^2cx^8+3a^2bx^9+3ac^2x^{11}+6abcx^{12}+3ab^2x^{13}+c^3x^{14}+3bc^2x^{15}+3b^2\right.\right.\\ &= a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^{8/3}+\frac{18}{17}b^2cx\end{aligned}$$

Mathematica [A] time = 0.076006, size = 114, normalized size = 1.

$$\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+a^3x+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Maple [A] time = 0.003, size = 86, normalized size = 0.8

$$\frac{2c^3}{5}x^{\frac{5}{2}} + 3c^2\left(3/8\,bx^{8/3} + 1/2\,ax^2\right) + 3c\left(\frac{6b^2}{17}x^{\frac{17}{6}} + \frac{12ab}{13}x^{\frac{13}{6}} + 2/3\,a^2x^{3/2}\right) + a^3x + \frac{b^3x^3}{3} + \frac{9a^2b}{5}x^{\frac{5}{3}} + \frac{9ab^2}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^3,x)
```

```
[Out] 2/5*c^3*x^(5/2)+3*c^2*(3/8*b*x^(8/3)+1/2*a*x^2)+3*c*(6/17*b^2*x^(17/6)+12/1
3*a*b*x^(13/6)+2/3*a^2*x^(3/2))+a^3*x+1/3*b^3*x^3+9/5*a^2*b*x^(5/3)+9/7*a*b
^2*x^(7/3)
```

Maxima [A] time = 1.10472, size = 115, normalized size = 1.01

$$\frac{1}{3}b^3x^3 + \frac{18}{17}b^2cx^{\frac{17}{6}} + \frac{9}{8}bc^2x^{\frac{8}{3}} + \frac{2}{5}c^3x^{\frac{5}{2}} + a^3x + \frac{1}{5}\left(9\,bx^{\frac{5}{3}} + 10\,cx^{\frac{3}{2}}\right)a^2 + \frac{3}{182}\left(78\,b^2x^{\frac{7}{3}} + 168\,bcx^{\frac{13}{6}} + 91\,c^2x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 168*
b*c*x^(13/6) + 91*c^2*x^2)*a
```

Fricas [A] time = 1.70376, size = 243, normalized size = 2.13

$$\frac{1}{3}b^3x^3 + \frac{18}{17}b^2cx^{\frac{17}{6}} + \frac{9}{7}ab^2x^{\frac{7}{3}} + \frac{36}{13}abcx^{\frac{13}{6}} + \frac{3}{2}ac^2x^2 + a^3x + \frac{9}{40}\left(5\,bc^2x^2 + 8\,a^2bx\right)x^{\frac{2}{3}} + \frac{2}{5}\left(c^3x^2 + 5\,a^2cx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/
6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5*(
c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

Sympy [A] time = 2.75444, size = 116, normalized size = 1.02

$$a^3x + \frac{9a^2bx^{\frac{5}{3}}}{5} + 2a^2cx^{\frac{3}{2}} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{36abcx^{\frac{13}{6}}}{13} + \frac{3ac^2x^2}{2} + \frac{b^3x^3}{3} + \frac{18b^2cx^{\frac{17}{6}}}{17} + \frac{9bc^2x^{\frac{8}{3}}}{8} + \frac{2c^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 36
*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 18*b**2*c*x**(17/6)/1
7 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

Giac [A] time = 1.1156, size = 113, normalized size = 0.99

$$\frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^2x^2+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+2\,a^2cx^{\frac{3}{2}}+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5/3
) + 2*a^2*c*x^(3/2) + a^3*x
```

3.811 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rubi [A] time = 0.0568747, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx &= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}x}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b}\\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.0609258, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqr
t[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))
]*x^2)
```

Maple [B] time = 0.016, size = 102, normalized size = 1.8

$$\frac{1}{bx^2}\sqrt{ax^2-bx^2+b}\left(\ln\left(\sqrt{-b+a}x+\sqrt{ax^2-bx^2+b}\right)bx+\sqrt{ax^2-bx^2+b}\sqrt{-b+a}\right)\frac{1}{\sqrt{\frac{ax^2-bx^2+b}{x^2}}}\frac{1}{\sqrt{-b+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln((-b+a)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-
b*x^2+b)^(1/2)*(-b+a)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(-b+a)^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70727, size = 390, normalized size = 6.72

$$\left[\frac{\sqrt{a-b}b\log\left(-2(a-b)x^2-2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\left(ab-b^2\right)}, \frac{\sqrt{-a+b}b\arctan\left(-\frac{\sqrt{-a+b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}}{(a-b)x^2+b}\right)}{ab-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

Sympy [A] time = 2.44019, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b = 0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(a - b + b/x^2)*x^3), x)
```

3.812 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rubi [A] time = 0.137406, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1978, 514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 1978

```
Int[(Pq_)*(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*Pq*Expand
ToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && PolyQ[Pq, x] && BinomialQ[u, x]
 &&  !BinomialMatchQ[u, x]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx &= \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx \\
&= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}x}\,dx \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.0074315, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqr
t[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))
]*x^2)
```

Maple [B] time = 0.008, size = 102, normalized size = 1.8

$$\frac{1}{bx^2}\sqrt{ax^2-bx^2+b}\left(\ln\left(\sqrt{-b+a}\,x+\sqrt{ax^2-bx^2+b}\right)bx+\sqrt{ax^2-bx^2+b}\sqrt{-b+a}\right)\frac{1}{\sqrt{\frac{ax^2-bx^2+b}{x^2}}}\frac{1}{\sqrt{-b+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln((-b+a)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-
b*x^2+b)^(1/2)*(-b+a)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(-b+a)^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81721, size = 390, normalized size = 6.72

$$\left[\frac{\sqrt{a-b}b\log\left(-2(a-b)x^2-2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\left(ab-b^2\right)},\frac{\sqrt{-a+b}b\arctan\left(-\frac{\sqrt{-a+b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}}{(a-b)x^2+b}\right)-}{ab-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

Sympy [A] time = 7.91614, size = 70, normalized size = 1.21

$$-\frac{\begin{cases}-\frac{1}{\sqrt{ax^2}} & \text{for } b=0\\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise}\end{cases}}{2}-\frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{b\left(\frac{1}{x^2}-1\right)+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(b*(1/x^2 - 1) + a)*x^3), x)
```

3.813 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}} \, dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/S
qrt[5]]/Sqrt[5]
```

Rubi [A] time = 0.0307952, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1010, 377, 203, 444, 63, 207}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/S
qrt[5]]/Sqrt[5]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx &= \int \frac{1}{(4+x^2)\sqrt{9+x^2}}\,dx + \int \frac{x}{(4+x^2)\sqrt{9+x^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(4+x)\sqrt{9+x}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{4+5x^2}\,dx, x, \frac{x}{\sqrt{9+x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} + \text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{9+x^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [C] time = 0.0455921, size = 64, normalized size = 1.21

$$-\frac{(2+i)\tanh^{-1}\left(\frac{9-2ix}{\sqrt{5}\sqrt{x^2+9}}\right) + (2-i)\tanh^{-1}\left(\frac{9+2ix}{\sqrt{5}\sqrt{x^2+9}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] -((2 + I)*ArcTanh[(9 - (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])] + (2 - I)*ArcTanh[
(9 + (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])])/(4*Sqrt[5])
```

Maple [A] time = 0.013, size = 39, normalized size = 0.7

$$\frac{\sqrt{5}}{10}\arctan\left(\frac{x\sqrt{5}}{2}\frac{1}{\sqrt{x^2+9}}\right) - \frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{\sqrt{5}}{5}\sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+4)/(x^2+9)^(1/2),x)
```

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+9}\left(x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

Fricas [B] time = 1.7695, size = 574, normalized size = 10.83

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}\left(x+\sqrt{5}\right)+\sqrt{5}x+9}+\frac{1}{2}x+\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x + sqrt(5)) + sqr
t(5)*x + 9) + 1/2*x + 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/5*sqrt(5)*arctan
(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x - sqrt(5)) - sqrt(5)*x + 9) + 1/2*
x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log(50*x^2 - 50*sqrt(x^
2 + 9)*(x + sqrt(5)) + 50*sqrt(5)*x + 450) - 1/10*sqrt(5)*log(50*x^2 - 50*s
qrt(x^2 + 9)*(x - sqrt(5)) - 50*sqrt(5)*x + 450)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+1}{\left(x^2+4\right)\sqrt{x^2+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

Giac [B] time = 1.25997, size = 528, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/40*(9*sqrt(5)*arctan(2/(sqrt(5) + 3)) + 9*sqrt(5)*arctan(2/(sqrt(5) - 3))
 + 49*sqrt(5)*log(3/2*sqrt(5) + 9/2) - 49*sqrt(5)*log(-3/2*sqrt(5) + 9/2) -
 15*arctan(2/(sqrt(5) + 3)) + 15*arctan(2/(sqrt(5) - 3)) - 105*log(3/2*sqrt
(5) + 9/2) - 105*log(-3/2*sqrt(5) + 9/2))*sgn(x) - 1/10*(7*sqrt(5) + 15)*lo
g((sqrt(9/x^2 + 1) - 3/x)^2 + 1/2*(3*sqrt(5)*sgn(x) + 7*sgn(x))/sgn(x))*sgn
(x)/(7*abs(sgn(x))*sgn(x) + 3*sqrt(5)) + 1/10*(7*sqrt(5) - 15)*log((sqrt(9/
x^2 + 1) - 3/x)^2 - 1/2*(3*sqrt(5)*sgn(x) - 7*sgn(x))/sgn(x))*sgn(x)/(7*abs
(sgn(x))*sgn(x) - 3*sqrt(5)) - 1/20*(5*(sqrt(5) + 3)*abs(sgn(x)) + 3*(3*sqr
t(5) + 5)*sgn(x))*arctan(2*sqrt(1/2)*(sqrt(9/x^2 + 1) - 3/x)/sqrt((3*sqrt(5
)*sgn(x) + 7*sgn(x))/sgn(x)))/(7*abs(sgn(x))*sgn(x) + 3*sqrt(5)) + 1/20*(5*
```

```
(sqrt(5) - 3)*abs(sgn(x)) + 3*(3*sqrt(5) - 5)*sgn(x))*arctan(2*sqrt(1/2)*(s
qrt(9/x^2 + 1) - 3/x)/sqrt(-(3*sqrt(5)*sgn(x) - 7*sgn(x))/sgn(x)))/(7*abs(s
gn(x))*sgn(x) - 3*sqrt(5))
```

3.814 $\int x \left(1+\sqrt{1-x^2}\right) \, dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3} \left(1-x^2\right)^{3/2}$$

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rubi [A] time = 0.0071066, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {14, 261}

$$\frac{x^2}{2}-\frac{1}{3} \left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x \left(1+\sqrt{1-x^2}\right) \, dx &= \int \left(x+x \sqrt{1-x^2}\right) \, dx\\
&= \frac{x^2}{2}+\int x \sqrt{1-x^2} \, dx\\
&= \frac{x^2}{2}-\frac{1}{3} \left(1-x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0115721, size = 23, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{3} \left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Maple [A] time = 0.002, size = 18, normalized size = 0.8

$$\frac{x^2}{2}-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Maxima [A] time = 1.12028, size = 23, normalized size = 1.

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 1.69514, size = 54, normalized size = 2.35

$$\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

Sympy [A] time = 0.181158, size = 27, normalized size = 1.17

$$\frac{x^2\sqrt{1-x^2}}{3}+\frac{x^2}{2}-\frac{\sqrt{1-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x**2+1)**(1/2)),x)
```

```
[Out] x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

Giac [A] time = 1.10405, size = 24, normalized size = 1.04

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

3.815 $\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rubi [A] time = 0.008017, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 261}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx &= \int\left(x+x\sqrt{1-x^2}\right)\,dx\\ &= \frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\ &= \frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0040071, size = 23, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Maple [A] time = 0.003, size = 26, normalized size = 1.1

$$\frac{x^2-1}{3}\sqrt{1-x}\sqrt{1+x}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x)
```

```
[Out] 1/3*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)+1/2*x^2
```

Maxima [A] time = 1.54099, size = 23, normalized size = 1.

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 1.63926, size = 68, normalized size = 2.96

$$\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [A] time = 71.2532, size = 105, normalized size = 4.57

$$-x+\frac{(x+1)^2}{2}-2\left(\left\{\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}\quad\text{for}\ x\geq-1\wedge x<1\right)+2\left(\left\{\frac{x\sqrt{1-x}\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

```
[Out] -x + (x + 1)**2/2 - 2*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)
*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 2*Piecewise((x*sqrt(1 - x)*sqrt(
x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2,
 (x >= -1) & (x < 1))) - 1
```

Giac [A] time = 1.156, size = 39, normalized size = 1.7

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}+\frac{1}{2}\,(x+1)^2-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/2*(x + 1)^2 - x - 1
```

3.816 $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rubi [A] time = 0.0154203, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {14, 80, 54, 215}

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)dx &= \int\left(x+\frac{x}{\sqrt{2+x}\sqrt{3+x}}\right)dx\\ &= \frac{x^2}{2}+\int\frac{x}{\sqrt{2+x}\sqrt{3+x}}dx\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-\frac{5}{2}\int\frac{1}{\sqrt{2+x}\sqrt{3+x}}dx\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\,\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}dx,x,\sqrt{2+x}\right)\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\sinh^{-1}\left(\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0117544, size = 33, normalized size = 1.

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Maple [B] time = 0.012, size = 58, normalized size = 1.8

$$-\frac{1}{2}\sqrt{2+x}\sqrt{3+x}\left(-2\sqrt{x^2+5x+6}+5\ln\left(5/2+x+\sqrt{x^2+5x+6}\right)\right)\frac{1}{\sqrt{x^2+5x+6}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x)
```

```
[Out] -1/2*(2+x)^(1/2)*(3+x)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(5/2+x+(x^2+5*x+6)^(
1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2
```

Maxima [A] time = 1.04535, size = 49, normalized size = 1.48

$$\frac{1}{2}x^2+\sqrt{x^2+5x+6}-\frac{5}{2}\log\left(2x+2\sqrt{x^2+5x+6}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)
```

Fricas [A] time = 1.7657, size = 111, normalized size = 3.36

$$\frac{1}{2}x^2+\sqrt{x+3}\sqrt{x+2}+\frac{5}{2}\log\left(2\sqrt{x+3}\sqrt{x+2}-2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*x
 - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\sqrt{x+2}\sqrt{x+3}+1\right)}{\sqrt{x+2}\sqrt{x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
```

```
[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)
```

Giac [A] time = 1.16425, size = 54, normalized size = 1.64

$$\frac{1}{2}(x+3)^2+\sqrt{x+3}\sqrt{x+2}-3x+5\log\left(\left|-\sqrt{x+3}+\sqrt{x+2}\right|\right)-9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(abs(-sqrt(x + 3) + sq
rt(x + 2))) - 9
```

3.817 $\int \frac{x-\sqrt{x^6}}{x(1-x^4)} \, dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rubi [A] time = 0.158017, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.0671234, size = 27, normalized size = 0.6

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

Maple [A] time = 0.005, size = 35, normalized size = 0.8

$$\frac{\ln(x-1)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{x^6}+\frac{\operatorname{Artanh}(x)}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/x/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

Maxima [A] time = 1.6714, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 1.65722, size = 15, normalized size = 0.33

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.096463, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
```

```
[Out] atan(x)
```

Giac [A] time = 1.09433, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\operatorname{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

3.818 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rubi [A] time = 0.0552958, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.0130317, size = 27, normalized size = 0.6

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

Maple [A] time = 0.003, size = 35, normalized size = 0.8

$$\frac{\ln(x-1)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{x^6}+\frac{\mathrm{Artanh}(x)}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(x^6)^(1/2)/x)/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

Maxima [A] time = 1.67937, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 1.65794, size = 15, normalized size = 0.33

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.094229, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

```
[Out] atan(x)
```

Giac [A] time = 1.12625, size = 42, normalized size = 0.93

$$\frac{1}{2}(\operatorname{sgn}(x)+1)\arctan(x)-\frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x+1|)+\frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

3.819 $\int \frac{x-\sqrt{x^6}}{x-x^5} \, dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rubi [A] time = 0.0981848, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx &= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx\\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.0128539, size = 27, normalized size = 0.6

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

Maple [A] time = 0.003, size = 35, normalized size = 0.8

$$\frac{\ln(x-1)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{x^6}+\frac{\mathrm{Artanh}(x)}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/(-x^5+x),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(1+x)+2*arctan(x))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

Maxima [A] time = 1.72333, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 1.82952, size = 15, normalized size = 0.33

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.094702, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

```
[Out] atan(x)
```

Giac [A] time = 1.12683, size = 42, normalized size = 0.93

$$\frac{1}{2}(\operatorname{sgn}(x)+1)\arctan(x)-\frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x+1|)+\frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

3.820 $\int \frac{x}{x+\sqrt{x^6}} \, dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rubi [A] time = 0.133012, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6729, 1584, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{x+\sqrt{x^6}}\,dx &= \int \frac{x\left(x-\sqrt{x^6}\right)}{x^2-x^6}\,dx\\
&= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx\\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.0341553, size = 27, normalized size = 0.6

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[x^6]),x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

Maple [A] time = 0.007, size = 27, normalized size = 0.6

$$\arctan\left(\sqrt{\frac{1}{x^3}\sqrt{x^6}}x\right)\frac{1}{\sqrt{\frac{1}{x^3}\sqrt{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+(x^6)^(1/2)),x)
```

```
[Out] 1/((x^6)^(1/2)/x^3)^(1/2)*arctan(((x^6)^(1/2)/x^3)^(1/2)*x)
```

Maxima [A] time = 1.70077, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 1.68473, size = 15, normalized size = 0.33

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.091828, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x**6)**(1/2)),x)
```

```
[Out] atan(x)
```

Giac [A] time = 1.11389, size = 16, normalized size = 0.36

$$\frac{\arctan\left(x\sqrt{\operatorname{sgn}(x)}\right)}{\sqrt{\operatorname{sgn}(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))
```

3.821 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rubi [A] time = 0.183135, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx &= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx\\
&= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx\\
&= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0603503, size = 49, normalized size = 0.94

$$
\frac{\left(x^{3/2}+\sqrt{x^3}\right)\tan^{-1}\left(\sqrt{x}\right)+\left(x^{3/2}-\sqrt{x^3}\right)\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

Maple [A] time = 0.006, size = 41, normalized size = 0.8

$$\operatorname{Artanh}\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{x^3}\left(\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+2\arctan\left(\sqrt{x}\right)\right)x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x)
```

```
[Out] arctanh(x^(1/2))+arctan(x^(1/2))+1/2*(x^3)^(1/2)*(ln(-1+x^(1/2))-ln(1+x^(1/
2))+2*arctan(x^(1/2)))/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\arctan\left(\sqrt{x}\right)-\int\frac{\sqrt{x}}{2\,(x+1)}\,dx+\int\frac{1}{4\left(\sqrt{x}+1\right)}\,dx+\int\frac{1}{4\left(\sqrt{x}-1\right)}\,dx+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(x
) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2*l
og(sqrt(x) - 1)
```

Fricas [A] time = 1.9677, size = 26, normalized size = 0.5

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{x}}{x^3-x}\,dx-\int-\frac{\sqrt{x^3}}{x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)
```

Giac [A] time = 1.10667, size = 8, normalized size = 0.15

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.822 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rubi [A] time = 0.125848, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx &= \int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx\\
&= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx\\
&= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx\\
&= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0274004, size = 49, normalized size = 0.94

$$
\frac{\left(x^{3/2}+\sqrt{x^3}\right)\tan^{-1}\left(\sqrt{x}\right)+\left(x^{3/2}-\sqrt{x^3}\right)\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

Maple [A] time = 0.007, size = 30, normalized size = 0.6

$$2\arctan\left(\sqrt{x}\sqrt{\frac{\sqrt{x^3}}{x^{3/2}}}\right)\frac{1}{\sqrt{\frac{\sqrt{x^3}}{x^{3/2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x^3)^(1/2)),x)
```

```
[Out] 2/((x^3)^(1/2)/x^(3/2))^(1/2)*arctan(x^(1/2)*((x^3)^(1/2)/x^(3/2))^(1/2))
```

Maxima [A] time = 1.71265, size = 8, normalized size = 0.15

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

Fricas [A] time = 1.99446, size = 26, normalized size = 0.5

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

Giac [A] time = 1.09506, size = 8, normalized size = 0.15

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.823 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/
2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 +
 x)^(3/2)
```

Rubi [A] time = 0.155333, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/
2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 +
 x)^(3/2)
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx,x,-1+x\right)\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx,x,-1+x\right)+\text{Subst}\left(\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx,x,-1+x\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{-1+x}\right)\right)+\frac{\sqrt{(-1+x)^3}\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx,x,-1+x\right)}{(-1+x)^{3/2}}\\
&= \frac{\left(2\sqrt{(-1+x)^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{-1+x}\right)+\\
&= \tan^{-1}\left(\sqrt{-1+x}\right)+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\\
&= \tan^{-1}\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\tan^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\,\text{t}}{(-1}
\end{aligned}
$$

Mathematica [A] time = 0.158107, size = 64, normalized size = 0.94

$$\left(\frac{\sqrt{(x-1)^3}}{(x-1)^{3/2}}+1\right)\tan^{-1}\left(\sqrt{x-1}\right)+\frac{\left((x-1)^{3/2}-\sqrt{(x-1)^3}\right)\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] (1 + Sqrt[(-1 + x)^3]/(-1 + x)^(3/2))*ArcTan[Sqrt[-1 + x]] + (((-1 + x)^(3/
2) - Sqrt[(-1 + x)^3])*ArcTanh[Sqrt[-1 + x]])/(-1 + x)^(3/2)
```

Maple [A] time = 0.01, size = 40, normalized size = 0.6

$$2\arctan\left(\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{3/2}}}\sqrt{x-1}\right)\frac{1}{\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{3/2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+((x-1)^3)^(1/2)),x)
```

```
[Out] 2/(((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*arctan(((((x-1)^3)^(1/2)/(x-1)^(3/2))^
(1/2)*(x-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\sqrt{x-1}-\int\frac{\sqrt{x-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)
```

Fricas [A] time = 1.9407, size = 31, normalized size = 0.46

$$2\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1} + \sqrt{(x-1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

Giac [A] time = 1.12214, size = 11, normalized size = 0.16

$$2\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

3.824 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.0152564, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {803}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && N
eQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g,
 0]
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx &= \frac{3}{5(4+5x)}-\int \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\, dx\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}\end{aligned}$$

Mathematica [A] time = 0.189383, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] time = 0.009, size = 32, normalized size = 1.

$$\frac{1}{5}\sqrt{-\left(x+\frac{4}{5}\right)^{2}+\frac{8x}{5}+\frac{41}{25}}\left(x+\frac{4}{5}\right)^{-1}+\frac{3}{20+25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

Maxima [A] time = 1.57145, size = 36, normalized size = 1.16

$$\frac{\sqrt{-x^2+1}}{5x+4}+\frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

Fricas [A] time = 1.67726, size = 65, normalized size = 2.1

$$\frac{25x+20\sqrt{-x^2+1}+32}{20(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{}{25x^2\sqrt{1-x^2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{4x+5}{\sqrt{-x^2+1}(5x+4)^2} - \frac{3}{(5x+4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(-(4*x + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2) - 3/(5*x + 4)^2, x)
```

3.825 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.287747, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {6742, 731, 725, 206, 807}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \int \left(-\frac{3}{(4+5x)^2}-\frac{5}{(4+5x)^2\sqrt{1-x^2}}-\frac{4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx$$
$$= \frac{3}{5(4+5x)}-4\int \frac{x}{(4+5x)^2\sqrt{1-x^2}}\,dx-5\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] time = 0.145681, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] time = 0.001, size = 32, normalized size = 1.

$$\frac{1}{5}\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}\left(x+\frac{4}{5}\right)^{-1}+\frac{3}{20+25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

Maxima [A] time = 1.27767, size = 34, normalized size = 1.1

$$\frac{5\sqrt{x+1}\sqrt{-x+1}+3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

Fricas [A] time = 1.74197, size = 65, normalized size = 2.1

$$\frac{25x+20\sqrt{-x^2+1}+32}{20(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{}{25x^2\sqrt{1-x^2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

Giac [B] time = 1.13998, size = 74, normalized size = 2.39

$$-\frac{1}{5}\,i\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)+\frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] -1/5*i*sgn(1/(5*x + 4)) + 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(
5*x + 4)) + 3/5/(5*x + 4)
```

3.826 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.140378, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3\left(1-x^2\right)}\,dx &= \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right)dx\\ &= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}\,dx-5\int \frac{1}{(4\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx-\frac{1}{2}\int \frac{}{\sqrt{}}\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}\end{aligned}$$

Mathematica [A] time = 0.0902564, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [B] time = 0.032, size = 81, normalized size = 2.6

$$\frac{3}{20+25x}+\frac{1}{18}\sqrt{-(x-1)^2-2x+2}+\frac{5}{9}\left(-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}\left(x+\frac{4}{5}\right)^{-1}+\frac{5x}{9}\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}-\frac{1}{2}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x)
```

```
[Out] 3/5/(4+5*x)+1/18*(-(x-1)^2-2*x+2)^(1/2)+5/9/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25
)^(3/2)+5/9*x*(-(x+4/5)^2+8/5*x+41/25)^(1/2)-1/2*(-(1+x)^2+2*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 + \sqrt{-x^2+1}(4x+5) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

Fricas [A] time = 1.62223, size = 65, normalized size = 2.1

$$\frac{25x + 20\sqrt{-x^2+1} + 32}{20(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 + 4x\sqrt{1-x^2} + 5\sqrt{1-x^2} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

Giac [B] time = 1.11442, size = 92, normalized size = 2.97

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - 4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 2\right)} + \frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.827 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.125123, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx &= \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right)dx\\
&= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}\,dx-5\int\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx-\frac{1}{2}\int\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}
\end{aligned}$$

Mathematica [A] time = 0.0581363, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [B] time = 0.002, size = 81, normalized size = 2.6

$$\frac{3}{20+25x}+\frac{1}{18}\sqrt{-(x-1)^2-2x+2}+\frac{5}{9}\left(-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}\left(x+\frac{4}{5}\right)^{-1}+\frac{5x}{9}\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}-\frac{1}{2}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x)
```

```
[Out] 3/5/(4+5*x)+1/18*(-(x-1)^2-2*x+2)^(1/2)+5/9/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25
)^(3/2)+5/9*x*(-(x+4/5)^2+8/5*x+41/25)^(1/2)-1/2*(-(1+x)^2+2*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 + 4\sqrt{-x^2+1}x + 5\sqrt{-x^2+1} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="max
ima")
```

```
[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)
```

Fricas [A] time = 1.61212, size = 65, normalized size = 2.1

$$\frac{25x + 20\sqrt{-x^2+1} + 32}{20(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fri
cas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 + 4x\sqrt{1-x^2} + 5\sqrt{1-x^2} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

Giac [B] time = 1.12394, size = 92, normalized size = 2.97

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - 4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 2\right)} + \frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="gia
c")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.828 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.654172, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 13, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.302, Rules used = {6742, 277, 216, 266, 50, 63, 206, 733, 844, 725, 735, 264, 731}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx = \int \left(\frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2} - \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\right) dx$$

$$= \int \frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx - \int \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx$$

$$= -\int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{15}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{1}{2x^2\sqrt{1-x^2}} - \frac{1}{x\sqrt{1-x^2}} + \frac{9}{2(4+5x)^2\sqrt{1-}}\right.$$

$$= \frac{3}{5(4+5x)} - \frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}\,dx + \frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2}\,dx - \frac{9}{2}\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx - 5$$

$$= \frac{3}{5(4+5x)} + \sqrt{1-x^2} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, x\right.$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\sin^{-1}(x) + \frac{5}{3}\tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \frac{3}{10}\int \frac{1}{\sqrt{1-x^2}}\,dx - \frac{1}{2}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} + \tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{6}{5}\,\mathrm{Subst}\left(\int \frac{1}{9-}\right.$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] time = 0.18001, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),
x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] time = 0.004, size = 32, normalized size = 1.

$$\frac{1}{5}\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}\left(x+\frac{4}{5}\right)^{-1}+\frac{3}{20+25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{56}\sqrt{7}\log\left(\frac{3x-2\sqrt{7}-2}{3x+2\sqrt{7}-2}\right) - \int -\frac{100x^7}{8\left(21x^9+278x^8+283x^7-2022x^6-3632x^5+2256x^4+7424x^3+1536x^2-8(9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="maxima")
```

```
[Out] -1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) - integrate(
-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 - 2
022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7 - 1
01*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*sqrt(x + 1)*sq
rt(-x + 1) - 4096*x - 2048), x) - 1/24*log(x + 2) + 1/16*log(x + 1) - 1/48*
log(x - 1)
```

Fricas [A] time = 1.62737, size = 65, normalized size = 2.1

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x^2}-1}{\sqrt{-(x-1)\,(x+1)}\left(x-2\sqrt{1-x^2}+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2
),x)
```

```
[Out] Integral((sqrt(1 - x**2) - 1)/(sqrt(-(x - 1)*(x + 1))*(x - 2*sqrt(1 - x**2)
 + 2)**2), x)
```

Giac [B] time = 1.23758, size = 92, normalized size = 2.97

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.829 $\int \frac{a+bx^{-1+n}}{cx+dx^n} \, dx$

Optimal. Leaf size=43

$$\frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rubi [A] time = 0.0668804, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1593, 514, 446, 72}

$$\frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = \int \frac{x^{-n}\left(a+bx^{-1+n}\right)}{d+cx^{1-n}}\,dx$$
$$= \int \frac{b+ax^{1-n}}{x\left(d+cx^{1-n}\right)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+ax}{x(d+cx)}\,dx, x, x^{1-n}\right)}{1-n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b}{dx}+\frac{-bc+ad}{d(d+cx)}\right)dx, x, x^{1-n}\right)}{1-n}$$
$$= \frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}$$

Mathematica [A] time = 0.0408059, size = 38, normalized size = 0.88

$$\frac{\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{c(n-1)}+b\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] (b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d
```

Maple [A] time = 0.015, size = 73, normalized size = 1.7

$$\frac{\ln(x)\,an}{c(-1+n)} - \frac{\ln(x)\,b}{d(-1+n)} - \frac{\ln\left(cx+d\mathrm{e}^{n\ln(x)}\right)a}{c(-1+n)} + \frac{\ln\left(cx+d\mathrm{e}^{n\ln(x)}\right)b}{d(-1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(-1+n))/(c*x+d*x^n),x)
```

```
[Out] 1/c/(-1+n)*ln(x)*a*n-1/d/(-1+n)*ln(x)*b-1/c/(-1+n)*ln(c*x+d*exp(n*ln(x)))*a
+1/d/(-1+n)*ln(c*x+d*exp(n*ln(x)))*b
```

Maxima [B] time = 1.01548, size = 115, normalized size = 2.67

$$b\left(\frac{\log(x)}{d} - \frac{n\log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right) + a\left(\frac{n\log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a*
(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))
```

Fricas [A] time = 1.74653, size = 93, normalized size = 2.16

$$\frac{(bc - ad)\log(cx + dx^n) + (adn - bc)\log(x)}{cdn - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)
```

Sympy [A] time = 9.88533, size = 212, normalized size = 4.93

$$\begin{cases} \tilde{\infty}(a+b)\log(x) & \text{for}\ c = 0 \wedge d = 0 \wedge n = 1 \\ \frac{-\frac{anx}{n^2x^n - nx^n} + \frac{bn^2x^n\log(x)}{n^2x^n - nx^n} - \frac{bnx^n\log(x)}{n^2x^n - nx^n} - \frac{bnx^n}{n^2x^n - nx^n}}{d} & \text{for}\ c = 0 \\ \frac{\frac{anx\log(x)}{nx - x} - \frac{ax\log(x)}{nx - x} + \frac{bx^n}{nx - x}}{c} & \text{for}\ d = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for}\ n = 1 \\ \frac{adn\log(x)}{cdn - cd} - \frac{ad\log\left(x + \frac{dx^n}{c}\right)}{cdn - cd} - \frac{bc\log(x)}{cdn - cd} + \frac{bc\log\left(x + \frac{dx^n}{c}\right)}{cdn - cd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*n*x/(n
**2*x**n - n*x**n) + b*n**2*x**n*log(x)/(n**2*x**n - n*x**n) - b*n*x**n*log
(x)/(n**2*x**n - n*x**n) - b*n*x**n/(n**2*x**n - n*x**n))/d, Eq(c, 0)), ((a
*n*x*log(x)/(n*x - x) - a*x*log(x)/(n*x - x) + b*x**n/(n*x - x))/c, Eq(d, 0
)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/(c*d*n - c*d) - a*d*l
og(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n - c*d) + b*c*log(x + d*x
**n/c)/(c*d*n - c*d), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^{n-1} + a}{cx + dx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

3.830 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}} dx$

Optimal. Leaf size=42

$$\frac{\sqrt{2x^2+1}}{2x} + x - \frac{1}{2x} - \frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.129511, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6740, 6742, 277, 215}

$$\frac{\sqrt{2x^2+1}}{2x} + x - \frac{1}{2x} - \frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx &= \int \left(1+\frac{1}{-1-\sqrt{1+2x^2}}\right)dx \\
&= x+\int \frac{1}{-1-\sqrt{1+2x^2}}\,dx \\
&= x+\int \left(\frac{1}{2x^2}-\frac{\sqrt{1+2x^2}}{2x^2}\right)dx \\
&= -\frac{1}{2x}+x-\frac{1}{2}\int \frac{\sqrt{1+2x^2}}{x^2}\,dx \\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\int \frac{1}{\sqrt{1+2x^2}}\,dx \\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0378985, size = 42, normalized size = 1.

$$
\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] time = 0.004, size = 45, normalized size = 1.1

$$
x-\frac{1}{2\,x}+\frac{1}{2\,x}\left(2\,x^2+1\right)^{\frac{3}{2}}-x\sqrt{2\,x^2+1}-\frac{\mathrm{Arcsinh}\left(x\sqrt{2}\right)\sqrt{2}}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x)
```

```
[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(x*2^(1/2))*2^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
x-\int \frac{1}{\sqrt{2\,x^2+1}+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

Fricas [A] time = 1.67909, size = 111, normalized size = 2.64

$$\frac{\sqrt{2}x\log\left(\sqrt{2}x - \sqrt{2\,x^2+1}\right) + 2\,x^2 + \sqrt{2\,x^2+1} - 1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1) -
 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

Giac [A] time = 1.13607, size = 77, normalized size = 1.83

$$\frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}x + \sqrt{2\,x^2+1}\right) + x - \frac{\sqrt{2}}{\left(\sqrt{2}x - \sqrt{2\,x^2+1}\right)^2 - 1} - \frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x - s
qrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

3.831 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqr
t[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rubi [A] time = 0.13286, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6742, 444, 50, 63, 207, 388}

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqr
t[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx &= \int \left(-\frac{x\sqrt{-1+4x^2}}{-1+3x^2}+\frac{-1+4x^2}{-1+3x^2}\right)dx\\
&= -\int \frac{x\sqrt{-1+4x^2}}{-1+3x^2}\,dx+\int \frac{-1+4x^2}{-1+3x^2}\,dx\\
&= \frac{4x}{3}+\frac{1}{3}\int \frac{1}{-1+3x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+4x}}{-1+3x}\,dx,x,x^2\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{(-1+3x)\sqrt{-1+4x}}\,dx,x,x^2\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}+\frac{3x^2}{4}}\,dx,x,\sqrt{-1+4x^2}\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{-1+4x^2}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.0473054, size = 54, normalized size = 0.83

$$\frac{1}{9}\left(-3\sqrt{4x^2-1}+\sqrt{3}\tanh^{-1}\left(\sqrt{12x^2-3}\right)+12x-\sqrt{3}\tanh^{-1}\left(\sqrt{3}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (12*x - 3*Sqrt[-1 + 4*x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x] + Sqrt[3]*ArcTanh[S
qrt[-3 + 12*x^2]])/9
```

Maple [B] time = 0.032, size = 262, normalized size = 4.

$$\frac{4\,x}{3}-\frac{\text{Artanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{1}{18}\sqrt{36\left(x-1/3\sqrt{3}\right)^2+24\sqrt{3}\left(x-1/3\sqrt{3}\right)+3}-\frac{\sqrt{3}\sqrt{4}}{18}\ln\left(x\sqrt{4}+\sqrt{4\left(x-1/3\sqrt{3}\right)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x)
```

```
[Out] 4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)-1/18*(36*(x-1/3*3^(1/2))^2+24*3^(1/2)*
(x-1/3*3^(1/2))+3)^(1/2)-1/18*3^(1/2)*ln(x*4^(1/2)+(4*(x-1/3*3^(1/2))^2+8/3
```

```
*3^(1/2)*(x-1/3*3^(1/2))+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3+
8/3*3^(1/2)*(x-1/3*3^(1/2)))*3^(1/2)/(36*(x-1/3*3^(1/2))^2+24*3^(1/2)*(x-1/
3*3^(1/2))+3)^(1/2))-1/18*(36*(x+1/3*3^(1/2))^2-24*3^(1/2)*(x+1/3*3^(1/2))+
3)^(1/2)+1/18*3^(1/2)*ln(x*4^(1/2)+(4*(x+1/3*3^(1/2))^2-8/3*3^(1/2)*(x+1/3*
3^(1/2))+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3-8/3*3^(1/2)*(x+1
/3*3^(1/2)))*3^(1/2)/(36*(x+1/3*3^(1/2))^2-24*3^(1/2)*(x+1/3*3^(1/2))+3)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x - \int \frac{x}{\sqrt{2\,x+1}\sqrt{2\,x-1}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)
```

Fricas [A] time = 1.654, size = 212, normalized size = 3.26

$$\frac{1}{18}\,\sqrt{3}\log\left(\frac{6\,x^2+\sqrt{3}\sqrt{4\,x^2-1}-1}{3\,x^2-1}\right)+\frac{1}{18}\,\sqrt{3}\log\left(\frac{3\,x^2-2\,\sqrt{3}x+1}{3\,x^2-1}\right)+\frac{4}{3}\,x-\frac{1}{3}\,\sqrt{4\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*log((6*x^2 + sqrt(3)*sqrt(4*x^2 - 1) - 1)/(3*x^2 - 1)) + 1/18*
sqrt(3)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + 4/3*x - 1/3*sqrt(4*x^2
 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(2x-1)\,(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

```
[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

Giac [B] time = 1.15428, size = 180, normalized size = 2.77

$$\frac{1}{18}\,\sqrt{3}\log\left(\frac{\left|6\,x-2\,\sqrt{3}\right|}{\left|6\,x+2\,\sqrt{3}\right|}\right)-\frac{1}{18}\,\sqrt{3}\log\left(-\frac{\left|-12\,x-4\,\sqrt{3}+6\,\sqrt{4\,x^2-1}+\frac{6}{2\,x-\sqrt{4\,x^2-1}}\right|}{2\left(6\,x-2\,\sqrt{3}-3\,\sqrt{4\,x^2-1}-\frac{3}{2\,x-\sqrt{4\,x^2-1}}\right)}\right)+\frac{4}{3}\,x-\frac{1}{3}\,\sqrt{4\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2*x - sqrt(4*x^2 -
1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(2*x - sqrt(4*x^2 - 1)))) + 4
/3*x - 1/3*sqrt(4*x^2 - 1)
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\left(d^2-e^2\right)^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+\right.}{2\left(d^2-e^2\right)^{5/2}}$$

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e*x)^2) + (
(c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqrt[-1 + x^2])/(2*e*(d
^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Arc
Tanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rubi [A] time = 0.206653, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1651, 807, 725, 206}

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\left(d^2-e^2\right)^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+\right.}{2\left(d^2-e^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e*x)^2) + (
(c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqrt[-1 + x^2])/(2*e*(d
^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Arc
Tanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} - \frac{\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2\left(d^2-e^2\right)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)} - \frac{(3bd}{}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)} + \frac{(3bd}{}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)} - \frac{(3bd}{}$$

Mathematica [A] time = 0.323999, size = 240, normalized size = 1.23

$$\frac{1}{2}\left(\frac{\sqrt{x^2-1}\left(ae\left(-4d^2-3dex+e^2\right)+b\left(d^2ex+2d^3+de^2+2e^3x\right)+cd\left(d^2x-3de-4e^2x\right)\right)}{\left(d^2-e^2\right)^2(d+ex)^2} - \frac{\log\left(-\sqrt{x^2-1}\sqrt{d^2-e^2}+\right.}{(d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] ((Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^2*x - 4*e^
2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/((d^2 - e^2)^2*(d + e*x)^2)
+ ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*Log[d + e*x])/((d - e)^2*
(d + e)^2*Sqrt[d^2 - e^2]) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2)
)*Log[e + d*x - Sqrt[d^2 - e^2]*Sqrt[-1 + x^2]])/((d - e)^2*(d + e)^2*Sqrt[
d^2 - e^2]))/2
```

Maple [B] time = 0.028, size = 1407, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x)
```

```
[Out] -c/e^3/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)
/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))-1/2/e/(
d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a+1/2/e^2/
(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b*d-1/2/e
^3/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*c*d^2-
3/2*d/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a+3
/2/e*d^2/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*
```

```
b-3/2/e^2*d^3/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(
1/2)*c-3/2/e*d^2/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/
e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(
1/2))/(x+d/e))*a+3/2/e^2*d^3/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e
^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2
-e^2)/e^2)^(1/2))/(x+d/e))*b-3/2/e^3*d^4/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*
ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*
(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c+1/2/e/(d^2-e^2)/((d^2-e^2)/e^2)^(1
/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*
d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*a-3/2/e^2/(d^2-e^2)/((d^2-e^2)/e
^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e
)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b*d+5/2/e^3/(d^2-e^2)/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c*d^2-1/e/(d^2-e^2
)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b+2/e^2/(d^2-e^2)/(
x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99121, size = 2431, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + sqrt(d^2 - e^2)*(d*x + e) + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*s
qrt(x^2 - 1))/(e*x + d)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b
*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*
d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 +
b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*
x)*sqrt(x^2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3
*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e
^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c
)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*
d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3
*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*
x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(-d^2 +
e^2)*arctan(-(sqrt(-d^2 + e^2)*sqrt(x^2 - 1)*e - sqrt(-d^2 + e^2)*(e*x + d)
)/(d^2 - e^2)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4 +
 (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 -
```

```
b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3
- (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x^
2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 +
 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{\sqrt{(x-1)(x+1)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

Giac [B] time = 1.16424, size = 724, normalized size = 3.71

$$\frac{\left(2\,ad^2+cd^2-3\,bde+ae^2+2\,ce^2\right)\arctan\left(-\frac{\left(x-\sqrt{x^2-1}\right)e+d}{\sqrt{-d^2+e^2}}\right)}{\left(d^4-2\,d^2e^2+e^4\right)\sqrt{-d^2+e^2}}+\frac{2\,cd^4\left(x-\sqrt{x^2-1}\right)^3e+2\,cd^5\left(x-\sqrt{x^2-1}\right)^2+2\,bd^4\left(x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-((x - sqrt(x^2 - 1))*
e + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*c*
d^4*(x - sqrt(x^2 - 1))^3*e + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*(x -
sqrt(x^2 - 1))^2*e - 2*a*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 5*c*d^2*(x - sqrt(
x^2 - 1))^3*e^3 - 6*a*d^3*(x - sqrt(x^2 - 1))^2*e^2 - 7*c*d^3*(x - sqrt(x^2
 - 1))^2*e^2 + 2*c*d^4*(x - sqrt(x^2 - 1))*e + 3*b*d*(x - sqrt(x^2 - 1))^3*
e^4 + 5*b*d^2*(x - sqrt(x^2 - 1))^2*e^3 + 4*b*d^3*(x - sqrt(x^2 - 1))*e^2 -
 a*(x - sqrt(x^2 - 1))^3*e^5 - 3*a*d*(x - sqrt(x^2 - 1))^2*e^4 - 4*c*d*(x -
 sqrt(x^2 - 1))^2*e^4 - 10*a*d^2*(x - sqrt(x^2 - 1))*e^3 - 11*c*d^2*(x - sq
rt(x^2 - 1))*e^3 + c*d^3*e^2 + 2*b*(x - sqrt(x^2 - 1))^2*e^5 + 5*b*d*(x - s
qrt(x^2 - 1))*e^4 + b*d^2*e^3 + a*(x - sqrt(x^2 - 1))*e^5 - 3*a*d*e^4 - 4*c
*d*e^4 + 2*b*e^5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*((x - sqrt(x^2 - 1))^2*e + 2
*d*(x - sqrt(x^2 - 1)) + e)^2)
```

3.833 $\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rubi [A] time = 0.0143167, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = \frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right)$$
$$= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)$$
$$= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)$$
$$= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

Mathematica [A] time = 0.0139714, size = 28, normalized size = 1.

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A] time = 0.025, size = 29, normalized size = 1.

$$-\frac{1}{4}\frac{1}{\sqrt{x^8+1}} + \frac{1}{4}\ln\left(\left(\sqrt{x^8+1}-1\right)\frac{1}{\sqrt{x^8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)/x/(x^8+1)^(3/2),x)
```

```
[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

Maxima [A] time = 1.53085, size = 46, normalized size = 1.64

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Fricas [B] time = 1.7639, size = 140, normalized size = 5.

$$-\frac{\left(x^8+1\right)\log\left(\sqrt{x^8+1}+1\right) - \left(x^8+1\right)\log\left(\sqrt{x^8+1}-1\right) + 2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

Sympy [A] time = 12.2368, size = 37, normalized size = 1.32

$$\frac{\log\left(\sqrt{x^8+1}-1\right)}{8}-\frac{\log\left(\sqrt{x^8+1}+1\right)}{8}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
```

```
[Out] log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1)
)
```

Giac [A] time = 1.12574, size = 46, normalized size = 1.64

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

3.834 $\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rubi [A] time = 0.0583902, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1586, 1593, 446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]

[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx &= \int \frac{1+2x^8}{\sqrt{1+x^8}\left(x+x^9\right)}\,dx \\
&= \int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx \\
&= \frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)
\end{aligned}$$

Mathematica [A] time = 0.00577, size = 28, normalized size = 1.

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A] time = 0.02, size = 29, normalized size = 1.

$$-\frac{1}{4}\frac{1}{\sqrt{x^8+1}} + \frac{1}{4}\ln\left(\left(\sqrt{x^8+1}-1\right)\frac{1}{\sqrt{x^8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x)
```

```
[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^8+1\right)\sqrt{x^8+1}}{x^{17}+2\,x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

Fricas [B] time = 1.78793, size = 140, normalized size = 5.

$$-\frac{(x^8+1)\log\left(\sqrt{x^8+1}+1\right)-(x^8+1)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\,(x^8+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="fricas")
```

```
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09357, size = 46, normalized size = 1.64

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

3.835 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx$

Optimal. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rubi [A] time = 0.0050603, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {261}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\, dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A] time = 0.0073615, size = 22, normalized size = 1.

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A] time = 0.002, size = 19, normalized size = 0.9

$$x-3\,x^3-\frac{1}{9}\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-9*x^2+x/(-9*x^2+1)^(1/2),x)
```

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Maxima [A] time = 1.14436, size = 24, normalized size = 1.09

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Fricas [A] time = 1.75207, size = 47, normalized size = 2.14

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Sympy [A] time = 0.13385, size = 17, normalized size = 0.77

$$-3x^3+x-\frac{\sqrt{1-9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

Giac [A] time = 1.10133, size = 24, normalized size = 1.09

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.836 $\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx$

Optimal. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rubi [A] time = 0.0820809, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {6742, 261}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx &= \int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)dx\\ &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A] time = 0.0018521, size = 22, normalized size = 1.

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A] time = 0.003, size = 19, normalized size = 0.9

$$x-3\,x^{3}-\frac{1}{9}\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x)
```

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Maxima [A] time = 1.10861, size = 24, normalized size = 1.09

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Fricas [A] time = 1.80464, size = 47, normalized size = 2.14

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Sympy [A] time = 1.15003, size = 17, normalized size = 0.77

$$-3x^{3}+x-\frac{\sqrt{1-9x^{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

Giac [A] time = 1.13827, size = 24, normalized size = 1.09

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.837 $\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rubi [A] time = 0.0613476, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2034, 629}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx,x,\sqrt{x}\right)\\ &= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}\end{aligned}$$

Mathematica [A] time = 0.0158369, size = 17, normalized size = 1.

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [A] time = 0.006, size = 12, normalized size = 0.7

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x)
```

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-3\sqrt{x}\right)^{\frac{2}{3}}\left(2\sqrt{x}-3\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

Fricas [A] time = 2.07424, size = 36, normalized size = 2.12

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [B] time = 1.23684, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}+\frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
```

```
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5
```

Giac [A] time = 1.09796, size = 15, normalized size = 0.88

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

3.838 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rubi [A] time = 0.0466631, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2043, 1631, 629}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]

[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 2043

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{d =
 Denominator[n]}, Dist[d, Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x ->
 x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x]] /; FreeQ[{a, b, j
, n, p}, x] && PolyQ[Pq, x^n] &&  !IntegerQ[p] && NeQ[n, j] && RationalQ[j,
 n] && IntegerQ[j/n] && LtQ[-1, n, 1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)\\ &= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}\end{aligned}$$

Mathematica [A] time = 0.0247953, size = 17, normalized size = 1.

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [C] time = 0.059, size = 125, normalized size = 7.4

$$\frac{18\,3^{2/3}}{5}\sqrt[3]{-\mathrm{signum}\left(-1+\frac{1}{3}\sqrt{x}\right)}x^{\frac{5}{6}}{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{1}{3}\sqrt{x}\right)\frac{1}{\sqrt[3]{\mathrm{signum}\left(-1+\frac{1}{3}\sqrt{x}\right)}}+\frac{4\,3^{2/3}}{11}\sqrt[3]{-\mathrm{signum}\left(-1+\frac{1}{3}\sqrt{x}\right)}x^{\frac{11}{6}}{}_2F$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x)
```

```
[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x
^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*x^
(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3,11/3],
[14/3],1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/
3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

Fricas [A] time = 2.36866, size = 36, normalized size = 2.12

$$\frac{6}{5}\left(x-3\,\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-9\sqrt{x}+2x+9}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
```

```
[Out] Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

3.839 $\int \frac{1}{\sqrt{4-9x^2}} dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi [A] time = 0.0014107, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [A] time = 0.004271, size = 10, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Maple [A] time = 0.003, size = 7, normalized size = 0.7

$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^2+4)^(1/2),x)
```

```
[Out] 1/3*arcsin(3/2*x)
```

Maxima [A] time = 1.65214, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] time = 1.49721, size = 58, normalized size = 5.8

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A] time = 0.133744, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**2+4)**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

Giac [A] time = 1.12351, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(3/2*x)
```

3.840 $\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi [A] time = 0.002131, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]

[Out] ArcSin[(3*x)/2]/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0043825, size = 10, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]

[Out] ArcSin[(3*x)/2]/3
```

Maple [B] time = 0.007, size = 34, normalized size = 3.4

$$\frac{1}{3}\sqrt{(2-3x)(2+3x)}\,\arcsin\left(\frac{3x}{2}\right)\frac{1}{\sqrt{2-3x}}\frac{1}{\sqrt{2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x)
```

```
[Out] 1/3*((2-3*x)*(2+3*x))^(1/2)/(2-3*x)^(1/2)/(2+3*x)^(1/2)*arcsin(3/2*x)
```

Maxima [A] time = 1.62172, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] time = 1.50983, size = 74, normalized size = 7.4

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{3x+2}\sqrt{-3x+2}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

Sympy [B] time = 1.04167, size = 51, normalized size = 5.1

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for}\ \frac{3\left|x+\frac{2}{3}\right|}{4}>1 \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, 3*Abs(x + 2/3)/4 > 1), (2
*asin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

Giac [A] time = 1.1356, size = 16, normalized size = 1.6

$$\frac{2}{3} \arcsin\left(\frac{1}{2} \sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arcsin(1/2*sqrt(3*x + 2))
```

3.841 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi [A] time = 0.0037278, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1972, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] ArcSin[(3*x)/2]/3
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0040046, size = 10, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] ArcSin[(3*x)/2]/3
```

Maple [A] time = 0.006, size = 7, normalized size = 0.7

$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-3*x)*(2+3*x))^(1/2),x)
```

```
[Out] 1/3*arcsin(3/2*x)
```

Maxima [A] time = 1.67563, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] time = 1.42755, size = 58, normalized size = 5.8

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A] time = 1.24983, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

Giac [A] time = 1.18089, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(3/2*x)
```

3.842 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi [A] time = 0.0066258, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[15 - 2*x - x^2],x]

[Out] -ArcSin[(-1 - x)/4]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{15-2x-x^2}}\,dx &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)\end{aligned}$$

Mathematica [A] time = 0.0061172, size = 12, normalized size = 1.

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[15 - 2*x - x^2],x]

[Out] -ArcSin[(-1 - x)/4]
```

Maple [A] time = 0.002, size = 7, normalized size = 0.6

$$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+15)^(1/2),x)
```

```
[Out] arcsin(1/4+1/4*x)
```

Maxima [A] time = 1.7208, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{4}x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [B] time = 1.47849, size = 77, normalized size = 6.42

$$-\arctan\left(\frac{\sqrt{-x^2-2x+15}(x+1)}{x^2+2x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-2x+15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 15), x)
```

Giac [A] time = 1.19498, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{1}{4}x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x + 1/4)
```

3.843 $\int \frac{1}{\sqrt{3-x}\sqrt{5+x}} dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi [A] time = 0.0074626, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]

[Out] -ArcSin[(-1 - x)/4]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x}\sqrt{5+x}} dx &= \int \frac{1}{\sqrt{15-2x-x^2}} dx \\
&= -\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} dx, x, -2-2x\right)\right) \\
&= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)
\end{aligned}$$

Mathematica [A] time = 0.011606, size = 21, normalized size = 1.75

$$-2\sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]
```

Maple [B] time = 0.006, size = 31, normalized size = 2.6

$$\sqrt{(3-x)(5+x)}\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)\frac{1}{\sqrt{3-x}}\frac{1}{\sqrt{5+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(1/2)/(5+x)^(1/2),x)
```

```
[Out] ((3-x)*(5+x))^(1/2)/(3-x)^(1/2)/(5+x)^(1/2)*arcsin(1/4+1/4*x)
```

Maxima [A] time = 1.60581, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{4}x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [B] time = 1.44129, size = 81, normalized size = 6.75

$$-\arctan\left(\frac{\sqrt{x+5}(x+1)\sqrt{-x+3}}{x^2+2x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))
```

Sympy [B] time = 1.01498, size = 41, normalized size = 3.42

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for}\ \frac{|x+5|}{8}>1 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5)/8 > 1), (2*asin(sq
rt(2)*sqrt(x + 5)/4), True))
```

Giac [B] time = 1.14132, size = 18, normalized size = 1.5

$$2 \arcsin\left(\frac{1}{4}\sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

3.844 $\int \frac{1}{\sqrt{(3-x)(5+x)}} dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi [A] time = 0.008832, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3 - x)*(5 + x)],x]

[Out] -ArcSin[(-1 - x)/4]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(3-x)(5+x)}} \, dx &= \int \frac{1}{\sqrt{15-2x-x^2}} \, dx \\
&= -\left(\frac{1}{8} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} \, dx, x, -2-2x\right)\right) \\
&= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)
\end{aligned}$$

Mathematica [A] time = 0.002709, size = 21, normalized size = 1.75

$$-2\sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]
```

Maple [A] time = 0.005, size = 7, normalized size = 0.6

$$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3-x)*(5+x))^(1/2),x)
```

```
[Out] arcsin(1/4+1/4*x)
```

Maxima [A] time = 2.10107, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{4}x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [B] time = 1.45716, size = 77, normalized size = 6.42

$$-\arctan\left(\frac{\sqrt{-x^2-2x+15}(x+1)}{x^2+2x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(3-x)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((3 - x)*(x + 5)), x)
```

Giac [A] time = 1.32539, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{1}{4}x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x + 1/4)
```

3.845 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] ArcSin[4 + x]
```

Rubi [A] time = 0.0051104, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-15 - 8*x - x^2],x]

[Out] ArcSin[4 + x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,-8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [A] time = 0.0055258, size = 4, normalized size = 1.

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-15 - 8*x - x^2],x]

[Out] ArcSin[4 + x]
```

Maple [A] time = 0.003, size = 5, normalized size = 1.3

$$\arcsin(4+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-8*x-15)^(1/2),x)
```

```
[Out] arcsin(4+x)
```

Maxima [B] time = 1.89915, size = 11, normalized size = 2.75

$$-\arcsin\left(-x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [B] time = 1.50147, size = 77, normalized size = 19.25

$$-\arctan\left(\frac{\sqrt{-x^2-8\,x-15}(x+4)}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^2-8x-15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-8*x-15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 8*x - 15), x)
```

Giac [A] time = 1.13153, size = 5, normalized size = 1.25

$$\arcsin\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x + 4)
```

3.846 $\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] ArcSin[4 + x]
```

Rubi [A] time = 0.0059976, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] ArcSin[4 + x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\
&= \sin^{-1}(4+x)
\end{aligned}$$

Mathematica [B] time = 0.0103522, size = 18, normalized size = 4.5

$$-2\sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]
```

Maple [B] time = 0.006, size = 29, normalized size = 7.3

$$\arcsin\left(4+x\right)\sqrt{\left(-3-x\right)\left(5+x\right)}\frac{1}{\sqrt{-3-x}}\frac{1}{\sqrt{5+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-x)^(1/2)/(5+x)^(1/2),x)
```

```
[Out] ((-3-x)*(5+x))^(1/2)/(-3-x)^(1/2)/(5+x)^(1/2)*arcsin(4+x)
```

Maxima [B] time = 2.03335, size = 11, normalized size = 2.75

$$-\arcsin\left(-x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [B] time = 1.51496, size = 81, normalized size = 20.25

$$-\arctan\left(\frac{\sqrt{x+5}(x+4)\sqrt{-x-3}}{x^2+8x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))
```

Sympy [B] time = 1.01342, size = 41, normalized size = 10.25

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for}\ \frac{|x+5|}{2}>1 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5)/2 > 1), (2*asin(sq
rt(2)*sqrt(x + 5)/2), True))
```

Giac [B] time = 1.1544, size = 18, normalized size = 4.5

$$2 \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

3.847 $\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] ArcSin[4 + x]
```

Rubi [A] time = 0.0070078, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [B] time = 0.0027375, size = 18, normalized size = 4.5

$$-2\sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]
```

Maple [A] time = 0.004, size = 5, normalized size = 1.3

$$\arcsin(4 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-3-x)*(5+x))^(1/2),x)
```

```
[Out] arcsin(4+x)
```

Maxima [B] time = 2.18335, size = 11, normalized size = 2.75

$$-\arcsin(-x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [B] time = 1.48181, size = 77, normalized size = 19.25

$$-\arctan\left(\frac{\sqrt{-x^2 - 8x - 15}(x + 4)}{x^2 + 8x + 15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(-x - 3)(x + 5)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 3)*(x + 5)), x)
```

Giac [A] time = 1.13548, size = 5, normalized size = 1.25

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x + 4)
```

3.848 $\int \left(1-\sqrt{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

[Out] x - (2*x^(3/2))/3

Rubi [A] time = 0.0010962, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3

Rubi steps

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.0012431, size = 11, normalized size = 1.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$x-\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1-x^(1/2),x)

[Out] x-2/3*x^(3/2)

Maxima [A] time = 1.40866, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A] time = 1.40489, size = 24, normalized size = 2.18

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A] time = 0.052709, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

Giac [A] time = 1.12633, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

3.849 $\int \frac{1-x}{1+\sqrt{x}} dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x - (2*x^(3/2))/3
```

Rubi [A] time = 0.008115, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1398, 26, 43}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{1+\sqrt{x}} dx &= 2\,\text{Subst}\left(\int \frac{x\left(1-x^2\right)}{1+x} dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int (1-x)x\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(x-x^2\right) dx, x, \sqrt{x}\right)\\
&= x-\frac{2x^{3/2}}{3}
\end{aligned}$$

Mathematica [A] time = 0.000415, size = 11, normalized size = 1.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Maple [A] time = 0.003, size = 8, normalized size = 0.7

$$x-\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(1+x^(1/2)),x)
```

```
[Out] x-2/3*x^(3/2)
```

Maxima [A] time = 1.28941, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A] time = 1.45331, size = 24, normalized size = 2.18

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A] time = 0.13158, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

Giac [A] time = 1.16135, size = 9, normalized size = 0.82

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

3.850 $\int \sqrt{\frac{1}{1-x^2}}\,dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rubi [A] time = 0.014481, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6720, 216}

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x^2)^(-1)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1}{1-x^2}}\,dx &= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0076001, size = 27, normalized size = 1.

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x^2)^(-1)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Maple [A] time = 0.004, size = 30, normalized size = 1.1

$$\sqrt{-\left(x^2-1\right)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(-x^2+1))^(1/2),x)
```

```
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-1/(x^2 - 1)), x)
```

Fricas [A] time = 1.46854, size = 65, normalized size = 2.41

$$2\arctan\left(\frac{\left(x^2-1\right)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

Sympy [A] time = 0.854086, size = 7, normalized size = 0.26

$$\begin{cases}\operatorname{asin}(x) & \text{for}\ x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x**2+1))**(1/2),x)
```

```
[Out] Piecewise((asin(x), (x > -1) & (x < 1)))
```

Giac [A] time = 1.15724, size = 14, normalized size = 0.52

$$-\arcsin(x)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(x)*sgn(x^2 - 1)
```

3.851 $\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rubi [A] time = 0.0206499, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6688, 6720, 216}

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx &= \int \sqrt{\frac{1}{1-x^2}}\,dx\\ &= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0049208, size = 27, normalized size = 1.

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Maple [A] time = 0.003, size = 30, normalized size = 1.1

$$\sqrt{-\left(x^2-1\right)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(-x^4+1))^(1/2),x)
```

```
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)
```

Fricas [A] time = 1.47535, size = 65, normalized size = 2.41

$$2\arctan\left(\frac{\left(x^2-1\right)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x^2+1}{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(-x**4+1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

Giac [A] time = 1.13012, size = 14, normalized size = 0.52

$$-\arcsin(x)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(x)*sgn(x^2 - 1)
```

3.852 $\int \sqrt{\frac{1}{-1+x^2}}\,dx$

Optimal. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

Rubi [A] time = 0.014449, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1}{-1+x^2}}\,dx &= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx\\ &= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0198312, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

Maple [A] time = 0.003, size = 28, normalized size = 1.1

$$\sqrt{\left(x^2-1\right)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2-1))^(1/2),x)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 1.2255, size = 19, normalized size = 0.76

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 1.42682, size = 35, normalized size = 1.4

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A] time = 1.18868, size = 15, normalized size = 0.6

$$\begin{cases}\log\left(x+\sqrt{x^2-1}\right) & \text{for}\: x > -1 \wedge x < 1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x**2-1))**(1/2),x)
```

```
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

Giac [A] time = 1.15246, size = 20, normalized size = 0.8

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))
```

3.853 $\int \sqrt{\frac{1+x^2}{-1+x^4}} \, dx$

Optimal. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

Rubi [A] time = 0.0216021, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6688, 6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx &= \int \sqrt{\frac{1}{-1+x^2}}\,dx \\
&= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\
&= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.0033965, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

Maple [A] time = 0.003, size = 28, normalized size = 1.1

$$\sqrt{\left(x^2-1\right)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(x^4-1))^(1/2),x)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)
```

Fricas [A] time = 1.4318, size = 35, normalized size = 1.4

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(x**4-1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

Giac [A] time = 1.16616, size = 28, normalized size = 1.12

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)
```

3.854 $\int \frac{1}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

[Out] -2*Sqrt[1 - x]

Rubi [A] time = 0.0009113, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - x],x]

[Out] -2*Sqrt[1 - x]

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

Mathematica [A] time = 0.0027051, size = 11, normalized size = 1.

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - x],x]

[Out] -2*Sqrt[1 - x]

Maple [A] time = 0., size = 10, normalized size = 0.9

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-x)^(1/2),x)

[Out] -2*(1-x)^(1/2)

Maxima [A] time = 1.40839, size = 12, normalized size = 1.09

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x + 1)
```

Fricas [A] time = 1.4112, size = 23, normalized size = 2.09

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x + 1)
```

Sympy [A] time = 0.052253, size = 8, normalized size = 0.73

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2),x)
```

```
[Out] -2*sqrt(1 - x)
```

Giac [A] time = 1.14489, size = 12, normalized size = 1.09

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x + 1)
```

3.855 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x]
```

Rubi [A] time = 0.001246, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {26, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1-x}}\,dx\\ &= -2\sqrt{1-x}\end{aligned}$$

Mathematica [B] time = 0.02142, size = 23, normalized size = 2.09

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] (2*(-1 + x)*Sqrt[1 + x])/Sqrt[1 - x^2]
```

Maple [B] time = 0.002, size = 20, normalized size = 1.8

$$2\frac{(x-1)\sqrt{1+x}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2*(x-1)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.42184, size = 16, normalized size = 1.45

$$\frac{2(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)/sqrt(-x + 1)
```

Fricas [C] time = 1.45357, size = 42, normalized size = 3.82

$$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.08708, size = 20, normalized size = 1.82

$$2\sqrt{2}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

3.856 $\int \frac{1}{\sqrt{1+x}} dx$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

```
[Out] 2*Sqrt[1 + x]
```

Rubi [A] time = 0.0007305, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}} dx = 2\sqrt{1+x}$$

Mathematica [A] time = 0.0018038, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Maple [A] time = 0.001, size = 8, normalized size = 0.9

$$2\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2),x)
```

```
[Out] 2*(1+x)^(1/2)
```

Maxima [A] time = 1.57436, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

Fricas [A] time = 1.44447, size = 20, normalized size = 2.22

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)
```

Sympy [A] time = 0.051254, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x + 1)
```

Giac [A] time = 1.13343, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 1)
```

3.857 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

```
[Out] 2*Sqrt[1 + x]
```

Rubi [A] time = 0.0011453, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1+x}}\,dx \\ &= 2\sqrt{1+x} \end{aligned}$$

Mathematica [B] time = 0.0226464, size = 25, normalized size = 2.78

$$\frac{2\sqrt{1-x}(x+1)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] (2*Sqrt[1 - x]*(1 + x))/Sqrt[1 - x^2]
```

Maple [B] time = 0.002, size = 22, normalized size = 2.4

$$2\frac{(1+x)\sqrt{1-x}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2*(1+x)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.47036, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

Fricas [C] time = 1.43803, size = 54, normalized size = 6.

$$-\frac{2\sqrt{-x^2+1}\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.12649, size = 18, normalized size = 2.

$$-2\sqrt{2}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] -2*sqrt(2) + 2*sqrt(x + 1)
```

3.858 $\int \sqrt{1-x}\,dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rubi [A] time = 0.0008786, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

Mathematica [A] time = 0.0030709, size = 13, normalized size = 1.

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Maple [A] time = 0.003, size = 10, normalized size = 0.8

$$-\frac{2}{3}(1-x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2),x)
```

```
[Out] -2/3*(1-x)^(3/2)
```

Maxima [A] time = 1.15072, size = 12, normalized size = 0.92

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

Fricas [A] time = 1.41788, size = 35, normalized size = 2.69

$$\frac{2}{3}\,(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Sympy [A] time = 0.052573, size = 10, normalized size = 0.77

$$-\frac{2\,(1-x)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2),x)
```

```
[Out] -2*(1 - x)**(3/2)/3
```

Giac [A] time = 1.10317, size = 12, normalized size = 0.92

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

3.859 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rubi [A] time = 0.0013626, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {26, 32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} \, dx &= \int \sqrt{1-x} \, dx \\ &= -\frac{2}{3}(1-x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.0196129, size = 25, normalized size = 1.92

$$\frac{2(x-1)\sqrt{1-x^2}}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (2*(-1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 + x])
```

Maple [B] time = 0.003, size = 20, normalized size = 1.5

$$\frac{2x-2}{3}\sqrt{-x^2+1}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 2/3*(x-1)*(-x^2+1)^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 1.11251, size = 16, normalized size = 1.23

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Fricas [B] time = 1.46389, size = 54, normalized size = 4.15

$$\frac{2\sqrt{-x^2+1}(x-1)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-x^2 + 1)*(x - 1)/sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

Giac [A] time = 1.1058, size = 20, normalized size = 1.54

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}+\frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

3.860 $\int \sqrt{1+x}\,dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0006516, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A] time = 0.0021514, size = 11, normalized size = 1.

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$\frac{2}{3}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2),x)
```

```
[Out] 2/3*(1+x)^(3/2)
```

Maxima [A] time = 0.992441, size = 9, normalized size = 0.82

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 1.41257, size = 26, normalized size = 2.36

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Sympy [A] time = 0.051079, size = 8, normalized size = 0.73

$$\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(3/2)/3
```

Giac [A] time = 1.10494, size = 9, normalized size = 0.82

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

3.861 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0010932, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx &= \int \sqrt{1+x} \, dx \\ &= \frac{2}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [B] time = 0.0216299, size = 27, normalized size = 2.45

$$\frac{2(x+1)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 - x])
```

Maple [B] time = 0.003, size = 22, normalized size = 2.

$$\frac{2+2x}{3}\sqrt{-x^2+1}\frac{1}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1-x)^(1/2),x)
```

```
[Out] 2/3*(1+x)*(-x^2+1)^(1/2)/(1-x)^(1/2)
```

Maxima [A] time = 0.986706, size = 9, normalized size = 0.82

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Fricas [B] time = 1.46537, size = 68, normalized size = 6.18

$$-\frac{2\sqrt{-x^2+1}(x+1)\sqrt{-x+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+1)}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(1 - x), x)
```

Giac [A] time = 1.10436, size = 18, normalized size = 1.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-\frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

3.862 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rubi [A] time = 0.006951, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 54, 215}

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx &= \sqrt{1+x}\sqrt{2+3x}-\frac{1}{2}\int \frac{1}{\sqrt{1+x}\sqrt{2+3x}}\,dx\\
&= \sqrt{1+x}\sqrt{2+3x}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+3x}\right)}{\sqrt{3}}\\
&= \sqrt{1+x}\sqrt{2+3x}-\frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0188439, size = 49, normalized size = 1.4

$$\frac{3\sqrt{x+1}(3x+2)-\sqrt{9x+6}\sinh^{-1}\left(\sqrt{3x+2}\right)}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

Maple [B] time = 0.005, size = 67, normalized size = 1.9

$$\sqrt{1+x}\sqrt{2+3x}-\frac{\sqrt{3}}{6}\sqrt{(1+x)(2+3x)}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x+2}\right)\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2+3*x)^(1/2)-1/6*((1+x)*(2+3*x))^(1/2)/(2+3*x)^(1/2)/(1+x)^(1/
2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.61212, size = 55, normalized size = 1.57

$$-\frac{1}{6}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqrt(3*x^2 +
5*x + 2)
```

Fricas [A] time = 1.48094, size = 157, normalized size = 4.49

$$\frac{1}{12}\sqrt{3}\log\left(-4\sqrt{3}(6x+5)\sqrt{3x+2}\sqrt{x+1}+72x^2+120x+49\right)+\sqrt{3x+2}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 +
120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)
```

Sympy [A] time = 1.61436, size = 97, normalized size = 2.77

$$\begin{cases} \frac{3(x+1)^{\frac{3}{2}}}{\sqrt{3x+2}} - \frac{\sqrt{x+1}}{\sqrt{3x+2}} - \frac{\sqrt{3}\operatorname{acosh}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{for } 3|x+1| > 1 \\ i\sqrt{-3x-2}\sqrt{x+1} + \frac{\sqrt{3}i\operatorname{asin}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((3*(x + 1)**(3/2)/sqrt(3*x + 2) - sqrt(x + 1)/sqrt(3*x + 2) - sqr
t(3)*acosh(sqrt(3)*sqrt(x + 1))/3, 3*Abs(x + 1) > 1), (I*sqrt(-3*x - 2)*sqr
t(x + 1) + sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3, True))
```

Giac [A] time = 1.08706, size = 53, normalized size = 1.51

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3x+3}\sqrt{3x+2} + \log\left(\sqrt{3x+3} - \sqrt{3x+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x + 2
)))
```

3.863 $\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rubi [A] time = 0.0066087, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {26, 50, 54, 215}

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$$
$$= \sqrt{1+x}\sqrt{2+3x} - \frac{1}{2}\int \frac{1}{\sqrt{1+x}\sqrt{2+3x}}\,dx$$
$$= \sqrt{1+x}\sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}}$$
$$= \sqrt{1+x}\sqrt{2+3x} - \frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0076575, size = 49, normalized size = 1.4

$$\frac{3\sqrt{x+1}(3x+2) - \sqrt{9x+6}\sinh^{-1}\left(\sqrt{3x+2}\right)}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

Maple [B] time = 0.012, size = 86, normalized size = 2.5

$$\frac{1}{6x-6}\sqrt{1-x}\sqrt{2+3x}\sqrt{-x^2+1}\left(\ln\left(\frac{5\sqrt{3}}{6}+x\sqrt{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}-6\sqrt{3x^2+5x+2}\right)\frac{1}{\sqrt{3x^2+5x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/6*(1-x)^(1/2)*(2+3*x)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+x*3^(1/2)+(3*x
^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(x-1)/(3*x^2+5*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

Fricas [B] time = 1.58799, size = 252, normalized size = 7.2

$$\frac{\sqrt{3}(x-1)\log\left(-\frac{72\,x^3+4\,\sqrt{3}\sqrt{-x^2+1}(6\,x+5)\sqrt{3\,x+2}\sqrt{-x+1}+48\,x^2-71\,x-49}{x-1}\right)-12\,\sqrt{-x^2+1}\sqrt{3\,x+2}\sqrt{-x+1}}{12\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sqr
t(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1)*
sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x}\sqrt{3x+2}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

3.864 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} dx$

Optimal. Leaf size=43

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

Rubi [A] time = 0.0129731, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {98, 21, 105, 41, 216, 92, 206}

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - 2\int \frac{-\frac{1}{2}+\frac{x}{2}}{\sqrt{1-x}x\sqrt{1+x}}\,dx \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} + \int \frac{\sqrt{1-x}}{x\sqrt{1+x}}\,dx \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx + \int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x}\sqrt{1+x}\right) \\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0381312, size = 61, normalized size = 1.42

$$\frac{2\left(\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+2x+2\right)}{\sqrt{1-x^2}} - \tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (2*(2 + 2*x + Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x^2] - A
rcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.015, size = 70, normalized size = 1.6

$$\frac{1}{x-1}\left(-\arcsin(x)\,x - \text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x + \arcsin(x) + \text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - 4\sqrt{-x^2+1}\right)\sqrt{1-x}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(3/2)/x,x)
```

```
[Out] (-arcsin(x)*x-arctanh(1/(-x^2+1)^(1/2))*x+arcsin(x)+arctanh(1/(-x^2+1)^(1/2
))-4*(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/(x-1)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.56226, size = 72, normalized size = 1.67

$$\frac{4x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

Fricas [B] time = 1.50558, size = 201, normalized size = 4.67

$$\frac{2(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + (x-1)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 4x - 4\sqrt{x+1}\sqrt{-x+1} - 4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}}{x(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.865 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\sin^{-1}(x)$$

```
[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0606364, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1805, 844, 216, 266, 63, 206}

$$\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]

[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx &= \frac{4(1+x)}{\sqrt{1-x^2}}-\int \frac{-1+x}{x\sqrt{1-x^2}}\,dx\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\int \frac{1}{\sqrt{1-x^2}}\,dx+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.0211454, size = 47, normalized size = 1.34

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-x^2\right)-\sqrt{1-x^2}\sin^{-1}(x)+4x+3}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (3 + 4*x - Sqrt[1 - x^2]*ArcSin[x] + Hypergeometric2F1[-1/2, 1, 1/2, 1 - x^
2])/Sqrt[1 - x^2]
```

Maple [A] time = 0.008, size = 41, normalized size = 1.2

$$4\,\frac{x}{\sqrt{-x^2+1}}-\arcsin(x)+4\,\frac{1}{\sqrt{-x^2+1}}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/x/(-x^2+1)^(3/2),x)
```

```
[Out] 4*x/(-x^2+1)^(1/2)-arcsin(x)+4/(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.5344, size = 72, normalized size = 2.06

$$\frac{4\,x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin(x)-\log\left(\frac{2\,\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

Fricas [B] time = 1.48969, size = 161, normalized size = 4.6

$$\frac{2\,(x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+4\,x-4\,\sqrt{-x^2+1}-4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) - 1
)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^3}{x\,(-(x-1)\,(x+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

Giac [A] time = 1.12818, size = 59, normalized size = 1.69

$$\frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1}-\arcsin(x)+\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(
x))
```

3.866 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)$$

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]]
```

Rubi [A] time = 0.0236908, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {98, 21, 105, 41, 216, 92, 208}

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\int \frac{-\frac{a}{2}+\frac{a^2x}{2}}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a}\\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}+\int \frac{\sqrt{1-ax}}{x\sqrt{1+ax}}\,dx\\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-a\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx+\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx\\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-a\,\mathrm{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)\\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)\end{aligned}$$

Mathematica [A] time = 0.0595692, size = 72, normalized size = 1.41

$$\frac{2\left(\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+2ax+2\right)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (2*(2 + 2*a*x + Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/Sqrt[1 -
a^2*x^2] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Maple [C] time = 0.036, size = 134, normalized size = 2.6

$$\frac{\mathrm{csgn}\,(a)}{ax-1}\left(-\mathrm{csgn}\,(a)\,\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)xa-\arctan\left(\mathrm{csgn}\,(a)\,xa\frac{1}{\sqrt{-(ax+1)(ax-1)}}\right)xa+\mathrm{csgn}\,(a)\,\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x)
```

```
[Out] (-csgn(a)*arctanh(1/(-a^2*x^2+1)^(1/2))*x*a-arctan(csgn(a)*a*x/(-(a*x+1)*(a
*x-1))^(1/2))*x*a+csgn(a)*arctanh(1/(-a^2*x^2+1)^(1/2))-4*(-a^2*x^2+1)^(1/2
)*csgn(a)+arctan(csgn(a)*a*x/(-(a*x+1)*(a*x-1))^(1/2)))*csgn(a)*(-a*x+1)^(1
/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.4994, size = 105, normalized size = 2.06

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}} - \frac{a\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{4}{\sqrt{-a^2x^2+1}} - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) - a*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 4/sqrt(-a^
2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [B] time = 1.51141, size = 234, normalized size = 4.59

$$\frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{x}\right) - 4\sqrt{ax+1}\sqrt{-ax+1} - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a*
x - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x) - 4*sqrt(a*x + 1)*sqrt(-a*
x + 1) - 4)/(a*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^{\frac{3}{2}}}{x\,(-ax+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.867 $\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0910878, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1805, 844, 216, 266, 63, 208}

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]

[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \int \frac{-1+ax}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2} \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.0376443, size = 59, normalized size = 1.31

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-a^2x^2\right) - \sqrt{1-a^2x^2}\sin^{-1}(ax) + 4ax + 3}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (3 + 4*a*x - Sqrt[1 - a^2*x^2]*ArcSin[a*x] + Hypergeometric2F1[-1/2, 1, 1/2
, 1 - a^2*x^2])/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.012, size = 75, normalized size = 1.7

$$4\frac{ax}{\sqrt{-a^2x^2+1}} - a\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}} + 4\frac{1}{\sqrt{-a^2x^2+1}} - \text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 4*a*x/(-a^2*x^2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1
/2))+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.70773, size = 105, normalized size = 2.33

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}} - \frac{a\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{4}{\sqrt{-a^2x^2+1}} - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) - a*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) + 4/sqrt(-a^
2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [B] time = 1.53943, size = 193, normalized size = 4.29

$$\frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\,\sqrt{-a^2x^2+1}-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax+1)^3}{x\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.18501, size = 117, normalized size = 2.6

$$-\frac{a\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))
```

3.868 $\int \frac{1}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] ArcSin[x]
```

Rubi [A] time = 0.0010183, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x)$$

Mathematica [A] time = 0.0036128, size = 2, normalized size = 1.

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

Maple [A] time = 0.003, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2),x)
```

```
[Out] arcsin(x)
```

Maxima [A] time = 1.85818, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [B] time = 1.40812, size = 47, normalized size = 23.5

$$-2 \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.124319, size = 2, normalized size = 1.

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2),x)
```

```
[Out] asin(x)
```

Giac [A] time = 1.12543, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x)
```

3.869 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] ArcSin[x]
```

Rubi [A] time = 0.0013687, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0254332, size = 32, normalized size = 16.

$$-\tan^{-1}\left(\frac{x\sqrt{x^2+1}\sqrt{1-x^4}}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

Maple [B] time = 0.013, size = 29, normalized size = 14.5

$$\arcsin(x)\sqrt{-x^4+1}\,\frac{1}{\sqrt{x^2+1}}\,\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Fricas [B] time = 1.47706, size = 66, normalized size = 33.

$$-\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.870 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.0007175, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.0033069, size = 2, normalized size = 1.

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$\text{Arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)

[Out] arcsinh(x)
```

Maxima [A] time = 1.49112, size = 3, normalized size = 1.5

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] time = 1.43456, size = 35, normalized size = 17.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.123759, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] time = 1.12184, size = 19, normalized size = 9.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

3.871 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.0011891, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {26, 215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSinh[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \sinh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0233328, size = 42, normalized size = 21.

$$\log\left(1-x^2\right)-\log\left(x^3+\sqrt{1-x^2}\sqrt{1-x^4}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

Maple [B] time = 0.009, size = 29, normalized size = 14.5

$$\mathrm{Arcsinh}\,(x)\,\sqrt{-x^4+1}\,\frac{1}{\sqrt{-x^2+1}}\,\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Fricas [B] time = 1.46115, size = 171, normalized size = 85.5

$$-\frac{1}{2}\,\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)+\frac{1}{2}\,\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(x
^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.872 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0026651, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\,dx &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0051649, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$\frac{\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.67506, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 1.47047, size = 74, normalized size = 3.22

$$\frac{1}{2}\sqrt{-x^2+1}x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.184459, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Giac [A] time = 1.12368, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.873 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0030979, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {26, 195, 216}

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx &= \int \sqrt{1-x^2}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0411808, size = 50, normalized size = 2.17

$$\frac{1}{2}\left(\frac{\sqrt{1-x^4}x}{\sqrt{x^2+1}}+\tan^{-1}\left(\frac{x\sqrt{x^2+1}}{\sqrt{1-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[1 - x^4]])
/2
```

Maple [B] time = 0.008, size = 42, normalized size = 1.8

$$\frac{1}{2}\sqrt{-x^4+1}\left(x\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{x^2+1}}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*(x*(-x^2+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

Fricas [B] time = 1.45123, size = 144, normalized size = 6.26

$$\frac{\sqrt{-x^4+1}\sqrt{x^2+1}x-\left(x^2+1\right)\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4 + 1)*sqrt(
x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

3.874 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi [A] time = 0.0020896, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\,dx &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0040904, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{\operatorname{Arcsinh}(x)}{2}+\frac{x}{2}\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Maxima [A] time = 1.48696, size = 20, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^{2}+1}x+\frac{1}{2}\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Fricas [A] time = 1.41558, size = 69, normalized size = 3.29

$$\frac{1}{2}\sqrt{x^{2}+1}x-\frac{1}{2}\log\left(-x+\sqrt{x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.181181, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^{2}+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Giac [A] time = 1.14834, size = 34, normalized size = 1.62

$$\frac{1}{2}\sqrt{x^{2}+1}x-\frac{1}{2}\log\left(-x+\sqrt{x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.875 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi [A] time = 0.0025367, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {26, 195, 215}

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx &= \int \sqrt{1+x^2}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0533573, size = 70, normalized size = 3.33

$$\frac{1}{2}\left(\frac{\sqrt{1-x^4}x}{\sqrt{1-x^2}}+\log\left(1-x^2\right)-\log\left(x^3+\sqrt{1-x^2}\sqrt{1-x^4}-x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x
^2]*Sqrt[1 - x^4]])/2
```

Maple [B] time = 0.007, size = 47, normalized size = 2.2

$$-\frac{1}{2x^2-2}\sqrt{-x^4+1}\sqrt{-x^2+1}\left(x\sqrt{x^2+1}+\mathrm{Arcsinh}(x)\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*(x*(x^2+1)^(1/2)+arcsinh(x))/(x^2-1)/(x^
2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

Fricas [B] time = 1.38679, size = 258, normalized size = 12.29

$$-\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}x+\left(x^2-1\right)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)-\left(x^2-1\right)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*sq
rt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

3.876 $\int \left(\frac{a+b+cx^2}{d}\right)^m dx$

Optimal. Leaf size=49

$$\frac{dx\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^{m+1}{}_2F_1\left(1,m+\frac{3}{2};\frac{3}{2};-\frac{cx^2}{a+b}\right)}{a+b}$$

```
[Out] (d*x*((a + b)/d + (c*x^2)/d)^(1 + m)*Hypergeometric2F1[1, 3/2 + m, 3/2, -((
c*x^2)/(a + b))])/(a + b)
```

Rubi [A] time = 0.0167575, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1972, 246, 245}

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a +
 b))])/(1 + (c*x^2)/(a + b))^m
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{a+b+cx^2}{d}\right)^m dx &= \int \left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m dx\\
&= \left(\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m\right)\int \left(1+\frac{cx^2}{a+b}\right)^m dx\\
&= x\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)
\end{aligned}$$

Mathematica [A] time = 0.0120852, size = 53, normalized size = 1.08

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b+cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b + c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a + b))
])/(1 + (c*x^2)/(a + b))^m
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2+a+b)/d)^m,x)
```

```
[Out] int(((c*x^2+a+b)/d)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{cx^2+a+b}{d}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="fricas")
```

```
[Out] integral(((c*x^2 + a + b)/d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x**2+a+b)/d)**m,x)
```

```
[Out] Integral(((a + b + c*x**2)/d)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{cx^2 + a + b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="giac")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

3.877 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rubi [A] time = 0.0091935, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2106, 30, 195, 215}

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{1+x^2}}\,dx &= -\int x\,dx-\int \sqrt{1+x^2}\,dx\\
&= -\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\
&= -\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0111033, size = 38, normalized size = 1.36

$$\frac{1}{2}\log\left(x-\sqrt{x^2+1}\right)-\frac{1}{4\left(x-\sqrt{x^2+1}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -1/(4*(x - Sqrt[1 + x^2])^2) + Log[x - Sqrt[1 + x^2]]/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$-\frac{x^2}{2}-\frac{\mathrm{Arcsinh}\,(x)}{2}-\frac{x}{2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x^2+1)^(1/2)),x)
```

```
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x-\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(x^2 + 1)), x)
```

Fricas [A] time = 1.43845, size = 84, normalized size = 3.

$$-\frac{1}{2}x^2-\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [B] time = 0.356448, size = 58, normalized size = 2.07

$$-\frac{x\,\mathrm{asinh}\,(x)}{2x-2\sqrt{x^2+1}}+\frac{x}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}\,\mathrm{asinh}\,(x)}{2x-2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x**2+1)**(1/2)),x)
```

```
[Out] -x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + x/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x*
*2 + 1)*asinh(x)/(2*x - 2*sqrt(x**2 + 1))
```

Giac [A] time = 1.13211, size = 41, normalized size = 1.46

$$-\frac{1}{2}x^2 - \frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

3.878 $\int \frac{1}{x-\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rubi [A] time = 0.0418386, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 260, 402, 216, 377, 207}

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x-\sqrt{1-x^2}}\,dx &= \int \left(\frac{x}{-1+2x^2}+\frac{\sqrt{1-x^2}}{-1+2x^2}\right)dx\\
&= \int \frac{x}{-1+2x^2}\,dx+\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx\\
&= \frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\left(-1+2x^2\right)}\,dx\\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}\log\left(1-2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0223991, size = 37, normalized size = 1.

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Maple [B] time = 0.031, size = 175, normalized size = 4.7

$$\frac{\ln\left(2x^2-1\right)}{4}-\frac{\sqrt{2}}{8}\sqrt{-4\left(x+1/2\sqrt{2}\right)^2+4\left(x+1/2\sqrt{2}\right)\sqrt{2}+2}-\frac{\arcsin(x)}{2}+\frac{1}{4}\text{Artanh}\left(\sqrt{2}\left(1+\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\frac{}{\sqrt{-4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/4*ln(2*x^2-1)-1/8*2^(1/2)*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)
+2)^(1/2)-1/2*arcsin(x)+1/4*arctanh((1+(x+1/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4
*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*2^(1/2)*(-4*(x-1
/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/4*arctanh((1-(x-1/2*2^(1
/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x-\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(-x^2 + 1)), x)
```

Fricas [B] time = 1.48303, size = 220, normalized size = 5.95

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(2x^2-1\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt(
-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) +
x - 1)/x^2)
```

Sympy [A] time = 0.152398, size = 17, normalized size = 0.46

$$\frac{\log\left(x-\sqrt{1-x^2}\right)}{2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] log(x - sqrt(1 - x**2))/2 - asin(x)/2
```

Giac [B] time = 1.13184, size = 189, normalized size = 5.11

$$-\frac{1}{4}\pi\operatorname{sgn}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{4}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) + 1/4*log(abs(x + 1/2*sqrt(2))) + 1/4*log(abs(x - 1/2*sqrt(2
))) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2)) +
1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))
```

3.879 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)$$

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0429, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 260, 402, 215, 377, 206}

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x-\sqrt{1+2x^2}}\,dx &= \int \left(-\frac{x}{1+x^2}-\frac{\sqrt{1+2x^2}}{1+x^2}\right)dx\\
&= -\int \frac{x}{1+x^2}\,dx-\int \frac{\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -\frac{1}{2}\log\left(1+x^2\right)-2\int \frac{1}{\sqrt{1+2x^2}}\,dx+\int \frac{1}{\left(1+x^2\right)\sqrt{1+2x^2}}\,dx\\
&= -\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)-\frac{1}{2}\log\left(1+x^2\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{1+2x^2}}\right)\\
&= -\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)+\tanh^{-1}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0392163, size = 74, normalized size = 1.85

$$
\frac{1}{4}\left(-2\log\left(x^2+1\right)-\log\left(3x^2-2\sqrt{2x^2+1}x+1\right)+\log\left(3x^2+2\sqrt{2x^2+1}x+1\right)-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (-4*Sqrt[2]*ArcSinh[Sqrt[2]*x] - 2*Log[1 + x^2] - Log[1 + 3*x^2 - 2*x*Sqrt[
1 + 2*x^2]] + Log[1 + 3*x^2 + 2*x*Sqrt[1 + 2*x^2]])/4
```

Maple [A] time = 0.009, size = 33, normalized size = 0.8

$$
\text{Artanh}\left(x\frac{1}{\sqrt{2\,x^2+1}}\right)-\frac{\ln\left(x^2+1\right)}{2}-\text{Arcsinh}\left(x\sqrt{2}\right)\sqrt{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{x-\sqrt{2\,x^2+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

Fricas [B] time = 1.43387, size = 236, normalized size = 5.9

$$\sqrt{2}\log\left(\sqrt{2}x - \sqrt{2\,x^2+1}\right) - \frac{1}{2}\,\log\left(x^2+1\right) - \frac{1}{2}\,\log\left(\frac{2\,x^2 - \sqrt{2\,x^2+1}(x+1) + x + 1}{x^2}\right) + \frac{1}{2}\,\log\left(\frac{2\,x^2 + \sqrt{2\,x^2+1}(x-1) -}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^
2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 + 1
)*(x - 1) - x + 1)/x^2)
```

Sympy [A] time = 0.196709, size = 27, normalized size = 0.68

$$-\log\left(x - \sqrt{2x^2+1}\right) - \sqrt{2}\,\operatorname{asinh}\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] -log(x - sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

Giac [B] time = 1.09438, size = 119, normalized size = 2.98

$$\sqrt{2}\log\left(-\sqrt{2}x + \sqrt{2\,x^2+1}\right) - \frac{1}{2}\,\log\left(x^2+1\right) - \frac{1}{2}\,\log\left(\frac{\left(\sqrt{2}x - \sqrt{2\,x^2+1}\right)^2 - 2\,\sqrt{2} + 3}{\left(\sqrt{2}x - \sqrt{2\,x^2+1}\right)^2 + 2\,\sqrt{2} + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log(((sq
rt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqrt(2) + 3)/((sqrt(2)*x - sqrt(2*x^2 + 1)
)^2 + 2*sqrt(2) + 3))
```

3.880 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/4
```

Rubi [A] time = 0.12824, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6725, 260, 266, 43, 478, 12, 377, 207}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/4
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx &= \int \left(\frac{x}{-1+x^2}-\frac{x^3}{2\left(-1+x^2\right)}+\frac{x^2\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{x^3}{-1+x^2}\,dx\right)+\frac{1}{2}\int \frac{x^2\sqrt{2-x^2}}{-1+x^2}\,dx+\int \frac{x}{-1+x^2}\,dx\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\int -\frac{2}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx-\frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{-1+x}\,dx,x,x^2\right)\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)dx,x,x^2\right)+\frac{1}{2}\int \frac{1}{\sqrt{2-x^2}\left(-1+x\right.}\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}}\right)\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0559274, size = 77, normalized size = 1.43

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

Maple [B] time = 0.012, size = 111, normalized size = 2.1

$$\frac{x}{4}\sqrt{-x^2+2}+\frac{1}{4}\sqrt{-(x-1)^2-2x+3}-\frac{1}{4}\operatorname{Artanh}\left(\frac{4-2x}{2}\frac{1}{\sqrt{-(x-1)^2-2x+3}}\right)-\frac{1}{4}\sqrt{-(1+x)^2+2x+3}+\frac{1}{4}\operatorname{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)+1/4*(-(x-1)^2-2*x+3)^(1/2)-1/4*arctanh(1/2*(4-2*x)/(-(
x-1)^2-2*x+3)^(1/2))-1/4*(-(1+x)^2+2*x+3)^(1/2)+1/4*arctanh(1/2*(4+2*x)/(-(
1+x)^2+2*x+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(1+x)
```

Maxima [B] time = 1.5322, size = 127, normalized size = 2.35

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}x+\frac{1}{4}\log\left(x^2-1\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x+2|}+\frac{2}{|2x+2|}+1\right)-\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x-2|}+\frac{2}{|2x-2|}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sqrt(-x^2 +
2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(-x^2 + 2)/abs(2*x -
2) + 2/abs(2*x - 2) - 1)
```

Fricas [A] time = 1.44158, size = 174, normalized size = 3.22

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}x+\frac{1}{4}\log\left(x^2-1\right)-\frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{2x}{x^2-1}\,dx+\int \frac{x^3}{x^2-1}\,dx+\int -\frac{x^2\sqrt{2-x^2}}{x^2-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-x
**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

Giac [B] time = 1.1314, size = 158, normalized size = 2.93

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

3.881 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=60

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4 +
Log[1 + x]/4
```

Rubi [A] time = 0.299584, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6742, 195, 216, 697, 402, 377, 207}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4 +
Log[1 + x]/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx &= \int \left(\frac{\sqrt{2-x^2}}{2}+\frac{2-x^2}{4(-1+x)}+\frac{2-x^2}{4(1+x)}+\frac{\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx\\
&= \frac{1}{4}\int \frac{2-x^2}{-1+x}\,dx+\frac{1}{4}\int \frac{2-x^2}{1+x}\,dx+\frac{1}{2}\int \sqrt{2-x^2}\,dx+\frac{1}{2}\int \frac{\sqrt{2-x^2}}{-1+x^2}\,dx\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\int \left(-1+\frac{1}{-1+x}-x\right)dx+\frac{1}{4}\int \left(1-x+\frac{1}{1+x}\right)dx+\frac{1}{2}\int \frac{1}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}}\right)\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0850813, size = 77, normalized size = 1.28

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

Maple [B] time = 0.005, size = 111, normalized size = 1.9

$$\frac{x}{4}\sqrt{-x^2+2}+\frac{1}{4}\sqrt{-(x-1)^2-2x+3}-\frac{1}{4}\text{Artanh}\left(\frac{4-2x}{2}\frac{1}{\sqrt{-(x-1)^2-2x+3}}\right)-\frac{1}{4}\sqrt{-(1+x)^2+2x+3}+\frac{1}{4}\text{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)+1/4*(-(x-1)^2-2*x+3)^(1/2)-1/4*arctanh(1/2*(4-2*x)/(-(
x-1)^2-2*x+3)^(1/2))-1/4*(-(1+x)^2+2*x+3)^(1/2)+1/4*arctanh(1/2*(4+2*x)/(-(
1+x)^2+2*x+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}x^2 - \int -\frac{x^2}{x - \sqrt{-x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

Fricas [A] time = 1.43754, size = 174, normalized size = 2.9

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2 + 2}x + \frac{1}{4}\log\left(x^2 - 1\right) - \frac{1}{8}\log\left(-\frac{\sqrt{-x^2 + 2}x + 1}{x^2}\right) + \frac{1}{8}\log\left(\frac{\sqrt{-x^2 + 2}x - 1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{2 - x^2}}{x - \sqrt{2 - x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)
```

Giac [B] time = 1.24137, size = 158, normalized size = 2.63

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2 + 2}x + \frac{1}{4}\log\left(\left|x^2 - 1\right|\right) - \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2} - \sqrt{-x^2 + 2}} - \frac{\sqrt{2} - \sqrt{-x^2 + 2}}{x} + 2\right|\right) + \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2} - \sqrt{-x^2 + 2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

3.882 $\int \frac{x}{-x+\sqrt{2x-x^2}} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rubi [A] time = 0.111184, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6742, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx$$
$$= -\frac{x}{2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right)$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

Mathematica [A] time = 0.0504605, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-\log(1-x)+\tanh^{-1}\left(\sqrt{-(x-2)x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

Maple [A] time = 0.005, size = 38, normalized size = 0.8

$$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{1}{2}\sqrt{-(x-1)^2+1}+\frac{1}{2}\text{Artanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x+(-x^2+2*x)^(1/2)),x)
```

```
[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x-\sqrt{-x^2+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

Fricas [A] time = 1.46842, size = 163, normalized size = 3.2

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log(x-1)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x-\sqrt{-x^2+2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
```

```
[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

Giac [A] time = 1.13862, size = 68, normalized size = 1.33

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

3.883 $\int \frac{x+\sqrt{2x-x^2}}{2-2x} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rubi [A] time = 0.0967327, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]

[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx &= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx\\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right)\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.0462925, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-\log(1-x)+\tanh^{-1}\left(\sqrt{-(x-2)x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{1}{2}\sqrt{-(x-1)^2+1}+\frac{1}{2}\text{Artanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-x^2+2*x)^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))
```

Maxima [A] time = 1.57894, size = 73, normalized size = 1.43

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A] time = 1.45869, size = 163, normalized size = 3.2

$$-\frac{1}{2}x - \frac{1}{2}\sqrt{-x^2+2x} - \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right) - \frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x))/2
```

Giac [A] time = 1.12349, size = 68, normalized size = 1.33

$$-\frac{1}{2}x - \frac{1}{2}\sqrt{-x^2+2x} - \frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

3.884 $\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rubi [A] time = 0.14782, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6688, 2115, 6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 2115

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
+ Coefficient[v, x, 0]*h^2), 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
```

```
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx &= \int \frac{x+\sqrt{-(-2+x)x}}{2-2x}\,dx\\
&= \int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx\\
&= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx\\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right)\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.0235279, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-\log(1-x)+\tanh^{-1}\left(\sqrt{-(x-2)x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

Maple [A] time = 0.008, size = 51, normalized size = 1.

$$-\frac{1}{2}\sqrt{2-x}\sqrt{x}\left(\sqrt{-x(-2+x)}-\text{Artanh}\left(\frac{1}{\sqrt{-x(-2+x)}}\right)\right)\frac{1}{\sqrt{-x(-2+x)}}-\frac{x}{2}-\frac{\ln(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(-2+x))^(1/2)*((-x*(-2+x))^(1/2)-arctanh(1/(-x
*(-2+x))^(1/2)))-1/2*x-1/2*ln(x-1)
```

Maxima [A] time = 1.61006, size = 73, normalized size = 1.43

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A] time = 1.46496, size = 180, normalized size = 3.53

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{x-1}\,dx+\int \frac{\sqrt{x}\sqrt{2-x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.885 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

Optimal. Leaf size=54

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)$$

```
[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 - x
]/2
```

Rubi [A] time = 0.0491659, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2105, 101, 12, 92, 206, 43}

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 - x
]/2
```

Rule 2105

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2
)*x), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^
2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && N
eQ[b*e^2 - d*f^2, 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx &= \int \frac{\sqrt{2-x}\sqrt{x}}{2-2x}\,dx+\int \frac{x}{2-2x}\,dx\\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x}+\frac{1}{2}\int \frac{2}{(2-2x)\sqrt{2-x}\sqrt{x}}\,dx+\int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}\right)dx\\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+\int \frac{1}{(2-2x)\sqrt{2-x}\sqrt{x}}\,dx\\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+2\,\mathrm{Subst}\left(\int \frac{1}{4-4x^2}\,dx,x,\sqrt{2-x}\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.139623, size = 82, normalized size = 1.52

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-\log\left(1-\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)+\tanh^{-1}\left(\frac{2-\sqrt{x}}{\sqrt{2-x}}\right)-\tanh^{-1}\left(\frac{\sqrt{x}+2}{\sqrt{2-x}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[(2 - Sqrt[x])/Sqrt[2 - x]] - ArcTanh[(2
 + Sqrt[x])/Sqrt[2 - x]] - Log[1 - Sqrt[x]] - Log[1 + Sqrt[x]])/2
```

Maple [A] time = 0.004, size = 51, normalized size = 0.9

$$-\frac{1}{2}\sqrt{2-x}\sqrt{x}\left(\sqrt{-x(-2+x)}-\mathrm{Artanh}\left(\frac{1}{\sqrt{-x(-2+x)}}\right)\right)\frac{1}{\sqrt{-x(-2+x)}}-\frac{x}{2}-\frac{\ln(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(-2+x))^(1/2)*((-x*(-2+x))^(1/2)-arctanh(1/(-x
*(-2+x))^(1/2)))-1/2*x-1/2*ln(x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt{-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

Fricas [A] time = 1.44088, size = 180, normalized size = 3.33

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\log\left(x-1\right)+\frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x}}{-\sqrt{x}+\sqrt{2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.886 $\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}} \, dx$

Optimal. Leaf size=27

$$-\frac{3\left(1-x^2\right)}{2\left(-(x+1)\left(1-x^2\right)\right)^{2/3}}$$

```
[Out] (-3*(1 - x^2))/(2*(-((1 + x)*(1 - x^2)))^(2/3))
```

Rubi [A] time = 0.0298237, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2067, 2064, 37}

$$-\frac{3(1-x)(x+1)}{2\left(x^3+x^2-x-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*(-1 + x^2))^(-2/3),x]
```

```
[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}} \, dx = \text{Subst}\left(\int \frac{1}{\left(-\frac{16}{27}-\frac{4x}{3}+x^3\right)^{2/3}} \, dx, x, \frac{1}{3}+x\right)$$
$$= \frac{\left(32\sqrt[3]{2}(-1-x)^{4/3}(-1+x)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{4/3}\left(-\frac{16}{9}+\frac{4x}{3}\right)^{2/3}} \, dx, x, \frac{1}{3}+x\right)}{9\left(-1-x+x^2+x^3\right)^{2/3}}$$
$$= -\frac{3(1-x)(1+x)}{2\left(-1-x+x^2+x^3\right)^{2/3}}$$

Mathematica [A] time = 0.0098332, size = 23, normalized size = 0.85

$$\frac{3(x-1)(x+1)}{2\left((x-1)(x+1)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

Maple [A] time = 0.001, size = 20, normalized size = 0.7

$$\frac{(3x-3)(1+x)}{2}\left((1+x)\left(x^2-1\right)\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+x)*(x^2-1))^(2/3),x)
```

```
[Out] 3/2*(x-1)*(1+x)/((1+x)*(x^2-1))^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x^2-1)(x+1)\right)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

Fricas [A] time = 1.39376, size = 53, normalized size = 1.96

$$\frac{3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x**2-1))**(2/3),x)
```

```
[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2-1\right)(x+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

3.887 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

Optimal. Leaf size=14

$$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$$

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rubi [A] time = 0.149441, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6719, 449}

$$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]

[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x\left(1+x^2\right)}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{x\left(1+x^2\right)}}\\ &= -\frac{2x}{\sqrt{x\left(1+x^2\right)}}\end{aligned}$$

Mathematica [A] time = 0.0257639, size = 12, normalized size = 0.86

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]

[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A] time = 0.006, size = 13, normalized size = 0.9

$$-2\,{\frac{x}{\sqrt{x\left(x^{2}+1\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x)

[Out] -2*x/(x*(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}-1}{\sqrt{\left(x^{2}+1\right)x}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

Fricas [A] time = 1.45797, size = 38, normalized size = 2.71

$$-\frac{2\sqrt{x^{3}+x}}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)(x+1)}{\sqrt{x\left(x^{2}+1\right)}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)

[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 - 1}{\sqrt{(x^2 + 1)x}(x^2 + 1)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

3.888 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

Optimal. Leaf size=12

$$-\frac{2x}{\sqrt{x^3+x}}$$

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rubi [A] time = 0.0701814, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x+x^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{x+x^3}}\\ &= -\frac{2x}{\sqrt{x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.0103464, size = 12, normalized size = 1.

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A] time = 0.004, size = 11, normalized size = 0.9

$$-2\frac{x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x)
```

```
[Out] -2*x/(x^3+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{x^3+x}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

Fricas [A] time = 1.50459, size = 38, normalized size = 3.17

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 - 1}{\sqrt{x^3 + x}(x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

3.889 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} \, dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)

Rubi [A] time = 0.139978, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6718, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]

[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} \, dx &= \frac{\left(\sqrt{x}\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}} \, dx}{-1+x^2} \\ &= \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2} \end{aligned}$$

Mathematica [A] time = 0.0196528, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

```
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A] time = 0.005, size = 34, normalized size = 0.9

$$-2\frac{x}{(x-1)(1+x)}\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x)
```

```
[Out] -2*x/(x-1)/(1+x)*((x^2-1)^2/x/(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

Fricas [A] time = 1.45234, size = 68, normalized size = 1.89

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

3.890 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2} \, dx$

Optimal. Leaf size=33

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rubi [A] time = 0.192576, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6719, 2056, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\sqrt{x+x^3}\right)\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx}{-1+x^2}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{-1+x^2}$$
$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

Mathematica [A] time = 0.0115261, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

```
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A] time = 0.003, size = 34, normalized size = 1.

$$-2\,\frac{x}{(x-1)(1+x)}\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x)
```

```
[Out] -2*x/(x-1)/(1+x)*((x^2-1)^2/x/(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

Fricas [A] time = 1.46514, size = 68, normalized size = 2.06

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{\left(x^2-1\right)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

3.891 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rubi [A] time = 0.0681262, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {435, 444, 63, 217, 206}

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```


```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```


Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx &= \frac{\sqrt{b+ax^2}\int \frac{x}{\sqrt{b+ax^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}x}\\ &= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax}\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{a+\frac{b}{x^2}}x}\\ &= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{bd}{a}+\frac{dx^2}{a}}}\,dx,x,\sqrt{b+ax^2}\right)}{a\sqrt{a+\frac{b}{x^2}}x}\\ &= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a}}\,dx,x,\frac{\sqrt{b+ax^2}}{\sqrt{c+dx^2}}\right)}{a\sqrt{a+\frac{b}{x^2}}x}\\ &= \frac{\sqrt{b+ax^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}x}\end{aligned}$$

Mathematica [A] time = 0.125109, size = 105, normalized size = 1.5

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{ac-bd}}\right)}{\sqrt{d}\sqrt{ax^2+b}\sqrt{ac-bd}\sqrt{\frac{a(c+dx^2)}{ac-bd}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b/x^2]*x*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*Sqrt[b + a*x^2])/Sqrt[a
*c - b*d]])/(Sqrt[d]*Sqrt[a*c - b*d]*Sqrt[b + a*x^2]*Sqrt[(a*(c + d*x^2))/(
a*c - b*d)])
```

Maple [B] time = 0.042, size = 117, normalized size = 1.7

$$\frac{ax^2+b}{2x}\ln\left(\frac{1}{2}\left(2\,adx^2+2\sqrt{adx^4+acx^2+bdx^2+bc}\sqrt{ad}+ac+bd\right)\frac{1}{\sqrt{ad}}\right)\sqrt{dx^2+c}\frac{1}{\sqrt{\frac{ax^2+b}{x^2}}}\frac{1}{\sqrt{ad}}\frac{1}{\sqrt{adx^4+acx^2+bdx^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*(a*d*x^4+a*c*x^2+
b*d*x^2+b*c)^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^
(1/2)/(a*d*x^4+a*c*x^2+b*d*x^2+b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

Fricas [A] time = 1.61428, size = 467, normalized size = 6.67

$$\left[\frac{\sqrt{ad}\log\left(8\,a^2d^2x^4+a^2c^2+6\,abcd+b^2d^2+8\left(a^2cd+abd^2\right)x^2+4\left(2\,adx^3+(ac+bd)x\right)\sqrt{dx^2+c}\sqrt{ad}\sqrt{\frac{ax^2+b}{x^2}}\right)}{4\,ad}, -\frac{\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*c
*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*d)
*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (a*c
 + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^4 +
a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

3.892 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqr
t[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rubi [A] time = 0.154027, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2056, 571, 83, 63, 203}

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqr
t[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx &= \frac{\sqrt{-2x^2+x^4}\int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)}\,dx}{x\sqrt{-2+x^2}}\\ &= \frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}}\\ &= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)}\,dx,x,x^2\right)}{6x\sqrt{-2+x^2}}+\frac{\left(2\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(2+x)}\,dx,x,x^2\right.}{3x\sqrt{-2+x^2}}\\ &= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}+\frac{\left(4\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{4+x^2}\,dx,x,\sqrt{-2+\right.}{3x\sqrt{-2+x^2}}\\ &= \frac{2\sqrt{-2x^2+x^4}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}-\frac{\sqrt{-2x^2+x^4}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}\end{aligned}$$

Mathematica [A] time = 0.061988, size = 52, normalized size = 0.63

$$-\frac{x\sqrt{x^2-2}\left(2\tan^{-1}\left(\frac{2}{\sqrt{x^2-2}}\right)+\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3\sqrt{x^2\left(x^2-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] -(x*Sqrt[-2 + x^2]*(2*ArcTan[2/Sqrt[-2 + x^2]] + ArcTan[Sqrt[-2 + x^2]]))/(
3*Sqrt[x^2*(-2 + x^2)])
```

Maple [A] time = 0.023, size = 63, normalized size = 0.8

$$-\frac{1}{6x}\sqrt{x^4-2x^2}\left(\arctan\left((-2+x)\frac{1}{\sqrt{x^2-2}}\right)-4\arctan\left(1/2\sqrt{x^2-2}\right)-\arctan\left((2+x)\frac{1}{\sqrt{x^2-2}}\right)\right)\frac{1}{\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x)
```

```
[Out] -1/6*(x^4-2*x^2)^(1/2)*(arctan((-2+x)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(
1/2))-arctan((2+x)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4-2x^2}}{\left(x^2+2\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)
```

Fricas [A] time = 1.48531, size = 97, normalized size = 1.17

$$-\frac{1}{3}\arctan\left(\frac{\sqrt{x^4-2x^2}}{x}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4-2x^2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(x^2-2\right)}}{(x-1)(x+1)\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

Giac [A] time = 1.1604, size = 65, normalized size = 0.78

$$\frac{1}{3}\left(\arctan\left(\sqrt{2}i\right)-2\arctan\left(\frac{1}{2}\sqrt{2}i\right)\right)\operatorname{sgn}(x)+\frac{1}{3}\left(2\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)-\arctan\left(\sqrt{x^2-2}\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/3*(arctan(sqrt(2)*i) - 2*arctan(1/2*sqrt(2)*i))*sgn(x) + 1/3*(2*arctan(1/
2*sqrt(x^2 - 2)) - arctan(sqrt(x^2 - 2)))*sgn(x)
```

3.893 $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2} \, dx$

Optimal. Leaf size=47

$$\frac{\left(1-x^2\right) \sqrt{1-\frac{1}{\left(1-x^2\right)^2}} \tan^{-1}\left(\sqrt{x^2-2}\right)}{x \sqrt{x^2-2}}$$

```
[Out] ((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2 + x^
2])
```

Rubi [A] time = 0.460336, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6722, 6725, 1990, 1146, 21, 261, 444, 50, 63, 203}

$$\frac{\left(1-x^2\right) \sqrt{x^4-2 x^2} \sqrt{1-\frac{1}{\left(1-x^2\right)^2}} \tan^{-1}\left(\sqrt{x^2-2}\right)}{x \sqrt{x^2-2} \sqrt{\left(x^2-1\right)^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2
]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx = \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{\left(2-x^2\right)\left(-1+x^2\right)}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \left(\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}+\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{-1+x^2}\right)dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{-1+x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{-1+x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{2-x^2}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{\ldots}}{\ldots}}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{-1+x}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} - \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\ldots}\right.}{x\sqrt{-2+x^2}\sqrt{-1+\left(\ldots\right.}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(1-x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

Mathematica [A] time = 0.11678, size = 91, normalized size = 1.94

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)(x+1)(x+2)\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}}{x\left(x^2-2\right)}\right)-\frac{1}{2}\tan^{-1}\left(\frac{(x-2)(x-1)(x+1)\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}}{x\left(x^2-2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] -ArcTan[((-2 + x)*(-1 + x)*(1 + x)*Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2])/(x*
(-2 + x^2))]/2 + ArcTan[((-1 + x)*(1 + x)*(2 + x)*Sqrt[(x^2*(-2 + x^2))/(-1
```

```
+ x^2)^2])/(x*(-2 + x^2))]/2
```

Maple [A] time = 0.016, size = 63, normalized size = 1.3

$$-\frac{x^2-1}{2x}\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}\left(\arctan\left((-2+x)\frac{1}{\sqrt{x^2-2}}\right)-\arctan\left((2+x)\frac{1}{\sqrt{x^2-2}}\right)\right)\frac{1}{\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x)
```

```
[Out] -1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((-2+x)/(x^2-2)^(1/2))-ar
ctan((2+x)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-\frac{1}{\left(x^2-1\right)^2}+1}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

Fricas [A] time = 1.46195, size = 81, normalized size = 1.72

$$-\arctan\left(\frac{\left(x^2-1\right)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] -arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{\frac{x^4}{x^4-2x^2+1}-\frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] -Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**2
 - 2), x)
```

Giac [A] time = 1.1152, size = 24, normalized size = 0.51

$$-\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

3.894 $\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2} \, dx$

Optimal. Leaf size=123

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}} - \frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/
(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan
[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rubi [A] time = 0.285672, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6719, 2056, 571, 83, 63, 203}

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}} - \frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/
(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan
[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
```

```
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx &= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{\sqrt{-2x^2+x^4}}\\
&= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{x\sqrt{-2+x^2}}\\
&= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}}\\
&= -\frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)}\,dx,x,x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-2+x}(2+\cdots}\right.}{3x\sqrt{-2+x^2}}\\
&= -\frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\mathrm{Subst}\left(\int \frac{1}{4+x^2}\,dx,\cdots\right.}{3x\sqrt{-2+x^2}}\\
&= -\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0202175, size = 70, normalized size = 0.57

$$
\frac{\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}\left(x^2-1\right)\left(2\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)-\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3x\sqrt{x^2-2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2]
 - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

Maple [A] time = 0.007, size = 75, normalized size = 0.6

$$-\frac{x^2-1}{6x}\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}\left(\arctan\left((-2+x)\frac{1}{\sqrt{x^2-2}}\right)-4\arctan\left(1/2\sqrt{x^2-2}\right)-\arctan\left((2+x)\frac{1}{\sqrt{x^2-2}}\right)\right)\frac{1}{\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x)
```

```
[Out] -1/6*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((-2+x)/(x^2-2)^(1/2))-4*
arctan(1/2*(x^2-2)^(1/2))-arctan((2+x)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{x^4-2x^2}{(x^2-1)^2}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)
```

Fricas [A] time = 1.52951, size = 178, normalized size = 1.45

$$-\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{x}\right)+\frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arctan
(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11271, size = 45, normalized size = 0.37

$$\frac{1}{3}\left(2\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)-\arctan\left(\sqrt{x^2-2}\right)\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/3*(2*arctan(1/2*sqrt(x^2 - 2)) - arctan(sqrt(x^2 - 2)))*sgn(x^3 - x)
```

3.895 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

Optimal. Leaf size=133

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3(x^2+1)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{(3x+4)(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh}{x+1}$$

```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1 + x)^3*Sqr
t[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)
/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x
])/(1 + x)
```

Rubi [A] time = 0.0737738, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6723, 970, 739, 819, 780, 215}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3(x^2+1)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{(3x+4)(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(5/2),x]
```


```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1 + x)^3*Sqr
t[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)
/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x
])/(1 + x)
```


Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\left(1+2x+x^2\right)^{5/2}}{\left(1+x^2\right)^{5/2}}\,dx}{\sqrt{1+2x+x^2}}\\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^5}{\left(1+x^2\right)^{5/2}}\,dx}{16(2+2x)}\\
&= -\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(24-8x)(2+2x)^3}{\left(1+x^2\right)^{3/2}}\,dx}{48(2+2x)}\\
&= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(96-288x)(2+}{\sqrt{1+x^2}}}{48(2+2x)}\\
&= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)}-\frac{(4+3x)\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{(10}{}\\
&= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)}-\frac{(4+3x)\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{5\sqrt{1}}{}
\end{aligned}
$$

Mathematica [A] time = 0.0728741, size = 64, normalized size = 0.48

$$
\frac{(x+1)\left(3x^4-8x^3-18x^2+15\left(x^2+1\right)^{3/2}\sinh^{-1}(x)-12x-17\right)}{3\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+1\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 + 15*(1 + x^2)^(3/2)*ArcSinh[
x]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

Maple [A] time = 0.015, size = 62, normalized size = 0.5

$$\frac{x^2+1}{3\,(1+x)^5}\left(\frac{x^2+2\,x+1}{x^2+1}\right)^{\frac{5}{2}}\left(15\,\mathrm{Arcsinh}\,(x)\left(x^2+1\right)^{3/2}+3\,x^4-8\,x^3-18\,x^2-12\,x-17\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(5/2),x)
```

```
[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(1+x)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/2
)+3*x^4-8*x^3-18*x^2-12*x-17)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

Fricas [A] time = 1.43611, size = 292, normalized size = 2.2

$$-\frac{8\,x^3+8\,x^2+15\left(x^3+x^2+x+1\right)\log\left(-\frac{x^2-\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-\left(3\,x^4-8\,x^3-18\,x^2-12\,x-17\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+8\,x+8}{3\left(x^3+x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^
2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17
)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(5/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

Giac [A] time = 1.13822, size = 116, normalized size = 0.87

$$\left(\sqrt{2}+5\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\left(x+1\right)-5\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}\left(x+1\right)+\frac{(((3\,x\mathrm{sgn}\left(x+1\right)-8\,\mathrm{sgn}\left(x+1\right))x-18\,\mathrm{sgn}\left(x+\right.}{3\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(x
 + 1) + 1/3*((((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*sg
n(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)
```

3.896 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx$

Optimal. Leaf size=90

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/
(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x]
)/(1 + x)
```

Rubi [A] time = 0.0478881, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6723, 970, 739, 517, 388, 215}

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/
(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x]
)/(1 + x)
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
```

```
Q[a2, 0]))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\left(1+2x+x^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}\\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^3}{\left(1+x^2\right)^{3/2}}\,dx}{4(2+2x)}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(8-8x)(2+2x)}{\sqrt{1+x^2}}\,dx}{4(2+2x)}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{16-16x^2}{\sqrt{1+x^2}}\,dx}{4(2+2x)}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{x\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\left(6\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{x\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}\end{aligned}$$

Mathematica [A] time = 0.0340606, size = 44, normalized size = 0.49

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+3\sqrt{x^2+1}\sinh^{-1}(x)-2x-1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 + 3*Sqrt[1 + x^2]*ArcSinh[x]))/(
1 + x)
```

Maple [A] time = 0.01, size = 49, normalized size = 0.5

$$\frac{x^2+1}{(1+x)^3}\left(\frac{x^2+2\,x+1}{x^2+1}\right)^{\frac{3}{2}}\left(3\,\mathrm{Arcsinh}\,(x)\,\sqrt{x^2+1}+x^2-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(3/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(1+x)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^2
-2*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(3/2), x)
```

Fricas [A] time = 1.48585, size = 203, normalized size = 2.26

$$-\frac{3\,(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-\left(x^2-2\,x-1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x +
 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

Giac [A] time = 1.19456, size = 90, normalized size = 1.

$$-\left(\sqrt{2}-3\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\,(x+1)-3\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}\,(x+1)+\frac{(x\mathrm{sgn}\,(x+1)-2\,\mathrm{sgn}\,(x+1))x-\mathrm{sgn}\,(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn(
x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)
```

3.897 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\left(x^2+1\right)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\sqrt{x^2+1}\sinh^{-1}(x)}{x+1}$$

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*
x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi [A] time = 0.0302999, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6723, 970, 641, 215}

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\left(x^2+1\right)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\sqrt{x^2+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*
x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}dx}{\sqrt{1+2x+x^2}}$$
$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\sqrt{1+x^2}}dx}{2+2x}$$
$$= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\left(2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x^2}}dx}{2+2x}$$
$$= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}$$

Mathematica [A] time = 0.0162468, size = 40, normalized size = 0.66

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+\sqrt{x^2+1}\sinh^{-1}(x)+1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 + Sqrt[1 + x^2]*ArcSinh[x]))/(1 + x)
```

Maple [A] time = 0.005, size = 42, normalized size = 0.7

$$\frac{1}{1+x}\sqrt{\frac{x^2+2x+1}{x^2+1}}\sqrt{x^2+1}\left(\sqrt{x^2+1}+\mathrm{Arcsinh}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(1/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(1+x)*(x^2+1)^(1/2)*((x^2+1)^(1/2)+arcsinh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{2x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1), x)
```

Fricas [A] time = 1.49756, size = 178, normalized size = 2.92

$$-\frac{(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1
)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{2x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)
```

Giac [A] time = 1.09261, size = 66, normalized size = 1.08

$$-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1)-\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1)+\sqrt{x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

3.898 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

Optimal. Leaf size=109

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqr
t[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1
+ x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rubi [A] time = 0.0648028, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6723, 970, 735, 844, 215, 725, 206}

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqr
t[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1
+ x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] && !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx &= \frac{\sqrt{1+2x+x^2}\int \frac{\sqrt{1+x^2}}{\sqrt{1+2x+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{(2+2x)\int \frac{\sqrt{1+x^2}}{2+2x}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}+\frac{(2+2x)\int \frac{2-2x}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(2+2x)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}+\frac{(2(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(2(2+2x))\,\mathrm{Subst}\left(\int \frac{1}{8-x^2}\,dx,x,\frac{2-2x}{\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{\sqrt{2}(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0282981, size = 72, normalized size = 0.66

$$\frac{(x+1)\left(\sqrt{x^2+1}-\sqrt{2}\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)-\sinh^{-1}(x)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2] - ArcSinh[x] - Sqrt[2]*ArcTanh[(1 - x)/(Sqrt[2]*Sqr
t[1 + x^2])]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

Maple [A] time = 0.03, size = 79, normalized size = 0.7

$$(1+x)\frac{1}{\sqrt{\frac{(1+x)^2}{x^2+1}}}+(1+x)\left(-\mathrm{Arcsinh}\,(x)-\sqrt{2}\mathrm{Artanh}\left(\frac{(2-2\,x)\sqrt{2}}{4}\frac{1}{\sqrt{(1+x)^2-2\,x}}\right)\right)\frac{1}{\sqrt{\frac{(1+x)^2}{x^2+1}}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x/(x^2+1))^(1/2),x)
```

```
[Out] 1/((1+x)^2/(x^2+1))^(1/2)*(1+x)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(2-2*x)*2^
(1/2)/((1+x)^2-2*x)^(1/2)))/((1+x)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{2\,x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)
```

Fricas [A] time = 1.49127, size = 360, normalized size = 3.3

$$\frac{\sqrt{2}(x+1)\log\left(-\frac{x^2+\sqrt{2}\left(x^2-1\right)+\left(2\,x^2+\sqrt{2}\left(x^2+1\right)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+(x+1)\log\left(-\frac{x^2-\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)+\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1)
 + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log(-
(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 + 1)*
sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

Giac [A] time = 1.17987, size = 119, normalized size = 1.09

$$\frac{\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}-2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}-2\right|}\right)}{\operatorname{sgn}(x+1)}+\frac{\log\left(-x+\sqrt{x^2+1}\right)}{\operatorname{sgn}(x+1)}+\frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(2
) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1) +
 sqrt(x^2 + 1)/sgn(x + 1)
```

3.899 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (
2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1
+ x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rubi [A] time = 0.0817175, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6723, 970, 733, 813, 844, 215, 725, 206}

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (
2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1
+ x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{\left(1+x^2\right)^{3/2}}{\left(1+2x+x^2\right)^{3/2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(4(2+2x))\int \frac{\left(1+x^2\right)^{3/2}}{(2+2x)^3}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= -\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(3(2+2x))\int \frac{x\sqrt{1+x^2}}{(2+2x)^2}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{-4+8x}{(2+2x)\sqrt{1+x^2}}\,dx}{8\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(9(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(9(2+2x))\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{2-2x}{\sqrt{1+x^2}}\right)}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{9(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

Mathematica [A] time = 0.0901986, size = 95, normalized size = 0.66

$$\frac{(x+1)\left(2\sqrt{x^2+1}\left(2x^2+9x+5\right)+9\sqrt{2}(x+1)^2\tanh^{-1}\left(\frac{x-1}{\sqrt{2}\sqrt{x^2+1}}\right)-12(x+1)^2\sinh^{-1}(x)\right)}{4\left(\frac{(x+1)^2}{x^2+1}\right)^{3/2}\left(x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*(2*Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) - 12*(1 + x)^2*ArcSinh[x] + 9*S
qrt[2]*(1 + x)^2*ArcTanh[(-1 + x)/(Sqrt[2]*Sqrt[1 + x^2])]))/(4*((1 + x)^2/
(1 + x^2))^(3/2)*(1 + x^2)^(3/2))
```

Maple [A] time = 0.01, size = 218, normalized size = 1.5

$$\frac{1+x}{8}\left(\left(x^2+1\right)^{\frac{5}{2}}x-\left(x^2+1\right)^{\frac{3}{2}}x^3-\left(x^2+1\right)^{\frac{5}{2}}+\left(x^2+1\right)^{\frac{3}{2}}x^2+18\sqrt{2}\text{Artanh}\left(1/2\,\frac{(x-1)\sqrt{2}}{\sqrt{x^2+1}}\right)x^2+5x\left(x^2+1\right)^{3/2}-6\sqrt{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x/(x^2+1))^(3/2),x)
```

```
[Out] 1/8/((x^2+2*x+1)/(x^2+1))^(3/2)*(1+x)*((x^2+1)^(5/2)*x-(x^2+1)^(3/2)*x^3-(x
^2+1)^(5/2)+(x^2+1)^(3/2)*x^2+18*2^(1/2)*arctanh(1/2*(x-1)*2^(1/2)/(x^2+1)^
```

```
(1/2))*x^2+5*x*(x^2+1)^(3/2)-6*(x^2+1)^(1/2)*x^3+36*2^(1/2)*arctanh(1/2*(x-
1)*2^(1/2)/(x^2+1)^(1/2))*x-24*arcsinh(x)*x^2+3*(x^2+1)^(3/2)+6*(x^2+1)^(1/
2)*x^2+18*2^(1/2)*arctanh(1/2*(x-1)*2^(1/2)/(x^2+1)^(1/2))-48*arcsinh(x)*x+
30*x*(x^2+1)^(1/2)-24*arcsinh(x)+18*(x^2+1)^(1/2))/(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

Fricas [A] time = 1.53229, size = 513, normalized size = 3.56

$$\frac{10\,x^3+9\,\sqrt{2}\left(x^3+3\,x^2+3\,x+1\right)\log\left(-\frac{x^2+\sqrt{2}\left(x^2-1\right)+\left(2\,x^2+\sqrt{2}\left(x^2+1\right)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+30\,x^2+12\left(x^3+3\,x^2+3\,x+1\right)\log\left(\right.}{4\left(x^3+3\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 -
1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(
x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 + 1)
*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x^2 +
 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x +
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

3.900 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

```
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rubi [A] time = 0.11607, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 970, 637}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]

[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}\\ &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\left(1+x^2\right)^{3/2}}\,dx}{2+2x}\\ &= -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}\end{aligned}$$

Mathematica [A] time = 0.0106533, size = 26, normalized size = 0.93

$$\frac{(x-1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] ((-1 + x)*Sqrt[(1 + x)^2/(1 + x^2)])/(1 + x)
```

Maple [A] time = 0.003, size = 28, normalized size = 1.

$$\frac{x-1}{1+x}\sqrt{\frac{x^2+2x+1}{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(1/2)/(x^2+1),x)
```

```
[Out] (x-1)/(1+x)*((x^2+2*x+1)/(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{2x}{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)
```

Fricas [A] time = 1.44015, size = 80, normalized size = 2.86

$$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

Giac [A] time = 1.10619, size = 41, normalized size = 1.46

$$\sqrt{2}\text{sgn}\,(x+1)+\frac{x\text{sgn}\,(x+1)-\text{sgn}\,(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)
```

3.901 $\int \sqrt{x-x^2}F(x)\,dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\sqrt{x-x^2}F(x),x\right)$$

```
[Out] CannotIntegrate[Sqrt[x - x^2]*F[x], x]
```

Rubi [A] time = 0.0337432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{x-x^2}F(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[x - x^2]*F[x],x]
```

```
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\int \sqrt{x-x^2}F(x)\,dx=\int \sqrt{x-x^2}F(x)\,dx$$

Mathematica [A] time = 0.0945659, size = 0, normalized size = 0.

$$\int \sqrt{x-x^2}F(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x - x^2]*F[x],x]
```

```
[Out] Integrate[Sqrt[x - x^2]*F[x], x]
```

Maple [A] time = 0.017, size = 0, normalized size = 0.

$$\int F(x)\sqrt{-x^2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(-x^2+x)^(1/2),x)
```

```
[Out] int(F(x)*(-x^2+x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + x}F(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^2 + x}F(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + x)*F(x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x(x-1)}F(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x**2+x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))*F(x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + x}F(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

3.902 $\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate[F[x]/Sqrt[x - x^2], x]
```

Rubi [A] time = 0.0376719, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x]/Sqrt[x - x^2],x]
```

```
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx=\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [A] time = 0.163957, size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x]/Sqrt[x - x^2],x]
```

```
[Out] Integrate[F[x]/Sqrt[x - x^2], x]
```

Maple [A] time = 0.018, size = 0, normalized size = 0.

$$\int F(x)\frac{1}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(-x^2+x)^(1/2),x)
```

```
[Out] int(F(x)/(-x^2+x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^2+x}F(x)}{x^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + x)*F(x)/(x^2 - x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{-x(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x**2+x)**(1/2),x)
```

```
[Out] Integral(F(x)/sqrt(-x*(x - 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

3.903 $\int \sqrt{1-x}\sqrt{x}F(x)\,dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\sqrt{x-x^2}F(x),x\right)$$

```
[Out] CannotIntegrate[Sqrt[x - x^2]*F[x], x]
```

Rubi [A] time = 0.103657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - x]*Sqrt[x]*F[x],x]

[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x-x^2}F(x)\,dx$$

Mathematica [A] time = 0.0254193, size = 0, normalized size = 0.

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x],x]

[Out] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x], x]
```

Maple [A] time = 0.016, size = 0, normalized size = 0.

$$\int F(x)\sqrt{1-x}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(1-x)^(1/2)*x^(1/2),x)

[Out] int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x}\sqrt{-x+1}F(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{1-x}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(1 - x)*F(x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

3.904 $\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate[F[x]/Sqrt[x - x^2], x]
```

Rubi [A] time = 0.112653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]

[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx=\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [A] time = 0.0199639, size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]

[Out] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]), x]
```

Maple [A] time = 0.015, size = 0, normalized size = 0.

$$\int F(x)\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(1-x)^(1/2)/x^(1/2),x)

[Out] int(F(x)/(1-x)^(1/2)/x^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x}\sqrt{-x+1}F(x)}{x^2-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x)*sqrt(-x + 1)*F(x)/(x^2 - x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x}\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(F(x)/(sqrt(x)*sqrt(1 - x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

3.905 $\int F\left(\frac{a+bx}{x}\right) dx$

Optimal. Leaf size=10

$$\text{CannotIntegrate}\left(F\left(\frac{a}{x}+b\right),x\right)$$

```
[Out] CannotIntegrate[F[b + a/x], x]
```

Rubi [A] time = 0.0123544, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F\left(\frac{a+bx}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[(a + b*x)/x],x]
```

```
[Out] Defer[Int][F[b + a/x], x]
```

Rubi steps

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(b+\frac{a}{x}\right) dx$$

Mathematica [A] time = 0.0051797, size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[(a + b*x)/x],x]
```

```
[Out] Integrate[F[(a + b*x)/x], x]
```

Maple [A] time = 0.013, size = 0, normalized size = 0.

$$\int F\left(\frac{bx+a}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((b*x+a)/x),x)
```

```
[Out] int(F((b*x+a)/x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx+a}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="maxima")
```

```
[Out] integrate(F((b*x + a)/x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F\left(\frac{bx+a}{x}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="fricas")
```

```
[Out] integral(F((b*x + a)/x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x)
```

```
[Out] Integral(F((a + b*x)/x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx+a}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="giac")
```

```
[Out] integrate(F((b*x + a)/x), x)
```

3.906 $\int F\left(\frac{a+bx^2}{x^2}\right) dx$

Optimal. Leaf size=10

$$\text{CannotIntegrate}\left(F\left(\frac{a}{x^2}+b\right),x\right)$$

```
[Out] CannotIntegrate[F[b + a/x^2], x]
```

Rubi [A] time = 0.0126367, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[(a + b*x^2)/x^2],x]
```

```
[Out] Defer[Int][F[b + a/x^2], x]
```

Rubi steps

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \int F\left(b+\frac{a}{x^2}\right) dx$$

Mathematica [A] time = 0.0053383, size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[(a + b*x^2)/x^2],x]
```

```
[Out] Integrate[F[(a + b*x^2)/x^2], x]
```

Maple [A] time = 0.007, size = 0, normalized size = 0.

$$\int F\left(\frac{bx^2+a}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((b*x^2+a)/x^2),x)
```

```
[Out] int(F((b*x^2+a)/x^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx^2+a}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="maxima")
```

```
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F\left(\frac{bx^2+a}{x^2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="fricas")
```

```
[Out] integral(F((b*x^2 + a)/x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx^2}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x**2+a)/x**2),x)
```

```
[Out] Integral(F((a + b*x**2)/x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx^2+a}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="giac")
```

```
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

3.907 $\int F\left(\frac{x}{a+bx}\right) dx$

Optimal. Leaf size=12

$$\text{CannotIntegrate}\left(F\left(\frac{x}{a+bx}\right),x\right)$$

```
[Out] CannotIntegrate[F[x/(a + b*x)], x]
```

Rubi [A] time = 0.0078925, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F\left(\frac{x}{a+bx}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x/(a + b*x)],x]
```

```
[Out] Defer[Int][F[x/(a + b*x)], x]
```

Rubi steps

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{a+bx}\right) dx$$

Mathematica [A] time = 0.0056554, size = 0, normalized size = 0.

$$\int F\left(\frac{x}{a+bx}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x/(a + b*x)],x]
```

```
[Out] Integrate[F[x/(a + b*x)], x]
```

Maple [A] time = 0.007, size = 0, normalized size = 0.

$$\int F\left(\frac{x}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x/(b*x+a)),x)
```

```
[Out] int(F(x/(b*x+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F\left(\frac{x}{bx+a}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(F(x/(b*x + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x)
```

```
[Out] Integral(F(x/(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

3.908 $\int F\left(\frac{x^2}{a+bx^2}\right) dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(F\left(\frac{x^2}{a+bx^2}\right),x\right)$$

```
[Out] CannotIntegrate[F[x^2/(a + b*x^2)], x]
```

Rubi [A] time = 0.008748, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x^2/(a + b*x^2)],x]
```

```
[Out] Defer[Int][F[x^2/(a + b*x^2)], x]
```

Rubi steps

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Mathematica [A] time = 0.0064459, size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x^2/(a + b*x^2)],x]
```

```
[Out] Integrate[F[x^2/(a + b*x^2)], x]
```

Maple [A] time = 0.005, size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{bx^2+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(b*x^2+a)),x)
```

```
[Out] int(F(x^2/(b*x^2+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{bx^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="maxima")
```

```
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F\left(\frac{x^2}{bx^2+a}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] integral(F(x^2/(b*x^2 + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**2/(b*x**2+a)),x)
```

```
[Out] Integral(F(x**2/(a + b*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{bx^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="giac")
```

```
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

3.909 $\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$

Optimal. Leaf size=14

$$\text{CannotIntegrate}\left(F\left(\frac{x^2}{(a+bx)^2}\right),x\right)$$

[Out] CannotIntegrate[F[x^2/(a + b*x)^2], x]

Rubi [A] time = 0.0084059, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is Not applicable to the result.

[In] Int[F[x^2/(a + b*x)^2],x]

[Out] Defer[Int][F[x^2/(a + b*x)^2], x]

Rubi steps

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Mathematica [A] time = 0.0085232, size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[F[x^2/(a + b*x)^2],x]

[Out] Integrate[F[x^2/(a + b*x)^2], x]

Maple [A] time = 0.007, size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F(x^2/(b*x+a)^2),x)

[Out] int(F(x^2/(b*x+a)^2),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(bx+a)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F\left(\frac{x^2}{b^2x^2+2\,abx+a^2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] integral(F(x^2/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**2/(b*x+a)**2),x)
```

```
[Out] Integral(F(x**2/(a + b*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(bx+a)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="giac")
```

```
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

3.910 $\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(F\left(\frac{x^4}{(a+bx^2)^2}\right),x\right)$$

```
[Out] CannotIntegrate[F[x^4/(a + b*x^2)^2], x]
```

Rubi [A] time = 0.0089925, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x^4/(a + b*x^2)^2],x]
```

```
[Out] Defer[Int][F[x^4/(a + b*x^2)^2], x]
```

Rubi steps

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Mathematica [A] time = 0.0086473, size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x^4/(a + b*x^2)^2],x]
```

```
[Out] Integrate[F[x^4/(a + b*x^2)^2], x]
```

Maple [A] time = 0.008, size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{(bx^2+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^4/(b*x^2+a)^2),x)
```

```
[Out] int(F(x^4/(b*x^2+a)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{\left(bx^2+a\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F\left(\frac{x^4}{b^2x^4+2\,abx^2+a^2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="fricas")
```

```
[Out] integral(F(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{\left(a+bx^2\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**4/(b*x**2+a)**2),x)
```

```
[Out] Integral(F(x**4/(a + b*x**2)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{\left(bx^2+a\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="giac")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

3.911 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Rubi [A] time = 0.10908, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-2bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [A] time = 0.022504, size = 47, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + \sqrt{b^2x^4 + a}}\frac{1}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

```
[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Fricas [A] time = 12.0447, size = 348, normalized size = 7.4

$$\left[\frac{\sqrt{2}\log\left(4\,b^2x^4 + 4\,\sqrt{b^2x^4 + a}bx^2 + 2\left(\sqrt{2}b^{\frac{3}{2}}x^3 + \sqrt{2}\sqrt{b^2x^4 + a}\sqrt{b}x\right)\sqrt{bx^2 + \sqrt{b^2x^4 + a}} + a\right)}{4\,\sqrt{b}}, -\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{b}}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2)
*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))
```

```
 + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + sqrt
(b^2*x^4 + a))*sqrt(-1/b)/x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.912 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

Rubi [A] time = 0.108093, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+2bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0194982, size = 48, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \sqrt{-bx^2+\sqrt{b^2x^4+a}}\frac{1}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

```
[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Fricas [A] time = 13.0062, size = 366, normalized size = 7.62

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4-4\,\sqrt{b^2x^4+a}bx^2+2\left(\sqrt{2}b^2x^3\sqrt{-\frac{1}{b}}-\sqrt{2}\sqrt{b^2x^4+a}bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}}+a\right),-\frac{\sqrt{2}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt
(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-b*
```

```
x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x^2
+ sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.913 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*
Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*A
rcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3]
+ (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

Rubi [A] time = 0.265768, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2133, 725, 204, 206}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*
Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*A
rcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3]
+ (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx$$

$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}d^2-x^2}\,dx, x, \frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{}{-2ic^2+}\right.$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}d^2}} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}d^2}}$$

Mathematica [F] time = 0.109194, size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\sqrt{2\,x^2+\sqrt{4\,x^4+3}}\frac{1}{\sqrt{4\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)\sqrt{4x^4 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2 + \sqrt{4\,x^4 + 3}}}{\sqrt{4\,x^4 + 3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

3.914 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=268

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}d\cdots}{\sqrt{\sqrt{3}+2ix^2}\cdots}\right)}{\left(\sqrt{3}d^2+2ic^2\right)^{3/\cdots}}$$

[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d
*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)
*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - S
qrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) +
((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*S
qrt[Sqrt[3] + (2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)

Rubi [A] time = 0.311041, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2133, 731, 725, 204, 206}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}d\cdots}{\sqrt{\sqrt{3}+2ix^2}\cdots}\right)}{\left(\sqrt{3}d^2+2ic^2\right)^{3/\cdots}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]

[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d
*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)
*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - S
qrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) +
((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*S
qrt[Sqrt[3] + (2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)

Rule 2133

Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]

Rule 731

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ

```
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}-2ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}+2ix^2}}\,dx$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} + \frac{((1+i)c)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx}{2c^2-i\sqrt{3}d^2} + \frac{((1-i)c)\int}{2c}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} + -\frac{((1+i)c)\,\mathrm{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}d^2-x^2}\,dx, x, \frac{\sqrt{3}}{\sqrt{}}\right.}{2c^2-i\sqrt{3}d^2}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} + \frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\left(2ic^2-\sqrt{3}d^2\right)^{3/2}} + \frac{(1-}{}$$

Mathematica [F] time = 0.0982352, size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\sqrt{2x^2+\sqrt{4x^4+3}}\frac{1}{\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)^2 \sqrt{4x^4 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**4 + 3)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

3.915 $\int \frac{-4+x}{(1+\sqrt[3]{x})\sqrt{x}} dx$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 30\sqrt[6]{x} + 30\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Rubi [A] time = 0.0451169, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1840, 1620, 50, 63, 203}

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 30\sqrt[6]{x} + 30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Rule 1840

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g =
Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1) - 1)*(Pq /. x -> x^g)*(a +
b*x^(g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, b, m, p}, x] && PolyQ[Pq, x
] && FractionQ[n]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{\sqrt{x}\left(-4+x^3\right)}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 3\,\text{Subst}\left(\int \left(\sqrt{x}-x^{3/2}+x^{5/2}-\frac{5\sqrt{x}}{1+x}\right)dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}-15\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+15\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0260594, size = 41, normalized size = 1.

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Maple [A] time = 0.003, size = 28, normalized size = 0.7

$$-30\sqrt[6]{x}-\frac{6}{5}x^{\frac{5}{6}}+\frac{6}{7}x^{\frac{7}{6}}+30\arctan\left(\sqrt[6]{x}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-4)/(1+x^(1/3))/x^(1/2),x)
```

```
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Maxima [A] time = 1.73269, size = 36, normalized size = 0.88

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-30x^{\frac{1}{6}}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

Fricas [A] time = 1.64817, size = 93, normalized size = 2.27

$$\frac{6}{7}(x-35)x^{\frac{1}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))
```

Sympy [A] time = 8.36068, size = 37, normalized size = 0.9

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-30\sqrt[6]{x}+2\sqrt{x}+30\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 30*x**(1/6) + 2*sqrt(x) + 30*atan(x**(1/6))
```

Giac [A] time = 1.16516, size = 36, normalized size = 0.88

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-30x^{\frac{1}{6}}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```



```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

3.916 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx$

Optimal. Leaf size=26

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rubi [A] time = 0.0400841, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1593, 1819, 1810, 635, 203, 260}

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx &= \int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)x^{5/6}}\,dx\\ &= 6\,\text{Subst}\left(\int \frac{1+x^3}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(x+\frac{1-x}{1+x^2}\right)dx, x, \sqrt[6]{x}\right)\\ &= 3\sqrt[3]{x}+6\,\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= 3\sqrt[3]{x}+6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)-6\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= 3\sqrt[3]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [C] time = 0.024554, size = 38, normalized size = 1.46

$$3\sqrt[3]{x}+(-3-3i)\log\left(-\sqrt[6]{x}+i\right)-(3-3i)\log\left(\sqrt[6]{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) - (3 + 3*I)*Log[I - x^(1/6)] - (3 - 3*I)*Log[I + x^(1/6)]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$3\sqrt[3]{x}+6\arctan\left(\sqrt[6]{x}\right)-3\ln\left(1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(x^(5/6)+x^(7/6)),x)
```

```
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

Maxima [A] time = 1.78767, size = 27, normalized size = 1.04

$$3\,x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Fricas [A] time = 1.75033, size = 70, normalized size = 2.69

$$3\,x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Sympy [A] time = 4.88963, size = 24, normalized size = 0.92

$$3\sqrt[3]{x} - 3\log\left(\sqrt[3]{x} + 1\right) + 6\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
```

```
[Out] 3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

Giac [A] time = 1.09701, size = 27, normalized size = 1.04

$$3\,x^{\frac{1}{3}} + 6\,\arctan\left(x^{\frac{1}{6}}\right) - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="giac")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

3.917 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=42

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

Rubi [A] time = 0.153555, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6688, 1593, 1802, 635, 203, 260}

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx &= \int \frac{1+\frac{1}{\sqrt{x}}}{1+\sqrt[3]{x}}\,dx \\
&= 6\,\mathrm{Subst}\left(\int \frac{x^2+x^5}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\,\mathrm{Subst}\left(\int \frac{x^2\left(1+x^3\right)}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\,\mathrm{Subst}\left(\int \left(1-x+x^3-\frac{1-x}{1+x^2}\right)dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\,\mathrm{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)+6\,\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [C] time = 0.0222615, size = 54, normalized size = 1.29

$$\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+(3+3i)\log\left(-\sqrt[6]{x}+i\right)+(3-3i)\log\left(\sqrt[6]{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 + (3 + 3*I)*Log[I - x^(1/6)] + (3 - 3
*I)*Log[I + x^(1/6)]
```

Maple [A] time = 0.012, size = 48, normalized size = 1.1

$$\ln(1+x)+\frac{3}{2}x^{\frac{2}{3}}-\ln\left(x^{\frac{2}{3}}-\sqrt[3]{x}+1\right)+2\ln\left(1+\sqrt[3]{x}\right)-3\sqrt[3]{x}+6\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x)
```

```
[Out] ln(1+x)+3/2*x^(2/3)-ln(x^(2/3)-x^(1/3)+1)+2*ln(1+x^(1/3))-3*x^(1/3)+6*x^(1/
6)-6*arctan(x^(1/6))
```

Maxima [A] time = 1.65704, size = 41, normalized size = 0.98

$$\frac{3}{2}x^{\frac{2}{3}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)+3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Fricas [A] time = 1.65724, size = 105, normalized size = 2.5

$$\frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right) + 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Sympy [A] time = 16.6806, size = 39, normalized size = 0.93

$$6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} + 1\right) - 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(1
/6))
```

Giac [A] time = 1.07974, size = 41, normalized size = 0.98

$$\frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right) + 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

3.918 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx$

Optimal. Leaf size=20

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rubi [A] time = 0.0081978, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {25, 335, 215}

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx &= \int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2} dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} dx, x, \frac{1}{x}\right) \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.0131397, size = 48, normalized size = 2.4

$$-\frac{x\sqrt{\frac{b}{x^2}+2}\tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*
x^2]))
```

Maple [B] time = 0.01, size = 50, normalized size = 2.5

$$-x\sqrt{\frac{2\,x^2+b}{x^2}}\ln\left(2\,\frac{\sqrt{b}\sqrt{2\,x^2+b}+b}{x}\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{2\,x^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+b/x^2)^(1/2)/(2*x^2+b),x)
```

```
[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(1
/2)+b)/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78051, size = 184, normalized size = 9.2

$$\left[\frac{\log\left(-\frac{x^2-\sqrt{b}x\sqrt{\frac{2\,x^2+b}{x^2}}+b}{x^2}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{2\,x^2+b}{x^2}}}{2\,x^2+b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b
)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

```
[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

Giac [B] time = 1.09049, size = 59, normalized size = 2.95

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b))
*sgn(x)/sqrt(-b)
```

3.919 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx$

Optimal. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rubi [A] time = 0.00904, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {25, 335, 216}

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx &= \int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2} dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} dx, x, \frac{1}{x}\right) \\
&= -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.0133659, size = 52, normalized size = 2.6

$$\frac{x\sqrt{2-\frac{b}{x^2}}\tan^{-1}\left(\frac{\sqrt{2x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{2x^2-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x
^2])
```

Maple [B] time = 0.008, size = 62, normalized size = 3.1

$$-x\sqrt{\frac{2\,x^2-b}{x^2}}\ln\left(2\,\frac{\sqrt{-b}\sqrt{2\,x^2-b}-b}{x}\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{2\,x^2-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-b/x^2)^(1/2)/(2*x^2-b),x)
```

```
[Out] -((2*x^2-b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x^2
-b)^(1/2)-b)/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55005, size = 186, normalized size = 9.3

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{x^2-\sqrt{-b}x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{b}x\sqrt{\frac{2\,x^2-b}{x^2}}}{2\,x^2-b}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -a
rctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
```

```
[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

Giac [B] time = 1.10405, size = 54, normalized size = 2.7

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*s
gn(x)/sqrt(b)
```

3.920 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh
[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/(d*e) - (Sqrt[c]*A
rcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rubi [A] time = 0.164957, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1444, 1475, 896, 266, 63, 208, 844, 217, 206, 725}

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh
[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/(d*e) - (Sqrt[c]*A
rcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{\left(e+\frac{d}{x}\right)x}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{ad-cex}{(e+dx)\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}$$
$$= -\frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, \frac{1}{x^2}\right)}{2e} + \left(\frac{ad}{e}+\frac{ce}{d}\right)\operatorname{Subst}\left(\int \frac{1}{(e+dx)\sqrt{a+cx}}\right.$$
$$= -\frac{c\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{c}{x^2}}x}\right)}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\frac{c}{x^2}}\right)}{ce} + \left(-\frac{ad}{e}-\frac{ce}{d}\right)\operatorname{Subst}\left(\int \frac{}{ad^2}\right.$$
$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}x}\right)}{d}$$

Mathematica [A] time = 0.0993253, size = 136, normalized size = 1.12

$$\frac{x\sqrt{a+\frac{c}{x^2}}\left(\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ce-adx}{\sqrt{ax^2+c}\sqrt{ad^2+ce^2}}\right)+\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+c}}\right)-\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{ax^2+c}}{\sqrt{c}}\right)\right)}{de\sqrt{ax^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a + c/x^2]*x*(Sqrt[a]*d*ArcTanh[(Sqrt[a]*x)/Sqrt[c + a*x^2]] + Sqrt[a
*d^2 + c*e^2]*ArcTanh[(c*e - a*d*x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[c + a*x^2])]
- Sqrt[c]*e*ArcTanh[Sqrt[c + a*x^2]/Sqrt[c]]))/(d*e*Sqrt[c + a*x^2])
```

Maple [B] time = 0.025, size = 244, normalized size = 2.

$$\frac{x}{de^2}\sqrt{\frac{ax^2+c}{x^2}}\left(\sqrt{a}d\ln\left(\left(\sqrt{ax^2+c}\sqrt{a}+ax\right)\frac{1}{\sqrt{a}}\right)e\sqrt{\frac{ad^2+ce^2}{e^2}}-\ln\left(2\frac{\sqrt{c}\sqrt{ax^2+c}+c}{x}\right)\sqrt{c}\sqrt{\frac{ad^2+ce^2}{e^2}}e^2+\ln\left(2\frac{1}{ex+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2)^(1/2)/(e*x+d),x)
```

```
[Out] ((a*x^2+c)/x^2)^(1/2)*x*(a^(1/2)*d*ln(((a*x^2+c)^(1/2)*a^(1/2)+a*x)/a^(1/2)
)*e*((a*d^2+c*e^2)/e^2)^(1/2)-ln(2*(c^(1/2)*(a*x^2+c)^(1/2)+c)/x)*c^(1/2)*(
(a*d^2+c*e^2)/e^2)^(1/2)*e^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2
)*e-a*x*d+c*e)/(e*x+d))*a*d^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/
2)*e-a*x*d+c*e)/(e*x+d))*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + c/x^2)/(e*x + d), x)
```

Fricas [A] time = 6.00252, size = 3367, normalized size = 27.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + s
qrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sqrt
(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e
^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e
^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqr
t((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt(
(a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d
^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2
 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sq
rt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e
*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2*sqrt(-a*d^
2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x
^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-a)
*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-
(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sqrt(-a*d^2 - c*
e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a
*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arct
an(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 -
 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 + c*e^2)*log((2*a*c*
d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*
x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(
d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 +
 c)) - 2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) -
sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a
*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2)
)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arctan(sqrt(-c)*x*sq
rt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*s
qrt((a*x^2 + c)/x^2) - c) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)
*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 +
 a*c*e^2)*x^2)))/(d*e), -(sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x
^2)/(a*x^2 + c)) - sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^
2 + c)) - sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2
)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*
e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.921 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

Optimal. Leaf size=181

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e - (Sqrt[
c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d - (Sqrt[a*d^
2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt[a*d^2 -
e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/(d*e)
```

Rubi [A] time = 0.272594, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1443, 1474, 895, 724, 206, 843, 621}

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e - (Sqrt[
c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d - (Sqrt[a*d^
2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt[a*d^2 -
e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/(d*e)
```

Rule 1443

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)
^(n2_.))^(p_.), x_Symbol] :> Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/
x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -
n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
```

```
&& GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx &= \int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{\left(e+\frac{d}{x}\right)x}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\text{Subst}\left(\int \frac{ad-be-cex}{(e+dx)\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} + \left(-b+\frac{ad}{e}+\frac{ce}{d}\right)\text{Subs}\\
&= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} + \left(2\left(b-\frac{ad}{e}-\frac{ce}{d}\right)\right)\text{Subst}\Bigg(\int\\
&= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} - \frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\Bigg(\frac{2ad-be+}{2\sqrt{ad^2-e(bd-c}}}{de}
\end{aligned}
$$

Mathematica [A] time = 0.275022, size = 189, normalized size = 1.04

$$-\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(\sqrt{ad^2-bde+ce^2}\tanh^{-1}\left(\frac{2adx+bd-bex-2ce}{2\sqrt{x(ax+b)+c}\sqrt{ad^2-bde+ce^2}}\right)-\sqrt{a}d\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+\sqrt{c}e\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)}}\right.\right.}{de\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] -((x*Sqrt[a + (c + b*x)/x^2]*(-(Sqrt[a]*d*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sq
rt[c + x*(b + a*x)])]) + Sqrt[c]*e*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c +
x*(b + a*x)])] + Sqrt[a*d^2 - b*d*e + c*e^2]*ArcTanh[(b*d - 2*c*e + 2*a*d*x
 - b*e*x)/(2*Sqrt[a*d^2 - b*d*e + c*e^2]*Sqrt[c + x*(b + a*x)])]))/(d*e*Sqr
t[c + x*(b + a*x)]))
```

Maple [B] time = 0.03, size = 397, normalized size = 2.2

$$-\frac{x}{de^2}\sqrt{\frac{ax^2+bx+c}{x^2}}\left(\sqrt{c}\ln\left(\frac{1}{x}\left(2c+bx+2\sqrt{c}\sqrt{ax^2+bx+c}\right)\right)\sqrt{\frac{ad^2-bde+ce^2}{e^2}}\sqrt{a}e^2-\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx+c}\sqrt{a}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)/(e*x+d),x)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*x*(c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(
1/2))/x)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*a^(1/2)*e^2-ln(1/2*(2*(a*x^2+b*x+c
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*a*d*e-ln(
(2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*x*d+x*b*e-b*d+
2*c*e)/(e*x+d))*a^(3/2)*d^2+ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/
e^2)^(1/2)*e-2*a*x*d+x*b*e-b*d+2*c*e)/(e*x+d))*a^(1/2)*b*d*e-ln((2*(a*x^2+b
*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*x*d+x*b*e-b*d+2*c*e)/(e*x
+d))*a^(1/2)*c*e^2)/(a*x^2+b*x+c)^(1/2)/d/e^2/a^(1/2)/((a*d^2-b*d*e+c*e^2)/
e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.922 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx$

Optimal. Leaf size=26

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

```
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Rubi [A] time = 0.0063937, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 15, 30}

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
```

```
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx &= \int \left(\frac{1}{\sqrt[3]{x}}+\frac{\sqrt[5]{x^3}}{\sqrt{x}}\right)dx\\
&= \frac{3x^{2/3}}{2}+\int \frac{\sqrt[5]{x^3}}{\sqrt{x}}\,dx\\
&= \frac{3x^{2/3}}{2}+\frac{\sqrt[5]{x^3}\int \sqrt[10]{x}\,dx}{x^{3/5}}\\
&= \frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\sqrt[5]{x^3}
\end{aligned}$$

Mathematica [A] time = 0.0117846, size = 26, normalized size = 1.

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
```

```
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Maple [A] time = 0.002, size = 17, normalized size = 0.7

$$\frac{3}{2}x^{\frac{2}{3}}+\frac{10}{11}\sqrt[5]{x^{3}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x)
```

```
[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

Maxima [A] time = 1.05393, size = 22, normalized size = 0.85

$$\frac{10}{11}\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Fricas [A] time = 1.89933, size = 55, normalized size = 2.12

$$\frac{10}{11}\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06577, size = 15, normalized size = 0.58

$$\frac{10}{11}x^{\frac{11}{10}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")
```

```
[Out] 10/11*x^(11/10) + 3/2*x^(2/3)
```

3.923 $\int \frac{2+x}{\sqrt{4x-x^2}}\,dx$

Optimal. Leaf size=26

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rubi [A] time = 0.0112103, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {640, 619, 216}

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/Sqrt[4*x - x^2],x]

[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\sqrt{4x-x^2}}\,dx &= -\sqrt{4x-x^2}+4\int \frac{1}{\sqrt{4x-x^2}}\,dx\\ &= -\sqrt{4x-x^2}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right)\\ &= -\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0295019, size = 27, normalized size = 1.04

$$-\sqrt{-(x-4)x}-8\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[-((-4 + x)*x)] - 8*ArcSin[Sqrt[1 - x/4]]
```

Maple [A] time = 0.006, size = 23, normalized size = 0.9

$$4 \arcsin (x/2-1)-\sqrt{-x^2+4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-x^2+4*x)^(1/2),x)
```

```
[Out] 4*arcsin(1/2*x-1)-(-x^2+4*x)^(1/2)
```

Maxima [A] time = 1.6664, size = 30, normalized size = 1.15

$$-\sqrt{-x^2+4x}-4 \arcsin \left(-\frac{1}{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

Fricas [A] time = 1.5946, size = 68, normalized size = 2.62

$$-\sqrt{-x^2+4x}-8 \arctan \left(\frac{\sqrt{-x^2+4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{\sqrt{-x(x-4)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral((x + 2)/sqrt(-x*(x - 4)), x)
```

Giac [A] time = 1.12249, size = 30, normalized size = 1.15

$$-\sqrt{-x^2+4x}+4 \arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

3.924 $\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

```
[Out] (3*(6*x + x^2)^(2/3))/4
```

Rubi [A] time = 0.0033134, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {629}

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(6*x + x^2)^(2/3))/4
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx=\frac{3}{4}\left(6x+x^2\right)^{2/3}$$

Mathematica [A] time = 0.0065614, size = 13, normalized size = 0.87

$$\frac{3}{4}(x(x+6))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(x*(6 + x))^(2/3))/4
```

Maple [A] time = 0.005, size = 16, normalized size = 1.1

$$\frac{3\,x\,(x+6)}{4}\frac{1}{\sqrt[3]{x^2+6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(x^2+6*x)^(1/3),x)
```

```
[Out] 3/4*x*(x+6)/(x^2+6*x)^(1/3)
```

Maxima [A] time = 1.04785, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Fricas [A] time = 1.41599, size = 31, normalized size = 2.07

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Sympy [A] time = 0.147264, size = 12, normalized size = 0.8

$$\frac{3\left(x^2+6x\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2+6*x)**(1/3),x)
```

```
[Out] 3*(x**2 + 6*x)**(2/3)/4
```

Giac [A] time = 1.09174, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

3.925 $\int \frac{4+x}{(6x-x^2)^{3/2}} dx$

Optimal. Leaf size=22

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

```
[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])
```

Rubi [A] time = 0.0047028, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {636}

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

Mathematica [A] time = 0.0100448, size = 19, normalized size = 0.86

$$\frac{7x-12}{9\sqrt{-(x-6)x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$-\frac{x(-6+x)(-12+7x)}{9}\left(-x^2+6x\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+x)/(-x^2+6*x)^(3/2),x)
```

```
[Out] -1/9*x*(-6+x)*(-12+7*x)/(-x^2+6*x)^(3/2)
```

Maxima [A] time = 1.03403, size = 38, normalized size = 1.73

$$\frac{7\,x}{9\,\sqrt{-x^2+6\,x}}-\frac{4}{3\,\sqrt{-x^2+6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

Fricas [A] time = 1.4443, size = 62, normalized size = 2.82

$$-\frac{\sqrt{-x^2+6\,x}(7\,x-12)}{9\,\left(x^2-6\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+4}{(-x\,(x-6))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x**2+6*x)**(3/2),x)
```

```
[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

Giac [A] time = 1.12045, size = 36, normalized size = 1.64

$$-\frac{\sqrt{-x^2+6\,x}(7\,x-12)}{9\,\left(x^2-6\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

3.926 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rubi [A] time = 0.0074974, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.0111596, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]
```

Maple [A] time = 0.005, size = 13, normalized size = 1.1

$$-\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x)^(1/2),x)
```

```
[Out] -arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A] time = 1.63703, size = 12, normalized size = 1.

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

Fricas [A] time = 1.46446, size = 49, normalized size = 4.08

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(x+2)}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

Giac [A] time = 1.11088, size = 23, normalized size = 1.92

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.927 $\int \frac{1}{(1+2x)\sqrt{x+x^2}} \, dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

```
[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rubi [A] time = 0.0081645, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+2x)\sqrt{x+x^2}} \, dx &= 4\,\text{Subst}\left(\int \frac{1}{2+8x^2} \, dx, x, \sqrt{x+x^2}\right)\\ &= \tan^{-1}\left(2\sqrt{x+x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.0120436, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcTan[Sqrt[x]/Sqrt[1 + x]])/Sqrt[x*(1 + x)]
```

Maple [A] time = 0.006, size = 15, normalized size = 1.3

$$-\arctan\left(\frac{1}{\sqrt{4\left(x+1/2\right)^{2}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)/(x^2+x)^(1/2),x)
```

```
[Out] -arctan(1/(4*(x+1/2)^2-1)^(1/2))
```

Maxima [A] time = 1.68146, size = 15, normalized size = 1.25

$$-\arcsin\left(\frac{1}{|2x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(2*x + 1))
```

Fricas [A] time = 1.48214, size = 51, normalized size = 4.25

$$2\arctan\left(-2x+2\sqrt{x^{2}+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\left(x+1\right)}\left(2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

Giac [A] time = 1.1142, size = 23, normalized size = 1.92

$$2\arctan\left(-2x+2\sqrt{x^{2}+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

3.928 $\int \frac{-1+x}{\sqrt{2x-x^2}} dx$

Optimal. Leaf size=15

$$-\sqrt{2x-x^2}$$

```
[Out] -Sqrt[2*x - x^2]
```

Rubi [A] time = 0.0038994, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {629}

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{-1+x}{\sqrt{2x-x^2}} dx = -\sqrt{2x-x^2}$$

Mathematica [A] time = 0.0061087, size = 12, normalized size = 0.8

$$-\sqrt{-(x-2)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[-((-2 + x)*x)]
```

Maple [A] time = 0.001, size = 17, normalized size = 1.1

$$x(-2+x)\frac{1}{\sqrt{-x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(-x^2+2*x)^(1/2),x)
```

```
[Out] x*(-2+x)/(-x^2+2*x)^(1/2)
```

Maxima [A] time = 1.10355, size = 18, normalized size = 1.2

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Fricas [A] time = 1.4182, size = 26, normalized size = 1.73

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Sympy [A] time = 0.122126, size = 10, normalized size = 0.67

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x**2+2*x)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 2*x)
```

Giac [A] time = 1.09644, size = 18, normalized size = 1.2

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

3.929 $\int \frac{\sqrt{x-x^2}}{1+x} \, dx$

Optimal. Leaf size=54

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]
*Sqrt[x - x^2])]
```

Rubi [A] time = 0.039589, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {734, 843, 619, 216, 724, 204}

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]
*Sqrt[x - x^2])]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x-x^2}}{1+x}\,dx &= \sqrt{x-x^2}-\frac{1}{2}\int \frac{1-3x}{(1+x)\sqrt{x-x^2}}\,dx\\ &= \sqrt{x-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx-2\int \frac{1}{(1+x)\sqrt{x-x^2}}\,dx\\ &= \sqrt{x-x^2}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)+4\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,\frac{-1+3x}{\sqrt{x-x^2}}\right)\\ &= \sqrt{x-x^2}-\frac{3}{2}\sin^{-1}(1-2x)+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0576623, size = 95, normalized size = 1.76

$$\sqrt{-(x-1)x}-\frac{3\sqrt{-(x-1)x}\sin^{-1}\left(\sqrt{1-x}\right)}{\sqrt{1-x}\sqrt{x}}+\frac{2\sqrt{2}\sqrt{-(x-1)x}\tanh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{x-1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[-((-1 + x)*x)] - (3*Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(Sqrt[1 -
 x]*Sqrt[x]) + (2*Sqrt[2]*Sqrt[-((-1 + x)*x)]*ArcTanh[Sqrt[-1 + x]/(Sqrt[2]
*Sqrt[x])])/(Sqrt[-1 + x]*Sqrt[x])
```

Maple [A] time = 0.007, size = 54, normalized size = 1.

$$\sqrt{-(1+x)^2+3x+1}+\frac{3\arcsin(2x-1)}{2}-\sqrt{2}\arctan\left(\frac{(3x-1)\sqrt{2}}{4}\frac{1}{\sqrt{-(1+x)^2+3x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(1/2)/(1+x),x)
```

```
[Out] (-(1+x)^2+3*x+1)^(1/2)+3/2*arcsin(2*x-1)-2^(1/2)*arctan(1/4*(3*x-1)*2^(1/2)
/(-(1+x)^2+3*x+1)^(1/2))
```

Maxima [A] time = 1.60678, size = 57, normalized size = 1.06

$$-\sqrt{2}\arcsin\left(\frac{3x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcsi
n(2*x - 1)
```

Fricas [A] time = 1.50108, size = 127, normalized size = 2.35

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+x}}{2x}\right)+\sqrt{-x^2+x}-3\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan(
sqrt(-x^2 + x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-x(x-1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

Giac [A] time = 1.13924, size = 72, normalized size = 1.33

$$2\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{-x^2+x}-1\right)}{2x-1}-1\right)\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + sq
rt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

3.930 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

Optimal. Leaf size=59

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]
/Sqrt[x^(1/4) + x]]/3
```

Rubi [A] time = 0.0685488, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2004, 2018, 2024, 2029, 206}

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(1/4) + x],x]
```

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]
/Sqrt[x^(1/4) + x]]/3
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sqrt[4]{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\frac{1}{4}\int \frac{\sqrt[4]{x}}{\sqrt{\sqrt[4]{x}+x}}\,dx \\
&= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\text{Subst}\left(\int \frac{x^4}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0348624, size = 57, normalized size = 0.97

$$\frac{2x^2+3x^{5/4}-\sqrt{x^{3/4}+1}\sqrt[8]{x}\sinh^{-1}\left(x^{3/8}\right)+\sqrt{x}}{3\sqrt{x+\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(1/4) + x],x]
```

```
[Out] (Sqrt[x] + 3*x^(5/4) + 2*x^2 - Sqrt[1 + x^(3/4)]*x^(1/8)*ArcSinh[x^(3/8)])/
(3*Sqrt[x^(1/4) + x])
```

Maple [C] time = 0.067, size = 342, normalized size = 5.8

$$\frac{2x}{3}\sqrt{\sqrt[4]{x}+x}+\frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{\frac{\frac{3}{2}+\frac{i}{2}\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}}\sqrt[4]{x}\left(1+\sqrt[4]{x}\right)^{-1}}\left(1+\sqrt[4]{x}\right)^{2}\sqrt{-\frac{1}{\frac{1}{2}-\frac{i}{2}\sqrt{3}}\left(\sqrt[4]{x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\left(1+\sqrt[4]{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4)+x)^(1/2),x)
```

```
[Out] 2/3*x*(x^(1/4)+x)^(1/2)+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+(-1/2-1/2*I*3^(1/2))*
((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)*(1+x^(1
/4))^2*(-(x^(1/4)-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)
*(-(x^(1/4)-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)/(3/2+
1/2*I*3^(1/2))/(x^(1/4)*(1+x^(1/4))*(x^(1/4)-1/2+1/2*I*3^(1/2))*(x^(1/4)-1/
2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I
*3^(1/2))/(1+x^(1/4)))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1
```

```
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/
2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x + x^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + x^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt[4]{x} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/4)+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**(1/4) + x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.931 $\int \sqrt{x+x^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) +
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rubi [A] time = 0.0515844, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2002, 2016, 2014}

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(3/2)],x]
```

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) +
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{x+x^{3/2}}\,dx = \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}} - \frac{4}{7}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx$$
$$= -\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x} + \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}} + \frac{8}{35}\int \frac{\sqrt{x+x^{3/2}}}{x}\,dx$$
$$= \frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}} - \frac{16\left(x+x^{3/2}\right)^{3/2}}{35x} + \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}$$

Mathematica [A] time = 0.0194226, size = 39, normalized size = 0.66

$$\frac{4\left(\sqrt{x}+1\right)\left(15x-12\sqrt{x}+8\right)\sqrt{x^{3/2}+x}}{105\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(3/2)],x]
```

```
[Out] (4*(1 + Sqrt[x])*(8 - 12*Sqrt[x] + 15*x)*Sqrt[x + x^(3/2)])/(105*Sqrt[x])
```

Maple [A] time = 0.007, size = 28, normalized size = 0.5

$$\frac{4}{105}\sqrt{x+x^{\frac{3}{2}}}\left(1+\sqrt{x}\right)\left(15\,x-12\,\sqrt{x}+8\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(3/2))^(1/2),x)
```

```
[Out] 4/105*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(15*x-12*x^(1/2)+8)/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(3/2) + x), x)
```

Fricas [A] time = 1.83732, size = 84, normalized size = 1.42

$$\frac{4\left(15\,x^{2}+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{3}{2}} + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(3/2) + x), x)
```

Giac [A] time = 1.08443, size = 45, normalized size = 0.76

$$\frac{4}{105}\left(15\left(\sqrt{x}+1\right)^{\frac{7}{2}} - 42\left(\sqrt{x}+1\right)^{\frac{5}{2}} + 35\left(\sqrt{x}+1\right)^{\frac{3}{2}} - 8\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^(
3/2) - 8)*sgn(x)
```

3.932 $\int x\sqrt{x+x^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (
256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x])
+ (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rubi [A] time = 0.0907418, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2016, 2002, 2014}

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (
256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x])
+ (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x\sqrt{x+x^{3/2}}\,dx = \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} - \frac{8}{11}\int \sqrt{x}\sqrt{x+x^{3/2}}\,dx$$
$$= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} + \frac{16}{33}\int \sqrt{x+x^{3/2}}\,dx$$
$$= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} + \frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} - \frac{64}{231}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx$$
$$= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} - \frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x} + \frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} + \frac{128\int \frac{\sqrt{x+x^{3/2}}}{x}\,dx}{1155}$$
$$= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} + \frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}} - \frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x} + \frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}$$

Mathematica [A] time = 0.0303241, size = 51, normalized size = 0.54

$$\frac{4\left(\sqrt{x}+1\right)\sqrt{x^{3/2}+x}\left(315x^2-280x^{3/2}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (4*(1 + Sqrt[x])*Sqrt[x + x^(3/2)]*(128 - 192*Sqrt[x] + 240*x - 280*x^(3/2)
 + 315*x^2))/(3465*Sqrt[x])
```

Maple [A] time = 0.003, size = 38, normalized size = 0.4

$$\frac{4}{3465}\sqrt{x+x^{\frac{3}{2}}}\left(1+\sqrt{x}\right)\left(315\,x^2-280\,x^{3/2}+240\,x-192\,\sqrt{x}+128\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+x^(3/2))^(1/2),x)
```

```
[Out] 4/3465*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(315*x^2-280*x^(3/2)+240*x-192*x^(1/2)
+128)/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{3}{2}}+x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(3/2) + x)*x, x)
```

Fricas [A] time = 1.9217, size = 116, normalized size = 1.23

$$\frac{4\left(315\,x^{3}-40\,x^{2}+\left(35\,x^{2}+48\,x+128\right)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3/
2) + x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**(3/2) + x), x)
```

Giac [A] time = 1.10183, size = 69, normalized size = 0.73

$$\frac{4}{3465}\left(315\left(\sqrt{x}+1\right)^{\frac{11}{2}}-1540\left(\sqrt{x}+1\right)^{\frac{9}{2}}+2970\left(\sqrt{x}+1\right)^{\frac{7}{2}}-2772\left(\sqrt{x}+1\right)^{\frac{5}{2}}+1155\left(\sqrt{x}+1\right)^{\frac{3}{2}}-128\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(315*(sqrt(x) + 1)^(11/2) - 1540*(sqrt(x) + 1)^(9/2) + 2970*(sqrt(x)
 + 1)^(7/2) - 2772*(sqrt(x) + 1)^(5/2) + 1155*(sqrt(x) + 1)^(3/2) - 128)*sg
n(x)
```

3.933 $\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}} \, dx$

Optimal. Leaf size=18

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rubi [A] time = 0.0488586, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6720, 383}

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}} \, dx &= \left(\sqrt{\frac{1}{2-x^2}} \sqrt{2-x^2}\right) \int \frac{1-x^2}{\sqrt{2-x^2}} \, dx\\ &= \frac{x}{2\sqrt{\frac{1}{2-x^2}}}\end{aligned}$$

Mathematica [A] time = 0.0089827, size = 18, normalized size = 1.

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Maple [A] time = 0.005, size = 20, normalized size = 1.1

$$-\frac{x\left(x^{2}-2\right)}{2}\sqrt{-\left(x^{2}-2\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(1/(-x^2+2))^(1/2),x)
```

```
[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(x^{2}-1\right)\sqrt{-\frac{1}{x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)
```

Fricas [A] time = 1.67516, size = 50, normalized size = 2.78

$$-\frac{1}{2}\left(x^{3}-2\,x\right)\sqrt{-\frac{1}{x^{2}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))
```

Sympy [B] time = 0.505023, size = 26, normalized size = 1.44

$$-\frac{x^{3}\sqrt{\frac{1}{2-x^{2}}}}{2}+x\sqrt{\frac{1}{2-x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
```

```
[Out] -x**3*sqrt(1/(2 - x**2))/2 + x*sqrt(1/(2 - x**2))
```

Giac [A] time = 1.12969, size = 24, normalized size = 1.33

$$-\frac{1}{2}\sqrt{-x^2+2}x\operatorname{sgn}\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)
```

3.934 $\int \sqrt{x^2+x^3-x^4}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{-x^4+x^3+x^2}(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

```
[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^2 + x^3 -
x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqr
t[1 + x - x^2])
```

Rubi [A] time = 0.0288431, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1903, 640, 612, 619, 216}

$$-\frac{\sqrt{-x^4+x^3+x^2}(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^2 + x^3 -
x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqr
t[1 + x - x^2])
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x^2+x^3-x^4}\,dx = \frac{\sqrt{x^2+x^3-x^4}\int x\sqrt{1+x-x^2}\,dx}{x\sqrt{1+x-x^2}}$$
$$= -\frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\sqrt{x^2+x^3-x^4}\int \sqrt{1+x-x^2}\,dx}{2x\sqrt{1+x-x^2}}$$
$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\left(5\sqrt{x^2+x^3-x^4}\right)\int \frac{1}{\sqrt{1+x-x^2}}\,dx}{16x\sqrt{1+x-x^2}}$$
$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} - \frac{\left(\sqrt{5}\sqrt{x^2+x^3-x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx, x\right.}{16x\sqrt{1+x-x^2}}$$
$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} - \frac{5\sqrt{x^2+x^3-x^4}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

Mathematica [A] time = 0.0275871, size = 84, normalized size = 0.79

$$\frac{\sqrt{-x^4+x^3+x^2}\left(2\sqrt{x^2-x-1}\left(8x^2-2x-11\right)-15\tanh^{-1}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)\right)}{48x\sqrt{x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) - 15*ArcTa
nh[(-1 + 2*x)/(2*Sqrt[-1 - x + x^2])]))/(48*x*Sqrt[-1 - x + x^2])
```

Maple [A] time = 0.005, size = 81, normalized size = 0.8

$$\frac{1}{48\,x}\sqrt{-x^4+x^3+x^2}\left(-16\left(-x^2+x+1\right)^{3/2}+12\,x\sqrt{-x^2+x+1}+15\arcsin\left(1/5\,(2\,x-1)\sqrt{5}\right)-6\sqrt{-x^2+x+1}\right)\frac{1}{\sqrt{-x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^3+x^2)^(1/2),x)
```

```
[Out] 1/48*(-x^4+x^3+x^2)^(1/2)*(-16*(-x^2+x+1)^(3/2)+12*x*(-x^2+x+1)^(1/2)+15*ar
csin(1/5*(2*x-1)*5^(1/2))-6*(-x^2+x+1)^(1/2))/x/(-x^2+x+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^3 + x^2), x)
```

Fricas [A] time = 1.63206, size = 147, normalized size = 1.37

$$-\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^4+x^3+x^2}}{x^2}\right)-\sqrt{-x^4+x^3+x^2}\left(8\,x^2-2\,x-11\right)+11\,x}{24\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 + x
^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)
```

Giac [A] time = 1.12201, size = 81, normalized size = 0.76

$$\frac{1}{48}\left(15\,\arcsin\left(\frac{1}{5}\,\sqrt{5}\right)+22\right)\mathrm{sgn}\left(x\right)+\frac{5}{16}\,\arcsin\left(\frac{1}{5}\,\sqrt{5}(2\,x-1)\right)\mathrm{sgn}\left(x\right)+\frac{1}{24}\left(2\left(4\,x\mathrm{sgn}\left(x\right)-\mathrm{sgn}\left(x\right)\right)x-11\,\mathrm{sgn}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x -
1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x + 1
)
```

3.935 $\int \frac{1}{\sqrt{\left(a^2+x^2\right)^3}} \, dx$

Optimal. Leaf size=25

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}$$

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rubi [A] time = 0.0164506, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6720, 191}

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\left(a^2+x^2\right)^3}} \, dx &= \frac{\left(a^2+x^2\right)^{3/2} \int \frac{1}{\left(a^2+x^2\right)^{3/2}} \, dx}{\sqrt{\left(a^2+x^2\right)^3}} \\ &= \frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}} \end{aligned}$$

Mathematica [A] time = 0.0193384, size = 25, normalized size = 1.

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{\left(a^2+x^2\right)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$\frac{x\left(a^2+x^2\right)}{a^2}\frac{1}{\sqrt{\left(a^2+x^2\right)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+x^2)^3)^(1/2),x)
```

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Maxima [A] time = 1.07124, size = 19, normalized size = 0.76

$$\frac{x}{\sqrt{a^2+x^2}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

Fricas [B] time = 1.65692, size = 131, normalized size = 5.24

$$\frac{a^4+2\,a^2x^2+x^4+\sqrt{a^6+3\,a^4x^2+3\,a^2x^4+x^6}x}{a^6+2\,a^4x^2+a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 +
2*a^4*x^2 + a^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\left(a^2+x^2\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+x**2)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

Giac [A] time = 1.11909, size = 19, normalized size = 0.76

$$\frac{x}{\sqrt{a^2 + x^2}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

3.936 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x} \, dx$

Optimal. Leaf size=42

$$2\sqrt{x} - \log\left(x + \sqrt{x} + 1\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Rubi [A] time = 0.0287223, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1357, 703, 634, 618, 204, 628}

$$2\sqrt{x} - \log\left(x + \sqrt{x} + 1\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}+2\,\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}-\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)-\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}-\log\left(1+\sqrt{x}+x\right)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\
&= 2\sqrt{x}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)
\end{aligned}
$$

Mathematica [A] time = 0.0141231, size = 42, normalized size = 1.

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Maple [A] time = 0.005, size = 34, normalized size = 0.8

$$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2\sqrt{x}\right)\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x+x^(1/2)),x)
```

```
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Maxima [A] time = 1.69182, size = 45, normalized size = 1.07

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

Fricas [A] time = 1.45813, size = 123, normalized size = 2.93

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x
+ sqrt(x) + 1)
```

Sympy [A] time = 0.247189, size = 49, normalized size = 1.17

$$2\sqrt{x}-\log\left(4\sqrt{x}+4x+4\right)-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 +
 sqrt(3)/3)/3
```

Giac [A] time = 1.12951, size = 45, normalized size = 1.07

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

3.937 $\int \frac{x}{1+\sqrt{x}+x} \, dx$

Optimal. Leaf size=32

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.023297, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 701, 618, 204}

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x+x^2}\right)dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}+x+2\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}+x-4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\ &= -2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.009687, size = 32, normalized size = 1.

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.003, size = 26, normalized size = 0.8

$$x+\frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2\sqrt{x}\right)\right)-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+x^(1/2)),x)
```

```
[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Maxima [A] time = 1.56694, size = 34, normalized size = 1.06

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

Fricas [A] time = 1.4492, size = 96, normalized size = 3.

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)
```

Sympy [A] time = 0.228774, size = 37, normalized size = 1.16

$$-2\sqrt{x}+x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x**(1/2)),x)
```

```
[Out] -2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.07596, size = 34, normalized size = 1.06

$$\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\sqrt{x}+1\right)\right)+x-2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

3.938 $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx$

Optimal. Leaf size=76

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqrt[x]))/(13
5*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405*Sqrt[1 + Sqrt[x] +
x])
```

Rubi [A] time = 0.0225249, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 614, 613}

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqrt[x]))/(13
5*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405*Sqrt[1 + Sqrt[x] +
x])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{7/2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{32}{15}\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{5/2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}} + \frac{256}{135}\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}} + \frac{512\left(1+2\sqrt{x}\right)}{405\sqrt{1+\sqrt{x}+x}}$$

Mathematica [A] time = 0.0159448, size = 49, normalized size = 0.64

$$\frac{4\left(2\sqrt{x}+1\right)\left(128x^2+256x^{3/2}+432x+304\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(40
5*(1 + Sqrt[x] + x)^(5/2))
```

Maple [A] time = 0.001, size = 53, normalized size = 0.7

$$\frac{4}{15}\left(1+2\sqrt{x}\right)\left(1+x+\sqrt{x}\right)^{-\frac{5}{2}} + \frac{64}{135}\left(1+2\sqrt{x}\right)\left(1+x+\sqrt{x}\right)^{-\frac{3}{2}} + \frac{512}{405}\left(1+2\sqrt{x}\right)\frac{1}{\sqrt{1+x+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x)
```

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^(
3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{x}+1\right)^{\frac{7}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

Fricas [A] time = 1.68286, size = 267, normalized size = 3.51

$$-\frac{4\left(128\,x^5+272\,x^4+455\,x^3+232\,x^2-\left(256\,x^5+736\,x^4+1366\,x^3+1427\,x^2+839\,x+101\right)\sqrt{x}-128\,x-203\right)\sqrt{x+}}{405\left(x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^4 + 1366*x
^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqrt(x + sqrt(x) + 1)/(
x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10589, size = 61, normalized size = 0.8

$$\frac{4\left(2\left(8\left(2\left(4\sqrt{x}\left(2\sqrt{x}+5\right)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")
```

```
[Out] 4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 355
)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

3.939 $\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Rubi [A] time = 0.0857062, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 277, 215, 1591, 190, 43}

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}\right)dx\\ &= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\ &= -\frac{1}{x}-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\ &= -\frac{1}{x}+\frac{\sqrt{1+x^2}}{x}-\int \frac{1}{\sqrt{1+x^2}}\,dx+\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx,x,\sqrt{1+x^2}\right)\\ &= -\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0348584, size = 46, normalized size = 1.

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Maple [A] time = 0.003, size = 53, normalized size = 1.2

$$-x^{-1}+\sqrt{x^2+1}-\operatorname{Artanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{1}{x}\left(x^2+1\right)^{\frac{3}{2}}-x\sqrt{x^2+1}-\operatorname{Arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -1/x+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-ln(x)+1/x*(x^2+1)^(3/2)-x*(x^2+
1)^(1/2)-arcsinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^2-\frac{1}{2}x-\int \frac{x^3-x^2}{2\left(x^2+2\sqrt{x^2+1}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x)
```

Fricas [A] time = 1.71427, size = 171, normalized size = 3.72

$$\frac{x\log\left(2\,x^2-\sqrt{x^2+1}(2\,x+1)+x+1\right)-x\log\left(x\right)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}(x+1)+x-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="fricas")
```

```
[Out] (x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sqr
t(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

Sympy [A] time = 2.22857, size = 48, normalized size = 1.04

$$\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\operatorname{asinh}\left(x\right)-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x**2+1)**(1/2)+1),x)
```

```
[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1/
x + 1/(x*sqrt(x**2 + 1))
```

Giac [A] time = 1.09697, size = 107, normalized size = 2.33

$$\sqrt{x^2+1}-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 + 1
)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2
 + 1) - 1))
```

3.940 $\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=20

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

```
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rubi [A] time = 0.0056524, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {651}

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]

[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

Mathematica [A] time = 0.030237, size = 23, normalized size = 1.15

$$\frac{3(x-1)\sqrt[3]{x+1}}{2\left(x^2-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]

[Out] (3*(-1 + x)*(1 + x)^(1/3))/(2*(-1 + x^2)^(2/3))
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{3x-3}{2}\sqrt[3]{1+x}\left(x^2-1\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(2/3)/(x^2-1)^(2/3),x)
```

```
[Out] 3/2*(x-1)*(1+x)^(1/3)/(x^2-1)^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-1\right)^{\frac{2}{3}} (x+1)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

Fricas [A] time = 1.68169, size = 47, normalized size = 2.35

$$\frac{3 \left(x^2-1\right)^{\frac{1}{3}}}{2 (x+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x-1)(x+1))^{\frac{2}{3}} (x+1)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
```

```
[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-1\right)^{\frac{2}{3}} (x+1)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.941 $\int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx$

Optimal. Leaf size=35

$$\frac{1}{5}x(1-x^6)^{2/3} - \frac{(1-x^6)^{2/3}}{5x^5}$$

```
[Out] -(1 - x^6)^(2/3)/(5*x^5) + (x*(1 - x^6)^(2/3))/5
```

Rubi [C] time = 0.0111609, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {245, 364}

$$x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right) - \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/(5*x^5) + x*Hypergeometric2F1[-2/3
, 1/6, 7/6, x^6]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx &= \int (1-x^6)^{2/3}\, dx + \int \frac{(1-x^6)^{2/3}}{x^6}\, dx\\ &= -\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)\end{aligned}$$

Mathematica [A] time = 0.007662, size = 18, normalized size = 0.51

$$-\frac{(1-x^6)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -(1 - x^6)^(5/3)/(5*x^5)
```

Maple [A] time = 0.008, size = 35, normalized size = 1.

$$\frac{(x-1)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{5x^5}\left(-x^6+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x)
```

```
[Out] 1/5*(-x^6+1)^(2/3)*(x^2-x+1)*(x^2+x+1)/x^5*(x-1)*(1+x)
```

Maxima [A] time = 1.68922, size = 51, normalized size = 1.46

$$\frac{\left(x^6-1\right)\left(x^2+x+1\right)^{\frac{2}{3}}\left(-x^2+x-1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x - 1
)^(2/3)/x^5
```

Fricas [A] time = 1.6479, size = 49, normalized size = 1.4

$$\frac{\left(x^6-1\right)\left(-x^6+1\right)^{\frac{2}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5
```

Sympy [C] time = 1.15054, size = 68, normalized size = 1.94

$$\frac{x\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle| x^6e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)}+\frac{\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{2}{3}\\ \frac{1}{6}\end{matrix}\middle| x^6e^{2i\pi}\right)}{6x^5\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

```
[Out] x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7/
6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*x*
*5*gamma(1/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^6+1\right)^{\frac{2}{3}} + \frac{\left(-x^6+1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)
```

3.942 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rubi [A] time = 0.0181481, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {12, 449}

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{2\left(a+bx^n\right)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{\left(a+bx^n\right)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [C] time = 0.177732, size = 111, normalized size = 7.4

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\left(b(2m-n)x^n\,{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)+2a(m+n)\,{}_2F_1\left(\frac{3}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
```

```
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^{-1+m}\left(2\,am+b\left(2\,m-n\right)x^{n}\right)}{2}\left(a+bx^{n}\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

```
[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

Maxima [A] time = 1.24732, size = 18, normalized size = 1.2

$$\frac{x^{m}}{\sqrt{bx^{n}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

Fricas [A] time = 1.53544, size = 39, normalized size = 2.6

$$\frac{xx^{m-1}}{\sqrt{bx^{n}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
fricas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b(2\,m - n)x^n + 2\,am)x^{m-1}}{2\,(bx^n + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

3.943 $\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx$

Optimal. Leaf size=53

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 -
 (4*(2 + 3*x)^(7/2))/567
```

Rubi [A] time = 0.0208054, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 772}

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 -
 (4*(2 + 3*x)^(7/2))/567
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx &= \int \frac{x\left(1-2x^2\right)}{\sqrt{2+3x}} \, dx \\
&= \int \left(-\frac{2}{27\sqrt{2+3x}}-\frac{5}{9}\sqrt{2+3x}+\frac{4}{9}(2+3x)^{3/2}-\frac{2}{27}(2+3x)^{5/2}\right) dx \\
&= -\frac{4}{81}\sqrt{2+3x}-\frac{10}{81}(2+3x)^{3/2}+\frac{8}{135}(2+3x)^{5/2}-\frac{4}{567}(2+3x)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.0250221, size = 28, normalized size = 0.53

$$-\frac{2\sqrt{3x+2}\left(270x^3-216x^2-123x+164\right)}{2835}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

Maple [A] time = 0.005, size = 25, normalized size = 0.5

$$-\frac{540 x^3 - 432 x^2 - 246 x + 328}{2835}\sqrt{2+3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^3+x)/(2+3*x)^(1/2),x)
```

```
[Out] -2/2835*(270*x^3-216*x^2-123*x+164)*(2+3*x)^(1/2)
```

Maxima [A] time = 1.0009, size = 50, normalized size = 0.94

$$-\frac{4}{567}(3x+2)^{\frac{7}{2}} + \frac{8}{135}(3x+2)^{\frac{5}{2}} - \frac{10}{81}(3x+2)^{\frac{3}{2}} - \frac{4}{81}\sqrt{3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

Fricas [A] time = 1.44073, size = 77, normalized size = 1.45

$$-\frac{2}{2835}\left(270 x^3 - 216 x^2 - 123 x + 164\right)\sqrt{3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

Sympy [A] time = 24.6686, size = 46, normalized size = 0.87

$$-\frac{4(3x+2)^{\frac{7}{2}}}{567} + \frac{8(3x+2)^{\frac{5}{2}}}{135} - \frac{10(3x+2)^{\frac{3}{2}}}{81} - \frac{4\sqrt{3x+2}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
```

```
[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81 -
 4*sqrt(3*x + 2)/81
```

Giac [A] time = 1.1018, size = 50, normalized size = 0.94

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

3.944 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=31

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rubi [A] time = 0.0152038, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2012, 1593, 266, 43}

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rule 2012

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx,x,1+x\right)$$
$$= \text{Subst}\left(\int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx,x,1+x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx,x,\sqrt[4]{1+x}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx,x,\sqrt[4]{1+x}\right)$$
$$= -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)$$

Mathematica [A] time = 0.011783, size = 31, normalized size = 1.

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Maple [A] time = 0.008, size = 26, normalized size = 0.8

$$-4\sqrt[4]{1+x}+4\ln\left(1+\sqrt[4]{1+x}\right)+2\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+x)^(1/4)+(1+x)^(1/2)),x)
```

```
[Out] -4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

Maxima [A] time = 1.1067, size = 34, normalized size = 1.1

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Fricas [A] time = 1.47412, size = 81, normalized size = 2.61

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Sympy [A] time = 0.220459, size = 27, normalized size = 0.87

$$-4\sqrt[4]{x+1}+2\sqrt{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
```

```
[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

Giac [A] time = 1.11456, size = 34, normalized size = 1.1

$$2\sqrt{x+1}-4(x+1)^{\frac{1}{4}}+4\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

3.945 $\int \frac{1+2x}{\sqrt{x+x^2}} dx$

Optimal. Leaf size=11

$$2\sqrt{x^2+x}$$

[Out] 2*Sqrt[x + x^2]

Rubi [A] time = 0.0030018, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {629}

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/Sqrt[x + x^2],x]

[Out] 2*Sqrt[x + x^2]

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{x+x^2}} dx = 2\sqrt{x+x^2}$$

Mathematica [A] time = 0.0054137, size = 11, normalized size = 1.

$$2\sqrt{x(x+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/Sqrt[x + x^2],x]

[Out] 2*Sqrt[x*(1 + x)]

Maple [A] time = 0.003, size = 14, normalized size = 1.3

$$2\,\frac{x(1+x)}{\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)/(x^2+x)^(1/2),x)

```
[Out] 2*x*(1+x)/(x^2+x)^(1/2)
```

Maxima [A] time = 0.980865, size = 12, normalized size = 1.09

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x^2 + x)
```

Fricas [A] time = 1.42915, size = 23, normalized size = 2.09

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + x)
```

Sympy [A] time = 0.123541, size = 8, normalized size = 0.73

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] 2*sqrt(x**2 + x)
```

Giac [A] time = 1.09767, size = 12, normalized size = 1.09

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x^2 + x)
```

3.946 $\int \frac{1}{2\sqrt{x}(1+x)} \, dx$

Optimal. Leaf size=6

$$\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0024988, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 63, 203}

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]*(1 + x)),x]

[Out] ArcTan[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sqrt{x}(1+x)} \, dx &= \frac{1}{2} \int \frac{1}{\sqrt{x}(1+x)} \, dx \\
&= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{x}\right) \\
&= \tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0030513, size = 6, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Maple [A] time = 0.003, size = 5, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)/x^(1/2),x)
```

```
[Out] arctan(x^(1/2))
```

Maxima [A] time = 1.52695, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))
```

Fricas [A] time = 1.50217, size = 23, normalized size = 3.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))
```

Sympy [A] time = 0.205102, size = 5, normalized size = 0.83

$$\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x**(1/2),x)
```

```
[Out] atan(sqrt(x))
```

Giac [A] time = 1.11072, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))
```

3.947 $\int \frac{1}{x\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sqrt{6x-x^2}}{3x}$$

```
[Out] -Sqrt[6*x - x^2]/(3*x)
```

Rubi [A] time = 0.0049185, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[6*x - x^2]),x]

[Out] -Sqrt[6*x - x^2]/(3*x)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6x-x^2}}{3x}$$

Mathematica [A] time = 0.0057691, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-(x-6)x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[6*x - x^2]),x]

[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$\frac{-6+x}{3}\frac{1}{\sqrt{-x^2+6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+6*x)^(1/2),x)
```

```
[Out] 1/3*(-6+x)/(-x^2+6*x)^(1/2)
```

Maxima [A] time = 1.4938, size = 22, normalized size = 1.1

$$-\frac{\sqrt{-x^2+6x}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Fricas [A] time = 1.45785, size = 34, normalized size = 1.7

$$-\frac{\sqrt{-x^2+6x}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-x(x-6)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)
```

Giac [A] time = 1.11879, size = 34, normalized size = 1.7

$$\frac{2}{3\left(\frac{\sqrt{-x^2+6x}-3}{x-3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

3.948 $\int \left(1+\sqrt{x}\right) \sqrt{x} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rubi [A] time = 0.0029993, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sqrt{x}\right) \sqrt{x} \, dx &= \int \left(\sqrt{x}+x\right) \, dx\\ &= \frac{2x^{3/2}}{3}+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.001945, size = 17, normalized size = 1.

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A] time = 0., size = 12, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1+x^(1/2)),x)
```

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Maxima [B] time = 1.00536, size = 35, normalized size = 2.06

$$\frac{1}{2}\left(\sqrt{x}+1\right)^{4}-\frac{4}{3}\left(\sqrt{x}+1\right)^{3}+\left(\sqrt{x}+1\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Fricas [A] time = 1.4953, size = 31, normalized size = 1.82

$$\frac{1}{2}x^{2}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Sympy [A] time = 0.124231, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

Giac [A] time = 1.12414, size = 15, normalized size = 0.88

$$\frac{1}{2}x^{2}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

3.949 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rubi [A] time = 0.0033615, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx&=\int \left(\frac{1}{\sqrt[3]{x}}-\sqrt[6]{x}\right)dx\\&=\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}\end{aligned}$$

Mathematica [A] time = 0.0033107, size = 19, normalized size = 1.

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A] time = 0., size = 12, normalized size = 0.6

$$\frac{3}{2}x^{\frac{2}{3}}-\frac{6}{7}x^{\frac{7}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Maxima [A] time = 1.11909, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Fricas [A] time = 1.39463, size = 38, normalized size = 2.

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A] time = 1.77602, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Giac [A] time = 1.13185, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

3.950 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Rubi [A] time = 0.010337, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {341, 50, 63, 203}

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{7/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= \frac{6x^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{x^{5/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{x^{3/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\ &= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0074931, size = 41, normalized size = 1.

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Maple [A] time = 0.001, size = 28, normalized size = 0.7

$$-6\sqrt[6]{x} - \frac{6}{5}x^{\frac{5}{6}} + \frac{6}{7}x^{\frac{7}{6}} + 6\arctan\left(\sqrt[6]{x}\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x^(1/3)),x)
```

```
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Maxima [A] time = 1.49019, size = 36, normalized size = 0.88

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

Fricas [A] time = 1.48156, size = 90, normalized size = 2.2

$$\frac{6}{7}(x-7)x^{\frac{1}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))
```

Sympy [A] time = 2.89894, size = 37, normalized size = 0.9

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + 2\sqrt{x} + 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(1/3)),x)
```

```
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**(1/6))
```

Giac [A] time = 1.12019, size = 36, normalized size = 0.88

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

3.951 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx$

Optimal. Leaf size=67

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rubi [A] time = 0.0302427, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 50, 57, 618, 204, 31}

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} + 2\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} - \frac{\log(x)}{2} - 3\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right) - 3\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2} + 6\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\sqrt{x}}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.0186368, size = 88, normalized size = 1.31

$$
6\sqrt[3]{\sqrt{x}+1} + 2\log\left(1-\sqrt[3]{\sqrt{x}+1}\right) - \log\left(\left(\sqrt{x}+1\right)^{2/3} + \sqrt[3]{\sqrt{x}+1}+1\right) - 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 2*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + Sq
rt[x])^(2/3)]
```

Maple [A] time = 0.009, size = 64, normalized size = 1.

$$
6\sqrt[3]{1+\sqrt{x}} + 2\ln\left(\sqrt[3]{1+\sqrt{x}}-1\right) - \ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}} + \sqrt[3]{1+\sqrt{x}}+1\right) - 2\arctan\left(1/3\left(1+2\sqrt[3]{1+\sqrt{x}}\right)\sqrt{3}\right)\sqrt{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))^(1/3)/x,x)
```

```
[Out] 6*(1+x^(1/2))^(1/3)+2*ln((1+x^(1/2))^(1/3)-1)-ln((1+x^(1/2))^(2/3)+(1+x^(1/
2))^(1/3)+1)-2*arctan(1/3*(1+2*(1+x^(1/2))^(1/3))*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.47563, size = 85, normalized size = 1.27

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log((sqrt(
x) + 1)^(1/3) - 1)
```

Fricas [A] time = 1.79766, size = 238, normalized size = 3.55

$$-2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\sqrt{x}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*arctan(2/3*sqrt(3)*(sqrt(x) + 1)^(1/3) + 1/3*sqrt(3)) + 6*(sqrt(
x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(
(sqrt(x) + 1)^(1/3) - 1)
```

Sympy [C] time = 1.12071, size = 39, normalized size = 0.58

$$-\frac{2\sqrt[6]{x}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x)
)/gamma(2/3)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.952 $\int \left(1-\sqrt{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

[Out] x - (2*x^(3/2))/3

Rubi [A] time = 0.0011952, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3

Rubi steps

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.0008729, size = 11, normalized size = 1.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3

Maple [A] time = 0., size = 8, normalized size = 0.7

$$x-\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1-x^(1/2),x)

[Out] x-2/3*x^(3/2)

Maxima [A] time = 0.99302, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A] time = 1.69338, size = 24, normalized size = 2.18

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A] time = 0.054076, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

Giac [A] time = 1.22417, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

3.953 $\int \left(1-\sqrt[4]{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

```
[Out] x - (4*x^(5/4))/5
```

Rubi [A] time = 0.0012273, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^(1/4),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rubi steps

$$\int \left(1-\sqrt[4]{x}\right) dx = x-\frac{4x^{5/4}}{5}$$

Mathematica [A] time = 0.0015435, size = 11, normalized size = 1.

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^(1/4),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Maple [A] time = 0., size = 8, normalized size = 0.7

$$x-\frac{4}{5}x^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/4),x)
```

```
[Out] x-4/5*x^(5/4)
```

Maxima [A] time = 1.00362, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="maxima")
```

```
[Out] -4/5*x^(5/4) + x
```

Fricas [A] time = 1.48727, size = 24, normalized size = 2.18

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="fricas")
```

```
[Out] -4/5*x^(5/4) + x
```

Sympy [A] time = 0.053714, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/4),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

Giac [A] time = 1.12614, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="giac")
```

```
[Out] -4/5*x^(5/4) + x
```

3.954 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} dx$

Optimal. Leaf size=11

$$x - \frac{4x^{5/4}}{5}$$

```
[Out] x - (4*x^(5/4))/5
```

Rubi [A] time = 0.0015401, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {26}

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} dx &= \int \left(1-\sqrt[4]{x}\right) dx \\ &= x - \frac{4x^{5/4}}{5} \end{aligned}$$

Mathematica [A] time = 0.0003907, size = 11, normalized size = 1.

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Maple [C] time = 0.009, size = 44, normalized size = 4.

$$-\frac{4}{5}x^{\frac{5}{4}} + x + 2\ln\left(1+\sqrt[4]{x}\right) + 2\ln\left(\sqrt[4]{x}-1\right) - \ln(x-1) - \ln\left(-1+\sqrt{x}\right) + \ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/(1+x^(1/4)),x)
```

```
[Out] -4/5*x^(5/4)+x+2*ln(1+x^(1/4))+2*ln(x^(1/4)-1)-ln(x-1)-ln(-1+x^(1/2))+ln(1+
x^(1/2))
```

Maxima [A] time = 0.999827, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4/5*x^(5/4) + x
```

Fricas [A] time = 1.42162, size = 24, normalized size = 2.18

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")
```

```
[Out] -4/5*x^(5/4) + x
```

Sympy [A] time = 5.64295, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/(1+x**(1/4)),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

Giac [A] time = 1.20375, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")
```

```
[Out] -4/5*x^(5/4) + x
```

3.955 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x +
 b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.0254436, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1981, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x +
 b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx &= \int \frac{1}{\sqrt{ac+(bc+ad)x+bdx^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{4bd-x^2}\,dx, x, \frac{bc+ad+2bdx}{\sqrt{ac+(bc+ad)x+bdx^2}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0628196, size = 95, normalized size = 1.56

$$\frac{2\sqrt{a+bx}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] (2*Sqrt[b*c - a*d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
])
```

Maple [A] time = 0.006, size = 49, normalized size = 0.8

$$\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{ac+(ad+bc)x+bdx^2}\right)\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(d*x+c))^(1/2),x)
```

```
[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b*
d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54732, size = 435, normalized size = 7.13

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,\sqrt{bdx^2+ac+(bc+ad)x}(2\,bdx+bc+ad)\sqrt{bd}+8\left(b^2cd+abd^2\right)x\right)}{2\,bd}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(b
*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*d
+ a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c + a*d
)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a
```

```
*b*d^2)*x))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [A] time = 1.27431, size = 92, normalized size = 1.51

$$-\frac{\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}bc-\sqrt{bd}ad\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")

[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d)
```

3.956 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
- b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rubi [A] time = 0.0249964, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1981, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
- b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx &= \int \frac{1}{\sqrt{ac+(bc-ad)x-bdx^2}}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{-4bd-x^2}\,dx, x, \frac{bc-ad-2bdx}{\sqrt{ac+(bc-ad)x-bdx^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0741378, size = 94, normalized size = 1.45

$$\frac{2\sqrt{a+bx}\sqrt{ad+bc}\sqrt{\frac{b(c-dx)}{ad+bc}}\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad+bc}}\right)}{b\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] (2*Sqrt[b*c + a*d]*Sqrt[a + b*x]*Sqrt[(b*(c - d*x))/(b*c + a*d)]*ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[b*c + a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)]
)
```

Maple [A] time = 0.01, size = 55, normalized size = 0.9

$$\arctan\left(\sqrt{bd}\left(x-\frac{-ad+bc}{2\,bd}\right)\frac{1}{\sqrt{ac+(-ad+bc)\,x-bdx^2}}\right)\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(-d*x+c))^(1/2),x)
```

```
[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-b
*d*x^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51656, size = 439, normalized size = 6.75

$$\left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,\sqrt{-bdx^2+ac+(bc-ad)x}(2\,bdx-bc+ad)\sqrt{-bd}-8\left(b^2cd-abd^2\right)x\right)}{2\,bd},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqrt
(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^2*
c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*c -
 a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*c*d
```

```
- a*b*d^2)*x))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23893, size = 80, normalized size = 1.23

$$-\frac{\log\left(\left|bc - ad + 2\sqrt{-bd}\left(\sqrt{-bd}x - \sqrt{-bdx^2 + bcx - adx + ac}\right)\right|\right)}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(b*c - a*d + 2*sqrt(-b*d)*(sqrt(-b*d)*x - sqrt(-b*d*x^2 + b*c*x - a
*d*x + a*c))))/sqrt(-b*d)
```

3.957 $\int \frac{1}{\sqrt{x}(1-x^2)}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0065155, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0036707, size = 13, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 - x^2)),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A] time = 0.004, size = 10, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)+\operatorname{Artanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)/x^(1/2),x)

[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [B] time = 1.66338, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] time = 1.49295, size = 85, normalized size = 6.54

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] time = 0.394427, size = 26, normalized size = 2.

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/x**(1/2),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Giac [B] time = 1.13098, size = 30, normalized size = 2.31

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x} + 1\right) - \frac{1}{2}\,\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.958 $\int \frac{\sqrt{x}}{x-x^3}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0106321, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1584, 329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{x-x^3}\,dx &= \int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0032254, size = 13, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right) + \text{Artanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^3+x),x)
```

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [B] time = 1.66253, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] time = 1.46895, size = 85, normalized size = 6.54

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] time = 0.573867, size = 26, normalized size = 2.

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**3+x),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Giac [B] time = 1.16499, size = 30, normalized size = 2.31

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.959 $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3
])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rubi [A] time = 0.103707, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {634, 618, 206, 628}

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3
])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \frac{1}{2}\int \frac{1+\sqrt{3}+2x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx + \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$$

$$= \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right) + \left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(2-3\sqrt{3}\right)-x^2}\,dx, x, 1\right.$$

$$= \sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right) + \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)$$

Mathematica [A] time = 0.0925707, size = 72, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+\sqrt{3}x+x-\sqrt{3}+2\right) + \frac{\left(1+\sqrt{3}\right)\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{6\sqrt{3}-4}}\right)}{\sqrt{6\sqrt{3}-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4 +
 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

Maple [A] time = 0.013, size = 82, normalized size = 1.1

$$\frac{\ln\left(x\sqrt{3}+x^2-\sqrt{3}+x+2\right)}{2} + \frac{\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\text{Artanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right) + \frac{1}{\sqrt{-4+6\sqrt{3}}}\text{Artanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x)
```

```
[Out] 1/2*ln(x*3^(1/2)+x^2-3^(1/2)+x+2)+1/(-4+6*3^(1/2))^(1/2)*arctanh((1+2*x+3^(
1/2))/(-4+6*3^(1/2))^(1/2))*3^(1/2)+1/(-4+6*3^(1/2))^(1/2)*arctanh((1+2*x+3
^(1/2))/(-4+6*3^(1/2))^(1/2))
```

Maxima [A] time = 1.83463, size = 104, normalized size = 1.44

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1}{2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1}\right)}{2\sqrt{6\sqrt{3}-4}} + \frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/(2*x + sqr
t(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(x^2 + x*(sqr
```

```
t(3) + 1) - sqrt(3) + 2)
```

Fricas [A] time = 1.58244, size = 320, normalized size = 4.44

$$\frac{1}{46}\sqrt{23}\sqrt{8\sqrt{3}+13}\log\left(-\frac{\sqrt{23}\sqrt{8\sqrt{3}+13}\left(5\sqrt{3}-11\right)-46x-23\sqrt{3}-23}{\sqrt{23}\sqrt{8\sqrt{3}+13}\left(5\sqrt{3}-11\right)+46x+23\sqrt{3}+23}\right)+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log(-(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*s
qrt(3) - 11) - 46*x - 23*sqrt(3) - 23)/(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*sq
rt(3) - 11) + 46*x + 23*sqrt(3) + 23)) + 1/2*log(x^2 + x*(sqrt(3) + 1) - sq
rt(3) + 2)
```

Sympy [B] time = 1.0805, size = 168, normalized size = 2.33

$$\left(\frac{1}{2}-\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\log\left(x-\frac{-521+287\sqrt{3}}{11+64\sqrt{3}}+\frac{\left(\frac{1}{2}-\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\left(269+459\sqrt{3}\right)}{214+139\sqrt{3}}\right)+\left(\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}+\frac{1}{2}\right)\log\left(x+-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*log(x - (-521 + 287*sq
rt(3))/(11 + 64*sqrt(3)) + (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3
))))*(269 + 459*sqrt(3))/(214 + 139*sqrt(3))) + (sqrt(11 + 64*sqrt(3))/(2*(
-31 + 12*sqrt(3))) + 1/2)*log(x + (269 + 459*sqrt(3))*(sqrt(11 + 64*sqrt(3)
)/(2*(-31 + 12*sqrt(3))) + 1/2)/(214 + 139*sqrt(3)) - (-521 + 287*sqrt(3))/
(11 + 64*sqrt(3)))
```

Giac [A] time = 1.19527, size = 108, normalized size = 1.5

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{\left|2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1\right|}{\left|2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1\right|}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(3) + 1)*log(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/abs(2*x
 + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(abs(x^
2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

3.960 $\int \sqrt{x^2+x^3}\,dx$

Optimal. Leaf size=37

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rubi [A] time = 0.026565, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2002, 2014}

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3],x]
```

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^2+x^3}\,dx &= \frac{2\left(x^2+x^3\right)^{3/2}}{5x^2}-\frac{2}{5}\int \frac{\sqrt{x^2+x^3}}{x}\,dx\\ &= -\frac{4\left(x^2+x^3\right)^{3/2}}{15x^3}+\frac{2\left(x^2+x^3\right)^{3/2}}{5x^2}\end{aligned}$$

Mathematica [A] time = 0.0084559, size = 23, normalized size = 0.62

$$\frac{2\left(x^2(x+1)\right)^{3/2}(3x-2)}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3],x]
```

```
[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

Maple [A] time = 0.002, size = 23, normalized size = 0.6

$$\frac{(2+2x)(3x-2)}{15x}\sqrt{x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/2),x)
```

```
[Out] 2/15*(1+x)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

Maxima [A] time = 1.03624, size = 20, normalized size = 0.54

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Fricas [A] time = 1.42531, size = 54, normalized size = 1.46

$$\frac{2\sqrt{x^3+x^2}\left(3x^2+x-2\right)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**3 + x**2), x)
```

Giac [A] time = 1.15465, size = 32, normalized size = 0.86

$$\frac{2}{15}\left(3(x+1)^{\frac{5}{2}}-5(x+1)^{\frac{3}{2}}\right)\mathrm{sgn}(x)+\frac{4}{15}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x + 1)^(5/2) - 5*(x + 1)^(3/2))*sgn(x) + 4/15*sgn(x)
```

3.961 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rubi [A] time = 0.0073857, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.0095188, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]
```

Maple [A] time = 0., size = 13, normalized size = 1.1

$$-\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x)^(1/2),x)
```

```
[Out] -arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A] time = 1.50601, size = 12, normalized size = 1.

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

Fricas [A] time = 1.48203, size = 49, normalized size = 4.08

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(x+2)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

Giac [A] time = 1.14065, size = 23, normalized size = 1.92

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.962 $\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/
12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[
5]])/64
```

Rubi [A] time = 0.0541581, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1357, 742, 640, 612, 619, 216}

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```



```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/
12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[
5]])/64
```


Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx &= 2\,\text{Subst}\left(\int x^2\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{2}\,\text{Subst}\left(\int\left(-1+\frac{5x}{2}\right)\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{9}{8}\,\text{Subst}\left(\int\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\,\text{Subst}\left(\int\frac{1}{\sqrt{1-x}}\right.\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{64}\left(9\sqrt{5}\right)\text{Subst}\left(\int\right.\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0350549, size = 60, normalized size = 0.63

$$\frac{1}{96}\sqrt{-x-\sqrt{x}+1}\left(48x^{3/2}+8x-34\sqrt{x}+67\right)-\frac{45}{64}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (45*ArcSi
n[(-1 - 2*Sqrt[x])/Sqrt[5]])/64
```

Maple [A] time = 0.003, size = 67, normalized size = 0.7

$$-\frac{1}{2}\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}+\frac{5}{12}\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}-\frac{9}{32}\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}+\frac{45}{64}\arcsin\left(\frac{2\sqrt{5}}{5}\left(\sqrt{x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1-x-x^(1/2))^(1/2),x)
```

```
[Out] -1/2*(1-x-x^(1/2))^(3/2)*x^(1/2)+5/12*(1-x-x^(1/2))^(3/2)-9/32*(-2*x^(1/2)-
1)*(1-x-x^(1/2))^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

Fricas [A] time = 8.25083, size = 250, normalized size = 2.63

$$\frac{1}{96}\left(2\,(24\,x-17)\sqrt{x}+8\,x+67\right)\sqrt{-x-\sqrt{x}+1}-\frac{45}{128}\arctan\left(-\frac{\left(8\,x^2-\left(16\,x^2-38\,x+11\right)\sqrt{x}-9\,x+3\right)\sqrt{-x-\sqrt{x}+}}{4\left(4\,x^3-13\,x^2+7\,x-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) - 45/128*arc
tan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x)
 + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-sqrt(x) - x + 1), x)
```

Giac [A] time = 1.15576, size = 69, normalized size = 0.73

$$\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+1\right)-17\right)\sqrt{x}+67\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x - sqrt(x) +
1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

3.963 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

Optimal. Leaf size=35

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rubi [A] time = 0.0111444, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 190, 43}

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{1+\sqrt{-3+x}}\,dx &= \text{Subst}\left(\int \sqrt[3]{1+\sqrt{x}}\,dx, x, -3+x\right)\\
&= 2\,\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt{-3+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)dx, x, \sqrt{-3+x}\right)\\
&= -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}
\end{aligned}$$

Mathematica [A] time = 0.0098442, size = 28, normalized size = 0.8

$$\frac{3}{14}\left(\sqrt{x-3}+1\right)^{4/3}\left(4\sqrt{x-3}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

Maple [A] time = 0.002, size = 24, normalized size = 0.7

$$-\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{\frac{4}{3}}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-3+x)^(1/2))^(1/3),x)
```

```
[Out] -3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

Maxima [A] time = 0.996148, size = 31, normalized size = 0.89

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

Fricas [A] time = 1.49347, size = 74, normalized size = 2.11

$$\frac{3}{14}\left(4x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

Sympy [B] time = 1.02531, size = 184, normalized size = 5.26

$$\frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^{2}}-\frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^{2}}+\frac{9(x-3)^{\frac{5}{2}}}{14(x-3)^{\frac{5}{2}}+14(x-3)^{2}}+\frac{15(x-3)^{3}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^{2}}-\frac{9(x-}{14(x-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)**(1/2))**(1/3),x)
```

```
[Out] 12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)*
*2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x
- 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x - 3
)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x -
3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x -
 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)
```

Giac [A] time = 1.23988, size = 31, normalized size = 0.89

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

3.964 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rubi [A] time = 0.0133593, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {247, 190, 43}

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{x}}}\,dx,x,-1+2x\right)\\ &= \operatorname{Subst}\left(\int \frac{x}{\sqrt{3+x}}\,dx,x,\sqrt{-1+2x}\right)\\ &= \operatorname{Subst}\left(\int \left(-\frac{3}{\sqrt{3+x}}+\sqrt{3+x}\right)dx,x,\sqrt{-1+2x}\right)\\ &= -6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0103312, size = 30, normalized size = 0.81

$$\frac{2}{3}\left(\sqrt{2x-1}-6\right)\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

Maple [A] time = 0.004, size = 28, normalized size = 0.8

$$\frac{2}{3}\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}-6\sqrt{3+\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

Maxima [A] time = 1.07937, size = 36, normalized size = 0.97

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

Fricas [A] time = 1.43197, size = 66, normalized size = 1.78

$$\frac{2}{3}\sqrt{\sqrt{2x-1}+3}\left(\sqrt{2x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

Sympy [B] time = 0.960737, size = 265, normalized size = 7.16

$$-\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}-\frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^{2}\sqrt{\cdots}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] -6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x -
 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x - 1/2)**(5/2)/(3*sqr
t(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 4*sqrt(3)*(x - 1/2)**3*sq
rt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x -
1/2)**2) - 36*sqrt(3)*(x - 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(
6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*sqrt(6)
*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

Giac [A] time = 1.20493, size = 43, normalized size = 1.16

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}+4\sqrt{3}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) + 4*sqrt(3) - 6*sqrt(sqrt(2*x - 1) + 3)
```

3.965 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}} \, dx$

Optimal. Leaf size=29

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

```
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rubi [A] time = 0.0258106, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1398, 785, 780, 216}

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

```
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1+x}\,dx, x, \sqrt{x}\right)\\
&= 2\,\mathrm{Subst}\left(\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(2-\sqrt{x}\right)\sqrt{1-x} - \mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(2-\sqrt{x}\right)\sqrt{1-x} - \sin^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0269454, size = 26, normalized size = 0.9

$$\left(\sqrt{x}-2\right)\sqrt{1-x} - \sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

```
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] - ArcSin[Sqrt[x]]
```

Maple [B] time = 0.005, size = 48, normalized size = 1.7

$$-\frac{1}{2}\sqrt{1-x}\sqrt{x}\left(-2\sqrt{-x(x-1)}+\arcsin(2x-1)\right)\frac{1}{\sqrt{-x(x-1)}}-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x^(1/2)),x)
```

```
[Out] -1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-x*(x-1))^(1/2)+arcsin(2*x-1))/(-x*(x-1))^(1/
2)-2*(1-x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}}{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

Fricas [A] time = 1.44996, size = 95, normalized size = 3.28

$$\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}+\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [C] time = 1.71087, size = 32, normalized size = 1.1

$$i\sqrt{x}\sqrt{x-1} - 2i\sqrt{x-1} + i\operatorname{asinh}\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x**(1/2)),x)
```

```
[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

Giac [A] time = 1.15073, size = 39, normalized size = 1.34

$$\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

3.966 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rubi [A] time = 0.0252974, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1398, 785, 780, 216}

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1-x}\,dx, x, \sqrt{x}\right)\\ &= -\left(2\,\text{Subst}\left(\int \frac{(-1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\right)\\ &= -\left(2+\sqrt{x}\right)\sqrt{1-x} + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\left(2+\sqrt{x}\right)\sqrt{1-x} + \sin^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.029478, size = 26, normalized size = 1.04

$$\sqrt{1-x}\left(-\sqrt{x}-2\right)+\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + ArcSin[Sqrt[x]]
```

Maple [B] time = 0.003, size = 48, normalized size = 1.9

$$-2\sqrt{1-x}+\frac{1}{2}\sqrt{1-x}\sqrt{x}\left(-2\sqrt{-x(x-1)}+\arcsin(2x-1)\right)\frac{1}{\sqrt{-x(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1-x^(1/2)),x)
```

```
[Out] -2*(1-x)^(1/2)+1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-x*(x-1))^(1/2)+arcsin(2*x-1))/
(-x*(x-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-x+1}}{\sqrt{x}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)
```

Fricas [A] time = 1.45369, size = 96, normalized size = 3.84

$$-\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A] time = 2.71859, size = 87, normalized size = 3.48

$$2\left(\begin{cases} -\sqrt{1-x} + \frac{i\operatorname{acosh}\left(\sqrt{1-x}\right)}{2} - \frac{i(1-x)^{\frac{3}{2}}}{2\sqrt{-x}} + \frac{i\sqrt{1-x}}{2\sqrt{-x}} & \text{for } |x-1| > 1 \\ \frac{\sqrt{x}\sqrt{1-x}}{2} - \sqrt{1-x} + \frac{\operatorname{asin}\left(\sqrt{1-x}\right)}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1-x**(1/2)),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x) + I*acosh(sqrt(1 - x))/2 - I*(1 - x)**(3/2)/(2*sq
rt(-x)) + I*sqrt(1 - x)/(2*sqrt(-x)), Abs(x - 1) > 1), (sqrt(x)*sqrt(1 - x)
/2 - sqrt(1 - x) + asin(sqrt(1 - x))/2, True))
```

Giac [A] time = 1.17513, size = 43, normalized size = 1.72

$$-\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="giac")
```

```
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))
```

3.967 $\int \frac{x}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0230596, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2106, 30, 261}

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-\sqrt{1+x^2}}\,dx &= -\int x^2\,dx-\int x\sqrt{1+x^2}\,dx\\ &= -\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0238108, size = 21, normalized size = 1.

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(x^2+1)^(1/2)),x)
```

```
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x-\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(x^2 + 1)), x)
```

Fricas [A] time = 1.4724, size = 43, normalized size = 2.05

$$-\frac{1}{3}x^3-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] time = 0.359005, size = 56, normalized size = 2.67

$$\frac{2x^2}{3x-3\sqrt{x^2+1}}-\frac{x\sqrt{x^2+1}}{3x-3\sqrt{x^2+1}}+\frac{1}{3x-3\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1))
 + 1/(3*x - 3*sqrt(x**2 + 1))
```

Giac [A] time = 1.20324, size = 20, normalized size = 0.95

$$-\frac{1}{3}x^3 - \frac{1}{3}\left(x^2 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

3.968 $\int \frac{x}{x-\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0535978, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2107, 321, 206, 444, 50, 63, 207}

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/(2*Sqrt[2])
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{x-\sqrt{1-x^2}}\,dx &= -\int \frac{x^2}{1-2x^2}\,dx-\int \frac{x\sqrt{1-x^2}}{1-2x^2}\,dx\\
&= \frac{x}{2}-\frac{1}{2}\int \frac{1}{1-2x^2}\,dx-\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{1-2x}\,dx,x,x^2\right)\\
&= \frac{x}{2}+\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(1-2x)\sqrt{1-x}}\,dx,x,x^2\right)\\
&= \frac{x}{2}+\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{x}{2}+\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0525904, size = 54, normalized size = 0.83

$$\frac{1}{4}\left(2\left(\sqrt{1-x^2}+x\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{2-2x^2}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{2}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] (2*(x + Sqrt[1 - x^2]) - Sqrt[2]*ArcTanh[Sqrt[2]*x] - Sqrt[2]*ArcTanh[Sqrt[
2 - 2*x^2]])/4
```

Maple [B] time = 0.01, size = 175, normalized size = 2.7

$$\frac{x}{2}-\frac{\operatorname{Artanh}\left(x\sqrt{2}\right)\sqrt{2}}{4}+\frac{1}{8}\sqrt{-4\left(x+1/2\sqrt{2}\right)^2+4\left(x+1/2\sqrt{2}\right)\sqrt{2}+2}-\frac{\sqrt{2}}{8}\operatorname{Artanh}\left(\sqrt{2}\left(1+\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\frac{}{\sqrt{-4\left(x\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x-1/4*arctanh(x*2^(1/2))*2^(1/2)+1/8*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(
1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh((1+(x+1/2*2^(1/2))*2^(1/2))*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*(-4*(x-1/
2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh((1-(x-1
/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2
)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x - \sqrt{-x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)
```

Fricas [B] time = 1.4693, size = 252, normalized size = 3.88

$$\frac{1}{8}\sqrt{2}\log\left(\frac{6x^2 - 2\sqrt{2}(2x^2 - 3) + 2\sqrt{-x^2 + 1}(3\sqrt{2} - 4) - 9}{2x^2 - 1}\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{2x^2 - 2\sqrt{2}x + 1}{2x^2 - 1}\right) + \frac{1}{2}x + \frac{1}{2}\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2
) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x^2
 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x - \sqrt{1 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(1 - x**2)), x)
```

Giac [B] time = 1.2391, size = 142, normalized size = 2.18

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|4x-2\sqrt{2}\right|}{\left|4x+2\sqrt{2}\right|}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*lo
g(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqr
t(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

3.969 $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=31

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rubi [A] time = 0.0423509, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2107, 321, 203, 444, 50, 63}

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-\sqrt{1+2x^2}}\,dx &= -\int \frac{x^2}{1+x^2}\,dx - \int \frac{x\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -x - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+2x}}{1+x}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+2x}}\,dx, x, x^2\right)\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{1+2x^2}\right)\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \tan^{-1}\left(\sqrt{1+2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.032193, size = 31, normalized size = 1.

$$-\sqrt{2x^2+1} + \tan^{-1}\left(\sqrt{2x^2+1}\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Maple [A] time = 0.007, size = 28, normalized size = 0.9

$$-x + \arctan(x) + \arctan\left(\sqrt{2\,x^2+1}\right) - \sqrt{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x-\sqrt{2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

Fricas [A] time = 1.44933, size = 104, normalized size = 3.35

$$-x-\sqrt{2\,x^2+1}+\arctan\left(x\right)-\arctan\left(-\frac{x^2-\sqrt{2\,x^2+1}+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x-\sqrt{2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

Giac [B] time = 1.16244, size = 85, normalized size = 2.74

$$-\frac{1}{2}\,\pi-x-\sqrt{2\,x^2+1}+\arctan\left(x\right)+\arctan\left(-\frac{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2+1}{2\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)*x - sqrt(
2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

3.970 $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sq
rt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]])/32
```

Rubi [A] time = 0.0436464, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sq
rt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]])/32
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int x^2\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{4}\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} + \frac{5}{8}\text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{32}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0651486, size = 58, normalized size = 0.71

$$\frac{1}{96}\sqrt{x+\sqrt{x}}\left(48x^{3/2}+8x-10\sqrt{x}-\frac{15\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}+15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2) - (15*ArcSinh[x^(1/4
)])/(Sqrt[1 + Sqrt[x]]*x^(1/4))))/96
```

Maple [A] time = 0.003, size = 54, normalized size = 0.7

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{\frac{3}{2}}-\frac{5}{12}\left(x+\sqrt{x}\right)^{\frac{3}{2}}+\frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}-\frac{5}{64}\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x+x^(1/2))^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(x+x^(1/2))^(3/2)-5/12*(x+x^(1/2))^(3/2)+5/32*(1+2*x^(1/2))*(x+
x^(1/2))^(1/2)-5/64*ln(x^(1/2)+1/2+(x+x^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{x}}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x))*sqrt(x), x)
```

Fricas [A] time = 4.61749, size = 174, normalized size = 2.12

$$\frac{1}{96}\left(2\,(24\,x-5)\sqrt{x}+8\,x+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{128}\,\log\left(4\,\sqrt{x+\sqrt{x}}\left(2\,\sqrt{x}+1\right)-8\,x-8\,\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*log(4*sqrt
(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) + x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.971 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=74

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Rubi [A] time = 0.111235, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx &= 6\,\text{Subst}\left(\int \frac{x^5+x^7}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \frac{x^5\left(1+x^2\right)}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(-x+x^2+x^4+\frac{(1-x)x}{1+x^3}\right)dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+6\,\text{Subst}\left(\int \frac{(1-x)x}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+2\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-4\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)+3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-\text{Subst}\Bigg(\int \frac{-1+2x}{1-x+x^2}\,dx, x,\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)-6\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0368795, size = 74, normalized size = 1.

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Maple [A] time = 0.007, size = 56, normalized size = 0.8

$$\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3\sqrt[3]{x}-\ln\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)+2\sqrt{3}\arctan\left(1/3\left(2\sqrt[6]{x}-1\right)\sqrt{3}\right)-4\ln\left(1+\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/3))/(1+x^(1/2)),x)
```

```
[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-ln(1-x^(1/6)+x^(1/3))+2*3^(1/2)*arctan(1/3*
(2*x^(1/6)-1)*3^(1/2))-4*ln(1+x^(1/6))
```

Maxima [A] time = 1.49011, size = 74, normalized size = 1.

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Fricas [A] time = 1.49043, size = 190, normalized size = 2.57

$$2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt(
x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Sympy [C] time = 2.78487, size = 155, normalized size = 2.09

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)}-\frac{8\sqrt[3]{x}\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}+2\sqrt{x}-2\log\left(\sqrt{x}+1\right)-\frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16\log\left(-\sqrt[6]{x}e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{\frac{2i\pi}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

```
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
+ 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_pola
r(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi
) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_pola
r(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))
```

Giac [A] time = 1.15059, size = 74, normalized size = 1.

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

3.972 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}} \, dx$

Optimal. Leaf size=115

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan$$

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/1
2))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]
```

Rubi [A] time = 0.149687, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1593, 1836, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/1
2))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx &= 12\,\text{Subst}\left(\int \frac{x^{11}+x^{15}}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\,\text{Subst}\left(\int \frac{x^{11}\left(1+x^4\right)}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= \frac{12x^{13/12}}{13}+\frac{12}{13}\text{Subst}\left(\int \frac{(13-13x)x^{11}}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= \frac{12x^{13/12}}{13}+\frac{12}{13}\text{Subst}\left(\int \left(13+13x^2-13x^3-13x^5+13x^6+13x^8-13x^9-\frac{13\left(1+x^2\right)}{1+x^3}\right)dx, x, \sqrt[12]{x}\right)\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}-12\,\text{Subst}\left(\int \frac{1+x^2}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}-4\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[12]{x}\right)-\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}-8\log\left(1+\sqrt[12]{x}\right)-2\,\text{Subst}\left(\int \frac{-1+}{1-x}\right.\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}-8\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{}\right.\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)-8\log\left(1+\right.
\end{aligned}$$

Mathematica [A] time = 0.0806612, size = 123, normalized size = 1.07

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}+4\left(\sqrt[3]{-1}-1\right)\log\left(\sqrt[3]{-1}-\sqrt[12]{x}\right)-4\left(1+(-1)^{2/3}\right)\log\left(-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*(-1 + (-1)^(1/3))*Log[(-1)^(
1/3) - x^(1/12)] - 4*(1 + (-1)^(2/3))*Log[-(-1)^(2/3) - x^(1/12)] - 8*Log[1
 + x^(1/12)]
```

Maple [A] time = 0.006, size = 81, normalized size = 0.7

$$\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12x^{1/12}-2\ln\left(1-x^{1/12}+\sqrt[6]{x}\right)-4\sqrt{3}\arctan\left(1/3\left(2x^{1/12}-1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/3))/(1+x^(1/4)),x)
```

```
[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+4
*x^(1/4)+12*x^(1/12)-2*ln(1-x^(1/12)+x^(1/6))-4*3^(1/2)*arctan(1/3*(2*x^(1/
12)-1)*3^(1/2))-8*ln(1+x^(1/12))
```

Maxima [A] time = 1.67745, size = 108, normalized size = 0.94

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Fricas [A] time = 1.46299, size = 290, normalized size = 2.52

$$-4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{12}{13}(x+13)x^{\frac{1}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(1
/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) +
4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Sympy [C] time = 4.03954, size = 221, normalized size = 1.92

$$\frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)}+\frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)}+\frac{64\sqrt[12]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-\frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{16\sqrt[3]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-2\sqrt{x}-4\log\left(\sqrt[4]{x}+1\right)+\frac{64e^{-\frac{i\pi}{3}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

```
[Out] 64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(16/3)/(7*ga
mma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*x**(5/6)*gamma(16/3)
/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4) - 16*x**(1/3)*gamma(16/3)/gamm
a(19/3) - 2*sqrt(x) - 4*log(x**(1/4) + 1) + 64*exp(-I*pi/3)*log(-x**(1/12)*
exp_polar(I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)*exp_
polar(I*pi) + 1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x**(1/12
)*exp_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

Giac [A] time = 1.1758, size = 108, normalized size = 0.94

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

3.973 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=4

$$x+\sin^{-1}(x)$$

```
[Out] x + ArcSin[x]
```

Rubi [A] time = 0.0428694, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2156, 8, 216}

$$x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] x + ArcSin[x]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx &= -\int -1\,dx+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0588855, size = 4, normalized size = 1.

$$x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] x + ArcSin[x]
```

Maple [B] time = 0.006, size = 51, normalized size = 12.8

$$x+\frac{\ln \left(x -1\right)}{2}-\frac{\ln \left(1+x \right)}{2}+\operatorname{Artanh}\left(x \right)-\frac{1}{2}\sqrt{-\left(x -1\right)^{2}-2 x +2}+\arcsin \left(x \right)+\frac{1}{2}\sqrt{-\left(1+x \right)^{2}+2 x +2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] x+1/2*ln(x-1)-1/2*ln(1+x)+arctanh(x)-1/2*(-(x-1)^2-2*x+2)^(1/2)+arcsin(x)+1
/2*(-(1+x)^2+2*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{x^{2}+\sqrt{-x^{2}+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

Fricas [B] time = 1.43393, size = 51, normalized size = 12.75

$$x-2 \arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{x^{2}+\sqrt{1-x^{2}}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)
```

Giac [A] time = 1.21582, size = 5, normalized size = 1.25

$$x+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x + arcsin(x)
```

3.974 $\int \sqrt{\frac{1+x}{x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{\frac{1}{x}+1}x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi [A] time = 0.0095616, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1972, 242, 47, 63, 207}

$$\sqrt{\frac{1}{x}+1}x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x}}\,dx &= \int \sqrt{1+\frac{1}{x}}\,dx\\ &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}x - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\\ &= \sqrt{1+\frac{1}{x}}x + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0066558, size = 22, normalized size = 1.

$$\sqrt{\frac{1}{x}+1}x + \tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [B] time = 0.005, size = 41, normalized size = 1.9

$$\frac{x}{2}\sqrt{\frac{1+x}{x}}\left(2\sqrt{x^2+x}+\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/x)^(1/2),x)
```

```
[Out] 1/2*((1+x)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(1/2+x+(x^2+x)^(1/2)))/(x*(1+x))^(
1/2)
```

Maxima [B] time = 1.10705, size = 68, normalized size = 3.09

$$\frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sq
rt((x + 1)/x) - 1)
```

Fricas [B] time = 1.471, size = 109, normalized size = 4.95

$$x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
- 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x+1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x), x)
```

Giac [A] time = 1.21978, size = 42, normalized size = 1.91

$$-\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x)+\sqrt{x^2+x}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

3.975 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rubi [A] time = 0.0089003, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 47, 63, 203}

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1-x}{x}}\,dx &= \int \sqrt{-1+\frac{1}{x}}\,dx\\ &= -\mathrm{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{-1+\frac{1}{x}}x - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{-1+\frac{1}{x}}x - \mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right)\\ &= \sqrt{-1+\frac{1}{x}}x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0074142, size = 24, normalized size = 1.

$$\sqrt{\frac{1}{x}-1}x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Maple [A] time = 0.004, size = 40, normalized size = 1.7

$$\frac{x}{2}\sqrt{-\frac{x-1}{x}}\left(2\sqrt{-x^2+x}+\arcsin(2x-1)\right)\frac{1}{\sqrt{-x(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(x-1))^(1/2)
```

Maxima [A] time = 1.63696, size = 50, normalized size = 2.08

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Fricas [A] time = 1.47866, size = 63, normalized size = 2.62

$$x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{1-x}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/x), x)
```

Giac [A] time = 1.17534, size = 38, normalized size = 1.58

$$\frac{1}{4}\,\pi\mathrm{sgn}\left(x\right) + \frac{1}{2}\,\arcsin\left(2\,x - 1\right)\mathrm{sgn}\left(x\right) + \sqrt{-x^2 + x}\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

3.976 $\int \sqrt{\frac{-1+x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{x-1}\sqrt{x}-\sinh^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] Sqrt[-1 + x]*Sqrt[x] - ArcSinh[Sqrt[-1 + x]]
```

Rubi [A] time = 0.0112379, antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1972, 242, 47, 63, 206}

$$\sqrt{\frac{x-1}{x}}x-\tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-1+x}{x}}\,dx &= \int \sqrt{1-\frac{1}{x}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}x + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}x - \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-1+x}{x}}\right) \\
&= \sqrt{\frac{-1+x}{x}}x - \tanh^{-1}\left(\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0137291, size = 38, normalized size = 1.58

$$\frac{\sqrt{x}(x-1)+\sqrt{1-x}\sin^{-1}\left(\sqrt{1-x}\right)}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/x],x]
```

```
[Out] ((-1 + x)*Sqrt[x] + Sqrt[1 - x]*ArcSin[Sqrt[1 - x]])/Sqrt[-1 + x]
```

Maple [B] time = 0.005, size = 47, normalized size = 2.

$$\frac{x}{2}\sqrt{\frac{x-1}{x}}\left(2\sqrt{x^2-x}-\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)\right)\frac{1}{\sqrt{x(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/x)^(1/2),x)
```

```
[Out] 1/2*((x-1)/x)^(1/2)*x*(2*(x^2-x)^(1/2)-ln(x-1/2+(x^2-x)^(1/2)))/(x*(x-1))^(
1/2)
```

Maxima [B] time = 1.05021, size = 69, normalized size = 2.88

$$-\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(s
qrt((x - 1)/x) - 1)
```

Fricas [B] time = 1.43424, size = 109, normalized size = 4.54

$$x\sqrt{\frac{x-1}{x}} - \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x)
- 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x-1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/x), x)
```

Giac [A] time = 1.14224, size = 47, normalized size = 1.96

$$\frac{1}{2}\,\log\left(\left|-2\,x + 2\,\sqrt{x^2 - x} + 1\right|\right)\mathrm{sgn}\left(x\right) + \sqrt{x^2 - x}\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)
```

3.977 $\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx$

Optimal. Leaf size=24

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi [A] time = 0.0170135, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 266, 50, 63, 207}

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x]/x,x]

[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 1973

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{x}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{1+\frac{1}{x}} - \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{1+\frac{1}{x}} - 2\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\\
&= -2\sqrt{1+\frac{1}{x}} + 2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0055733, size = 24, normalized size = 1.

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right) - 2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x]/x,x]
```

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [B] time = 0.004, size = 60, normalized size = 2.5

$$-\frac{1}{x}\sqrt{\frac{1+x}{x}}\left(2\left(x^2+x\right)^{3/2} - 2x^2\sqrt{x^2+x} - \ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)x^2\right)\frac{1}{\sqrt{x\left(1+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/x)^(1/2)/x,x)
```

```
[Out] -((1+x)/x)^(1/2)/x*(2*(x^2+x)^(3/2)-2*x^2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1
/2))*x^2)/(x*(1+x))^(1/2)
```

Maxima [A] time = 1.14745, size = 51, normalized size = 2.12

$$-2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}}+1\right) - \log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

Fricas [A] time = 1.47816, size = 100, normalized size = 4.17

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

Sympy [A] time = 3.05388, size = 32, normalized size = 1.33

$$-2\sqrt{1+\frac{1}{x}}-\log\left(\sqrt{1+\frac{1}{x}}-1\right)+\log\left(\sqrt{1+\frac{1}{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2)/x,x)
```

```
[Out] -2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)
```

Giac [A] time = 1.32662, size = 51, normalized size = 2.12

$$-\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x)+\frac{2\operatorname{sgn}(x)}{x-\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x))
```

3.978 $\int \sqrt{\frac{x}{1+x}} \, dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.004786, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{x}{1+x}}\,dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\
&= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.015082, size = 42, normalized size = 1.91

$$
\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

Maple [B] time = 0., size = 45, normalized size = 2.1

$$
\frac{1+x}{2}\sqrt{\frac{x}{1+x}}\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(1+x))^(1/2),x)
```

```
[Out] 1/2*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)))/(x*(1+x
))^(1/2)
```

Maxima [B] time = 1.11973, size = 69, normalized size = 3.14

$$
-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

Fricas [B] time = 1.43171, size = 117, normalized size = 5.32

$$
(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

Giac [B] time = 1.25616, size = 47, normalized size = 2.14

$$\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\mathrm{sgn}\left(x+1\right)+\sqrt{x^2+x}\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

3.979 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}} dx$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right) - x\sqrt{-\frac{x+1}{x}}$$

```
[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rubi [A] time = 0.0118638, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 51, 63, 204}

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right) - x\sqrt{-\frac{x+1}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-1 - x)/x],x]

[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx &= \int \frac{1}{\sqrt{-1-\frac{1}{x}}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-xx^2}}\,dx, x, \frac{1}{x}\right) \\
&= -x\sqrt{-\frac{1+x}{x}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-xx}}\,dx, x, \frac{1}{x}\right) \\
&= -x\sqrt{-\frac{1+x}{x}} - \operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{1+x}{x}}\right) \\
&= -x\sqrt{-\frac{1+x}{x}} + \tan^{-1}\left(\sqrt{-\frac{1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0132626, size = 43, normalized size = 1.48

$$\frac{\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\sqrt{-\frac{x+1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] (Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])/(Sqrt[x]*Sqrt[-((1 + x)/x)
])
```

Maple [A] time = 0.005, size = 44, normalized size = 1.5

$$\frac{1+x}{2}\left(2\sqrt{-x^2-x} + \arcsin(1+2x)\right)\frac{1}{\sqrt{-\frac{1+x}{x}}}\frac{1}{\sqrt{-x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(1+x)*(2*(-x^2-x)^(1/2)+arcsin(1+2*x))/(-(1+x)/x)^(1/2)/(-x*(1+x))^(1/2
)
```

Maxima [A] time = 1.61727, size = 47, normalized size = 1.62

$$-\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

Fricas [A] time = 1.46477, size = 65, normalized size = 2.24

$$-x\sqrt{-\frac{x+1}{x}} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{-x-1}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 1)/x), x)
```

Giac [A] time = 1.19008, size = 47, normalized size = 1.62

$$\frac{1}{4}\,\pi \mathrm{sgn}\left(x\right) - \frac{\arcsin\left(2\,x + 1\right)}{2\,\mathrm{sgn}\left(x\right)} - \frac{\sqrt{-x^2 - x}}{\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) - 1/2*arcsin(2*x + 1)/sgn(x) - sqrt(-x^2 - x)/sgn(x)
```

3.980 $\int \sqrt{(4-x)x}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{4x-x^2}(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]
```

Rubi [A] time = 0.0116904, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1979, 612, 619, 216}

$$-\frac{1}{2}\sqrt{4x-x^2}(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(4 - x)*x],x]
```

```
[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{(4-x)x}\,dx &= \int \sqrt{4x-x^2}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}+2\int \frac{1}{\sqrt{4x-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right)\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}-2\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0365776, size = 32, normalized size = 0.97

$$\frac{1}{2}(x-2)\sqrt{-(x-4)x}-4\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(4 - x)*x],x]
```

```
[Out] ((-2 + x)*Sqrt[-((-4 + x)*x)])/2 - 4*ArcSin[Sqrt[1 - x/4]]
```

Maple [A] time = 0.003, size = 28, normalized size = 0.9

$$-\frac{4-2\,x}{4}\sqrt{-x^2+4\,x}+2\,\arcsin\left(x/2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4-x)*x)^(1/2),x)
```

```
[Out] -1/4*(4-2*x)*(-x^2+4*x)^(1/2)+2*arcsin(1/2*x-1)
```

Maxima [A] time = 1.53334, size = 49, normalized size = 1.48

$$\frac{1}{2}\sqrt{-x^2+4\,x}x-\sqrt{-x^2+4\,x}-2\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

Fricas [A] time = 1.44239, size = 82, normalized size = 2.48

$$\frac{1}{2}\sqrt{-x^2+4\,x}(x-2)-4\,\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x\left(4-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(4 - x)), x)
```

Giac [A] time = 1.15543, size = 34, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+4x}(x-2)+2\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

3.981 $\int \frac{1}{\sqrt{(1-x)x}} \, dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] -ArcSin[1 - 2*x]
```

Rubi [A] time = 0.0049172, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1979, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x)*x],x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(1-x)x}} \, dx &= \int \frac{1}{\sqrt{x-x^2}} \, dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, 1-2x\right)\\ &= -\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.0076131, size = 12, normalized size = 1.5

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x)*x],x]
```

[Out] -2*ArcSin[Sqrt[1 - x]]

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((1-x)*x)^(1/2),x)

[Out] arcsin(2*x-1)

Maxima [A] time = 1.54528, size = 8, normalized size = 1.

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")

[Out] arcsin(2*x - 1)

Fricas [B] time = 1.4491, size = 39, normalized size = 4.88

$$-2 \arctan\left(\frac{\sqrt{-x^2 + x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")

[Out] -2*arctan(sqrt(-x^2 + x)/x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(1 - x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)**(1/2),x)

[Out] Integral(1/sqrt(x*(1 - x)), x)

Giac [A] time = 1.20982, size = 8, normalized size = 1.

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(2*x - 1)
```

3.982 $\int \frac{x}{(x(2+x))^{3/2}} dx$

Optimal. Leaf size=13

$$\frac{x}{\sqrt{x^2+2x}}$$

```
[Out] x/Sqrt[2*x + x^2]
```

Rubi [A] time = 0.0126261, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1980, 636}

$$\frac{x}{\sqrt{x^2+2x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[2*x + x^2]
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(x(2+x))^{3/2}} dx &= \int \frac{x}{\left(2x+x^2\right)^{3/2}} dx \\ &= \frac{x}{\sqrt{2x+x^2}} \end{aligned}$$

Mathematica [A] time = 0.0033524, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[x*(2 + x)]
```

Maple [A] time = 0.005, size = 15, normalized size = 1.2

$$x^2 (2 + x) (x (2 + x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(2+x))^(3/2),x)
```

```
[Out] x^2*(2+x)/(x*(2+x))^(3/2)
```

Maxima [A] time = 1.08519, size = 15, normalized size = 1.15

$$\frac{x}{\sqrt{x^2 + 2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 2*x)
```

Fricas [A] time = 1.4272, size = 47, normalized size = 3.62

$$\frac{x + \sqrt{x^2 + 2x} + 2}{x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x (x + 2))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))**(3/2),x)
```

```
[Out] Integral(x/(x*(x + 2))**(3/2), x)
```

Giac [A] time = 1.15854, size = 22, normalized size = 1.69

$$\frac{2}{x - \sqrt{(x + 2)x} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")
```

```
[Out] 2/(x - sqrt((x + 2)*x) + 2)
```

3.983 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} \, dx$

Optimal. Leaf size=22

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rubi [A] time = 0.0321021, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1446, 1469, 627, 63, 207}

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rule 1446

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{\left(-1+\frac{1}{x^2}\right)x^2}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{-1+x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(2\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\right)$$
$$= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.0212112, size = 22, normalized size = 1.

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Maple [B] time = 0.012, size = 41, normalized size = 1.9

$$\frac{x\sqrt{2}}{2}\sqrt{\frac{1+x}{x}}\operatorname{Artanh}\left(\frac{(1+3x)\sqrt{2}}{4}\frac{1}{\sqrt{x^2+x}}\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1),x)
```

```
[Out] 1/2*((1+x)/x)^(1/2)*x/(x*(1+x))^(1/2)*2^(1/2)*arctanh(1/4*(1+3*x)*2^(1/2)/(
x^2+x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{\frac{1}{x}+1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(1/x + 1)/(x^2 - 1), x)
```

Fricas [A] time = 1.50179, size = 90, normalized size = 4.09

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}}+3x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

Giac [B] time = 1.23347, size = 99, normalized size = 4.5

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\operatorname{sgn}(x)-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+x}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+x}+2\right|}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(-
2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 +
x) + 2))*sgn(x)
```

3.984 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2} \, dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rubi [A] time = 0.027343, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6, 203}

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2} \, dx &= \int \frac{1}{1+\sqrt{5}+\left(-1+\sqrt{5}\right)x^2} \, dx \\
&= \frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)
\end{aligned}$$

Mathematica [C] time = 0.0240908, size = 39, normalized size = 1.62

$$\frac{1}{4}i\log\left(-2ix+\sqrt{5}+1\right)-\frac{1}{4}i\log\left(2ix+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]
```

Maple [B] time = 0.014, size = 32, normalized size = 1.3

$$4\frac{1}{\left(\sqrt{5}-1\right)\left(2+2\sqrt{5}\right)}\arctan\left(4\frac{x}{2+2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x^2+5^(1/2)+5^(1/2)*x^2),x)
```

```
[Out] 4/(5^(1/2)-1)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
```

Maxima [A] time = 1.58004, size = 15, normalized size = 0.62

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

Fricas [A] time = 1.43206, size = 45, normalized size = 1.88

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

Sympy [A] time = 0.203215, size = 14, normalized size = 0.58

$$\frac{\operatorname{atan}\left(x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
```

```
[Out] atan(x*(-1/2 + sqrt(5)/2))/2
```

Giac [A] time = 1.15564, size = 18, normalized size = 0.75

$$\frac{1}{2}\arctan\left(\frac{2x}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*x/(sqrt(5) + 1))
```

3.985 $\int \frac{1}{\sqrt{ax+bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0095421, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax+bx^2}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] time = 0.0198093, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a*x)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47185, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{ax+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**2), x)
```

Giac [A] time = 1.17164, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.986 $\int \frac{1}{\sqrt{x(a+bx)}} \, dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0113654, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x(a+bx)}} \, dx &= \int \frac{1}{\sqrt{ax+bx^2}} \, dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.0033909, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.006, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a))^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47828, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a + b*x)), x)
```

Giac [A] time = 1.20509, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.987 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0123569, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b + a/x)*x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] time = 0.0037719, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.003, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b+a/x)*x^2)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50496, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a/x + b)), x)
```

Giac [A] time = 1.23985, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.988 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0128981, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.0038001, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.003, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/x^2+b/x)*x^3)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48585, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3\left(\frac{a}{x^2}+\frac{b}{x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3*(a/x**2 + b/x)), x)
```

Giac [A] time = 1.21856, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.989 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0120017, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx &= \int \frac{1}{\sqrt{ax+bx^2}} dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.0038968, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.003, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a*x^2)/x)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4745, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**2 + b*x**3)/x), x)
```

Giac [A] time = 1.26372, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.990 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rubi [A] time = 0.0129292, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx &= \int \frac{1}{\sqrt{ax+bx^2}} dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] time = 0.0037886, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.004, size = 29, normalized size = 1.

$$\ln\left(\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a*x^3)/x^2)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49528, size = 154, normalized size = 5.5

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**3 + b*x**4)/x**2), x)
```

Giac [A] time = 1.20491, size = 47, normalized size = 1.68

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.991 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rubi [A] time = 0.0169132, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{acx+bcx^2}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0158734, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.003, size = 37, normalized size = 0.9

$$\ln\left(\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c*x^2+a*c*x)^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50532, size = 197, normalized size = 4.92

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^{2}+acx}\sqrt{bc}\right)}{bc},\,-\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^{2}+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{acx+bcx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*c*x + b*c*x**2), x)
```

Giac [A] time = 1.25144, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

3.992 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rubi [A] time = 0.0172882, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0040466, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.9

$$\ln\left(\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bc x^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b*x^2+a*x))^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46868, size = 197, normalized size = 4.92

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc},-\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*(a*x + b*x**2)), x)
```

Giac [A] time = 1.21455, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

3.993 $\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rubi [A] time = 0.0167859, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx(a+bx)}}\,dx &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0036817, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.004, size = 37, normalized size = 0.9

$$\ln\left(\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(b*x+a))^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53323, size = 197, normalized size = 4.92

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x*(a + b*x)), x)
```

Giac [A] time = 1.22376, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

3.994 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rubi [A] time = 0.0189369, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0042189, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.003, size = 37, normalized size = 0.9

$$\ln\left(\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b+a/x)*x^2)^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46291, size = 197, normalized size = 4.92

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc},\ -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x**2*(a/x + b)), x)
```

Giac [A] time = 1.24971, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

3.995 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}x+1}\left(\sqrt{x^2-1}+3x\right)+\frac{3\sin^{-1}\left(x-\sqrt{x^2-1}\right)}{4\sqrt{2}}$$

```
[Out] ((3*x + Sqrt[-1 + x^2])*Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]])/4 + (3*ArcSin[x -
 Sqrt[-1 + x^2]])/(4*Sqrt[2])
```

Rubi [F] time = 0.0266277, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Mathematica [F] time = 0.0257292, size = 0, normalized size = 0.

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$\int \sqrt{1-x^2+x\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

```
[Out] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

Fricas [A] time = 5.35895, size = 190, normalized size = 3.02

$$\frac{1}{4}\sqrt{-x^2 + \sqrt{x^2 - 1}x + 1}\left(3\,x + \sqrt{x^2 - 1}\right) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2 + \sqrt{x^2 - 1}x + 1}}{2\sqrt{x^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + x\sqrt{x^2 - 1} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

3.996 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{x+1}\right)\sqrt{\sqrt{x}\sqrt{x+1}-x}-\frac{3\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[Out] ((Sqrt[x] + 3*Sqrt[1 + x])*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]])/2 - (3*ArcSin[Sq
rt[x] - Sqrt[1 + x]])/(2*Sqrt[2])
```

Rubi [F] time = 0.138587, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x, Sqrt[1 +
 x]]
```

Rubi steps

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx=2\,\text{Subst}\left(\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx,x,\sqrt{1+x}\right)$$

Mathematica [B] time = 0.522494, size = 180, normalized size = 2.73

$$-\frac{(x+1)\left(2x-2\sqrt{x+1}\sqrt{x}+1\right)^2\left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}\left(-2x+2\sqrt{x+1}\sqrt{x}-3\right)+3\sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2}\log\left(2\sqrt{\sqrt{x}\cdots\right.\right.}{4\left(\sqrt{x+1}-\sqrt{x}\right)^3\left(x-\sqrt{x+1}\sqrt{x}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -((1 + x)*(1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x])^2*(2*Sqrt[-x + Sqrt[x]*Sqrt[1 +
 x]]*(-3 - 2*x + 2*Sqrt[x]*Sqrt[1 + x]) + 3*Sqrt[-2 - 4*x + 4*Sqrt[x]*Sqrt[
1 + x]]*Log[2*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] + Sqrt[-2 - 4*x + 4*Sqrt[x]*Sq
rt[1 + x]]]))/(4*(-Sqrt[x] + Sqrt[1 + x])^3*(1 + x - Sqrt[x]*Sqrt[1 + x])^2
)
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \sqrt{-x+\sqrt{x}\sqrt{1+x}}\frac{1}{\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] int((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

3.997 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=78

$$\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\left(\sqrt{x^2+1}+x\right)\right)-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\sqrt{5}-2}\left(\sqrt{x^2+1}+x\right)\right)$$

```
[Out] -(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[-2 + Sqrt[5]]*(x + Sqrt[1 + x^2])]) + S
qrt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[2 + Sqrt[5]]*(x + Sqrt[1 + x^2])]
```

Rubi [B] time = 0.568053, antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {6742, 261, 1130, 203, 207, 1251, 824, 707, 1093, 1247, 699, 1279}

$$-\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sq
rt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcT
an[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1
 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*S
qrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqr
t[1 + x^2]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\int\left(\frac{x}{x+x^3+\sqrt{1+x^2}}+\frac{2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\right)dx$$

$$= -\left(2\int\frac{\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx\right)-\int\frac{x}{x+x^3+\sqrt{1+x^2}}\,dx$$

$$= -\left(2\int\left(1+\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}-\frac{x^2\left(1+x^2\right)}{-1+x^2+x^4}\right)dx\right)-\int\left(\frac{x}{\sqrt{1+x^2}}+\frac{x^2}{-1+x^2+x^4}-\frac{x^3\sqrt{1+}}{-1+x^2}\right.$$

$$= -2x-2\int\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}\,dx+2\int\frac{x^2\left(1+x^2\right)}{-1+x^2+x^4}\,dx-\int\frac{x}{\sqrt{1+x^2}}\,dx-\int\frac{x^2}{-1+x^2+x^4}\,dx$$

$$= -\sqrt{1+x^2}+\frac{1}{2}\,\text{Subst}\left(\int\frac{x\sqrt{1+x}}{-1+x+x^2}\,dx,x,x^2\right)+2\int\frac{1}{-1+x^2+x^4}\,dx+\frac{1}{10}\left(-5+\sqrt{5}\right)\int$$

$$= -\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)+\frac{1}{2}\,\text{Subst}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-2\sqrt{\frac{2}{5\left(-1+\right.}}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-\sqrt{\frac{2}{5}\left(-1+\right.}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-\sqrt{\frac{2}{5\left(-1+\right.}}$$

Mathematica [F] time = 0.416731, size = 34, normalized size = 0.44

$$-\int\frac{2\sqrt{x^2+1}+x}{x^3+\sqrt{x^2+1}+x}\,dx$$

Antiderivative was successfully verified.

```
[In] Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]

[Out] -Integrate[(x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2]), x]
```

Maple [B] time = 0.147, size = 438, normalized size = 5.6

$$-\frac{\sqrt{5}}{\sqrt{2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{1}{\sqrt{2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{\sqrt{-2+2\sqrt{5}}}\text{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x)

[Out] -5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2)
)^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan
h(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2
))^(1/2))-1/2*(x^2+1)^(1/2)-1/2*x+3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*arctanh((
```

```
-x+(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arctanh((-x+(x
^2+1)^(1/2))/(-2+5^(1/2))^(1/2))+3/10*5^(1/2)/(2+5^(1/2))^(1/2)*arctan((-x+
(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctan((-x+(x^2+1)^
(1/2))/(2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arctan((-x+(x^2+1)^(1/2))/
(-2+5^(1/2))^(1/2))+1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan((-x+(x^2+1)^(1/2)
)/(-2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctanh((-x+(x^2+1)^(1/2))/(2+5
^(1/2))^(1/2))+1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh((-x+(x^2+1)^(1/2))/(2+
5^(1/2))^(1/2))+1/2/(-x+(x^2+1)^(1/2))+2/5*(-2+5^(1/2))^(1/2)*5^(1/2)*arcta
nh((-x+(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))-2/5*(2+5^(1/2))^(1/2)*5^(1/2)*arc
tan((-x+(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x-\frac{1}{2}\arctan\left(x\right)+\int\frac{2\,x^{6}+3\,x^{4}-x^{2}-1}{2\left(x^{6}+2\,x^{4}+2\,x^{2}+2\left(x^{3}+x\right)\sqrt{x^{2}+1}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4 +
 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

Fricas [B] time = 1.94273, size = 1173, normalized size = 15.04

$$\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{4}\sqrt{2}\sqrt{4\,x^{4}+4\,x^{2}+\sqrt{5}\left(2\,x^{2}+1\right)-2\left(2\,x^{3}+\sqrt{5}x+x\right)\sqrt{x^{2}+1}+1}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^{2}+1}\right)\sqrt{\sqrt{5}+1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/4*sqrt(2)*sqrt(4*x^4 + 4*x^2 + sqrt(5)*(
2*x^2 + 1) - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1)*(sqrt(2)*x + sqrt
(2)*sqrt(x^2 + 1))*sqrt(sqrt(5) + 1) - 1/2*sqrt(2)*sqrt(x^2 + 1)*sqrt(sqrt(
5) + 1)) + sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)
*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - s
qrt(2)*x)*sqrt(sqrt(5) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*
sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqr
t(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*l
og(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*
sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(s
qrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5)
 - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35034, size = 294, normalized size = 3.77

$$-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))
)/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5)
+ 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5) +
 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt(1/2
*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(5) -
1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5
) + 2) - 1/(x - sqrt(x^2 + 1))))
```

3.998 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=126

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5
] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]
```

Rubi [A] time = 0.156796, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1036, 1030, 207, 203}

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5
] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x}{\left(1+x^2\right)\sqrt{2+2x+x^2}}\,dx = -\frac{\int \frac{-5-\sqrt{5}-2\sqrt{5}x}{\left(1+x^2\right)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{-5+\sqrt{5}+2\sqrt{5}x}{\left(1+x^2\right)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}}$$
$$= \left(2\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{20\left(1-\sqrt{5}\right)+2x^2}\,dx, x, \frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{2+2x+x^2}}\right) + \left(2\left(5+\sqrt{5}\right)\right)\text{Su}$$
$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{2}{\sqrt{10\left(-\right.}}\right.$$

Mathematica [C] time = 0.0361806, size = 87, normalized size = 0.69

$$\frac{1}{2}i\left(\sqrt{1+2i}\tanh^{-1}\left(\frac{(1+i)x+(2+i)}{\sqrt{1+2i}\sqrt{x^2+2x+2}}\right)-\sqrt{1-2i}\tanh^{-1}\left(\frac{(2-2i)x+(4-2i)}{2\sqrt{1-2i}\sqrt{x^2+2x+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] (I/2)*(Sqrt[1 + 2*I]*ArcTanh[((2 + I) + (1 + I)*x)/(Sqrt[1 + 2*I]*Sqrt[2 +
2*x + x^2])] - Sqrt[1 - 2*I]*ArcTanh[((4 - 2*I) + (2 - 2*I)*x)/(2*Sqrt[1 -
2*I]*Sqrt[2 + 2*x + x^2])])
```

Maple [B] time = 0.075, size = 753, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x)
```

```
[Out] -1/2*(10*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1
/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)*(3*5^(1/2)*(-10+10*
5^(1/2))^(1/2)*(-22+10*5^(1/2))^(1/2)*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((
5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)+3))^(1/
2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(-1/2*5^(1/
2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(5^(1/2)-5)*(-1/2*5^(1/2)+
1/2+x)/(-1/2*5^(1/2)-1/2-x)/((-1/2*5^(1/2)+1/2+x)^4/(-1/2*5^(1/2)-1/2-x)^4+
3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))+5*(-10+10*5^(1/2))^(1/2
)*(-22+10*5^(1/2))^(1/2)*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2
*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1/2
)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(-1/2*5^(1/2)+1/2+x)^2/(
-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(5^(1/2)-5)*(-1/2*5^(1/2)+1/2+x)/(-1/2*
5^(1/2)-1/2-x)/((-1/2*5^(1/2)+1/2+x)^4/(-1/2*5^(1/2)-1/2-x)^4+3*(-1/2*5^(1/
2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))+20*arctanh((10*(-1/2*5^(1/2)+1/2+x)^
2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2
-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2))*5^(1/2)-60*arctanh((10*(-
1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^
2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/(-2*(
```

```
5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-5*(-1/2*5^(1/2)+1/2+x
)^2/(-1/2*5^(1/2)-1/2-x)^2-5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1
/2-x)+1)^2)^(1/2)/((-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)+1)/(5^(1/2)-5)
/(-10+10*5^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+2x+2}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

Fricas [B] time = 2.02381, size = 2399, normalized size = 19.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^
2 + 2*x + 2)*x - (2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*
sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)
*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(s
qrt(5) + 5) + 10*sqrt(10)*(sqrt(5) + 3)) + 1/10*sqrt(5)*(sqrt(5)*(2*x + 1)
+ 5) + 1/2*sqrt(5)*x + 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2))
- sqrt(x^2 + 2*x + 2)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt
(2)) + 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) -
 1/2*sqrt(x^2 + 2*x + 2)*(sqrt(5) + 3) + 1/2*x + 1) + 1/5*5^(3/4)*sqrt(2)*s
qrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^2 + 2*x + 2)*x + (2*5
^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5
*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)*(sqrt(10)*(5^(3/4)*(
sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(sqrt(5) + 5) - 10*sqrt
(10)*(sqrt(5) + 3)) - 1/10*sqrt(5)*(sqrt(5)*(2*x + 1) + 5) - 1/2*sqrt(5)*x
+ 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2)) - sqrt(x^2 + 2*x + 2)
*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2)) + 5^(1/4)*(sqrt(
5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 1/2*sqrt(x^2 + 2*x +
 2)*(sqrt(5) + 3) - 1/2*x - 1) + 1/40*5^(1/4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))
*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x + 1/10*(2*5^(3/4)*sq
rt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2))
)*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3) - 1/40*5^(1/4)*(sqrt(5)*sqrt(2) -
5*sqrt(2))*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x - 1/10*(2*
5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) -
5*sqrt(2)))*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)
```

Giac [B] time = 1.34969, size = 369, normalized size = 2.93

$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\left(\frac{2i}{\sqrt{5}-1}+1\right)\log\left(-16(i+1)\left(x-\sqrt{x^2+2x+2}\right)+16\sqrt{\sqrt{5}+2}\left(\frac{i}{\sqrt{5}+2}+1\right)-16i+16\right)+\frac{1}{4}\sqrt{2\sqrt{5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2*sqrt(5) - 2)*(2*i/(sqrt(5) - 1) + 1)*log(-16*(i + 1)*(x - sqrt(
x^2 + 2*x + 2)) + 16*sqrt(sqrt(5) + 2)*(i/(sqrt(5) + 2) + 1) - 16*i + 16) +
 1/4*sqrt(2*sqrt(5) - 2)*(2*i/(sqrt(5) - 1) + 1)*log(-16*(i + 1)*(x - sqrt(
x^2 + 2*x + 2)) - 16*sqrt(sqrt(5) + 2)*(i/(sqrt(5) + 2) + 1) - 16*i + 16) +
 1/4*sqrt(2*sqrt(5) - 2)*(2*i/(sqrt(5) - 1) - 1)*log(-16*(i + 1)*(x - sqrt(
x^2 + 2*x + 2)) + 16*sqrt(sqrt(5) - 2)*(i/(sqrt(5) - 2) + 1) + 16*i - 16) -
 1/4*sqrt(2*sqrt(5) - 2)*(2*i/(sqrt(5) - 1) - 1)*log(-16*(i + 1)*(x - sqrt(
x^2 + 2*x + 2)) - 16*sqrt(sqrt(5) - 2)*(i/(sqrt(5) - 2) + 1) + 16*i - 16)
```

3.999 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}} \, dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] time = 0.0623883, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}} \, dx &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\end{aligned}$$

Mathematica [A] time = 1.00432, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+1}\frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Fricas [B] time = 6.05857, size = 149, normalized size = 6.77

$$-\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+\left(7\,x^5-x\right)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.1000 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rubi [A] time = 0.135462, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2128, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^4\right)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.469075, size = 50, normalized size = 1.25

$$\frac{\sqrt{-\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{-\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}+cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[-c^(-1)]*ArcCot[(Sqrt[-c^(-1)]*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]])/x])/a
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{b x^{4}+a}\frac{1}{\sqrt{c x^{2}+d\sqrt{b x^{4}+a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{4}+a\right)\sqrt{c x^{2}+\sqrt{b x^{4}+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b x^{4}\right)\sqrt{c x^{2}+d\sqrt{a+b x^{4}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)\sqrt{cx^2 + \sqrt{bx^4 + a}d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.1001 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rubi [A] time = 0.14157, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2128, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.447791, size = 47, normalized size = 1.15

$$\frac{\sqrt{\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}-cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[c^(-1)]*ArcCot[(Sqrt[c^(-1)]*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]])/x])/
a
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{bx^4+a}\frac{1}{\sqrt{-cx^2+d\sqrt{bx^4+a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{-cx^2+\sqrt{bx^4+a}d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.1002 $\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal. Leaf size=184

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2*Sqrt[b]*d
^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(
Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x)
)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rubi [A] time = 0.232949, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1680, 1885, 220, 275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```


```
[Out] ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2*Sqrt[b]*d
^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(
Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x)
)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx &= \text{Subst}\left(\int \frac{-\frac{c}{d}+x}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right) \\
&= \text{Subst}\left(\int \left(-\frac{c}{d\sqrt{a+bd^4x^4}}+\frac{x}{\sqrt{a+bd^4x^4}}\right)dx, x, \frac{c}{d}+x\right) \\
&= -\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)}{d}+\text{Subst}\left(\int \frac{x}{\sqrt{a+bd^4x^4}}\,d\cdots\right. \\
&= -\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)\right|\cdots}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}} \\
&= -\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)\right|\cdots}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\cdots}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.57034, size = 330, normalized size = 1.79

$$
\frac{\sqrt[4]{-1}\sqrt{2}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\left(\sqrt{b}(c+dx)^2+i\sqrt{a}\right)\left(\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}c\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\sqrt[4]{b}(c+dx)+\sqrt[4]{-1}\sqrt[4]{a}\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\right)\right|-1\right)-2\sqrt[4]{-1}\sqrt[4]{a}\Pi\left(\cdots\right.}{\sqrt[4]{a}\sqrt{b}d^2\sqrt{\frac{\sqrt{b}(c+dx)^2+i\sqrt{a}}{\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2}}\sqrt{a+b(c+dx)^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((
-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)*(
((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^
(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]
 - 2*(-1)^(1/4)*a^(1/4)*EllipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4
) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(
a^(1/4)*Sqrt[b]*d^2*Sqrt[(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)/((-1)^(1/4)*a^(1
/4) - b^(1/4)*(c + d*x))^2]*Sqrt[a + b*(c + d*x)^4])
```

Maple [C] time = 0.382, size = 1528, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(
1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-
a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)
-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)
-c)/d))^(1/2)*((I/b*(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)
/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(
1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((
I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/
d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)
-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))+((1/b*(
-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*EllipticPi((((-I/b*(-a*b^3)^(1
/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2),((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)
^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a
*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b
*(-a*b^3)^(1/4)-c)/d))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

3.1003 $\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(Sqrt[a]
+ Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4
)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rubi [A] time = 0.100435, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {1106, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(Sqrt[a]
+ Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4
)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)\right.}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Mathematica [C] time = 0.0573804, size = 90, normalized size = 0.69

$$-\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4])
```

Maple [C] time = 0.019, size = 1036, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(
1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-
a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)
-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)
-c)/d))^(1/2)*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/
d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1
/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(-
1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/
d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4
)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

3.1004 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d - a*e])
```

Rubi [A] time = 0.253317, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2112, 208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d - a*e])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx &= a\,\text{Subst}\left(\int \frac{1}{ad-\left(abd-a^2e\right)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}\end{aligned}$$

Mathematica [C] time = 1.82404, size = 419, normalized size = 7.76

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae-\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae+\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\right.\right.}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e
^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/
(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [C] time = 0.034, size = 514, normalized size = 9.5

$$-\frac{\sqrt{2}}{4d}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(4-2/a*(-b+(-4*a*c+b^2)^
(1/2))*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/
(2*_alpha^2*c*d+a*e)*(-1/(_alpha^2/d*(-a*e+b*d))^(1/2)*arctanh(1/2*(2*_alph
a^2*c*x^2+_alpha^2*b+b*x^2+2*a)/(_alpha^2/d*(-a*e+b*d))^(1/2)/(c*x^4+b*x^2+
a)^(1/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/(1/a*(-b+(-4*a*c+b^2)^(1/
2)))^(1/2)*(2+b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2+b*x^2/a+1/a*x^2*
(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(1
/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(_alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alp
ha^2*b*c*d+(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*2^(1/2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2))),_alpha=RootOf(_Z^4*
c*d+_Z^2*a*e+a*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx^4-a}{\left(cdx^4+aex^2+ad\right)\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

Fricas [A] time = 111.351, size = 647, normalized size = 11.98

$$\left[\frac{\log\left(-\frac{c^2d^2x^8+2\left(4\,bcd^2-3\,acde\right)x^6-\left(8\,abde-a^2e^2-2\left(4\,b^2+ac\right)d^2\right)x^4+a^2d^2+2\left(4\,abd^2-3\,a^2de\right)x^2+4\left(cdx^5+(2\,bd-ae)x^3+adx\right)\sqrt{cx^4+bx^2+a}\sqrt{bd^2-ade}}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+\left(2\,acd^2+a^2e^2\right)x^4+a^2d^2}\right)}{4\sqrt{bd^2-ade}}, -\frac{\sqrt{-b\ldots}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*log(-(c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e - a^2*e
^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3*a^2*d*e)*x^2 + 4
*(c*d*x^5 + (2*b*d - a*e)*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(b*d^2 -
 a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^
2)*x^4 + a^2*d^2))/sqrt(b*d^2 - a*d*e), -1/2*sqrt(-b*d^2 + a*d*e)*arctan(2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + (2*b*d - a*e)*x^2
 + a*d))/(b*d^2 - a*d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{a}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\,dx-\int \frac{cx^4}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*
x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(
a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{cx^4-a}{\left(cdx^4+aex^2+ad\right)\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

3.1005 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)} dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

Rubi [A] time = 0.260311, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)} dx = a\,\text{Subst}\left(\int \frac{1}{ad-\left(-abd-a^2e\right)x^2} dx, x, \frac{x}{\sqrt{a-bx^2+cx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{bd+aex}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}$$

Mathematica [C] time = 1.42708, size = 416, normalized size = 7.85

$$\frac{i\sqrt{\frac{4cx^2}{\sqrt{b^2-4ac}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(-\Pi\left(\frac{\left(b-\sqrt{b^2-4ac}\right)d}{\sqrt{a}\sqrt{ae^2-4cd^2}-ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(\sqrt{b^2-4ac}-b\right)d}{ae+\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh\right.\right.}{2d\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}\sqrt{a-bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4
*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticPi[(
(b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e) + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*Arc
Sinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(
b + Sqrt[b^2 - 4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sq
rt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*
a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[c/(-b +
 Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

Maple [C] time = 0.053, size = 517, normalized size = 9.8

$$-\frac{\sqrt{2}}{4d}\sqrt{4-2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\frac{b(\cdots}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2/a*(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b/a*(
-b+(-4*a*c+b^2)^(1/2))/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_a
lpha^2*c*d+a*e)*(-1/(-_alpha^2*(a*e+b*d)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*c
*x^2-_alpha^2*b-b*x^2+2*a)/(-_alpha^2*(a*e+b*d)/d)^(1/2)/(c*x^4-b*x^2+a)^(1
/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(2-b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2-b*x^2/a+1/a*x^2*(-4*a*c+
b^2)^(1/2))^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),-1/2*(-_alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c*
d-(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*
e+a*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx^4-a}{\left(cdx^4+aex^2+ad\right)\sqrt{cx^4-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

Fricas [A] time = 118.877, size = 647, normalized size = 12.21

$$\left[-\frac{\sqrt{-bd^2 - ade}\log\left(-\frac{c^2d^2x^8 - 2\left(4\,bcd^2 + 3\,acde\right)x^6 + \left(8\,abde + a^2e^2 + 2\left(4\,b^2 + ac\right)d^2\right)x^4 + a^2d^2 - 2\left(4\,abd^2 + 3\,a^2de\right)x^2 + 4\left(cdx^5 - (2\,bd + ae)x^3 + adx\right)\sqrt{cx^4 - bx^2}}{c^2d^2x^8 + 2\,acdex^6 + 2\,a^2dex^2 + \left(2\,acd^2 + a^2e^2\right)x^4 + a^2d^2}\right.}{4\left(bd^2 + ade\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x^
6 + (8*a*b*d*e + a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*d^
2 + 3*a^2*d*e)*x^2 + 4*(c*d*x^5 - (2*b*d + a*e)*x^3 + a*d*x)*sqrt(c*x^4 - b
*x^2 + a)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^
2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/(b*d^2 + a*d*e), 1/2*arctan(2*sqr
t(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x^2 + a
*d))/sqrt(b*d^2 + a*d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{a}{ad\sqrt{a - bx^2 + cx^4} + aex^2\sqrt{a - bx^2 + cx^4} + cdx^4\sqrt{a - bx^2 + cx^4}}\,dx - \int \frac{cx^4}{ad\sqrt{a - bx^2 + cx^4} + aex^2\sqrt{a - bx^2 + cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*
x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sqrt(
a - b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 - bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

3.1006 $\int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)} \, dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

```
[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 - 3
*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x +
x^2]]/12
```

Rubi [A] time = 0.122149, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2074, 724, 206, 1025, 982, 203, 1024, 207}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 - 3
*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x +
x^2]]/12
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{5-2x+x^2}\left(8+x^3\right)}\,dx &= \int \left(\frac{1}{12(2+x)\sqrt{5-2x+x^2}}+\frac{4-x}{12\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\right)dx\\
&= \frac{1}{12}\int \frac{1}{(2+x)\sqrt{5-2x+x^2}}\,dx+\frac{1}{12}\int \frac{4-x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\\
&= -\left(\frac{1}{24}\int \frac{-2+2x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{14-6x}{\sqrt{5-2x+x^2}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx,x,\sqrt{5-2x+x^2}\right)+\text{Subst}\left(\int \frac{}{24}\right.\\
&= \frac{\tan^{-1}\left(\frac{-2+2x}{2\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{5-2x+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.309522, size = 160, normalized size = 1.9

$$\frac{1}{312}\left(-2\sqrt{13}\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)-13\left(\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{-2\sqrt[3]{-1}x+4x+5i\sqrt{3}+1}{\sqrt{2-2i\sqrt{3}}\sqrt{x^2-2x+5}}\right)+\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{2\left(2+\right.}{\sqrt{2+}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] (-13*((I + Sqrt[3])*ArcTan[(1 + (5*I)*Sqrt[3] + 4*x - 2*(-1)^(1/3)*x)/(Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x^2])] + (-I + Sqrt[3])*ArcTan[(1 - (5*I
```

```
)*Sqrt[3] + 2*(2 + (-1)^(2/3))*x)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x
^2])]) - 2*Sqrt[13]*ArcTanh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])])/312
```

Maple [A] time = 0.018, size = 69, normalized size = 0.8

$$-\frac{\sqrt{13}}{156}\mathrm{Artanh}\left(\frac{(14-6x)\sqrt{13}}{26}\frac{1}{\sqrt{(2+x)^2-6x+1}}\right)+\frac{1}{12}\mathrm{Artanh}\left(\sqrt{x^2-2x+5}\right)+\frac{\sqrt{3}}{12}\arctan\left(\frac{\sqrt{3}(2x-2)}{6}\frac{1}{\sqrt{x^2-2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x)
```

```
[Out] -1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((2+x)^2-6*x+1)^(1/2))+1/12*
arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2-2*x+5)^(1/2
)*(2*x-2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+8)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

Fricas [B] time = 1.75505, size = 481, normalized size = 5.73

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x-2)+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)+\frac{1}{156}\sqrt{13}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - 2) + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)) +
1/156*sqrt(13)*log(-(sqrt(13)*(3*x - 7) + sqrt(x^2 - 2*x + 5)*(3*sqrt(13) +
 13) + 9*x - 21)/(x + 2)) + 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*(x - 2) - 3*
x + 6) - 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*x - x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+2)(x^2-2x+4)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

Giac [B] time = 1.34712, size = 221, normalized size = 2.63

$$-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}-2\right)\right)+\frac{1}{156}\sqrt{13}\log\left(\frac{|-2x}{|-2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/12*sqrt(3)
*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1/156*sqrt(13)*log(ab
s(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)/abs(-2*x + 2*sqrt(13) + 2*
sqrt(x^2 - 2*x + 5) - 4)) + 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 - 4*x + 4*
sqrt(x^2 - 2*x + 5) + 7) - 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

3.1007 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

```
[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Rubi [A] time = 0.0044744, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1958, 15, 261}

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(1 + x^2)],x]

[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{x^2}{1+x^2}}\,dx &= \int \frac{\sqrt{x^2}}{\sqrt{1+x^2}}\,dx\\ &= \frac{\sqrt{x^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}\\ &= \frac{\sqrt{x^2}\sqrt{1+x^2}}{x}\end{aligned}$$

Mathematica [A] time = 0.0053422, size = 17, normalized size = 0.85

$$\frac{x}{\sqrt{\frac{x^2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(1 + x^2)],x]
```

```
[Out] x/Sqrt[x^2/(1 + x^2)]
```

Maple [A] time = 0.002, size = 23, normalized size = 1.2

$$\frac{x^{2}+1}{x}\sqrt{\frac{x^{2}}{x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2+1))^(1/2),x)
```

```
[Out] (x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

Maxima [A] time = 1.87452, size = 9, normalized size = 0.45

$$\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)
```

Fricas [A] time = 1.61264, size = 45, normalized size = 2.25

$$\frac{\left(x^{2}+1\right)\sqrt{\frac{x^{2}}{x^{2}+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

Sympy [B] time = 0.440631, size = 36, normalized size = 1.8

$$x\sqrt{x^{2}}\sqrt{\frac{1}{x^{2}+1}}+\frac{\sqrt{x^{2}}\sqrt{\frac{1}{x^{2}+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2+1))**(1/2),x)
```

```
[Out] x*sqrt(x**2)*sqrt(1/(x**2 + 1)) + sqrt(x**2)*sqrt(1/(x**2 + 1))/x
```

Giac [A] time = 1.23685, size = 20, normalized size = 1.

$$\sqrt{x^2+1}\,\mathrm{sgn}\,(x)-\mathrm{sgn}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

3.1008 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

Optimal. Leaf size=46

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2 +
 n)
```

Rubi [A] time = 0.0159383, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1958, 15, 364}

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2 +
 n)
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{x^n}{1+x^n}}\,dx &= \int \frac{\sqrt{x^n}}{\sqrt{1+x^n}}\,dx\\ &= \left(x^{-n/2}\sqrt{x^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.0119177, size = 38, normalized size = 0.83

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/(2
 + n)
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n/(1+x^n))^(1/2),x)
```

```
[Out] int((x^n/(1+x^n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**n/(1+x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(x**n/(x**n + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

3.1009 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tan^{-1}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sq
rt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Rubi [A] time = 0.249151, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {2084}

$$\frac{ef\tan^{-1}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 +
 a*x^4]),x]
```

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sq
rt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Rule 2084

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(a*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c),
 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[a^2*(2*a - c), 2]), x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \frac{ef\tan^{-1}\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}d}$$

Mathematica [C] time = 6.50512, size = 13884, normalized size = 157.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b
*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.125, size = 242984, normalized size = 2761.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\left(adx^2 + bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

Fricas [A] time = 31.0476, size = 698, normalized size = 7.93

$$\left[-\frac{\sqrt{-2\,a + c}ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x-\left(8\,a^4-a^2b^2-4\,a^3c\right)x^4-8\,a^4+a^2b^2+4\,a^3c+\left(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2-4\left(6\,a^3+ab^2\right)c\right)x^2-4\left(a^2bx^2+a^2b+\left(4\,a^3+ab^2-\right.\right.}{a^2x^4+2\,abx^3+2\,abx+\left(2\,a^2+b^2\right)x^2+a^2}\right.}{2\left(2\,a^2-ac\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^2 -
 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2*x^4
 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d), -sqrt
(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a - c)*
a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/((2*a^2 - a*c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx + \int -\frac{1}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)
**(1/2),x)
```

```
[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sq
rt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x
 + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x*
*2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*
x**3 + b*x + c*x**2)), x))/d
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1010 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*S
qrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*d)
```

Rubi [A] time = 0.329498, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2085}

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x
^3 - a*x^4]),x]
```

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*S
qrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*d)
```

Rule 2085

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> -Sim
p[(a*f*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[-(a^2*(2*a -
 c)), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[-(a^2*(2*a - c)), 2
]), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g
, 0] && NegQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\tanh^{-1}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}d}$$

Mathematica [C] time = 6.52871, size = 15147, normalized size = 172.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2
 + b*x^3 - a*x^4]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.135, size = 269221, normalized size = 3059.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}\left(adx^2 - bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

Fricas [A] time = 30.3305, size = 699, normalized size = 7.94

$$\left[\frac{\sqrt{2\,a + c}ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x+\left(8\,a^4-a^2b^2+4\,a^3c\right)x^4+8\,a^4-a^2b^2+4\,a^3c-\left(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2+4\,\left(6\,a^3+ab^2\right)c\right)x^2-4\,\left(a^2bx^2+a^2b-\left(4\,a^3+ab^2+2\,a^2c\right)x\right)}{a^2x^4-2\,abx^3-2\,abx+\left(2\,a^2+b^2\right)x^2+a^2}\right.}{2\left(2\,a^2 + ac\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4*
a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2
*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2 + 2
*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x^4 -
 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -sqrt(-
2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-2*a -
c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx + \int -\frac{1}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-
a)**(1/2),x)
```

```
[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sq
rt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c
*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a
 + b*x**3 + b*x + c*x**2)), x))/d
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1011 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rubi [A] time = 0.621526, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 215}

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \frac{\left(\sqrt{2}b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2}b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 1.07563, size = 148, normalized size = 3.22

$$\frac{\sqrt{2}x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\frac{1}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/
2),x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Fricas [A] time = 71.2977, size = 385, normalized size = 8.37

$$\left[\frac{\sqrt{2}b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\,\sqrt{a}}, -\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\cdots}}{2\,x\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
 - 1)/b**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

3.1012 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.623571, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 216}

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = -\frac{\left(\sqrt{2}b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 1.16242, size = 161, normalized size = 3.5

$$\frac{\sqrt{2}b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\tanh^{-1}\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}\right)}{a^2\left(-bx^2\sqrt{\frac{a(ax^2+1)}{b^2}}+ax^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2
))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(
a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)])/(a^2*(x + a*x^3 - b*x^2
*Sqrt[(a*(1 + a*x^2))/b^2]))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\frac{1}{\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2
),x)
```

```
[Out] int((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2
),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ax^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Fricas [A] time = 69.0537, size = 396, normalized size = 8.61

$$\left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),-\frac{\sqrt{2}b\,\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-x\left(ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-ax^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

3.1013 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rubi [A] time = 1.17126, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2131, 2130, 215}

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 2131

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=\frac{\left(\sqrt{2}b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.161254, size = 148, normalized size = 3.22

$$\frac{\sqrt{2}x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{x\left(ax+\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}b\right)}\frac{1}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x+(-a/b^2+a^2*x^2/b^2)^(1/2)*b))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/
2),x)
```

```
[Out] int((x*(a*x+(-a/b^2+a^2*x^2/b^2)^(1/2)*b))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/
2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Fricas [A] time = 71.9513, size = 385, normalized size = 8.37

$$\left[\frac{\sqrt{2}b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\sqrt{a}}, -\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+(-a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(-a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

3.1014 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 1.16756, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2131, 2130, 216}

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 2131

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=-\frac{\left(\sqrt{2}b\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.18469, size = 161, normalized size = 3.5

$$\frac{\sqrt{2}b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\tanh^{-1}\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}\right)}{a^2\left(-bx^2\sqrt{\frac{a(ax^2+1)}{b^2}}+ax^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2
))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(
a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)])/(a^2*(x + a*x^3 - b*x^2
*Sqrt[(a*(1 + a*x^2))/b^2]))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{x\left(\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}b-ax\right)}\frac{1}{\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a/b^2+a^2*x^2/b^2)^(1/2)*b-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2)
,x)
```

```
[Out] int((x*((a/b^2+a^2*x^2/b^2)^(1/2)*b-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2)
,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Fricas [A] time = 68.2958, size = 396, normalized size = 8.61

$$\left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),-\frac{\sqrt{2}b\,a}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*((a/b**2+a**2*x**2/b**2)**(1/2)*b-a*x))**(1/2)/x/(a/b**2+a**2*
x**2/b**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

3.1015 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx$

Optimal. Leaf size=19

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rubi [A] time = 0.556945, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6688, 1586, 6684}

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1
+ Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx &= \int \frac{\sqrt{-1+x}\left(-4+\sqrt{-4+x}\sqrt{-1+x}+x\right)}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx\\
&= \int \frac{-4+\sqrt{-4+x}\sqrt{-1+x}+x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(-4+x)\sqrt{-1+x}}\,dx\\
&= 2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)
\end{aligned}$$

Mathematica [B] time = 1.34478, size = 75, normalized size = 3.95

$$\frac{1}{2}\log\left(-5x-4\sqrt{x-4}\sqrt{x-1}+17\right)+\frac{1}{2}\log\left(-2x-2\sqrt{x-4}\sqrt{x-1}+5\right)-\tanh^{-1}\left(\sqrt{x-4}\right)+\tanh^{-1}\left(\frac{\sqrt{x-1}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x
)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] -ArcTanh[Sqrt[-4 + x]] + ArcTanh[Sqrt[-1 + x]/2] + Log[17 - 4*Sqrt[-4 + x]*
Sqrt[-1 + x] - 5*x]/2 + Log[5 - 2*Sqrt[-4 + x]*Sqrt[-1 + x] - 2*x]/2
```

Maple [B] time = 0.049, size = 147, normalized size = 7.7

$$\frac{\ln(x-5)}{2}+\frac{1}{2}\ln\left(-1+\sqrt{x-4}\right)-\frac{1}{2}\ln\left(1+\sqrt{x-4}\right)+\frac{1}{2}\ln\left(\sqrt{x-1}+2\right)-\frac{1}{2}\ln\left(-2+\sqrt{x-1}\right)+\frac{7}{4}\sqrt{x-4}\sqrt{x-1}\mathrm{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(x-1)^(1/2)+x*(x-1)^(1/2))/(x^2-5*x+4)/(1
+(x-4)^(1/2)+(x-1)^(1/2)),x)
```

```
[Out] 1/2*ln(x-5)+1/2*ln(-1+(x-4)^(1/2))-1/2*ln(1+(x-4)^(1/2))+1/2*ln((x-1)^(1/2)
+2)-1/2*ln(-2+(x-1)^(1/2))+7/4*(x-4)^(1/2)*(x-1)^(1/2)/(x^2-5*x+4)^(1/2)*ar
ctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2))+1/4*(x-4)^(1/2)*(x-1)^(1/2)*(2*ln(-5
/2+x+(x^2-5*x+4)^(1/2))-5*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2)))/(x^2-5*
x+4)^(1/2)
```

Maxima [B] time = 1.23879, size = 127, normalized size = 6.68

$$\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2x^2+2\left((x-1)\sqrt{x-4}+2x-6\right)\sqrt{x-1}+2(2x-3)\sqrt{x-4}-7x+3}{2\left((x-1)\sqrt{x-4}+2x-6\right)}\right)+\frac{1}{2}\log\left(\frac{(x-1)\sqrt{x-}}{x-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*x - 6)*sqr
t(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x -
6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))
```

Fricas [B] time = 1.87414, size = 317, normalized size = 16.68

$$-\frac{1}{2}\log\left(-(4x-11)\sqrt{x-1}\sqrt{x-4}+4x^2-21x+23\right)+\frac{1}{2}\log\left(\sqrt{x-1}\sqrt{x-4}-x+7\right)+\frac{1}{2}\log(x-5)+\frac{1}{2}\log\left(\sqrt{x-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23) + 1/2*log
(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/2*log(sqrt(x - 1) +
2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(x - 4) + 1) + 1/2*log(sqrt(x -
```

```
4) - 1)
```

Sympy [A] time = 171.598, size = 17, normalized size = 0.89

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/
(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] 2*log(sqrt(x - 4) + sqrt(x - 1) + 1)
```

Giac [B] time = 1.40703, size = 81, normalized size = 4.26

$$\log\left(\sqrt{x-1}+2\right)-\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}\right|\right)-\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-1\right|\right)+\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] log(sqrt(x - 1) + 2) - log(abs(-sqrt(x - 1) + sqrt(x - 4))) - log(abs(-sqrt
(x - 1) + sqrt(x - 4) - 1)) + log(abs(-sqrt(x - 1) + sqrt(x - 4) - 3))
```

3.1016 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}} dx$

Optimal. Leaf size=90

$$-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
- Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^(1
/3)]/(2*3^(1/3))
```

Rubi [A] time = 0.114392, antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {433, 431, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]/3^(5/6))
+ Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3
^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)
^(1/3)]/(6*3^(1/3))
```

Rule 433

```
Int[(u_)^(p_.)*(v_)^(q_.)*(x_)^(m_.), x_Symbol] :> Int[NormalizePseudoBinom
ial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
 && IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx &= \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}x}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{-2-\sqrt[3]{3}x}{1+\sqrt[3]{3}x+3^{2/3}x^2}\right.\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)-\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{6\sqrt[3]{3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{}\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+}{\sqrt[3]{2+(1+}}\right.}{6\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [A] time = 0.14659, size = 120, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)-\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)\right)-6\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))] + Sqrt[
3]*(2*Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)] - Log[1 + (3^(2/3)*(
1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)])
)/(6*3^(5/6))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3x+3\right)}\frac{1}{\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

```
[Out] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^2+3x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

Fricas [B] time = 79.788, size = 1281, normalized size = 14.23

$$-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(7x^4+28x^3+42x^2+30x+9\right)\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(31x^6+186x^5+465x^4+666x^3+6\right.}{x^6+6x^5+15x^4+18x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
```

```
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

3.1017 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\log\left(-x^3+2(1-x)^3+1\right)}{2\,2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x)\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)
 - Log[1 + 2*(1 - x)^3 - x^3]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x) + (1 - x
^3)^(1/3)])/(2*2^(2/3))
```

Rubi [F] time = 0.534065, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] -(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3]) - (1 + I*Sqrt[3])*Defer[Int][1/(
(-1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x] - (1 - I*Sqrt[3])*Defer[Int][1/
((-1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx &= \int \left(-\frac{1}{\left(1-x^3\right)^{2/3}}+\frac{2-x}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\right)dx\\
&= -\int \frac{1}{\left(1-x^3\right)^{2/3}}\,dx+\int \frac{2-x}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx\\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\int \left(\frac{-1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}+\frac{-1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}\right.\\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\left(-1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}\,dx+\left(-1+i\sqrt{3}\right)\int
\end{aligned}$$

Mathematica [F] time = 0.177865, size = 0, normalized size = 0.

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int \frac{-x^2+1}{x^2-x+1}\left(-x^3+1\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)
```

```
[Out] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

Fricas [B] time = 47.2642, size = 738, normalized size = 7.17

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\left(x^5-x^4-3\,x^3+3\,x^2+x-1\right)\left(-x^3+1\right)^{\frac{1}{3}}+4\left(x^4-4\,x^3+5\,x^2-4\,x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\right.}{6\left(3\,x^6-9\,x^5+6\,x^4-x^3+6\,x^2-9\,x+3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*x
^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-
x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))
/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2*4^(
1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*(-x^3
 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3)*log(
-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(
2/3))/(x^2 - x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\,dx-\int -\frac{1}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**
(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1
 - x**3)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

3.1018 $\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4}\tan^{-1}\left(\frac{x^2+1}{x\sqrt{x^4-1}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{1-x^2}{x\sqrt{x^4-1}}\right)$$

```
[Out] -ArcTan[(1 + x^2)/(x*Sqrt[-1 + x^4])]/4 - ArcTanh[(1 - x^2)/(x*Sqrt[-1 + x^
4])]/4
```

Rubi [C] time = 0.119633, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {490, 1211, 222, 1699, 206, 203}

$$\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*ArcTanh[((1 +
 I)*x)/Sqrt[-1 + x^4]]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-1+x^4}\left(1+x^4\right)}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\left(i-x^2\right)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1}{\left(i+x^2\right)\sqrt{-1+x^4}}\,dx\\
&= -\left(\frac{1}{4}i\int \frac{i-x^2}{\left(i+x^2\right)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{4}i\int \frac{i+x^2}{\left(i-x^2\right)\sqrt{-1+x^4}}\,dx\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{i-2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^4}}\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{i+2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^4}}\right)\\
&= \left(-\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0195402, size = 46, normalized size = 0.94

$$\frac{x^3\sqrt{1-x^4}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^4,-x^4\right)}{3\sqrt{x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (x^3*Sqrt[1 - x^4]*AppellF1[3/4, 1/2, 1, 7/4, x^4, -x^4])/(3*Sqrt[-1 + x^4]
)
```

Maple [B] time = 0.016, size = 88, normalized size = 1.8

$$\frac{1}{8}\arctan\left(1+\frac{1}{x}\sqrt{x^4-1}\right)-\frac{1}{8}\arctan\left(-\frac{1}{x}\sqrt{x^4-1}+1\right)+\frac{1}{16}\ln\left(\left(\frac{x^4-1}{2x^2}+\frac{1}{x}\sqrt{x^4-1}+1\right)\left(\frac{x^4-1}{2x^2}-\frac{1}{x}\sqrt{x^4-1}+1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)/(x^4-1)^(1/2),x)
```

```
[Out] 1/8*arctan(1+(x^4-1)^(1/2)/x)-1/8*arctan(-(x^4-1)^(1/2)/x+1)+1/16*ln((1/2*(
x^4-1)/x^2+(x^4-1)^(1/2)/x+1)/(1/2*(x^4-1)/x^2-(x^4-1)^(1/2)/x+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^4+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Fricas [A] time = 2.20115, size = 132, normalized size = 2.69

$$\frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2x^2+2\sqrt{x^4-1}x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 -
 1)*x - 1)/(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(x^4+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

3.1019 $\int \frac{a-cx^4}{\left(ae+cdx^2\right)\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.449093, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{\left(ae+cdx^2\right)\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= a\,\text{Subst}\left(\int \frac{1}{ade-\left(abde-a\left(cd^2+ae^2\right)\right)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [C] time = 0.798444, size = 383, normalized size = 4.79

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{2ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\right.\right.\right.}{\sqrt{2}de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
,x]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [C] time = 0.06, size = 555, normalized size = 6.9

$$-\frac{\sqrt{2}}{4\,de}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/e/d*2^(1/2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(4-2/a*(-b+(-4*a*c+b^2
)^(1/2))*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2
)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*
x^2/a+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(
1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),-2*a/(-b+(-4*a*c+b^2)^(1/
2))*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(1/a*(-b+(-4*a*c+b^2)
^(1/2)))^(1/2))+1/d/e*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*
x^2/a-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(-4*a*c+
b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(1/a*(-b+(
-4*a*c+b^2)^(1/2)))^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*c*d/e,(-1/2*(b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}\left(cdx^2+ae\right)\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{cx^4 - a}{\sqrt{cx^4 + bx^2 + a}\left(cdx^2 + ae\right)\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

3.1020 $\int \left(x+\frac{1-x^2}{1+x}\right) dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0014713, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

Rubi steps

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

Mathematica [A] time = 0.0001636, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

Maple [A] time = 0., size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+(-x^2+1)/(1+x),x)
```

```
[Out] x
```

Maxima [A] time = 1.01128, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.62357, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A] time = 0.052979, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x**2+1)/(1+x),x)
```

```
[Out] x
```

Giac [A] time = 1.13023, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="giac")
```

```
[Out] x
```

3.1021 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rubi [A] time = 0.154403, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6742, 1107, 618, 204, 1293, 216, 1174, 377, 205}

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
```

```
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx &= \int \left(\frac{x}{1-x^2+x^4}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right)dx\\
&= \int \frac{x}{1-x^2+x^4}\,dx-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+\int \frac{1}{\sqrt{1-x^2}}\,dx-\int \frac{1}{\sqrt{1-x^2}\left(1-x^2+x^4\right)}\,dx\\
&= \sin^{-1}(x)+\frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1-i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1+i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-\right.\\
&= \sin^{-1}(x)+\frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(2i)\,\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{(2i)\,\text{Subst}\left(\int \frac{}{-1-i\sqrt{}}\right.}{}\\
&= \sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [B] time = 4.14002, size = 1932, normalized size = 15.84

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] (24*ArcSin[x] - (2*(-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x +
 I*(7*I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqr
t[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 -
(2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt
[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 - I*Sqrt[3])/6] + (2*(I + Sqrt[3])*ArcTan[(x
*(7 - I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] +
 3*(I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 -
 (2*I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sq
rt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 + I*Sqrt[3])/6]
 - (2*(I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3]
)*x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]
]*Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x
^2]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/S
qrt[(1 + I*Sqrt[3])/6] + (2*(1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
*Sqrt[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 - I*Sqrt[3])/6] - (4*I)*Sqrt[3]*Log[-1/
2 - (I/2)*Sqrt[3] + x^2] + (4*I)*Sqrt[3]*Log[(I/2)*(I + Sqrt[3]) + x^2] - (
I*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 - I*Sqrt[3])/6] +
 (I*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6]
+ ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6
] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 - I*Sqrt[3])
/6] + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[
2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*S
qrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1
 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]
)/Sqrt[(1 - I*Sqrt[3])/6] - (I*(-I + Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqr
t[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sq
rt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 - (3*I)*Sqrt[3] - I*Sqrt[6 -
(6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I + 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sq
rt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 - I*Sqrt[3])/6] + ((1 - I*Sqrt[3])*Log[-3*I
 + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2
] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 - (5*I)*Sq
rt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x^3*(-3*I + 3*Sqrt
[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 + I*Sqrt[3])/6] + (I
*(I + Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I
)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 -
x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) +
 I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1
+ I*Sqrt[3])/6])/24
```

Maple [B] time = 0.032, size = 234, normalized size = 1.9

$$-2\arctan\left(\frac{-1+\sqrt{-x^2+1}}{x}\right)+\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(-1+\sqrt{-x^2+1}\right)^2+\frac{1+i\sqrt{3}}{x}\left(-1+\sqrt{-x^2+1}\right)-1\right)-\frac{i}{6}\sqrt{3}\ln\Bigg(\frac{1}{x^2}\Big(-1+\sqrt{-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x+(-x^2+1)^(1/2)),x)
```

```
[Out] -2*arctan((-1+(-x^2+1)^(1/2))/x)+1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2
+(1+I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2)
)^2/x^2+(1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)+1/6*I*3^(1/2)*ln((-1+(-x^2+1
```

```
)^(1/2))^2/x^2+(I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+
(-x^2+1)^(1/2))^2/x^2+(-I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)+1/3*3^(1/2)*a
rctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

Fricas [A] time = 1.73007, size = 204, normalized size = 1.67

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x*sqrt(1 - x**2) + 1), x)
```

Giac [B] time = 1.17403, size = 261, normalized size = 2.14

$$\frac{1}{2}\pi\mathrm{sgn}\left(x\right)-\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}\left(x\right)+2\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}\left(x\right)+2\arctan\left(\frac{\sqrt{3}x\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

3.1022 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rubi [A] time = 0.387729, antiderivative size = 149, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {6742, 1293, 216, 1174, 377, 205, 1251, 773, 618, 204}

$$-\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]
/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/S
qrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqr
t[3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
```

```
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \int \left(\frac{1}{2}(-1+x)+\frac{1+x}{2}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}+\frac{x^3\left(1-x^2\right)}{1-x^2+x^4}\right)dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx+\int \frac{x^3\left(1-x^2\right)}{1-x^2+x^4}\,dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2+\frac{1}{2}\,\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^2\right)+\int \frac{1}{\sqrt{1-x^2}}\,dx-\int \frac{1}{\sqrt{1-x^2}\,(1}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+\frac{(2i)\int \frac{}{\sqrt{1-x^2}(-1}}{\sqrt{3}}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{(2i)\,\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{(2i}{}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{\tan^{-1}}{}$$

Mathematica [B] time = 1.58702, size = 1910, normalized size = 15.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] ArcSin[x] - ((-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x + I*(7*
I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqrt[2 -
(2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 - (2*I)*
Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt[3]]*S
qrt[1 - x^2]))])/(2*Sqrt[6 - (6*I)*Sqrt[3]]) + ((I + Sqrt[3])*ArcTan[(x*(7
- I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] + 3*(
I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 - (2*
I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sqrt[3
] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*Sqrt[6 + (6*I)*Sqrt[3]])
 - ((I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3])*
x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]]*
Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2
]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*
Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/2)*Log[-1/2 -
 (I/2)*Sqrt[3] + x^2])/Sqrt[3] + ((I/2)*Log[(I/2)*(I + Sqrt[3]) + x^2])/Sqr
t[3] - ((I/4)*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 - (6*I
)*Sqrt[3]] + ((I/4)*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 +
 (6*I)*Sqrt[3]] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/(4*Sqr
t[6 - (6*I)*Sqrt[3]]) + ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/
(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sq
rt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + S
qrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6
 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*
Sqrt[3]]*Sqrt[1 - x^2])])/(4*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/4)*(-I + Sqrt[3
])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*S
```

```
qrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*
(3 - (3*I)*Sqrt[3] - I*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I +
 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[6 - (6*I)*Sqrt
[3]] + ((1 - I*Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt
[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*
Sqrt[1 - x^2]) + x*(3 - (5*I)*Sqrt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[
1 - x^2]) - I*x^3*(-3*I + 3*Sqrt[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]
)])/(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((I/4)*(I + Sqrt[3])*Log[-3*I + Sqrt[3] -
 (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^
2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*
Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 +
(6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[6 + (6*I)*Sqrt[3]]
```

Maple [B] time = 0.052, size = 234, normalized size = 1.9

$$-2\arctan\left(\frac{-1+\sqrt{-x^2+1}}{x}\right)+\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(-1+\sqrt{-x^2+1}\right)^2+\frac{1+i\sqrt{3}}{x}\left(-1+\sqrt{-x^2+1}\right)-1\right)-\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(-1+\sqrt{-x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x)
```

```
[Out] -2*arctan((-1+(-x^2+1)^(1/2))/x)+1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2
+(1+I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2)
)^2/x^2+(1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)+1/6*I*3^(1/2)*ln((-1+(-x^2+1
)^(1/2))^2/x^2+(I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+
(-x^2+1)^(1/2))^2/x^2+(-I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)+1/3*3^(1/2)*a
rctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2+\int-\frac{x^4-x^2}{x^3-x-\sqrt{x+1}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)
```

Fricas [A] time = 1.79936, size = 204, normalized size = 1.67

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{1-x^2}}{x^3-x-\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(x*sqrt(1 - x**2)/(x**3 - x - sqrt(1 - x**2)), x)
```

Giac [B] time = 1.23833, size = 261, normalized size = 2.14

$$\frac{1}{2}\,\pi\mathrm{sgn}\left(x\right)-\frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}\left(x\right)+2\,\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}\left(x\right)+2\,\arctan\left(\frac{\sqrt{3}x\left(\cdots\right.}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

3.1023 $\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$

Optimal. Leaf size=34

$$-\frac{(1-x)\left(x^3+x^2+x+1\right)^{-n}\left(1-x^4\right)^n}{n+1}$$

```
[Out] -(((1 - x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n))
```

Rubi [F] time = 0.0647157, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]
```

Rubi steps

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx=\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

Mathematica [A] time = 0.0339632, size = 31, normalized size = 0.91

$$\frac{(x-1)\left(x^3+x^2+x+1\right)^{-n}\left(1-x^4\right)^n}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)
```

Maple [A] time = 0., size = 32, normalized size = 0.9

$$\frac{(x-1)\left(-x^4+1\right)^n}{(1+n)\left(x^3+x^2+x+1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^n/((x^3+x^2+x+1)^n),x)

[Out] (x-1)/(1+n)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

Maxima [A] time = 1.51408, size = 22, normalized size = 0.65

$$\frac{(x-1)(-x+1)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")
```

```
[Out] (x - 1)*(-x + 1)^n/(n + 1)
```

Fricas [A] time = 1.75946, size = 73, normalized size = 2.15

$$\frac{\left(-x^4+1\right)^n(x-1)}{\left(x^3+x^2+x+1\right)^n(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")
```

```
[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))
```

Sympy [A] time = 169.789, size = 73, normalized size = 2.15

$$\begin{cases} \frac{x(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} - \frac{(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} & \text{for } n \neq -1 \\ -\log(x-1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

```
[Out] Piecewise((x*(1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x +
 1)**n) - (1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x + 1)
**n), Ne(n, -1)), (-log(x - 1), True))
```

Giac [B] time = 1.12934, size = 109, normalized size = 3.21

$$\frac{\frac{xe^{\left(n\log\left(x^3+x^2+x+1\right)+n\log(-x+1)\right)}}{\left(x^3+x^2+x+1\right)^n} - \frac{e^{\left(n\log\left(x^3+x^2+x+1\right)+n\log(-x+1)\right)}}{\left(x^3+x^2+x+1\right)^n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")
```

```
[Out] (x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^(
n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)
```

3.1024 $\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$

Optimal. Leaf size=177

$$\log\left(32462531054272512000b^2c^{10}x^6 + 21641687369515008000b^3c^9x^5 + 9510507144806400000b^4c^8x^4 + 2583100705996\right.$$

```
[Out] Log[20738073600000000*b^8*c^4 + 597005697024000000*b^6*c^6*x^2 + 2583100705
996800000*b^5*c^7*x^3 + 9510507144806400000*b^4*c^8*x^4 + 216416873695150080
00*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 14958734309808773529 6*
c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 +
5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*
x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^
6)]/(18432*c^2)
```

Rubi [A] time = 0.0888565, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2082}

$$\log\left(32462531054272512000b^2c^{10}x^6 + 21641687369515008000b^3c^9x^5 + 9510507144806400000b^4c^8x^4 + 2583100705996\right.$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x
^4],x]
```

```
[Out] Log[20738073600000000*b^8*c^4 + 597005697024000000*b^6*c^6*x^2 + 2583100705
996800000*b^5*c^7*x^3 + 9510507144806400000*b^4*c^8*x^4 + 216416873695150080
00*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 14958734309808773529 6*
c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 +
5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*
x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^
6)]/(18432*c^2)
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \frac{\log\left(20738073600000000b^8c^4 + 5970056970240000\right.}{}$$

Mathematica [C] time = 6.19329, size = 1671, normalized size = 9.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416
*c^4*x^4],x]
```

```
[Out] (2*(x - (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/
c)^2*(-((b*EllipticF[ArcSin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000
*#1^2 + 5308416*#1^4 & , 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 53
08416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4
 & , 4, 0]))/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4
 & , 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]
 - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))]], -(((
Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-4437
5 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 3, 0])*(Root[-44375 + 576000
*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0] - Root[-44375 + 576000*#1 + 5760
00*#1^2 + 5308416*#1^4 & , 4, 0]))/((-Root[-44375 + 576000*#1 + 576000*#1^2
 + 5308416*#1^4 & , 1, 0] + Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416
*#1^4 & , 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & ,
2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])))]*
Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c) + (Ellip
ticPi[(-((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])
/c) + (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c)
/(-((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c) +
 (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c), Arc
Sin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & ,
 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))/((c*x - b*R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])*(Root[-44375
 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0] - Root[-44375 + 576000*
#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))]], -(((Root[-44375 + 576000*#1
+ 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#
1^2 + 5308416*#1^4 & , 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &
 , 4, 0]))/((-Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0
] + Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 3, 0])*(Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 57
6000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])))]*(-(b*Root[-44375 + 57600
0*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Root[-44375 + 576000*#1 +
576000*#1^2 + 5308416*#1^4 & , 2, 0]))/c)*Sqrt[((-(b*Root[-44375 + 576000*#
1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Root[-44375 + 576000*#1 + 576
000*#1^2 + 5308416*#1^4 & , 2, 0])*(x - (b*Root[-44375 + 576000*#1 + 576000
*#1^2 + 5308416*#1^4 & , 3, 0])/c))/(c*(x - (b*Root[-44375 + 576000*#1 + 57
6000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*(-((b*Root[-44375 + 576000*#1 + 5760
00*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-44375 + 576000*#1 + 576000*
#1^2 + 5308416*#1^4 & , 3, 0])/c))]*Sqrt[((c*x - b*Root[-44375 + 576000*#1
+ 576000*#1^2 + 5308416*#1^4 & , 1, 0])*(Root[-44375 + 576000*#1 + 576000*#
1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 4, 0]))/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &
 , 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))
]*((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c - (
b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c)*Sqrt[(
(-(b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Ro
ot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])*(x - (b*Root[
-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))/(c*(x - (b*R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*(-((b*Roo
t[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))]))/(Sqrt[-4437
5*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(-((b*Root[-
```

```
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-443
75 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*((b*Root[-44375 +
 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c - (b*Root[-44375 + 576
000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))
```

Maple [C] time = 0.568, size = 1597, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x
)
```

```
[Out] 1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)
*b/c)^2*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+1
0*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)
*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/
c))^(1/2)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4
+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=
4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c))^(1/2)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^
4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2
)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(c^4*(x-5/48*RootOf(_
Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,ind
ex=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(x-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)*(5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=2)*b/c*EllipticF(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2),((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)/(5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*Roo
tOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2))+(5/48*RootOf(_Z^4+10*_Z^2+9
6*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*Ellipt
icPi(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/
c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))
^(1/2),(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c
-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c),((5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/4
8*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-
71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,in
dex=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{5308416\,c^4x^4+576000\,b^2c^2x^2+576000\,b^3cx-44375\,b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

Fricas [A] time = 2.57616, size = 537, normalized size = 3.03

$$\log\left(28179280429056\,c^8x^8+6115295232000\,b^2c^6x^6+4076863488000\,b^3c^5x^5+179159040000\,b^4c^4x^4+4866048000\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="fricas")
```

```
[Out] 1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488
000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112
464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*
b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*
c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*
c*x - 44375*b^4))/c^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375*
b**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 530
8416*c**4*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{5308416\,c^4x^4+576000\,b^2c^2x^2+576000\,b^3cx-44375\,b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

3.1025 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx$

Optimal. Leaf size=100

$$\frac{1}{16}\log\left(4096x^8+8192x^7+12288x^6+19456x^5+17024x^4+13440x^3+9280x^2+\sqrt{64x^4+64x^3+64x^2+120x+9}\,(\right.$$

```
[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6
 + 8192*x^7 + 4096*x^8 + Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(179 +
444*x + 744*x^2 + 1280*x^3 + 960*x^4 + 768*x^5 + 512*x^6)]/16
```

Rubi [B] time = 0.138549, antiderivative size = 243, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2083, 2082}

$$\frac{1}{16}\log\left(4096x^8+8192x^7+512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6+12288x^6+768\sqrt{64x^4+64x^3+64x^2+120x+9}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6
 + 8192*x^7 + 4096*x^8 + 179*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 4
44*x*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 744*x^2*Sqrt[9 + 120*x +
64*x^2 + 64*x^3 + 64*x^4] + 1280*x^3*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*
x^4] + 960*x^4*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 768*x^5*Sqrt[9
+ 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 512*x^6*Sqrt[9 + 120*x + 64*x^2 + 64*
x^3 + 64*x^4]]/16
```

Rule 2083

```
Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^
3 + (e_.)*(x_)^4], x_Symbol] :> Dist[B, Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^2
*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + ((d^3 - 4*c*d*e + 8*b*e^2)*x)/(8*e
^2) - ((3*d^2 - 8*c*e)*x^2)/(8*e) + e*x^4], x], x, d/(4*e) + x], x] /; Free
Q[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752*
c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c*
e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = 4\,\text{Subst}\left(\int \frac{x}{\sqrt{-\frac{71}{4}+96x+40x^2+64x^4}}\,dx, x, \frac{1}{4}+x\right)$$
$$= \frac{1}{16}\log\left(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5+12288x^6\right.$$

Mathematica [C] time = 6.10326, size = 2787, normalized size = 27.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] (8*(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])^2*(-(Ellip
ticF[ArcSin[Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1,
 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#
1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 6
4*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])
)]], -(((Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 +
 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2
 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#
1^4 & , 4, 0]))/((-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0]
+ Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1
 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#
1^3 + 64*#1^4 & , 4, 0])))]*Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 &
 , 2, 0]) + EllipticPi[(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/(-Root[9 +
 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2
 + 64*#1^3 + 64*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[9 + 120*#1 + 64*#1^
2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*
#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/
((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 4, 0]))]], -(((Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0]
)*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#
1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((-Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4
 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root
[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])))]*(-Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^
3 + 64*#1^4 & , 2, 0]))*Sqrt[(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*
#1^4 & , 3, 0])/((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2,
0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0]))]*(Root[9 + 120*#1 + 64*#1^2
+ 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1
^4 & , 4, 0])*Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
1, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 +
120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64
*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0
]))]*Sqrt[(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/((x
 - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*
#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64
*#1^3 + 64*#1^4 & , 4, 0]))])/(Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(
Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])) + (2*EllipticF[ArcSin[Sqrt[((x - R
oot[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(-Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^
3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4
 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Roo
t[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))]], ((Root[9 + 120*#1
 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#
```

```
1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((Root[9
 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1
^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64
*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))
]*(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])^2*Sqrt[(x -
 Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])/((x - Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 +
 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^
4 & , 3, 0]))]*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - R
oot[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])*Sqrt[(x - Root[9 +
120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/((x - Root[9 + 120*#1 + 64*
#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0
]))]*Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(-
Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^
4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))])/(
Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(-Root[9 + 120*#1 + 64*#1^2 + 64
*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 &
 , 4, 0]))
```

Maple [C] time = 0.816, size = 2992, normalized size = 29.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x)
```

```
[Out] 1/4*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=4))*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,i
ndex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*
_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2))^2*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60
*_Z+9,index=3)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Ro
otOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index
=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/
2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=2))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-
1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)*EllipticF(((1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(
1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+
```

```
8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_
Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))*(
(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))^2*
((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3)-1/2*RootOf(4*_Z^4
+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2
*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(
x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=4)))^(1/2)*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)*Ellipt
icF(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=2)))^(1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)
-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z
^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+
60*_Z+9,index=2))*EllipticPi(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootO
f(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2),(1/2*RootOf(4*_Z^4+8*_Z^3+
16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))
/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=2)),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^
4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

Fricas [A] time = 1.89448, size = 292, normalized size = 2.92

$$\frac{1}{16}\log\left(-4096\,x^8 - 8192\,x^7 - 12288\,x^6 - 19456\,x^5 - 17024\,x^4 - 13440\,x^3 - 9280\,x^2 - \left(512\,x^6 + 768\,x^5 + 960\,x^4 + 12\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*x
^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x +
 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x + 1}{\sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)
```

```
[Out] Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x + 1}{\sqrt{64\,x^4 + 64\,x^3 + 64\,x^2 + 120\,x + 9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```